

ii. Solve $9pqz^4 = 4(1+z^3)$.

(OR)

b. Solve the equation $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$.

29. a. Obtain the Fourier series of period $2l$ for the function

$$f(x) = l - x, \text{ in } 0 < x \leq l \\ = 0, \text{ in } l \leq x < 2l$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \infty = \frac{\pi^2}{8}$.

(OR)

b. Find the Fourier series of $y = f(x)$ in $(0, 2\pi)$ upto the third harmonic using the definition of y given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

30. a. A tightly stretched string of length π is fastened at both ends. The midpoint of the string is displaced by a distance d transversely and the string is released from rest in this position. Find the displacement of any point of the string at any subsequent time.

(OR)

b. A uniform bar of length l through which heat flows is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given by $k(lx - x^2)$ for $0 < x < l$, find the temperature distribution in the bar after time t .

31. a. Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence deduce $\int_0^\infty \left(\frac{\sin x}{x}\right)^4 dx = \frac{\pi}{3}$.

(OR)

b. Find Fourier sine and cosine transforms of e^{-x} . Hence evaluate $\int_0^\infty \frac{x^2}{(x^2+1)^2} dx$.

32. a.i. Find the Z transform of $(n+1)^2$ and $\sin(3n+5)$.

ii. Find the inverse Z-transform of $\frac{z^2}{(z-4)(z-3)}$.

(OR)

b. Solve the equation $y(k+2) + y(k) = 1$, $y(0) = y(1) = 0$, using Z-transform.

Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2019

3rd to 8th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

- The complete integral of $pq = 1$ is
(A) $az = a^2x + y + ac$ (B) $z = ax + ay + c$
(C) $az = x + y + c$ (D) $z = x + y + c$
- The partial differential equation formed by eliminating the arbitrary function from $z = f(x^2 + y^2)$ is
(A) $xp = yq$ (B) $xy = pq$
(C) $py = qx$ (D) $x + p = y + q$
- solve $(D^3 - 7DD'^2 - 6D'^3)z = 0$
(A) $z = f_1(y-x) + f_2(y-2x) + f_3(y+3x)$ (B) $z = f_1(y-x) + f_2(y+2x) + f_3(y-3x)$
(C) $z = f_1(y+x) + f_2(y-2x) + f_3(y+3x)$ (D) $z = f_1(y-x) + f_2(y-2x) + f_3(y-3x)$
- The particular integral of $(D^3 - 2D^2D')z = e^{x+2y}$ is
(A) $\frac{e^{x+2y}}{3}$ (B) $\frac{e^x}{3}$
(C) $\frac{e^{x+2y}}{3}$ (D) $\frac{-e^{x+2y}}{3}$
- The constant a_0 of the Fourier series for the function $f(x) = x^2$ in $(0, 2l)$
(A) $\frac{l^2}{3}$ (B) $\frac{4l^2}{3}$
(C) $\frac{8l^2}{3}$ (D) l^2
- The sum of the Fourier series of $f(x) = x + x^2$, in $-\pi < x < \pi$ at $x = \pi$ is
(A) π (B) π^2
(C) $\pi/2$ (D) $\pi^2/2$
- If $f(x) = x$ in $-l \leq x \leq l$, then a_n
(A) $\frac{-2l(-1)^n}{n\pi}$ (B) 0
(C) l (D) $2l^2/3$

8. The RMS value of $f(x) = x$ in $-1 \leq x \leq 1$ is
 (A) 1 (B) 0
 (C) $\frac{1}{\sqrt{3}}$ (D) -1
9. Classify the partial differential equation $4u_{xx} + 4u_{xy} + u_{yy} = 0$
 (A) Elliptic (B) Parabolic
 (C) Hyperbolic (D) Circular
10. The string is stretched between two fixed points $x=0$ and $x=l$, the boundary conditions are (t being positive)
 (A) $y(0, t) = 0, y(x, t) = 0$ (B) $y(x, 0) = 0, \left(\frac{\partial y}{\partial t}\right)(x, 0) = 0$
 (C) $y(0, t) = 0, y(l, t) = 0$ (D) $\left(\frac{\partial y}{\partial t}\right)(0, t) = 0, \left(\frac{\partial y}{\partial t}\right)(l, t) = 0$
11. The steady state temperature of a rod of length l whose ends are kept at 30°C and 40°C is
 (A) $u = \frac{10x}{l} + 30$ (B) $u = \frac{20x}{l} + 30$
 (C) $u = \frac{10x}{l} + 20$ (D) $u = \frac{10x}{l}$
12. One dimensional wave equation is used to find
 (A) Temperature (B) Displacement
 (C) Time (D) Mass
13. If $F\{f(x)\} = F(s)$, then $F\{e^{-iax}f(x)\}$ is
 (A) $F(s+a)$ (B) $F(s-a)$
 (C) $F(as)$ (D) $F(a/s)$
14. $F_c(x, f(x))$ is
 (A) $i \frac{dF_s(s)}{ds}$ (B) $\frac{-dF_s(s)}{ds}$
 (C) $i \frac{dF_s(s)}{ds}$ (D) $\frac{dF_s(s)}{ds}$
15. The Fourier cosine transform of e^{-ax} is
 (A) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + s^2}$ (B) $\sqrt{\frac{1}{\pi}} \frac{s}{s^2 + a^2}$
 (C) $\sqrt{\frac{1}{\pi}} \frac{a}{s^2 + a^2}$ (D) $\sqrt{\frac{2}{\pi}} \frac{a}{s^2 + a^2}$
16. $F(f(ax))$ is
 (A) $\frac{1}{s} F(s/a)$ (B) $\frac{1}{a} F(a/s)$
 (C) $\frac{1}{s} F(as/a+1)$ (D) $\frac{1}{|a|} F(s/a)$

