Statistical Quality Control

Introduction to Statistical Quality Control

In these days of tough businesss competition, it has become essential to maintain the quality of the goods manufactured and market them at reasonable price. If the consumers feel satisfied with regard to the quality, price, etc. of the product manufactured by a certain company, it will result in goodwill for the product and in increase in sales. If not and if proper attention is not given to the complaints of the consumers regarding quality, the manufacturer cannot push through his product in the market and ultimately he has to quit the market. Hence it is important to maintain and improve the quality of the manufactured products for the manufacturer to remain and flourish in his business.

Quality does not mean the highest standard of the manufactured product, but conforming to the prescribed standard of the product so as to satisfy the consumers, even if it may be below the highest absolute standard.

Though the quality standard might have been specified, it is not possible to avoid some variation in the quality of the product. For example a machine is set and hence expected to produce per day 10,000 bolts, each of length 2 cm. It is very unlikely that all the bolts are of length 2 cm exactly. Some of them may be slightly less than 2 cm and some slightly more than 2 cm in length. Such variation in quality of the product can be divided into two kinds, namely, *chance variation* and *assignable variation*.

Chance variation is the variation in the quality of the product which occurs due to many minor, but random causes, such as slight changes in temperature, pressure and metal hardness. *Assignable variation* is the variation that occurs due to non-random causes like poor quality of input raw material, mechanical faults, handling of machines by inexperienced operators, etc. Though no method is available by which the chance variation can be controlled or eliminated, assignable variation can be eliminated, if detected early during the production process.

Statistical Quality Control (SQC) is a statistical method for finding whether the variation in the quality of the product is due to random causes or assignable causes. SQC does not involve inspecting each and every item produced for quality standards, but involves inspection of samples of items produced and application of tests of significance.

Statistical Quality Control methods are applied to two distinct phases of manufacturing operation. Process control and Acceptance sampling or product control.

Process Control means control of the quality of the goods while they are in the process of production. To achieve process control, repeated random samples are taken from the population of items, as and when they are being produced, the sample results are subjected to statistical analysis by means of simple graphical device, known as control charts and the faults in the production process are rectified then and there.

Control Chart is a graphical device mainly used for the study and control of the manufacturing process. It is simple to construct and easy to interpret. The manufacturer can find out, at a glance, whether or not the process is under control so that the proportion of defective items is not excessive. Control chart is also called *shewhart chart*.

There are two types of control charts, namely,

1. Control charts of variables (Mean and range charts).
2. Control charts of attributes (p-chart and c-chart).

Variables are the quality characteristics of a product that are measurable e.g., diameter of a hole bored by a drilling machine. For the construction of control charts of a variable, a record of the actual measurements of that variable for the sampled items must be known.

Attributes are the quality characteristics of a product that are not amenable for measurement, but are identified by their presence or absence, e.g. the presence (and hence the number) of defective items in a sample.

Control Charts of Variables

Normally a production process is expected to turn out quality products or to be under control. So the production process is allowed to operate and to produce items. To ensure that the process continues to be in a state of control, a sample of the produced items is drawn periodically and tested for their quality. If the quality of the sampled items is satisfactory in the statistical sense, the production process is allowed to continue. Otherwise, corrective measures are taken to restore the quality of the items.

Instead of inspecting each item in a sample for its quality, it will be advantageous and prudent to arrive at a conclusion regarding the quality of the items on the basis of the sample mean. Hence the control chart for the sample mean is constructed and used for taking decisions regarding the quality of the items. The sample mean alone may not reflect the quality variations of the items, but the range of the sampled values is known to be a simple measure of the quality variations. Hence the control chart for the sample range is also constructed and used for taking decisions regarding the quality of the items.

Control Limits for the sample mean $\overline{X}$ and sample range R

Let X be the random variable that represents the measurable quality characteristic of the population of the items produced. We assume that X follows a normal distribution with mean μ and standard deviation σ.

Then $\overline{X}$, the mean of a sample of size n, is also a random variable that is normally distributed with mean μ and S.D. $\frac{\sigma}{\sqrt{n}}$.

We know, from the property of normal distribution that $P\left\{\mu - \frac{3\sigma}{\sqrt{n}} \le \overline{X} \le \mu + \frac{3\sigma}{\sqrt{n}}\right\} = 0.9973$. This means that 99.73% of the sample means (viz., almost all the sample mean values) will be within $\mu - \frac{3\sigma}{\sqrt{n}}$ and $\mu + \frac{3\sigma}{\sqrt{n}}$. The variation in the sample mean values within these limits is due to random causes. If an observed value of $\overline{X}$ lies outside $\mu \pm \frac{3\sigma}{\sqrt{n}}$, it indicates the presence of some assignable cause. $\mu \pm \frac{3\sigma}{\sqrt{n}}$ are called the *control limits* for the sample mean $\overline{X}$; $\mu - \frac{3\sigma}{\sqrt{n}}$ is called the *Lower Control Limit* (*LCL*) and $\mu + \frac{3\sigma}{\sqrt{n}}$ is called the *Upper Control Limit* (*UCL*).

Similarly R, the sample range follows a normal distribution with mean $\overline{R}$ and S.D. σ_R. Since $P\{\overline{R} - 3\sigma_R \le R \le \overline{R} + 3\sigma_R\} = 0.9973$, the lower and upper control limits for the sample range R are $\overline{R} \mp 3\sigma_R$, where $\overline{R} = \frac{1}{N}\sum R_i$, where N is the number of sample each of size n.

Now μ and σ are not known and computation of σ_R involves some numerical work. Hence they are estimated approximately by using $\mu = \overline{\overline{X}} = \frac{1}{N}\sum \overline{X}_i$, $\frac{3\sigma}{\sqrt{n}} = A_2\overline{R}$, σ_R (in LCL) $= \frac{1}{3}(1 - D_3)\,\overline{R}$ and σ_R (in UCL) $= \frac{1}{3}(D_4 - 1)\,\overline{R}$, where A_2, D_3 and D_4 are *control chart constants*, whose values depend on the sample size n and which are readily available in the table of control chart constants (Table 2.1), given at the end of the chapter. Using these approximations, the control limits are obtained as follows:

$$\text{LCL for } \overline{X} = \overline{\overline{X}} - A_2\overline{R} \text{ and UCL for } \overline{X} = \overline{\overline{X}} + A_2\overline{R}$$

$$\text{LCL for } R = D_3\,\overline{R} \text{ and UCL for } R = D_4\,\overline{R}$$

Procedure to draw the $\overline{X}$-chart and R-chart

1. The sample values in each of the N samples each of size n will be given. Let $\overline{X}_1, \overline{X}_2 \ldots \overline{X}_N$ be the means of the N samples and $R_1, R_2, \ldots, R_N$ be the

ranges of the N samples. By range of a sample, we mean the maximum sample value minus the minimum sample value in that sample.

2. We then compute $\bar{\bar{x}} = \frac{1}{N}(\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_N)$ and $\bar{R} = \frac{1}{N}(R_1 + R_2 + \ldots + R_N)$.
3. The values of A_2, D_3, D_4 for the given sample size n are taken from the table of control chart constants.
4. Then the values of the control limits $\bar{\bar{X}} \pm A_2\bar{R}$ (for the mean chart) and the control limits $D_3\,\bar{R}$ and $D_4\,\bar{R}$ (for the range chart) are computed.
5. On the ordinary graph sheet, the sample numbers are represented on the x-axis and the sample means on the y-axis (for the mean chart) and the sample ranges on the y-axis (for the range chart).
6. For drawing the mean chart, we draw the three lines $y = \bar{\bar{X}}$, $y = \bar{\bar{X}} - A_2\bar{R}$ and $y = \bar{\bar{X}} - A_2\bar{R}$ which represent respectively the central line, the LCL. line and UCL line. Also we plot the points whose coordinates are $(1, \bar{X}_1)$, $(2, \bar{X}_2)$, $\cdots$ $(N, \bar{X}_N)$ and join adjacent points by line segments. The graph thus obtained is the $\bar{X}$-chart.
7. For drawing the range chart, we draw the three line $y = \bar{R}$, $y = D_3\,\bar{R}$ and $y = D_4\,\bar{R}$ which represent respectively the central line, the LCL line and UCL line. Also we plot the points whose co-ordinates are $(1, R_1)$, $(2, R_2)$, $\cdots$ (N, R_N) and join adjacent points by line segments. The graph thus obtained is the R-chart.

Comments on State of Control of the Process

If the plotted points fall within the LCL and UCL lines, there is nothing to worry, as in such a case the variation between the samples is attributed to chance causes and the process is under control.

But when one or more plotted points lie outside the control lines, it is to be considered as a danger signal, indicating the variations between samples are caused by assignable causes and the process is out of control and that necessary corrective action should be taken at once.

Sometimes even though the plotted points may lie between the control lines, a sizeable number of successive points may show a tendency to lie on definite curves going towards the LCL or UCL lines or may lie on the same side of the central line. Such a pattern of sample points should also be considered as a danger signal, warranting a change in the production process.

Control Chart for Sample Standard Deviation or s-chart

Since the standard deviation is an ideal measure of dispersion, a combination of control charts for the sample mean $\bar{X}$ and the sample S.D. s is more appropriate than the combination of $\bar{X}$ and R charts for controlling process average and process variability. We know that s, the S.D. of a sample of size n, is a random variable that is normally distributed with mean σ and S.D. $\frac{s}{\sqrt{2n}}$, where σ is the S.D. of the population from which the sample is drawn.

Hence $P\left\{\sigma - \frac{3\sigma}{\sqrt{2n}} \le s \le \sigma + \frac{3\sigma}{\sqrt{2n}}\right\} = 0.9973$

$\therefore$ The lower and upper control limits for s are $\sigma - \frac{3\sigma}{\sqrt{2n}}$ and $\sigma + \frac{3\sigma}{\sqrt{2n}}$.

Since σ is not known, it is estimated approximately by $\bar{s} = \frac{1}{N}(s_1 + s_2 + \cdots + s_N)$, where s_i is the S.D. of the ith sample and N is the number of samples considered. Hence LCL for $s = \left(1 - \frac{3}{\sqrt{2n}}\right) s \simeq B_3\bar{s}$ and UCL for $s = \left(1 + \frac{3\sigma}{\sqrt{2n}}\right) s \simeq B_4\bar{s}$.

The values of B_3 and B_4 can be read for various values of sample size n from the table of control chart constants.

The procedure for drawing the s-chart is similar to that for $\bar{x}$ = chart and R - chart.

