Insert Markov pun here

A talk on Probabilistic Automata and Markov Chains

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Stochastic vectors and stochastic matrices

Stochastic vectors and stochastic matrices

A **Stochastic Vector** $\mathcal{S}$ is a vector in $\mathbb{R}^n$ which satisfies the following conditions:

- if $s \in \mathcal{S}$, then $0 < s < 1$
- the sum of all the elements of $\mathcal{S}$ is 1

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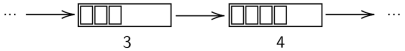
Stochastic vectors can be thought of as a form of **discrete probability distribution**: given a finite number of possible states, $\mathcal{S}$ gives the probability of transitioning to the ith state.

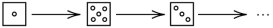
A short recap of NFA's

Deterministic and Nondeterministic Finite State Machines

Combining both these concepts, a **Deterministic Finite State Automata (DFA)** is one whose next state is uniquely determined by the current state. As an example, a loading bar at n% can only progress to a state at (n+1)%



In contrast, a **Nondeterministic Finite State Automata (NFA)** is one whose next state is not uniquely determined by it's current state. An example would be the output of a dice roll: it's a NFA, because the number of states are finite, and the next state is independent of the current state



Probabilistic Automata

A **Probabilistic Automata** (PA) generalizes the concept of a NFA, so that the transition probabilities are given by the transition function (the transition function for such an automata is a stochastic matrix / markov matrix)

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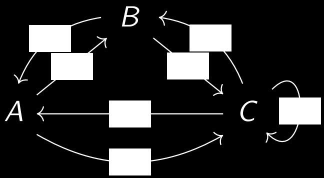

Modeling PA's: Markov Processes

PA's lend themselves to being modeled by markov matrices very well. As an example, the process in the previous slide can be modeled as follows:

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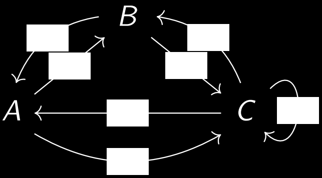

$$\mathcal{S}_A = \begin{bmatrix} 0 \\ 0.6 \\ 0.4 \end{bmatrix} \mathcal{S}_B = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix} \mathcal{S}_C = \begin{bmatrix} 0.5 \\ 0.1 \\ 0.4 \end{bmatrix}$$

$$\mathcal{S} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.6 & 0 & 0.1 \\ 0.4 & 0.5 & 0.4 \end{bmatrix}$$

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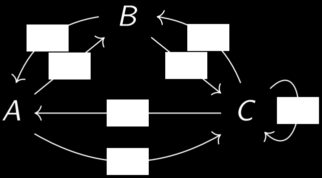

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Now, given an initial state a, $\mathcal{S}a$ would give the final state after one transition. By induction, $\mathcal{S}^n a$ would give the final state after n transitions.

More on Markov Matrices

Analysing the steady state