17. Z-transform of $\frac{1}{n!}$
 (A) $\frac{1}{e^z}$ (B) e^{z^2}
 (C) $e^{2/z}$ (D) e^{z^3}
18. $Z(n^2)$ is
 (A) $\frac{z}{(z-1)^3}$ (B) $\frac{z(z+1)}{(z)^3}$
 (C) $\frac{z(z+1)}{(z-1)^3}$ (D) $\frac{z+1}{(z-1)^3}$
19. $z\left(\sin \frac{n\pi}{2}\right)$ is
 (A) $\frac{z^2}{z-1}$ (B) $\frac{z}{z^2+4}$
 (C) $\frac{z}{z^2+1}$ (D) $\frac{z^2}{z^2+1}$
20. Poles of $\phi(z) = \frac{z^n}{(z-1)(z-2)}$ are
 (A) $z=1, z=0$ (B) $z=1, z=2$
 (C) $z=0, z=2$ (D) $z=0$

PART - B (5 × 4 = 20 Marks)
 Answer ANY FIVE Questions

21. Solve $p - q = \log(x+y)$
22. Find the Fourier series of $f(x) = x^2$ in $-\pi \leq x \leq \pi$.
23. Classify the PDE $(x+1)f_{xx} + 2(x+2)f_{xy} + (x+3)f_{yy} = 0$.
24. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} x & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$.
25. Find $z(na^n)$.
26. Solve $(D^2 - DD')z = \cos x \cos 2y$.
27. Find $Z(f(n))$ where $f(n) = an^2 + bn + c$.

PART - C (5 × 12 = 60 Marks)
 Answer ALL Questions

28. a.i. Find the partial differential equation of all planes which are at a constant distance k from the origin.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Solve (i) $z = px + qy + \sqrt{1 + p^2 + q^2}$ (ii) $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. Find also singular integral.

(OR)

- b. Solve (i) $(D^2 - 2DD' + D'^2)z = \cos(x - 3y)$ (ii) $(D^2 - DD'^2)z = e^{x+2y}$.
29. a. Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π . Hence deduce $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$.

(OR)

- b. Compute the first two harmonics of the fourier series $f(x)$ given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1

30. a. A tightly stretched string with fixed end point $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$. Find the displacement.

(OR)

- b. A rod of length l has its ends A and B kept at 0°C and 100°C respectively unit steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature $u(x, t)$.
31. a. Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence prove that

$$\int_0^\infty \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$$

(OR)

- b. Use transform method to evaluate $\int_0^\infty \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$
32. a.i. Find $Z(a^n)$ and $Z(n^2)$.
- ii. Using residues find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$.

(OR)

- b. Solve the equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$ by using Z-transform.

Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
3rd to 7th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
- (ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

- The partial differential equation formed by eliminating arbitrary constant a, b is $z = (x+a)(y+b)$
(A) $z = p + q$ (B) $z = p - q$
(C) $z = p/q$ (D) $z = pq$
- The complementary function of $(D^2 + 2DD' + D'^2)z = 0$ is
(A) $\phi_1(y-x) + \phi_2(y-x)$ (B) $\phi_1(y-x) + x\phi_2(y-x)$
(C) $\phi_1(y-x) + \phi_2(y+x)$ (D) $\phi_1(y-x) + x\phi_2(y+x)$
- The particular integral of $(D^2 - 2DD')z = e^{2x}$
(A) $e^{2x}/4$ (B) $e^{2x+y}/4$
(C) e^{2x} (D) $e^{2x}/2$
- The complete solution of $z = px + qy + p^2q^2$ is
(A) $z = ax + by^2 + ab^2$ (B) $z = ax^2 + by + ab^2$
(C) $z = ax + by + a^2b^2$ (D) $z = ax + by + c$
- $\sin x$ is a periodic function with period
(A) π (B) $\pi/2$
(C) 2π (D) 4π
- The constant a_0 of the Fourier series for the function $f(x) = k, 0 \leq x \leq 2\pi$ is
(A) k (B) $2k$
(C) 0 (D) $k/2$
- The RMS value of $f(x) = x$ in $-1 \leq x \leq 1$ is
(A) 1 (B) 0
(C) $1/\sqrt{3}$ (D) -1