If $\bar{x}$ values and s values only are given, then CL for $\bar{x} = \bar{\bar{x}}$, LCL for $\bar{x} = \bar{\bar{X}} - A_1\sqrt{\frac{n-1}{n}}\bar{s}$ and UCL for $\bar{X} = \bar{\bar{X}} + A_1\sqrt{\frac{n-1}{n}}\bar{s}$, when $n \le 25$.

Note *The difficulty of computation of s makes the use of s-chart almost impractical in most industrial situations. R-chart is preferred to s-chart because of its computational ease.*

Control Charts for Attributes

To control the quality of certain products whose attributes are available, the following control charts are used:

(*i*) p-chart for proportion of defectives
(*ii*) np-chart for number of defectives
(*iii*) c-chart for the number of defects in a unit.

Of the above, p-chart is used when all the samples drawn from the produced items are of the same size or of different size; np-chart is used only when all the samples are of the same size n; c-chart is used only when each sample consists of only one item.

np-chart

If the proportion of defectives (successes) in the population of items produced is p and the number of defectives in a sample of size n (viz., n trials) is X, then X follows a binomial distribution with mean np and S.D. $\sqrt{npq}$. When n is sufficiently large and when neither p nor q is very small, X follows a normal distribution with mean np and S.D. $\sqrt{npq}$.

Hence $P\{np - 3\sqrt{npq} \le X \le np + 3\sqrt{npq}\} = 0.9973$.

$\therefore$ The control limits for x, the number of defectives, are $np \mp 3\sqrt{npq}$.

As the population proportion of defectives p will not be known, it is estimated approximately as $\overline{p} = \frac{1}{N}(p_1 + p_2 + \cdots + p_N)$ or equivalently $n\overline{p} = \frac{1}{N}$ $(np_1 + np_2 \cdots + np_N)$, where all the samples are of the same size n, the number of samples is N and the number of defectives in the ith sample is np_i.

Hence the control limits for the number of sample defectives are $n\overline{p} \pm 3\sqrt{n\overline{p}(1-\overline{p})}$, where each sample is of size n. Here the target value or the central (line) value of np is $n\overline{p}$. To draw the np-chart, the sample number is represented on the x-axis and the number of defectives is represented on the y-axis.

The lines $y = n\overline{p}$, $y = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})}$ and $y = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}$, which represent respectively the central line, LCL line and UCL line are drawn. We plot the points $(1, np_1)$, $(2, np_2)$, $\cdots$, (N, np_N) on the graph sheet and adjacent points are joined by line segments. The state of control of the process is decided as before.

***p*-chart**

Under the same assumptions as those of np-chart, since X follows a normal distribution with mean np and S.D. $\sqrt{npq}$, the proportion of defectives, viz; $\frac{X}{n}$ follows a normal distribution with mean p and S.D. $\sqrt{\frac{pq}{n}}$.

$$\text{Hence} \quad P\left\{p - 3\sqrt{\frac{pq}{n}} \le \frac{X}{n} \le p + 3\sqrt{\frac{pq}{n}}\right\} = 0.9973.$$

$\therefore$ The control limits for $\frac{X}{n}$, the proportion of defectives, are $p \mp 3\sqrt{\frac{pq}{n}}$.

As in the previous case, p is estimated as $\overline{p} = \frac{1}{N}(p_1 + p_2 + \cdots + p_N)$, where the proportion of defectives in the ith sample is p_i.

Hence the control limits for the fraction or proportion of sample defective are

$$\overline{p} \mp 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{1}$$

The formula (1) holds good when all the samples are of the same size n.

If the size of the sample differs from sample to sample, then the formula for the control limits will be $\overline{p} \mp 3\sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}}$, where $\overline{n}$ is the average sample size given by $\bar{n} = \frac{1}{N}(n_1 + n_2 + \cdots + n_N)$, where n_i is the size of the ith sample. This holds good, when n_i values do not differ very much from $\bar{n}$. This method is applied if $0.75\,\bar{n} < n_i < 1.25\,\bar{n}$ for all i.

To draw the p-chart, the sample number is represented on the x-axis and the proportion of defectives is represented on the y-axis. The lines

$$y = \bar{p},\ y = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\left[\text{or } y = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}\right]$$

and
$$y = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\left[\text{or } y = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}\right]$$

which represent respectively the central line, LCL line and UCL line are drawn. We plot the points $(1, p_1), (2, p_2), \cdots, (N, p_N)$ on the graph paper and adjacent points are joined by line segments. The state of control of the process is decided as before.

Note *np-chart and p-chart are used when $\bar{p} \geq 0.05$ or $n\bar{p} \geq 4$.*

c-chart

When it is required to control (minimise) the number of defects per unit, c-chart is used. 'c' represents the number of defects in a unit. For construction of c-chart, a record of the number of defects in each of the N articles inspected should be known. Since the probability of occurrence of a defect in a unit is very small, the number X of defects in a unit follows a Poisson distribution with parameter λ, viz., with mean λ and S.D. $\sqrt{\lambda}$. In the limit, X follows a normal distribution with mean λ and S.D. $\sqrt{\lambda}$.

Hence $\quad P\,\{\lambda - 3\sqrt{\lambda} \leq X \leq \lambda + 3\sqrt{\lambda}\} = 0.9973$

$\therefore$ The control limits for X, the number of defects in a unit are $\lambda \mp 3\sqrt{\lambda}$.

As the value of λ will not be known, it is estimated approximately by $\bar{c}$, where $\bar{c} = \frac{1}{N}(c_1 + c_2 + \cdots + c_N)$, where c_i is the number of defects in the ith unit.

Hence the control limits for the number of defects c in a unit are $\bar{c} \mp 3\sqrt{\bar{c}}$.

To draw the c-chart, the item number is represented on the x-axis and the number of defects in a unit is represented on the y-axis. The lines $y = \bar{c}$, $y = \bar{c} - 3\sqrt{\bar{c}}$ and $y = \bar{c} + 3\sqrt{\bar{c}}$, which represent respectively the central line, LCL line and UCL line are drawn. We plot the points $(1, c_1), (2, c_2), \cdots, (N, c_N)$ on the graph sheet and the adjacent points are joined by line segments. The state of control of the process is decided as before.

Note *c-chart is used when $\bar{c} \geq 4$ or when $\bar{c}$ is small compared with the maximum number of defects given in the data.*

Specification Limits and Tolerance Limits

Though it is desirable to specify a single target value as the acceptable quality standard of a manufactured product, it is not possible to avoid some variation from the target value due to chance and assignable causes. Hence the quality of the product is taken as acceptable, if the measurement of the quality characteristic lies within an interval that encloses the target value. The end values of such an interval are called *specification limits*. For example when the target length of a bolt is 2.5 cm, the specification limits may be assumed as 2.4 cm and 2.6 cm, viz., 2.5 ± 0.1 cm.

Tolerance Limits of a quality characteristic are defined as those values between which nearly all the manufactured items will lie.

If the measurable quality characteristics X is assumed to be normally distributed with mean μ and S.D. σ, then the tolerance limits are usually taken as $\mu \pm 3\sigma$, since only 0.27% of all the items produced can be expected to fall outside these limits.

As μ and σ will not be known, we get the tolerance limits approximately using the control charts for $\overline{X}$ and R as explained below: N samples, each of size n, are taken from the population of items produced. Let $\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_N$ be the means of these samples and $R_1, R_2, \ldots, R_N$ be the ranges of these samples. The $\overline{X}$-chart and R-chart are constructed using these values. If the variations of the sample mean and range values are due to chance causes only, viz., if the process is under control with respect to both $\overline{X}$ and R, then the tolerance limits are computed as $\overline{\overline{X}} \pm 3\dfrac{\overline{R}}{d_2}$, since the estimates of the mean and S.D. of the population are given by $\hat{\mu} = \overline{\overline{X}}$ and $\hat{\sigma} = \dfrac{\overline{R}}{d_2}$, where d_2 is a control chart constant to be read from the table of control chart constants.

If the process is not under control with respect to $\overline{X}$ or R or both, then the samples whose means or ranges go out of control are removed and a new set of $\overline{\overline{X}}$ and $\overline{R}$ values are computed using the remaining samples. Using these values, a new set of control limits are computed and the control of the process is checked. This procedure is repeated until the process comes under control. After ascertaining that the process is under control with respect to both the sample mean and range, the tolerance limits are computed as $\hat{\mu} = \overline{\overline{X}} \pm \dfrac{3\overline{R}}{d_2}$, where $\overline{\overline{X}}$ and $\overline{R}$ are computed using the samples that remain under control ultimately.

If these tolerance limits are within the specification limits, then the process is assumed to operate at an acceptable level. If they do not fall within the specification limits, the process is bound to produce some defective (unacceptable) items, even though the process may be under control.

Worked Examples

Example 1 *Given below are the values of sample mean $\overline{X}$ and sample range R for 10 samples, each of size 5. Draw the appropriate mean and range charts and comment on the state of control of the process.*

Sample No. :	1	2	3	4	5	6	7	8	9	10
Mean :	43	49	37	44	45	37	51	46	43	47
Range :	5	6	5	7	7	4	8	6	4	6

$$\overline{\overline{X}} = \frac{1}{N}\sum \overline{x}_i$$

$$= \frac{1}{10}\ (43 + 49 + 37 + \ldots + 47)$$

$$= 44.2$$

$$\overline{R} = \frac{1}{N}\sum R_i$$

$$= \frac{1}{10}\ (5 + 6 + 5 \ldots + 6)$$

$$= 5.8$$

From the table of control chart constants, for sample size $n = 5$, we have

$$A_2 = 0.577,\ D_3 = 0 \text{ and } D_4 = 2.115$$

$$\frac{\text{Control limit for } \overline{X} = \text{chart:}}{\text{CL(central line)} = \overline{\overline{X}} = 44.2}$$

$$\text{LCL} = \overline{\overline{X}} - A_2\ \overline{R} = 44.2 - 0.577 \times 5.8 = 40.85$$

$$\text{UCL} = \overline{\overline{X}} + A_2\ \overline{R} = 44.2 + 0.577 \times 5.8 = 47.55$$

Control Limit for R-chart

CL = $\overline{R}$ = 5.8; LCL = $D_3\ \overline{R}$ = 0; UCL = $D_4\ \overline{R}$ = 2.115 × 5.8 = 12.27

The mean chart and range chart relevant to this problem are given in Fig. 11.1 (a) and (b):