8. Half range cosine series for $f(x)$ is $(0, \pi)$ is
 (A) $\sum_{n=1}^{\infty} a_n \cos nx$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$
 (C) $\sum_{n=1}^{\infty} b_n \sin nx$ (D) $\frac{a_0}{2} - \sum_{n=1}^{\infty} a_n \cos nx$
9. The proper solution of the problems of vibration of string is
 (A) $y(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})(Ce^{\lambda at} + De^{\lambda at})$ (B) $y(x,t) = (Ax + B)(ct + 1)$
 (C) $y(x,t) = (A \cos \lambda x + B \sin \lambda x)(C \cos \lambda at + D \sin \lambda at)$ (D) $y(x,t) = Ax + B$
10. The one dimensional wave equation is
 (A) $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ (B) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$
 (C) $\frac{\partial y}{\partial t} = a \frac{\partial^2 y}{\partial x^2}$ (D) $\frac{\partial^2 y}{\partial x^2} = a \frac{\partial^2 y}{\partial t^2}$
11. One dimensional heat equation is used to find
 (A) Density (B) Temperature distribution
 (C) Time (D) Displacement
12. A rod of length l has its ends A and B kept at 0° and 100° respectively, until steady state conditions prevail. Then the initial condition is given by
 (A) $u(x,0) = ax + b + 100l$ (B) $u(x,0) = \frac{100x}{l}$
 (C) $u(x,0) = 100xl$ (D) $u(x,0) = (x+l)100$
13. $F[e^{iax} f(x)]$
 (A) $F(s+a)$ (B) $F(s-a)$
 (C) $F(sa)$ (D) $F(s/a)$
14. $F[xf'(x)] =$
 (A) $\frac{dF(s)}{ds}$ (B) $i \frac{dF(s)}{ds}$
 (C) $-i \frac{dF(s)}{ds}$ (D) $\frac{dF(s)}{ds}$
15. The fourier cosine transform of $Fc[e^{-4x}]$
 (A) $\sqrt{\frac{2}{\pi}} \frac{4}{16+s^2}$ (B) $\sqrt{\frac{2}{\pi}} \frac{4}{4+s^2}$
 (C) $\sqrt{\frac{\pi}{2}} \frac{4}{16+s^2}$ (D) $\sqrt{\frac{\pi}{2}} \frac{4}{4+s^2}$

16. $F[f(x) * g(x)] =$
 (A) $F(s) + G(s)$ (B) $F(s) - G(s)$
 (C) $F(s)G(s)$ (D) $F(s)/G(s)$
17. What is $Z(7)$
 (A) $\frac{z}{z-1}$ (B) $7 \frac{z}{z-1}$
 (C) $\frac{1}{7} \frac{z}{z-1}$ (D) $\frac{z-1}{z}$
18. What is $Z[na^n]$
 (A) $\frac{az}{(z-a)^2}$ (B) $\frac{z}{(z-a)^2}$
 (C) $\frac{a}{(z-a)^2}$ (D) $\frac{z}{(z-a)^3}$
19. If $z[f(t)] = F(z)$ then $\lim_{z \rightarrow \infty} F(z) =$
 (A) $f(0)$ (B) $f(1)$
 (C) $\lim_{x \rightarrow \infty} f(t)$ (D) $f(\infty)$
20. $\phi(z) = \frac{z^n(2z+4)}{(z-2)^3}$ has a pole of order
 (A) 2 (B) 1
 (C) 3 (D) 4

PART - B (5 × 4 = 20 Marks)
 Answer ANY FIVE Questions

21. Form the Partial differential equation by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$.
22. Find the half range Fourier sine series for $f(x) = x$ in $0 < x < \pi$.
23. Write the one dimensional heat flow equation and all the possible solutions.
24. Find the Fourier sine transform of e^{-ax} $a > 0$.
25. Find Z-transform of $r^n \cos n\theta$.
26. Find $z^{-1} \left(\frac{1}{(z-1)(z-2)} \right)$ by convolution.
27. Solve $p^2 + q^2 = x + y$.

29. a. Express $f(x) = (\pi - x)^2$ as a Fourier series of periodicity 2π in $0 < x < 2\pi$ and hence deduce the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(OR)

- b. Compute the first three harmonics of the Fourier series of $f(x)$ given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

30. a. If a string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3 \frac{\pi x}{l}$, $0 < x < l$, determine the transverse displacement $y(x, t)$.

(OR)

- b. Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ subject to (i) $u(0, t) = 0$ for $t \geq 0$ (ii) $u(l, t) = 0$ for $t \geq 0$
 (iii) $u(x, 0) = \begin{cases} x & \text{for } 0 \leq x \leq l/2 \\ l-x & \text{for } l/2 \leq x \leq l \end{cases}$

31. a. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and hence evaluate

$$\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx.$$

(OR)

- b. Find Fourier cosine and sine transforms of e^{-ax} , $a > 0$ and evaluate $\int_0^{\infty} \frac{dx}{(a^2 + x^2)^2}$ and

$$\int_0^{\infty} \frac{x^2}{(a^2 + x^2)^2} dx \text{ if } a > 0.$$

- 32.a.i. Find the Z-transform of $\left\{ \frac{1}{n(n+1)} \right\}, n \geq 1$.

- ii. Find the inverse Z-transform of $x(z) = \frac{z^2}{(z-1/2)(z-1/4)}$ using Convolution theorem.

(OR)

- b. Solve $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$ given that $y_0 = 0, y_1 = 0$ using Z-transforms.

Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third to Seventh Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:

- (i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
 (ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

- The complete integral of $p = 2qx$
 (A) $z = ax^2 + ay + c$ (B) $z = ax + ay^2 + c$
 (C) $z = ax^2 - ay + c$ (D) $z = ax + by + c$
- The partial differential equation formed by eliminating arbitrary function in $z = f(x^2 + y^2)$ is
 (A) $xp = yq$ (B) $xy = pq$
 (C) $xq = yp$ (D) $x + p = y + q$
- Solve $(D^3 - 7DD'^2 - 6D'^2)z = 0$
 (A) $z = \phi_1(y-x) + \phi_2(y-2x) + \phi_3(y+3x)$ (B) $z = \phi_1(y-x) + \phi_2(y+2x) + \phi_3(y-3x)$
 (C) $z = \phi_1(y+x) + \phi_2(y-2x) + \phi_3(y+3x)$ (D) $z = \phi_1(y+x) + \phi_2(y+2x) + \phi_3(y+3x)$
- The general integral of $z = xp + yq$ is
 (A) $\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ (B) $\phi(x+y, y+z) = 0$
 (C) $\phi\left(x-y, \frac{x}{2}\right) = 0$ (D) $\phi\left(\frac{x}{y}, y+z\right) = 0$
- The constant a_0 of the Fourier series for the function $f(x) = k, 0 \leq x \leq 2\pi$
 (A) k (B) $2k$
 (C) 0 (D) $k/2$
- The RMS value of $f(x) = x$ in $-1 \leq x \leq 1$ is
 (A) 1 (B) 0
 (C) $1/\sqrt{3}$ (D) -1
- Find half-range cosine series of $f(x) = \cos x$ in $(0, \pi)$ the value of a_0 is
 (A) 4 (B) $2/\pi$
 (C) $4/\pi$ (D) 0

8. $\int_{-\pi}^{\pi} |x| dx$ is equal to

(A) $2 \int_0^{\pi} x dx$

(B) 0

(C) $2 \int_0^{\pi} (-x) dx$

(D) $4 \int_0^{\pi} x dx$

9. In wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) T/m

(B) k/c

(C) m/T

(D) k/m

10. One dimensional heat equation is used to find

(A) Temperature

(B) Displacement

(C) Time

(D) Mass

11. How many initial and boundary conditions are required to solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(A) Two

(B) Three

(C) Four

(D) Five

12. A rod of length l has its ends A and B are kept at 0°C and 100°C respectively, until steady state conditions prevail. Then the initial condition is given by

(A) $u(x, 0) = ax + b + 100l$

(B) $u(x, 0) = \frac{100x}{l}$

(C) $u(x, 0) = 100lx$

(D) $u(x, 0) = (x + l)100$

13. If $F\{f(x)\} = F(s)$, then $F\{f(x-a)\}$

(A) $e^{ias} F(s)$

(B) $e^{ias} F(a)$

(C) $e^{iax} F(a)$

(D) $e^{ias} F(x)$

14. The Fourier transform of $f(x) = e^{-x^2/2}$ is

(A) e^{-s^2}

(B) $\frac{1}{e^{s^2/2}}$

(C) $\frac{1}{e^{-s^2/2}}$

(D) $e^{-s^2/2}$

15. $F\{f(x) * g(x)\} =$

(A) $F(s) + G(s)$

(B) $F(s) - G(s)$

(C) $F(s).G(s)$

(D) $F(s)/G(s)$

16. Under Fourier cosine transform of $f(x) = 1/\sqrt{x}$ is

(A) Self-reciprocal function

(B) Cosine function

(C) Inverse function

(D) Complex function

17. $Z[(-1)^n]$

(A) $\frac{z+1}{z}$

(B) $\frac{z}{z-1}$

(C) $\frac{z}{z+1}$

(D) $\frac{-z}{z+1}$

18. If $Z[f(t)] = F(z)$, then $\lim_{z \rightarrow \infty} F(z)$

(A) $f(0)$

(B) $f(1)$

(C) $\lim_{t \rightarrow \infty} f(t)$

(D) $f(\infty)$

19. Find $Z^{-1}\left[\frac{z}{(z-1)^2}\right]$ is

(A) $n+1$

(B) n

(C) $n-1$

(D) $1/n$

20. The poles of $\phi(z) = \frac{z^n}{(z-1)(z-2)}$ are

(A) $z=1, z=2$

(B) $z=-1, z=-2$

(C) $z=1, z=-2$

(D) $z=0, z=2$

PART - B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

21. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$.

22. Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y}$.

23. Express $f(x) = x$ in half range sine series of periodicity $2l$ in the range $0 < x < l$

24. Write the possible solutions and correct solution of one dimensional heat equation.

25. Classify the equation $(1+x^2)f_{xx} + (5+2x^2)f_{xy} + (4+x^2)f_{yy} = 2\sin(x+y)$.

26. If $F\{f(x)\} = F(s)$ then $F\{f(x)\cos ax\} = \frac{1}{2}[F(s-a) + F(s+a)]$.

27. Find $Z\{\sin n\theta\}$.

PART - C (5 × 12 = 60 Marks)

Answer ALL Questions

28. a. Solve (i) $9(p^2z + q^2) = 4$ (ii) $x(y-z)p + y(z-x)q = z(x-y)$.

(OR)

b. Solve $(D^3 - 2D^2D')z = \sin(x+2y) + 3x^2y$.

29.a. Find the half-range cosine series for $f(x)=x, 0 \leq x \leq \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$

(OR)

b. Find the Fourier sine series upto third harmonic for the function $y=f(x)$ in $(0, \pi)$ from the table.

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y	2.34	2.2	1.6	0.83	0.51	0.88	2.34

30.a. A tightly stretched string of length l has its end fastened at $x=0, x=l$. At $t=0$, the string is in the form $f(x)=\lambda x(l-x)$ and then released. Find the displacement y at any time and at any distance from the end $x=0$.