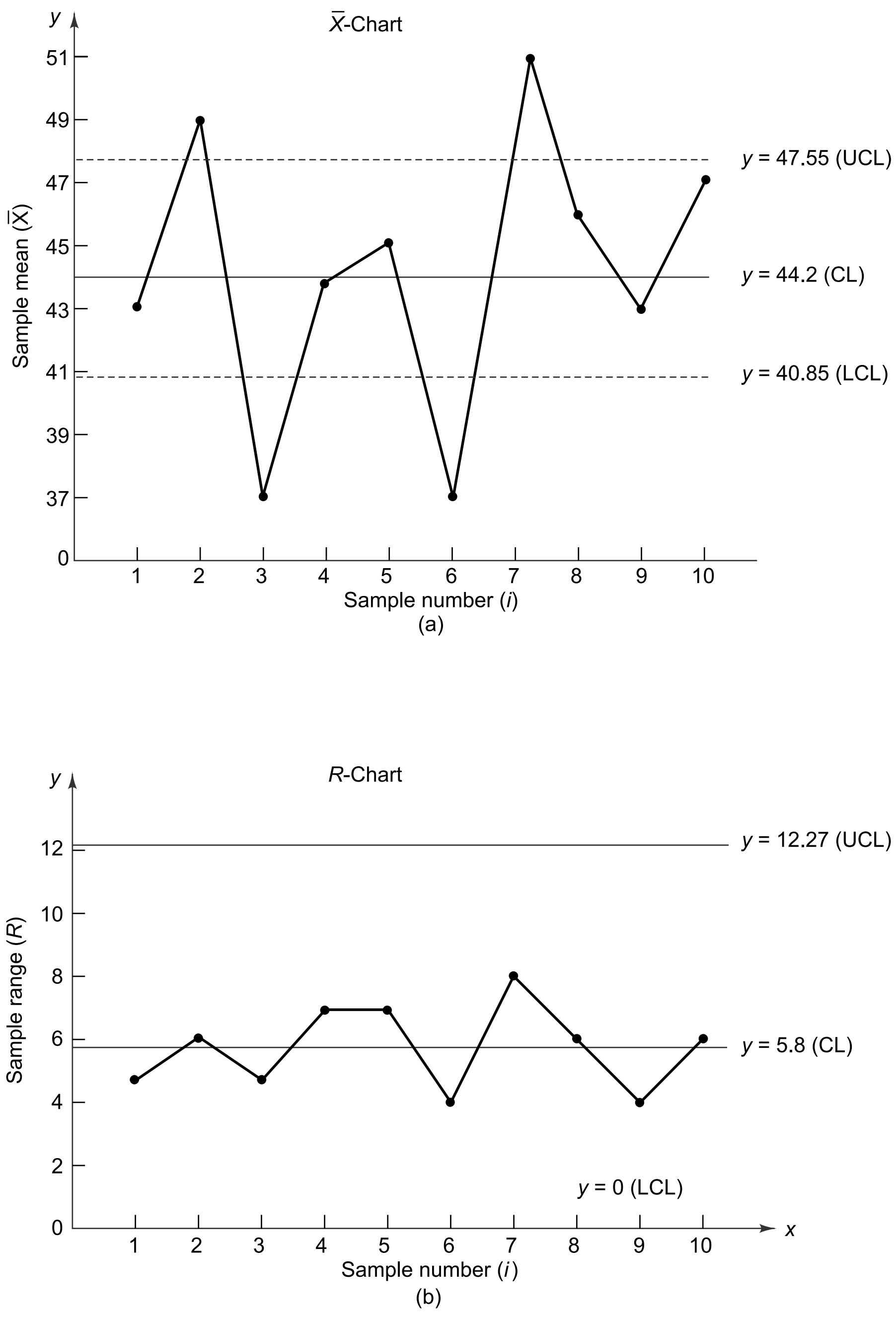

$\bar{X}$-Chart
y
Sample mean ($\bar{X}$)
51
49
47
45
43
41
39
37
0
1 2 3 4 5 6 7 8 9 10
y = 47.55 (UCL)
y = 44.2 (CL)
y = 40.85 (LCL)
Sample number (i)
(a)
R-Chart
y
Sample range (R)
12
10
8
6
4
2
0
1 2 3 4 5 6 7 8 9 10
x
y = 12.27 (UCL)
y = 5.8 (CL)
y = 0 (LCL)
Sample number (i)
(b)

Fig. 2.1

State of Control

All the sample points in the range chart lie within the control lines. Hence as far as the variability of the sample values is concerned, the process is under control. But in the mean chart, two points lie above the upper control line and two points lie below the lower control line. Hence as far as the average of the sample values is concerned, the process is not under control. On the whole, we conclude that the process is out of a control.

Note *Even though we could have arrived at this conclusion regarding the state of control without drawing the control charts, it is necessary to draw the control charts, as it is a part of the solution to the given problem.*

Example 2 *A machine fills boxes with dry cereal. 15 samples of 4 boxes are drawn randomly. The weights of the sampled boxes are shown as follows. Draw the control charts for the sample mean and sample range and determine whether the process is in a state of control.*

Sample Number	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
	10.0	10.3	11.5	11.0	11.3	10.7	11.3	12.3
Weights of boxes (X)	10.2	10.9	10.7	11.1	11.6	11.4	11.4	12.1
	11.3	10.7	11.4	10.7	11.9	10.7	11.1	12.7
	12.4	11.7	12.4	11.4	12.1	11.0	10.3	10.7

9	*10*	*11*	*12*	*13*	*14*	*15*
11.0	11.3	12.5	11.9	12.1	11.9	10.6
13.1	12.1	11.9	12.1	11.1	12.1	11.9
13.1	10.7	11.8	11.6	12.1	13.1	11.7
12.4	11.5	11.3	11.4	11.7	12.0	12.1

As the $\overline{X}$-chart and R-chart are to be drawn, we first compute the means and ranges of the given samples.

Sample No. (i)	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
ΣX	43.9	43.6	46.0	44.2	46.9	43.8	44.1	47.8	49.6	45.6
$\overline{X}_i$	11.0	10.9	11.5	11.1	11.7	11.0	11.0	12.0	12.4	11.4
R_i	2.4	1.4	1.7	0.7	0.8	0.7	1.1	2.0	2.1	1.4

11	*12*	*13*	*14*	*15*
47.5	47.0	47.0	49.1	46.3
11.9	11.8	11.8	12.3	11.6
1.2	0.7	1.0	1.2	1.5

Now $$\bar{\bar{X}} = \frac{1}{N}\sum \bar{X}_i = \frac{1}{15}\,(11.0 + 10.9 + 11.5 + \ldots + 11.6)$$

$$= \frac{173.4}{15} = 11.56$$

$$\bar{R} = \frac{1}{N}\sum R_i = \frac{1}{15}\,(2.4 + 1.4 + 1.7 + \ldots + 1.5)$$

$$= \frac{19.9}{15} = 1.33$$

From the table of control chart constants, for the sample size $n = 4$, we have

$$A_2 = 0.729,\ D_3 = 0 \text{ and } D_4 = 2.282$$

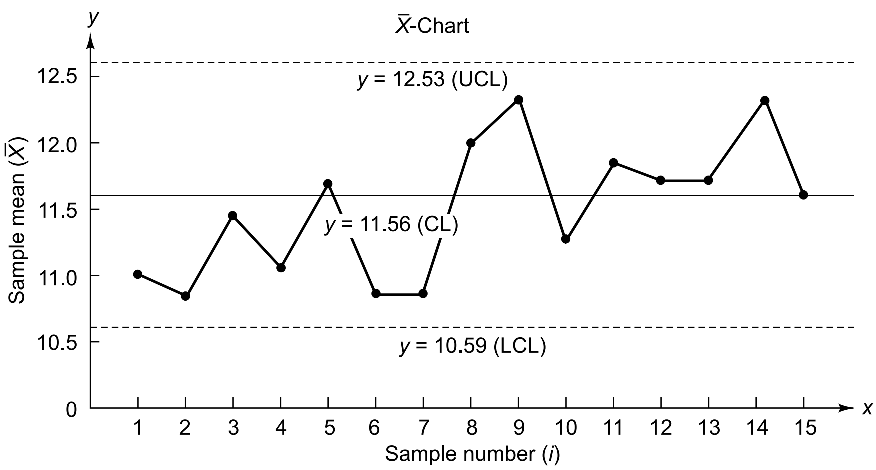
Control Limits for $\bar{X}$-chart

$$\text{CL} = \bar{\bar{X}} = 11.56;\ \text{LCL} = \bar{\bar{X}} - A_2\,\bar{R} = 11.56 - 0.729 \times 1.33 = 10.59$$

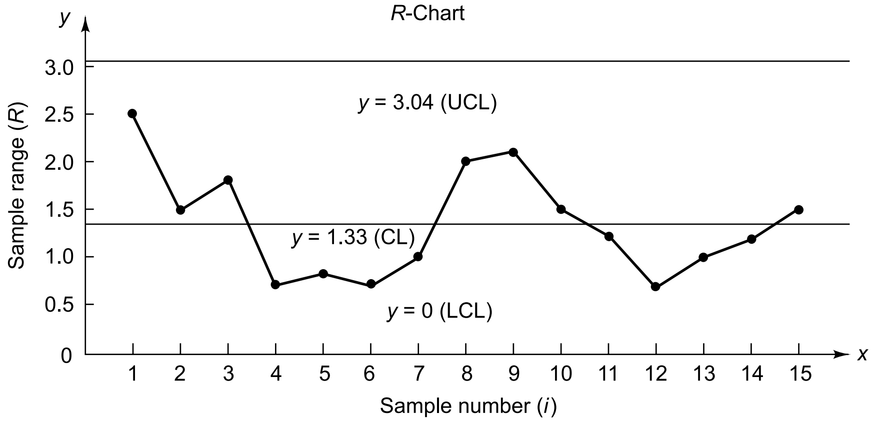
$$\text{UCL} = \bar{\bar{X}} + A_2\,\bar{R} = 11.56 + 0.729 \times 1.33 = 12.53$$

Control Limits for R-chart

$$\text{CL} = \bar{R} = 1.33;\ \text{LCL} = D_3\,\bar{R} = 0 \text{ and UCL} = D_4\,\bar{R} = 2.282 \times 1.33 = 3.04$$



(a)



(b)

State of Control

Since all the sample points lie within upper and lower control lines both in the $\overline{X}$-chart and in the R-chart, the process is under control.