(OR)

b. Find the solution of the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions.

(i) $u(0, t) = 0$ (ii)

(ii) $u(l, t) = 0$ for $t > 0$

(iii) $u(x, 0) = \begin{cases} x, 0 \leq x \leq l/2 \\ l-x, l/2 \leq x \leq l \end{cases}$

31.a. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} a^2 - x^2, & \text{if } |x| < a \\ 0 & \text{if } |x| > a > 0 \end{cases}$ hence prove that

$$\int_0^a \left(\frac{\sin x - x \cos x}{x^3} \right) dx = \frac{\pi}{4}.$$

(OR)

b.i. Find the Fourier transform of $e^{-a|x|}$ and hence evaluate $\int_0^{\infty} \frac{1}{(x^2 + a^2)^2} dx, a > 0$. (8 Marks)

ii. If $F[f(x)] = F(s)$, then $F(f(x) \cos ax) = \frac{1}{2} [F(s+a) + F(s-a)]$. (4 Marks)

32.a.i. Find $Z^{-1} \left[\frac{z^2}{(z-a)(z-b)} \right]$ using convolution theorem.

ii. Find $Z^{-1} \left[\frac{z^2}{(z+2)(z^2+4)} \right]$ by method of partial fraction.

(OR)

b. Using Z-transform solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$.

Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
First to Eighth Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:

- (i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. Find the complete integral of $p^2 + q^2 = 1$

(A) $z = ax + by + c$

(B) $z = ax + by$

(C) $z = a(x+y) + b$

(D) $z = ax - by + a$

2. Solve $pq = xy$

(A) $z = k \frac{x}{2} + \frac{1}{k} y/2 + c$

(B) $z = k \frac{x^2}{2} + \frac{1}{k} \frac{y^2}{2} + c$

(C) $z = k(x+y/2) + c$

(D) $z = k(x/2 - y) + c$

3. Solve $(D^3 - 3DD'^2 + 2D^3)z = 0$

(A) $z = \phi_1(y+2x) + \phi_2(y-x) + \phi_3(y+x)$ (B) $z = \phi_1(y+6x) + \phi_2(y-x) + x\phi_3(y-2x)$

(C) $z = \phi_1(y-2x) + \phi_2(y+x) + x\phi_3(y+x)$ (D) $z = \phi_1(y+3x) + \phi_2(y-2x) + \phi_3(y+4x)$

4. Find the particular integral of $(D^2 + 3DD' + 4D'^2)z = e^{x-y}$

(A) $\frac{e^{x+y}}{2}$

(B) $\frac{e^{2x+y}}{2}$

(C) $\frac{e^{x-2y}}{2}$

(D) $\frac{e^{x-y}}{2}$

5. $\int_{-1}^1 |x| dx$ is equal to

(A) $\int_0^1 x dx$

(B) $2 \int_0^1 x dx$

(C) $\frac{1}{2} \int_0^1 (-x) dx$

(D) $\frac{\pi/2}{4} \int_0^1 (-x) dx$

6. The constant a_0 of the Fourier series for the function $f(x)=x$ is $0 \leq x \leq 2\pi$

(A) π

(B) 3π

(C) 2π

(D) 0

7. The RMS value of $f(x)=x$ in $-1 \leq x \leq 1$ is
 (A) 1 (B) 0
 (C) -1 (D) $1/\sqrt{3}$
8. For half range cosine series of $f(x)=\cos x$ in $(0, \pi)$ the value of a_0 is
 (A) 0 (B) 4
 (C) $2/\pi$ (D) $4/\pi$
9. The partial differential equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ of the form.
 (A) Elliptic (B) Parabolic
 (C) Hyperbolic (D) None of these
10. The proper solution of $u_t = \alpha^2 u_{xx}$ is
 (A) $u = (Ax + B)C$ (B) $u = (A \cos \lambda x + B \sin \lambda x)e^{-\alpha^2 \lambda^2 t}$
 (C) $u = (Ae^{\lambda x} + Be^{-\lambda x})e^{\alpha^2 \lambda^2 t}$ (D) $u = At + B$
11. One dimensional wave equation is used to find
 (A) Temperature (B) Time
 (C) Displacement (D) Mass
12. The amount of heat required to produce a given temperature change in a body is proportional to
 (A) Weight of the body (B) Mass of the body
 (C) Density of the body (D) Tension of the body
13. The steady state temperature of a rod of length l whose ends are kept at 30 and 40 is
 (A) $u = \frac{10x}{l} + 30$ (B) $u = \frac{20x}{l} + 30$
 (C) $u = \frac{10x}{l} + 20$ (D) $u = \frac{10x}{l}$
14. The Fourier cosine transform of e^{-ax} is
 (A) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + x^2}$ (B) $\sqrt{\frac{1}{\pi}} \frac{s}{s^2 + a^2}$
 (C) $\sqrt{\frac{1}{\pi}} \frac{a}{s^2 - a^2}$ (D) $\sqrt{\frac{2}{\pi}} \frac{a}{(s^2 + a^2)}$
15. $F[xf'(x)] =$
 (A) $\frac{dF(s)}{ds}$ (B) $i \frac{dF(s)}{ds}$
 (C) $-i \frac{dF(s)}{ds}$ (D) $\frac{dF(s)}{ds}$
16. $F[f(x) * g(x)] =$
 (A) $F(s) + G(s)$ (B) $F(s) - G(s)$
 (C) $F(s)G(s)$ (D) $F(s) / G(s)$