Example 3 *The values of sample mean $\overline{X}$ and sample standard deviation s for 15 samples, each of size 4, drawn from a production process are given below. Draw the appropriate control charts for the process average and process variability. Comment on the state of control.*

Sample No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Mean	15.0	10.0	12.5	13.0	12.5	13.0	13.5	11.5	13.5	13.0	14.5	9.5	12.0	10.5	11.5
S.D.	3.1	2.4	3.6	2.3	5.2	5.4	6.2	4.3	3.4	4.1	3.9	5.1	4.7	3.3	3.3

$$\overline{\overline{X}} = \frac{1}{N}\sum \overline{X}_i = \frac{1}{15} \times 185.5 = 12.36$$

$$\bar{s} = \frac{1}{N}\sum s_i = \frac{1}{15} \times 60.3 = 4.02$$

From the table of control chart constants, for sample size $n = 4$, we have $A_1 = 1.880$, $B_3 = 0$ and $B_4 = 2.266$

Control Limits for $\overline{X}$-chart

$$\text{CL} = \overline{\overline{X}} = 12.36$$

$$\text{LCL} = \overline{\overline{X}} - A_1\sqrt{\frac{n}{n-1}}\bar{s} = 12.36 - 1.880\sqrt{\frac{4}{3}} \times 4.02 = 3.63$$

$$\text{UCL} = \overline{\overline{X}} + A_1\sqrt{\frac{n}{n-1}}\bar{s} = 12.36 + 1.880\sqrt{\frac{4}{3}} \times 4.02 = 21.09$$

Control Limits for s-chart

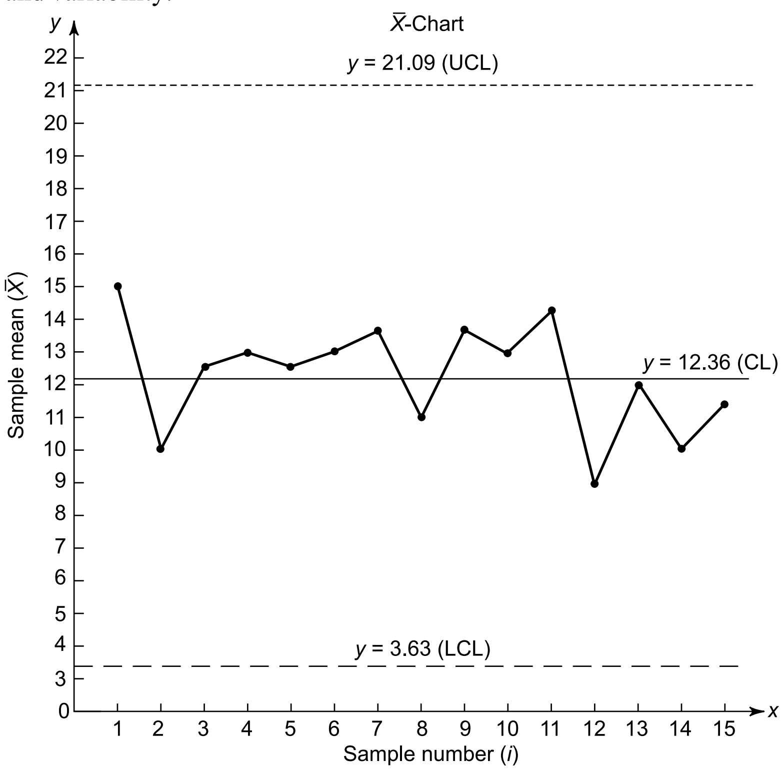
$$\text{CL} = \bar{s} = 4.02;\ \text{LCL} = B_3\,\bar{s} = 0;$$

$$\text{UCL} = B_4\,\bar{s} = 2.266 \times 4.02 = 9.11$$

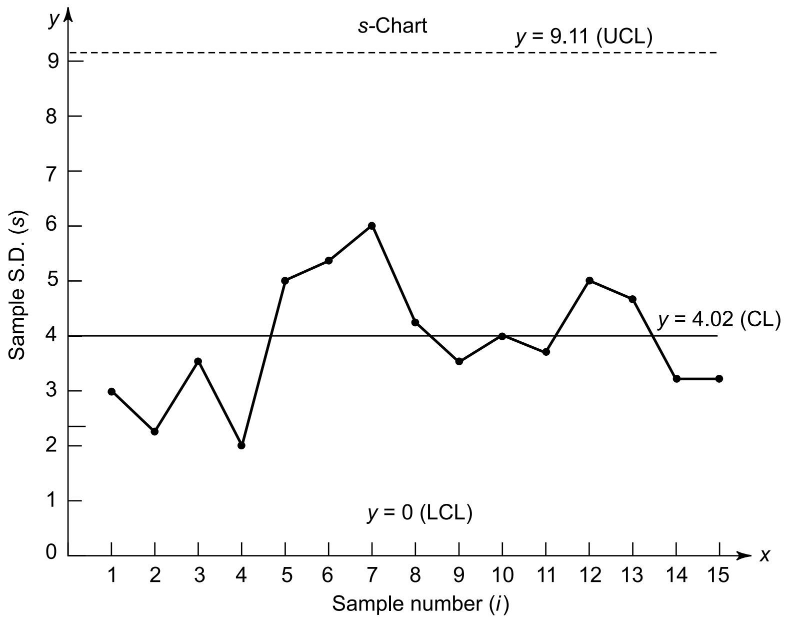
The mean chart and S.D. chart relevant to this problem are given in Fig. 11.3:

State of Control

Even before drawing the control charts, we observe that the given sample mean values lie between 3.63 and 21.09 and that the given S.D. values fall within 0 and 9.11. Hence the process is under control with respect to average and variability.



(a)



(b)

Example 4 *The following data given the coded measurements of 10 samples each of size 5, drawn from a production process at intervals of 1 hour. Calculate the sample means and S.D.'s and draw the control charts for $\overline{X}$ and s.*

Sample Number	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Coded meas-	9	10	10	8	7	12	9	15	10	16
urements (X)	15	11	13	13	9	15	9	15	13	14
	14	13	8	11	10	7	9	10	14	12
	9	6	12	10	4	16	13	13	7	14
	13	10	7	13	5	10	5	17	11	14

We first compute mean and S.D. for each sample.

Sample Number	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
$\sum X$	60	50	50	55	35	60	45	70	55	70
$\overline{X}$	12	10	10	11	7	12	9	14	11	14
$\sum\left(X-\overline{X}\right)^2$	32	26	26	18	26	54	32	28	30	8
s	2.5	2.3	2.3	1.9	2.3	3.3	2.5	2.4	2.4	1.3

$$\overline{\overline{X}} = \frac{1}{N}\sum \overline{X}_i = \frac{1}{10} \times (12 + 10 + 10 + \ldots + 14) = \frac{110}{10} = 11$$

$$\overline{s} = \frac{1}{N}\sum s_i = \frac{1}{10} \times (2.5 + 2.3 + \ldots + 1.3) = \frac{23.2}{10} = 2.32$$

From the table of control chart constants, for sample size $n = 5$, we have

$A_1 = 1.596$; $B_3 = 0$ and $B_4 = 2.089$

Control Limits for $\overline{X}$-chart

$$\text{CL} = \overline{\overline{X}} = 11; \text{ LCL} = \overline{\overline{X}} - \text{A}_1 \cdot \sqrt{\frac{n}{n-1}}\overline{s} = 11 - 1.596\sqrt{\frac{5}{4}} \times 2.32$$

$$= 6.86$$

$$\text{UCL} = \overline{\overline{X}} + \text{A}_1 \cdot \sqrt{\frac{n}{n-1}}\overline{s} = 11 + 1.596\sqrt{\frac{5}{4}} \times 2.32 = 15.14$$

Control Limits for s-chart

$$\text{CL} = \overline{s} = 2.32$$

$$\text{LCL} = B_3\,\overline{s} = 0; \text{ UCL} = B_4\,\overline{s} = 2.089 \times 2.32 = 4.85$$

(s –chart is given on page 11.18)

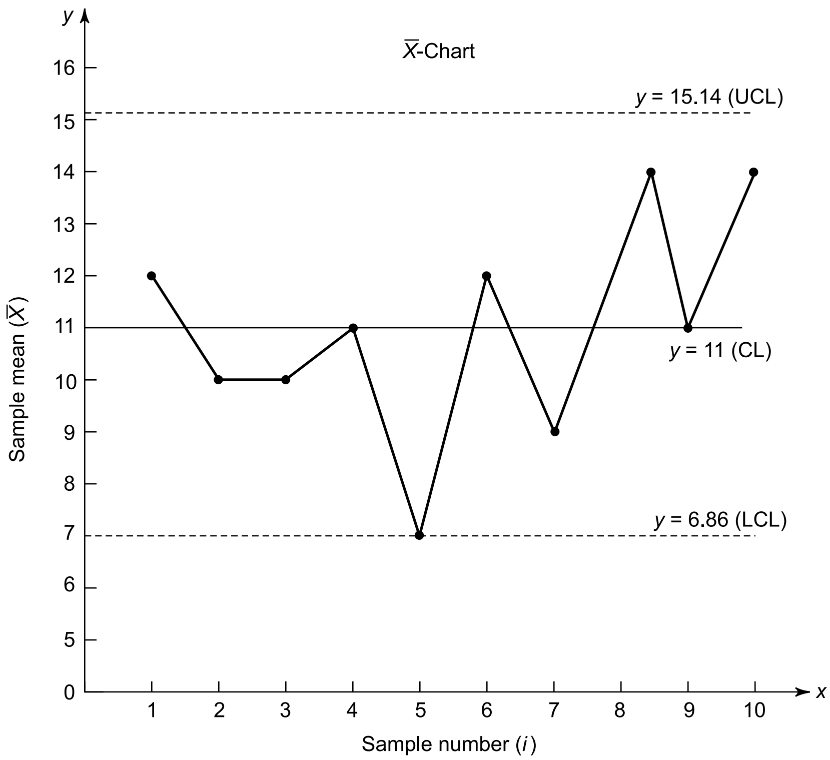
State of Control

The given sample mean ($\overline{X}$) values lie between 6.86 and 15.14 and the given S.D. (s) values between 0 and 4.85. Hence the process is under control with respect to average and variability.

Note *Had we computed LCL and UCL for $\overline{X}$ chart using ($\overline{\overline{X}} \mp A_2\overline{R}$), the process would be out of control with respect to the mean.*

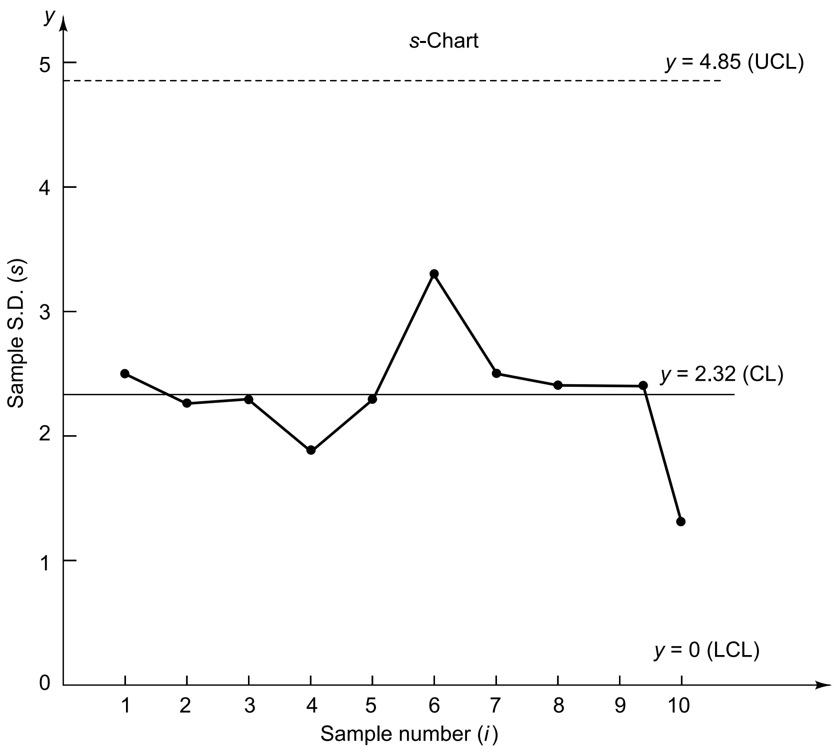
Example 5 *In a factory producing spark plugs, the number of defectives found in the inspection of 15 lots of 100 each is given below: Draw the control chart for the number of defectives and comment on the state of control.*

Sample number (i)	:	1	2	3	4	5	6	7	8	9	10
Number of defective (np)	:	5	10	12	8	6	4	6	3	4	5
	:	11	12	13	14	15					
	:	4	7	9	3	4					



$\overline{X}$-Chart
y = 15.14 (UCL)
y = 11 (CL)
y = 6.86 (LCL)
Sample mean ($\overline{X}$)
y
x
16
15
14
13
12
11
10
9
8
7
6
5
0
1 2 3 4 5 6 7 8 9 10
Sample number (i)

(a)



s-Chart
y = 4.85 (UCL)
y = 2.32 (CL)
y = 0 (LCL)
Sample S.D. (s)
y
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10
Sample number (i)

(b)

$$\sum np = 5 + 10 + 12 + \ldots + 3 + 4 = 90$$

$$\therefore \qquad n\bar{p} = \frac{1}{N}\sum np = \frac{90}{15} = 6$$

and $\bar{p} = \frac{1}{n} \times 6 = \frac{1}{100} \times 6 = 0.06$ ($\because$ each sample contains 100 items)

For the np-chart

$$\text{CL} = n\bar{p} = 6$$

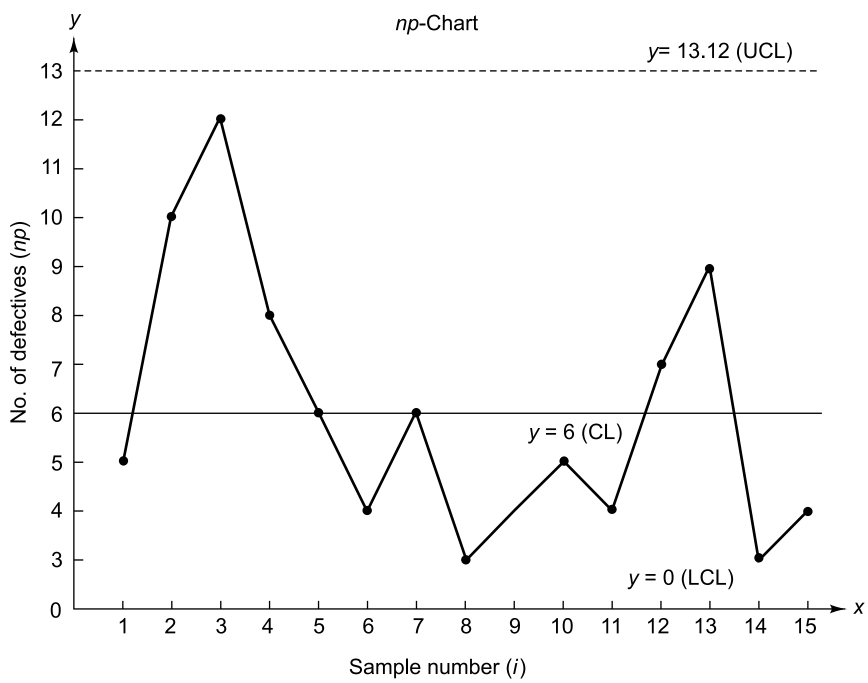
$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 6 - 3\sqrt{6 \times 0.94} = -1.12$$

Since LCL cannot be negative, LCL = 0

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 6 + 3\sqrt{6 \times 0.94} = 13.12$$



Since all the sample points lie between the upper and lower control lines, the process is under control.

Example 6 *15 samples of 200 items each were drawn from the output of a process. The number of defective items in the samples are given below. Prepare a control chart for the fraction defective and comment on the state of control.*

Sample No. (i)	:	1	2	3	4	5	6	7	8	9	10
No. of defective (np)	:	12	15	10	8	19	15	17	11	13	20
	:	11	12	13	14	15					
	:	10	8	9	5	8					

$$\Sigma\, np = 12 + 15 + 10 + \ldots + 5 + 8 = 180$$

$$\therefore \quad n\overline{p} = \frac{1}{N}\sum np = \frac{180}{15} = 12$$

$$\therefore \quad \overline{p} = \frac{12}{200} \quad (\because \text{ each sample contains 200 items})$$

$$= 0.06$$

For the p-chart

$$\text{CL} = \overline{p} = 0.06$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.06 - 3\sqrt{\frac{0.06 \times 0.94}{200}} = 0.01$$

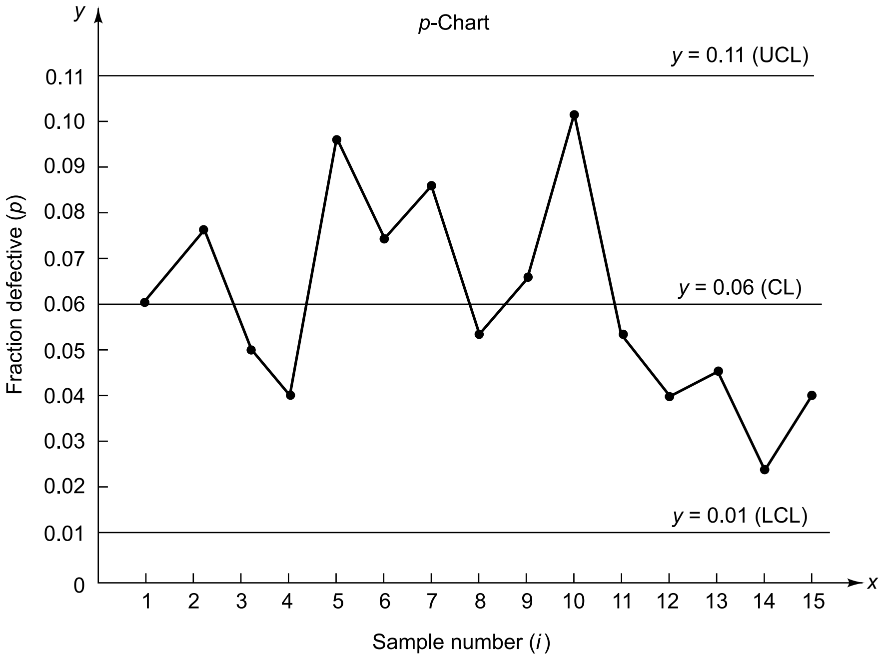
$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.11$$

The fraction defectives (values of p) for the given samples are:
0.06, 0.075, 0.05, 0.04, 0.095, 0.075, 0.085, 0.055, 0.065, 0.1, 0.05, 0.04, 0.045, 0.025, 0.04 (1) p-chart is given on page 11.21.

Since all the sample points lie between the LCL and UCL lines, the process is under control.

Example 7 *10 samples each of size 50 were inspected and the number of defectives in the inspection were: 2, 1, 1, 2, 3, 5, 5, 1, 2, 3. Draw the appropriate control chart for defectives.*

Since the number of defectives in 10 samples, each of size 50, are given, we may construct either number of defectives (np) chart or proportion of defectives (p) chart. We shall construct both and compare the charts.



$$\sum np = 2 + 1 + 1 + \ldots + 2 + 3 = 25$$

$$\therefore \qquad n\bar{p} = \frac{1}{N}\sum np = \frac{25}{10} = 2.5$$

and $$\bar{p} = \frac{1}{n} \times n\bar{p} = \frac{1}{50} \times 2.5 = 0.05$$

For the* np*-chart

$$\text{CL} = n\bar{p} = 2.5;\ \text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 2.5 - 3\ \sqrt{2.5 \times 0.95} = -\ 2.12$$

Since LCL cannot be negative, we take LCL = 0.