17. What is Z-transform of na^n ?
 (A) $\frac{az}{(z-a)^2}$ (B) $\frac{z}{(z-a)^2}$
 (C) $\frac{a}{(z-a)^2}$ (D) $\frac{z}{(z-a)^3}$
18. Region of convergence of a $Z[a^n]$ is
 (A) $|z| < a$ (B) $|z| > a$
 (C) $|z| > |a|$ (D) $|z| < |a|$
19. Find $Z^{-1}\left[\frac{z}{z-a}\right]$
 (A) a^{n+1} (B) a
 (C) a^n (D) a^{n-1}
20. The difference equation formed by eliminating 'a' in $u_n = a 2^{n+1}$ is
 (A) $u_{n+1} - 2u_n = 0$ (B) $u_{n+1} = 0$
 (C) $u_{n+1} - u_n = 0$ (D) $u_n = 0$

PART - B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

21. Form a partial differential equation by eliminating arbitrary constants 'a' and 'b' from
 $z = (x+a)^2 \cdot (y+b)^2$.
22. Find the general solution of $(5D^2 - 12DD' - 9D'^2)z = 0$.
23. Find a Fourier sine series for the function $f(x) = 1, 0 < x < \pi$.
24. Find the RMS value of $f(x) = 1 - x$ in $0 < x < 1$.
25. What are all the solutions of one dimensional wave equation?
26. Prove that $F(e^{iax} f(x)) = F(s+a)$, where $F(f(x)) = F(s)$.
27. Find the Z-transform of $(n+1)(n+2)$.

PART - C (5 × 12 = 60 Marks)

Answer ALL Questions

- 28.a.i. Form the partial differential equation by eliminating f from $xyz = f(x^2 + y^2 - z^2)$.
- ii. Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$.
- (OR)
- b. Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Solve (i) $z = px + qy + \sqrt{1 + p^2 + q^2}$ (ii) $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. Find also singular integral.

(OR)

- b. Solve (i) $(D^2 - 2DD' + D'^2)z = \cos(x - 3y)$ (ii) $(D^2 - DD'^2)z = e^{x+2y}$.
29. a. Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π . Hence deduce $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$.

(OR)

- b. Compute the first two harmonics of the fourier series $f(x)$ given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1

30. a. A tightly stretched string with fixed end point $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$. Find the displacement.

(OR)

- b. A rod of length l has its ends A and B kept at 0°C and 100°C respectively unit steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature $u(x, t)$.
31. a. Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence prove that

$$\int_0^\infty \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$$

(OR)

- b. Use transform method to evaluate $\int_0^\infty \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$
32. a.i. Find $Z(a^n)$ and $Z(n^2)$.
- ii. Using residues find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$.

(OR)

- b. Solve the equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$ by using Z-transform.

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Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
3rd to 7th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
- (ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

- The partial differential equation formed by eliminating arbitrary constant a, b is $z = (x+a)(y+b)$
(A) $z = p + q$ (B) $z = p - q$
(C) $z = p/q$ (D) $z = pq$
- The complementary function of $(D^2 + 2DD' + D'^2)z = 0$ is
(A) $\phi_1(y-x) + \phi_2(y-x)$ (B) $\phi_1(y-x) + x\phi_2(y-x)$
(C) $\phi_1(y-x) + \phi_2(y+x)$ (D) $\phi_1(y-x) + x\phi_2(y+x)$
- The particular integral of $(D^2 - 2DD')z = e^{2x}$
(A) $e^{2x}/4$ (B) $e^{2x+y}/4$
(C) e^{2x} (D) $e^{2x}/2$
- The complete solution of $z = px + qy + p^2q^2$ is
(A) $z = ax + by^2 + ab^2$ (B) $z = ax^2 + by + ab^2$
(C) $z = ax + by + a^2b^2$ (D) $z = ax + by + c$
- $\sin x$ is a periodic function with period
(A) π (B) $\pi/2$
(C) 2π (D) 4π
- The constant a_0 of the Fourier series for the function $f(x) = k, 0 \leq x \leq 2\pi$ is
(A) k (B) $2k$
(C) 0 (D) $k/2$
- The RMS value of $f(x) = x$ in $-1 \leq x \leq 1$ is
(A) 1 (B) 0
(C) $1/\sqrt{3}$ (D) -1