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 2.5 + 3\ \sqrt{2.5 \times 0.95} = 7.12$$

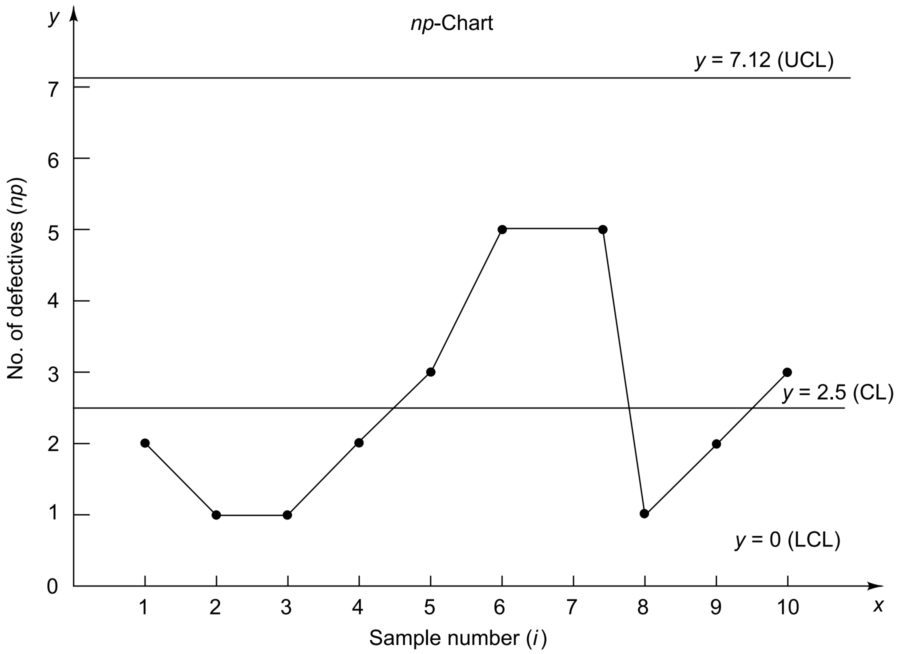
For the* p*-chart

$$\text{CL} = \bar{p} = 0.05;\ \text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.05 - 3\ \sqrt{\frac{0.05 \times 0.95}{50}}$$

$$= -\ 0.042$$

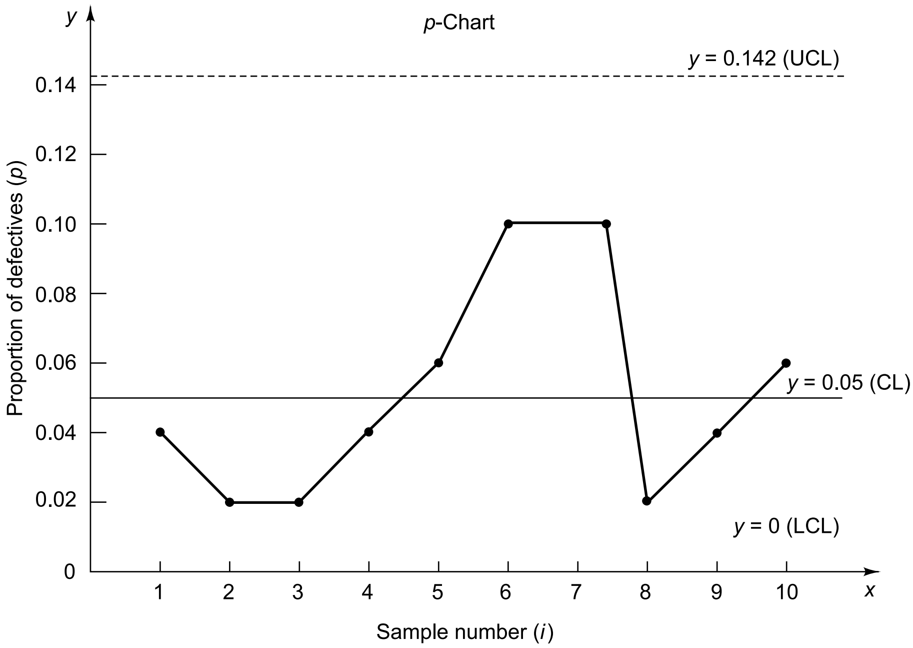
As LCL cannot be negative, we take LCL = 0.

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.05 + 3\sqrt{\frac{0.05 \times 0.95}{50}} = 0.142$$



(a)

The proportion of defecties (p) for the given samples are 0.04, 0.02, 0.02, 0.04, 0.06, 0.10, 0.10, 0.02, 0.04, 0.06.



Since all the sample points lie within the LCL and UCL lines in both *np* and *p*-charts, the process is under control. If we use suitable scales, we may see that the *np*-chart and *p*-chart are identical.

Example 8 *Construct a control chart for defectives for the following data:*

Sample No.	:	1	2	3	4	5	6	7	8	9	10
No. inspected	:	90	65	85	70	80	80	70	95	90	75
No. of defectives	:	9	7	3	2	9	5	3	9	6	7

We note that the size of the sample varies from sample to sample. Hence we cannot construct the *np*-chart. We can construct *p*-chart, provided $0.75\,\bar{n} < n_i < 1.25\,\bar{n}$, for all i

Here
$$\bar{n} = \frac{1}{N}\sum n_i = \frac{1}{10} \times (90 + 65 + \ldots + 90 + 75)$$
$$= \frac{1}{10} \times 800 = 80$$

Hence
$$0.75\,\bar{n} = 60 \text{ and } 1.25\,\bar{n} = 100$$

The values of n_i be between 60 and 100. Hence *p*-chart, given in Fig. 11.8, can be drawn by the method given below.

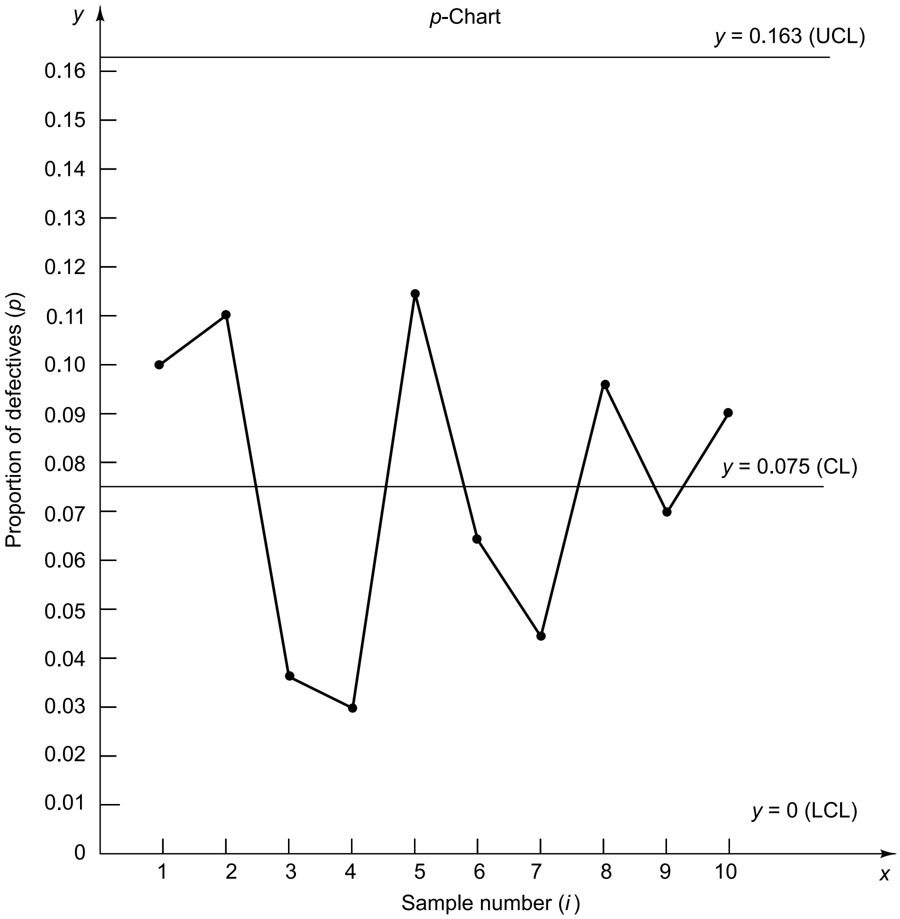
Note *If the condition ($0.75\,\bar{n} < n_i < 1.25\,\bar{n}$) is not satisfied, other available methods may be used. They are beyond the scope of this book.*

Now
$$\bar{p} = \frac{\text{Total no. of defectives}}{\text{Total no. of items inspected}}$$
$$= \frac{60}{800} = 0.075$$

Hence for the *p*-chart to be constructed,
$$\text{CL} = \bar{p} = 0.075$$
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.075 - 3\sqrt{\frac{0.075 \times 0.925}{80}}$$
$$= -\,0.013$$

Since LCL cannot be negative, it is taken as 0.
$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} = 0.075 + 3\sqrt{\frac{0.075 \times 0.925}{80}}$$
$$= 0.163$$

The values of p_i for the various samples are 0.100, 0.108, 0.035, 0.029, 0.113, 0.063, 0.043, 0.095, 0.067, 0.093.



Since all the sample points lie within the control lines, the process is under control.

Example 9 *15 tape-recorders were examined for quality control test. The number of defects in each tape-recorder is recorded below. Draw the appropriate control chart and comment on the state of control.*

Unit no. (i)	:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defects (c)	:	2	4	3	1	1	2	5	3	6	7	3	1	4	2	1

Since the number of defects per sample containing only one item is given, we can draw the c-chart (Fig. 11.9).

$$\bar{c} = \frac{1}{N}\sum c_i = \frac{1}{15}\,(2 + 4 + 3 + \ldots + 2 + 1)$$

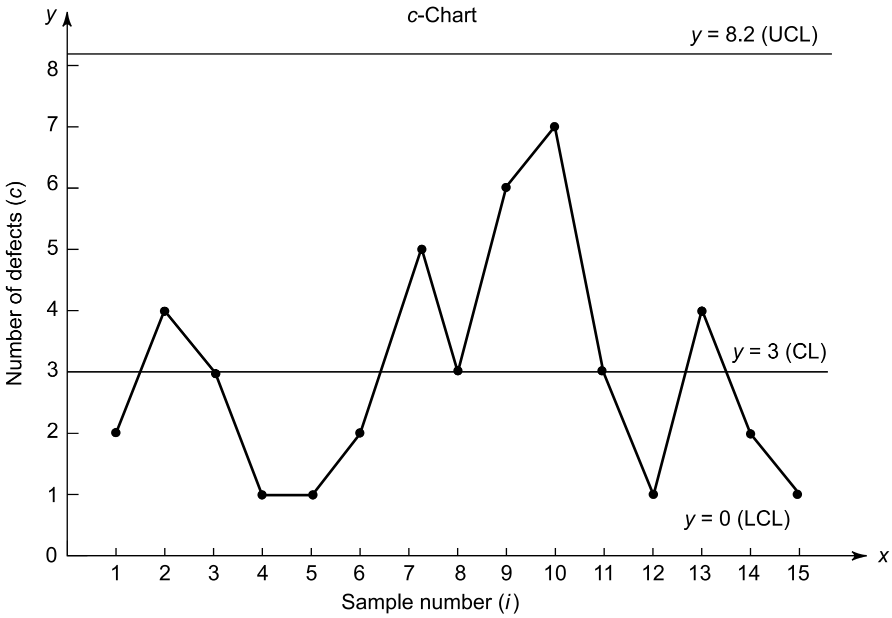
$$= \frac{45}{15} = 3$$

Note *Even though $\bar{c} < 4$, we draw the c-chart, as it is the only possible chart.*

$$CL = \bar{c} = 3;\ LCL = \bar{c} - 3\sqrt{\bar{c}} = 3 - 3\sqrt{3} - 2.20$$

Since LCL cannot be negative, we take LCL = 0

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 3 + 3\sqrt{\bar{c}} = 8.20$$



Since all the sample points lie within the LCL and UCL lines, the process is under control.