8. Half range cosine series for $f(x)$ is $(0, \pi)$ is
 (A) $\sum_{n=1}^{\infty} a_n \cos nx$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$
 (C) $\sum_{n=1}^{\infty} b_n \sin nx$ (D) $\frac{a_0}{2} - \sum_{n=1}^{\infty} a_n \cos nx$
9. The proper solution of the problems of vibration of string is
 (A) $y(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})(Ce^{\lambda at} + De^{\lambda at})$ (B) $y(x,t) = (Ax + B)(ct + 1)$
 (C) $y(x,t) = (A \cos \lambda x + B \sin \lambda x)(C \cos \lambda at + D \sin \lambda at)$ (D) $y(x,t) = Ax + B$
10. The one dimensional wave equation is
 (A) $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ (B) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$
 (C) $\frac{\partial y}{\partial t} = a \frac{\partial^2 y}{\partial x^2}$ (D) $\frac{\partial^2 y}{\partial x^2} = a \frac{\partial^2 y}{\partial t^2}$
11. One dimensional heat equation is used to find
 (A) Density (B) Temperature distribution
 (C) Time (D) Displacement
12. A rod of length l has its ends A and B kept at 0° and 100° respectively, until steady state conditions prevail. Then the initial condition is given by
 (A) $u(x,0) = ax + b + 100l$ (B) $u(x,0) = \frac{100x}{l}$
 (C) $u(x,0) = 100xl$ (D) $u(x,0) = (x+l)100$
13. $F[e^{iax} f(x)]$
 (A) $F(s+a)$ (B) $F(s-a)$
 (C) $F(sa)$ (D) $F(s/a)$
14. $F[xf'(x)] =$
 (A) $\frac{dF(s)}{ds}$ (B) $i \frac{dF(s)}{ds}$
 (C) $-i \frac{dF(s)}{ds}$ (D) $\frac{dF(s)}{ds}$
15. The fourier cosine transform of $Fc[e^{-4x}]$
 (A) $\sqrt{\frac{2}{\pi}} \frac{4}{16+s^2}$ (B) $\sqrt{\frac{2}{\pi}} \frac{4}{4+s^2}$
 (C) $\sqrt{\frac{\pi}{2}} \frac{4}{16+s^2}$ (D) $\sqrt{\frac{\pi}{2}} \frac{4}{4+s^2}$

16. $F[f(x) * g(x)] =$
 (A) $F(s) + G(s)$ (B) $F(s) - G(s)$
 (C) $F(s)G(s)$ (D) $F(s)/G(s)$
17. What is $Z(7)$
 (A) $\frac{z}{z-1}$ (B) $7 \frac{z}{z-1}$
 (C) $\frac{1}{7} \frac{z}{z-1}$ (D) $\frac{z-1}{z}$
18. What is $Z[na^n]$
 (A) $\frac{az}{(z-a)^2}$ (B) $\frac{z}{(z-a)^2}$
 (C) $\frac{a}{(z-a)^2}$ (D) $\frac{z}{(z-a)^3}$
19. If $z[f(t)] = F(z)$ then $\lim_{z \rightarrow \infty} F(z) =$
 (A) $f(0)$ (B) $f(1)$
 (C) $\lim_{x \rightarrow \infty} f(t)$ (D) $f(\infty)$
20. $\phi(z) = \frac{z^n(2z+4)}{(z-2)^3}$ has a pole of order
 (A) 2 (B) 1
 (C) 3 (D) 4

PART - B (5 × 4 = 20 Marks)
 Answer ANY FIVE Questions

21. Form the Partial differential equation by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$.
22. Find the half range Fourier sine series for $f(x) = x$ in $0 < x < \pi$.
23. Write the one dimensional heat flow equation and all the possible solutions.
24. Find the Fourier sine transform of e^{-ax} $a > 0$.
25. Find Z-transform of $r^n \cos n\theta$.
26. Find $z^{-1} \left(\frac{1}{(z-1)(z-2)} \right)$ by convolution.
27. Solve $p^2 + q^2 = x + y$.

PART - C (5 × 12 = 60 Marks)

Answer ALL Questions

28. a. Solve (i) $x^2p - y^2q - (x - y)z$ (ii) $p^2 + q^2 = z$.

(OR)

b. Solve the equation $(D^2 + 4DD' - 5D'^2)z = xy + \sin(2x + 3y)$.

29. a. Find the Fourier series expansion of period 2 for the function

$$f(x) = \begin{cases} \pi x & \text{in } 0 \leq x \leq 1 \\ \pi(2 - x) & \text{in } 1 \leq x \leq 2 \end{cases} \text{ Deduce the sum } \sum_{n=1,3,\dots}^{\infty} \frac{1}{n^2}.$$

(OR)

b. Find the Fourier series upto the second harmonic from the data.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

30. a. The ends of an uniform string of length $2l$ are fixed. The initial displacement is $y(x, 0) = kx(2l - x)$, $0 < x < 2l$ while the initial velocity is zero. Find the displacement at any distance x from the end $x = 0$ at any time t .

(OR)

b. Find the solution of the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the condition

$$u(0, t) = 0 \text{ and } u(l, t) = 0 \text{ for } t \geq 0 \text{ and } u(x, 0) = \begin{cases} x, & \text{for } 0 < x < \frac{l}{2} \\ l - x & \text{for } \frac{l}{2} < x < l \end{cases}$$

31. a. Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x|, & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and hence prove that

$$\int_0^{\infty} \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}.$$

(OR)

b. Show that $e^{-x^2/2}$ is self-reciprocal under Fourier transform by finding the Fourier transform of $e^{-a^2x^2}$, $a > 0$.

32. a. Find (i) $Z\left(2^n \cos \frac{n\pi}{2}\right)$ (ii) $Z^{-1}\left(\frac{z(z+1)}{(z-1)^3}\right)$ using the method of residues.

(OR)

b. Solve using Z-transform $y_{n+2} - 3y_{n+1} - 10y_n = 0$, given that $y_0 = y_1 = 0$.

Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2019

1st to 7th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. The partial differential equation formed by eliminating the arbitrary function 'f' from

$$z = f(x^2 - y^2) \text{ is}$$

(A) $qx + py = 0$

(B) $qx = py$

(C) $qx = p$

(D) $py = q$

2. The complete integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$ is

(A) $z = ax - by$

(B) $z = ax + by$

(C) $z = ax + by + \sqrt{1 + a^2 + b^2}$

(D) $z = ax + by - \sqrt{1 + a^2 + b^2}$

3. General solution of $(D^2 + 4DD' - 5D'^2)z = 0$

(A) $z = f_1(y - 5x) + f_2(y + x)$

(B) $z = f_1(y + 5x) + f_2(y + x)$

(C) $z = f_1(y - 5x) + f_2(y - x)$

(D) $z = f_1(y) + f_2(y - x)$

4. The particular integral of $(D^2 + 2DD' + D'^2)e^{x-y}$ is

(A) e^{x-y}

(B) $\frac{x^2}{2}e^{x-y}$

(C) $\frac{x}{2}e^{x-y}$

(D) $\frac{x^2}{2}e^{x+y}$

5. If $f(x) = |x|$ in $(-\pi, \pi)$ then the constant term a_0 of the Fourier series is

(A) 2π

(B) 0

(C) $\pi/2$

(D) π

6. If $f(x) = |\sin x|$ then its period is

(A) π

(B) 2π

(C) 0

(D) $\pi/2$

7. The root mean square value of $f(x) = x$ in $-1 \leq x \leq 1$ is

(A) 1

(B) 0

(C) $\frac{1}{\sqrt{3}}$

(D) -1

8. Half-Range sine series for $f(x)$ in $(0, \pi)$ is

(A) $\sum_{n=1}^{\infty} b_n \sin nx$

(B) $\sum_{n=1}^{\infty} a_n \cos nx$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

(D) $\frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$

9. In wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) $\frac{m}{T}$

(B) $\frac{T}{m}$

(C) Tm

(D) T^m

10. The steady state solution of $u_t = \alpha^2 u_{xx}$ is

(A) $u = c_1 x$

(B) $u = c_1 + c_2 t$

(C) $u = c_1 x + c_2$

(D) $u = \text{zero}$

11. Classify $u_{xx} + 2u_{xy} + u_{yy} = 0$

(A) Parabolic

(B) Elliptic

(C) Hyperbolic

(D) Geodesic

12. The number of initial and boundary conditions to solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ are

(A) Three

(B) Two

(C) Four

(D) One

13. If $F\{f(x)\} = F(s)$, then $F\{f(ax)\} =$

(A) $\frac{1}{|a|} F\left(\frac{s}{a}\right)$

(B) $aF(s)$

(C) $aF\left(\frac{s}{a}\right)$

(D) $F(as)$

14. The Fourier sine transform of e^{-ax} ($a > 0$) is

(A) $\sqrt{\frac{2}{\pi}} \left(\frac{s}{s^2 + a^2} \right)$

(B) $\sqrt{\frac{2}{\pi}} \left(\frac{a}{s^2 + a^2} \right)$

(C) $\sqrt{\frac{2}{\pi}} \left(\frac{s}{s^2 - a^2} \right)$

(D) $\sqrt{\frac{2}{\pi}} \left(\frac{a}{s^2 - a^2} \right)$

15. $F^{-1}[F(s)G(s)] =$

(A) $f(x)g(x)$

(B) $f(x) * g(x)$

(C) $f(x) + g(x)$

(D) $f(x) - g(x)$

16. If $F\{f(x)\} = F(s)$ then $\int_{-\infty}^{\infty} |f(x)|^2 dx =$

(A) $\int_0^{\infty} |F(s)|^2 ds$

(B) $\int_{-\infty}^{\infty} |F(x)|^2 dx$

(C) $\int_{-\infty}^{\infty} |F(s)|^2 ds$

(D) $\int_0^{\infty} |F(x)|^2 dx$

17. $Z\{(-1)^n\} =$

(A) $\frac{z}{z+1}$

(B) $\frac{z}{z-1}$

(C) $z(z+1)$

(D) $\frac{1}{z+1}$

18. $Z[na^n] =$

(A) $\frac{z}{(z-a)^3}$

(B) $\frac{a}{(z-a)^2}$

(C) $\frac{z}{(z-a)^2}$

(D) $\frac{az}{(z-a)^2}$

19. $z\left(\sin \frac{n\pi}{2}\right) =$

(A) $\frac{z}{z^2+1}$

(B) $\frac{z}{z^2-1}$

(C) $\frac{z^2}{z^2-4}$

(D) $\frac{z^2}{z^2+1}$

20. Poles of $f(z) = \frac{z^n}{(z+1)(z+2)}$ are

(A) $z = 1, 2$

(B) $z = -1, -2$

(C) $z = 1, -2$

(D) $z = -1, 2$

PART - B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

21. Form a partial differential equation by eliminating arbitrary constants a, b , from $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$.

22. Find half-range sine series of $f(x) = a$ in $(0, l)$.

23. Write down the three mathematically possible solutions of one dimensional heat flow equation.

24. Find the Fourier transform of $f(x)$ defined as $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$.

25. Find the Z-transform of $\frac{1}{n(n-1)}$.

26. Solve the equation $pq + p + q = 0$.

27. Find the Fourier sine transform of $f(x)$ defined as $f(x) = \begin{cases} \sin x & \text{when } 0 < x < a \\ 0 & \text{when } x > a \end{cases}$.