Example 10 *A plant produces paper for newsprint and rolls of paper are inspected for defects. The results of inspection of 20 rolls of papers are given below: Draw the c-chart and comment on the state of control.*

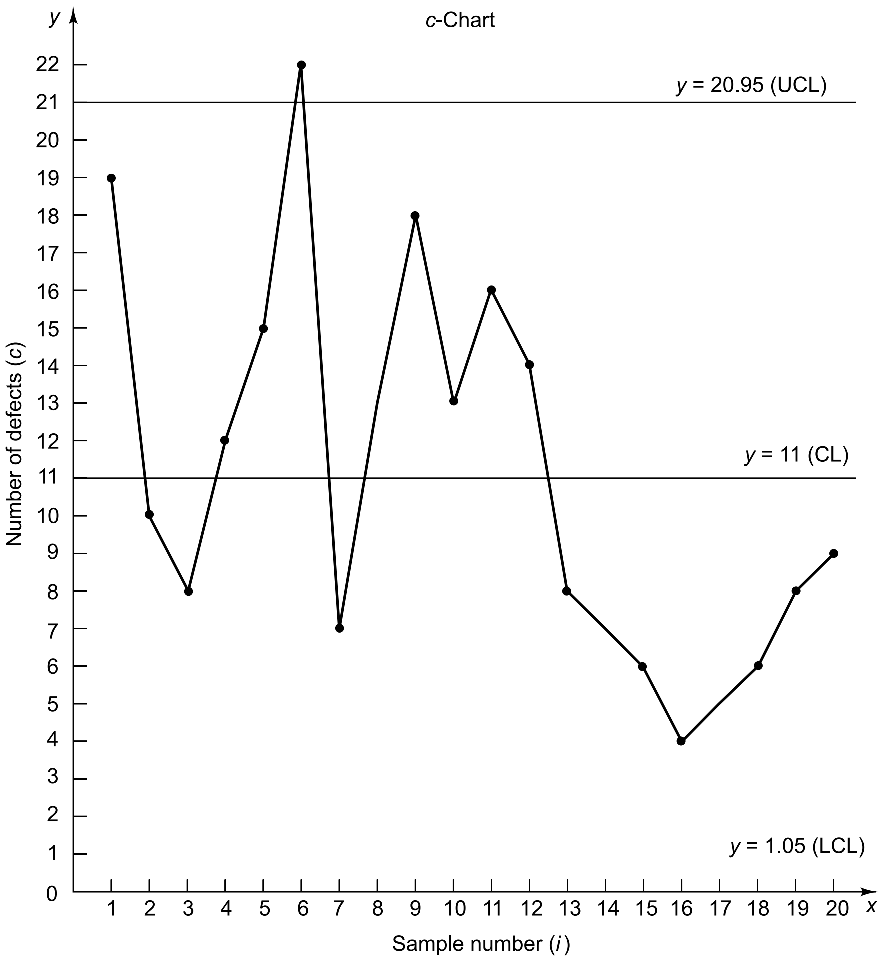
Roll No. (i):	1	2	3	4	5	6	7	8	9	10
No. of defects (c):	19	10	8	12	15	22	7	13	18	13
(i)	11	12	13	14	15	16	17	18	19	20
(c)	16	14	8	7	6	4	5	6	8	9

$$\bar{c} = \frac{1}{N}\sum c_i = \frac{1}{20} \times 220 = 11$$

For the c-chart

$$CL = \bar{c} = 11;\ LCL = \bar{c} - 3\sqrt{\bar{c}} = 11 - 3\sqrt{11} = 1.05$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 11 + 3\sqrt{11} = 20.95$$



Since one point falls outside the control lines, the process is out of control.

Example 11 *The specifications for a certain quality characteristic area 15.0 ± 6.0 (in coded values). 15 samples of 4 readings each gave the following values for $\overline{X}$ and R.*

Sample No. (i):	1	2	3	4	5	6	7
$\overline{X}$:	16.1	15.2	14.2	13.9	15.4	15.7	15.2
R:	3.0	2.1	5.6	2.4	4.1	2.7	2.3

i:	8	9	10	11	12	13	14	15
$\overline{X}$:	15.0	16.5	14.9	15.3	17.8	15.9	14.6	15.2
R:	3.8	5.0	2.9	13.8	14.2	4.8	5.0	2.2

Compute the control limits for $\overline{X}$ and R-charts using the above data for all the samples. Hence examine if the process is in control. If not, remove the doubtful samples and recompute the values of $\overline{\overline{X}}$ and $\overline{R}$. After testing the state of control, estimate the tolerance limits and find if the process will meet the required specifications.

Let us consider all the 15 samples given.

$$\bar{\bar{X}} = \frac{1}{N}\sum \bar{X} = \frac{1}{15}\,(16.1 + 15.2 + \ldots + 15.2)$$

$$= \frac{1}{15} \times 230.9 = 15.39$$

$$\bar{R} = \frac{1}{N}\sum R = \frac{1}{15}\,(3.0 + 2.1 + \ldots + 2.2)$$

$$= \frac{1}{15} \times 73.9 = 4.93$$

For the $\bar{\bar{X}}$-chart

$$\text{CL} = \bar{\bar{X}} = 15.39;\ \text{LCL} = \bar{\bar{X}} - A_2\,\bar{R} = 15.39 - 0.729 \times 4.93 = 11.80$$

$$\text{UCL} = \bar{\bar{X}} + A_2\,\bar{R} = 15.39 + 0.729 \times 4.93 = 18.98$$

For the* R*-chart

$$\text{CL} = \bar{R} = 4.93;\ \text{LCL} = D_3\,\bar{R} = 0;\ \text{UCL} = D_4\,\bar{R} = 2.282 \times 4.93 = 11.25$$

The process is under control with respect to the average ($\bar{X}$-chart), but it is not under control with respect to variability (R-chart), since R_{11} (R value for the sample No. 11) = 13.8 and R_{12} = 14.2 exceed UCL = 11.25.

Hence the process is not under control. So we remove the samples numbered 11 and 12 from the given data.

Let us now recompute $\bar{\bar{X}}$ and $\bar{R}$ based on the remaining 13 samples.

$$\bar{\bar{X}} = \frac{1}{13}\,(16.1 + 15.2 + \ldots + 14.9 + 15.9 + 14.6 + 15.2)$$

$$= \frac{1}{13} \times 197.8 = 15.22$$

$$\bar{R} = \frac{1}{13}\,(3.0 + 2.1 + \ldots + 2.9 + 4.8 + 5.0 + 2.2)$$

$$= \frac{1}{13} \times 45.9 = 3.53$$

Let us now recompute the revised control limits for $\bar{X}$ and R charts.

For the $\overline{X}$-chart,

$$\text{CL} = \overline{\overline{X}} = 15.22;\ \text{LCL} = \overline{\overline{X}} - A_2\,\overline{R} = 15.22 - 0.729 \times 3.53 = 12.65$$

$$\text{UCL} = \overline{\overline{X}} + A_2\,\overline{R} = 15.22 + 0.729 \times 3.53 = 17.79$$

For the R-chart

$$\text{CL} = \overline{R} = 3.53;\ \text{LCL} = D_3\,\overline{R} = 0;\ \text{UCL} = D_4\,\overline{R} = 2.282 \times 3.53 = 8.06$$

We see that the process is under control with respect to the 13 samples considered.

Now we can compute the tolerance limits using the revised values of $\overline{\overline{X}}$ and $\overline{R}$.

The tolerance limits are given by

$$\overline{\overline{X}} \mp \frac{3\overline{R}}{d_2}$$

$$= 15.22 \mp \frac{3 \times 3.53}{2.059}$$

(The value of d_2 is read from the table of control chart constants for n = 4)

$$= 15.22 \mp 5.14$$

Thus the required tolerance limits are (10.08, 20.36)

Since these tolerance limits lie within the specification limits (9.0, 2.10), the process meets the required specifications.

Example 12 *The specifications for a certain quality characteristic are (60 ± 24) in coded values. The table given below gives the measurements obtained in 10 samples. Find the tolerance limits for the process and test if the process meets the specifications*.

Sample No. (i)	1	2	3	4	5	6	7	8	9	10
Measurements (X)	75	48	57	61	55	49	74	67	66	62
	66	79	55	71	68	98	63	70	65	68
	50	53	53	66	58	65	62	68	58	66
	62	61	61	69	62	64	57	56	52	68
	52	49	72	77	75	66	62	61	58	73
	70	56	63	53	63	64	64	66	50	68

The values of $\overline{X}$ and R computed for all the samples are tabulated below:

i :	1	2	3	4	5	6	7	8	9	10
ΣX_i :	375	346	361	397	381	406	382	388	349	405
$\overline{X}_i$:	62.5	57.7	60.2	66.2	63.5	67.7	63.7	64.7	58.2	67.5
R_i :	25	31	19	24	20	49	17	14	16	11

Now $\overline{\overline{X}} = \frac{1}{N}\sum \overline{X}_i = \frac{1}{10} \times 631.9 = 63.19$

$\overline{R} = \frac{1}{N}\sum R_i = \frac{1}{10} \times 226 = 22.6$

For the $\overline{X}$-chart

$$\text{CL} = \overline{\overline{X}} = 63.19;\ \text{LCL} = \overline{\overline{X}} - A_2\,\overline{R} = 63.19 - 0.483 \times 22.6$$
$$= 52.27$$
$$\text{UCL} = \overline{\overline{X}} + A_2\,\overline{R} = 63.19 + 0.483 \times 22.6 = 74.11$$

For the R-chart

$$\text{CL} = \overline{R} = 22.6;\ \text{LCL} = D_3\,\overline{R} = 0;\ \text{UCL} = D_4\,\overline{R} = 2.004 \times 22.6$$
$$= 45.29$$

We note that

LCL (= 52.27) < $\overline{X}_i$ < UCL (= 74.11),
but R_6 (sample 6) > 45.29.

Hence the process is not under control.

Now we remove sample No. 6 from the data and recompute the values of $\overline{\overline{X}}$ and $\overline{R}$ based on the remaining 9 samples.

$$\overline{\overline{X}} = \frac{1}{9} \times (62.5 + 57.7 + \ldots + 63.5 + 63.7 + \ldots + 67.5)$$

$$= \frac{1}{9} \times 564.2 = 62.69$$

$$\overline{R} = \frac{1}{9} \times (25 + 31 + \ldots + 20 + 17 + \ldots\ 11)$$

$$= \frac{1}{9} \times 177 = 19.67$$

Let us now recompute the revised control limits for $\overline{X}$ - and R-charts.

For the $\overline{X}$-chart

$$CL = \overline{\overline{X}} = 62.69;\ LCL = \overline{\overline{X}} - A_2\,\overline{R} = 62.69 - 0.483 \times 19.67 = 53.19$$

$$UCL = \overline{\overline{X}} + A_2\,\overline{R} = 62.69 + 0.483 \times 19.67 = 72.19$$

For the R-chart

$$CL = \overline{R} = 19.67,\ LCL = D_3\,\overline{R} = 0;\ UCL = D_4\,\overline{R} = 2.004 \times 19.67 = 39.42$$

Now $53.19 < \overline{X}_i < 72.19$ and

$0 < R_i < 39.42$, for all i ($\neq 6$)

Hence the process is under control.

The tolerance limits are given by

$$\overline{\overline{X}} \mp \frac{3\overline{R}}{d_2}$$

$$= 62.69 \mp \frac{3 \times 19.67}{2.534} \qquad (\because\ d_2 = 2.534, \text{ for } n = 6)$$

$$= 62.69 \mp 23.29$$

i.e., the tolerance limits are (39.40, 85.98)

The specification limits are (36, 84).

Since the upper tolerance limit exceeds the upper specification limit, the process does not meet the specifications.

Exercise

Part A (Short–answer questions)

1. What is meant by 'quality' in the term Statistical Quality Control?
2. What do you mean by Statistical Quality Control?
3. What is the difference between chance variation and assignable variation?
4. What do you understand by process control?
5. What is control chart? Name the types of control charts.
6. Distinguish between variables and attributes in connection with the quality characteristics of a product.
7. Name any two control charts, for each, of variables and attributes.
8. Find the lower and upper control limits for $\overline{X}$-chart and R-chart, when

each sample is of size 4 and $\overline{\overline{X}} = 10.80$ and $\overline{R} = 0.46$.

9. When do you say that a process is out of control?
10. Find the lower and upper control limits for $\overline{X}$-chart and s-chart, if $n = 5$ $\overline{\overline{X}} = 15$ and $\overline{s} = 2.5$.
11. Under what situations p-chart is drawn instead of np-chart?
12. When n is constant, will the p-chart and np-chart lead to the same conclusions regarding the process of control?
13. Find the lower and upper control limits for p-chart and np-chart, when $n = 100$ and $\overline{p} = 0.085$.
14. Distinguish between p-chart and c-chart.
15. Find the lower and upper control limits for the c-chart, when $\overline{c} = 6$.
16. What do you mean by specification limits in a manufacturing process?
17. What is meant by tolerance limits?
18. Distinguish between control limits and tolerance limits.
19. When the process is under control and if $n = 5$, $\overline{\overline{X}} = 1.1126$ and $\overline{R} = 0.0054$, find the tolerance limits.
20. How will you decide whether a process is operating at an acceptable level?

Part B

21. The following data give the average life in hours and range in hours of 12 samples each of 5 lamps. Construct the control charts for $\overline{X}$ and R and comment on the state of control.

$\overline{X}$:	120	127	152	157	160	134	137	123	140	144	120	127
R:	30	44	60	34	38	35	45	62	39	50	35	41

22. Draw the mean chart and range chart using the following data relating to 15 samples each of size 5 and comment on the state of control.

$\overline{X}$:	65.0	64.6	64.1	68.5	68.4	67.9	65.0	64.6
$\overline{X}$:	64.1	63.2	62.9	62.4	67.0	66.6	66.1	
R:	9.8	9.8	8.4	3.9	7.6	8.7	0.1	9.7
R:	7.7	7.5	1.2	9.8	6.4	0.6	6.3	

23. A food company puts mango juice in cans, each of which is advertised to contain 10 ounces of the juice. The weights of the juice drained from cans immediately after filling 20 samples each of 4 cans are taken by random sampling method (at an interval of 30 minutes) and given in the following table in units of 0.01 ounce in excess of 10 ounces. To control the excess weights of mango juice drained while filling, draw the $\overline{X}$-chart and R-chart and comment on the nature of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
	15	10	8	12	18	20	15	13	9	6
Weights drained	12	8	15	17	13	16	19	23	8	10
	13	8	17	11	15	14	23	14	18	24
	20	14	10	12	4	20	17	16	5	20

Sample No.	11	12	13	14	15	16	17	18	19	20
	5	3	6	12	15	18	13	10	5	6
	12	15	18	9	15	17	16	20	15	14
Weights	20	18	12	15	6	8	5	8	10	12
drained	15	18	10	18	16	15	4	10	12	14

24. The table below gives the (coded) measurements obtained in 20 samples. Construct control charts based on mean and range and comment on the state of control:

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13
	2	0	1	1	– 1	– 1	– 1	1	1	1	1	1	1
Values of X	1	1	0	0	0	2	0	2	– 1	– 2	– 3	– 1	– 3
	0	0	0	– 1	0	0	– 2	– 1	0	2	2	0	2
	1	1	1	0	– 1	– 2	1	0	0	1	1	0	1

Sample No.	14	15	16	17	18	19	20
	0	– 1	1	2	2	0	3
	0	2	– 1	1	0	2	– 3
Values of X	– 1	1	2	– 1	1	1	–1
	0	1	0	0	0	–1	1
	1	2	2	0	1	1	2

25. The following data give the coded values of the crushing strengths of concrete blocks obtained from 20 samples each of size. 5. Draw the $\overline{X}$ and R-charts and comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9
	11.1	9.6	9.7	10.1	12.4	10.1	11.0	11.2	10.6
	9.4	10.8	10.0	8.4	10.0	10.2	11.5	10.0	10.4
Values of X	11.2	10.1	10.0	10.2	10.7	10.2	11.8	10.9	10.5
	10.4	10.8	9.8	9.4	10.1	11.2	11.0	11.2	10.5
	10.1	11.0	10.4	11.0	11.3	10.1	11.3	11.0	10.9

Sample No.	10	11	12	13	14	15	16	17	18	19	20
	8.3	10.6	10.8	10.7	11.3	11.4	10.1	10.7	11.9	10.8	12.4
	10.2	9.9	10.2	10.7	11.4	11.2	10.1	12.8	11.9	12.1	11.1
Values of X	9.8	10.7	10.5	10.8	10.4	11.4	9.7	11.2	11.6	11.8	10.8
	9.5	10.2	8.4	8.6	10.6	10.1	9.8	11.2	12.4	9.4	11.0
	9.8	11.4	9.9	11.4	11.1	11.6	10.5	11.3	11.4	11.6	11.9

26. The following data gives the measurements of 10 samples each of size 5, in a production process taken at intervals of 2 hours. Draw the control charts for the mean and range and comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
	47	52	48	49	50	55	50	54	49	53
	49	55	53	49	53	55	51	54	55	50
Measurements	50	47	51	49	48	50	53	52	54	54
(X)	44	56	50	53	52	53	46	54	49	47
	45	50	53	45	47	57	50	56	53	51

27. The following table gives the sample means and S.D.'s for 15 samples, each size 4, in the production of a certain component. Draw the $\overline{X}$ and s-charts and comment on the state of control.

Sample No. (i):	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15			
$\overline{X}$:	1.75	1.32	1.18	0.48	2.30	1.25	1.52	1.78	1.90
	1.72	2.40	3.20	2.52	2.05	1.68			
s:	0.36	0.53	0.14	0.18	0.55	0.74	0.38	0.45	0.87
	0.83	0.76	0.99	0.65	0.22	0.14			

28. The following data give the mean and S.D. values of 10 samples, each of size 5 drawn from a production process taken at intervals of one hour. Construct the mean and S.D. charts and comment on the state of control.

Sample No.:	1	2	3	4	5	6	7	8	9	10
$\overline{X}$:	54	51	54	49	52	47	51	50	50	52
s :	3.3	2.4	3.8	3.3	3.4	4.6	1.9	2.5	2.5	2.9

29. The mean and S.D. values of 15 samples each of size 5 are given in the following table: Draw the mean and S.D. charts and comment on the state of control.

Sample No. (i):	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15			
$\overline{X}$:	11.0	12.4	8.6	12.2	9.2	10.6	10.0	9.4	10.6
	11.4	14.8	10.6	11.6	9.0	12.8			
s:	3.63	3.14	2.87	3.49	1.47	3.01	2.45	3.98	2.15
	2.15	2.14	3.01	3.14	1.67	2.56			

30. Draw the mean and S.D. charts using the data values for the first 10 samples given in Exercise 24 and comment on the state of control.
31. 15 samples of 4 machined items were taken periodically from production and inside diameter of whole in each item was measured. The measurements (coded) are given in the following table. Draw the $\overline{X}$ and s-charts and comment on the state of control:

Sample No. (i):	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Measurements (X)		14	16	11	14	7	7	10	9	11	16				
	13	6	9	5	10	13	6	9	5	10	11				
	7	16	10	14	10	15	12	46	15	15	9	15			
	7	11	7	10	10	7	8	12	14	9	10				

32. Fifteen samples each of size 50 were inspected and the number of defectives in the inspection were:

    2, 3, 4, 2, 3, 0, 1, 2, 2, 3, 5, 5, 1, 2, 3

    Draw the control chart for the number of defectives and comment on the state of control.
33. In a manufacturing company where spark-plugs are produced, the number rejected by inspection of 20 lots of 100 plugs each is given below: Construct the *np*-chart and comment on the state of control.

Lot No. (i):	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
No. rejected (np):	10	6	5	10	8	5	12	8	4	3
	6	5	2	8	7	6	3	3	5	4

34. Using the following data, construct the *np*-chart and comment on the state of control. Assume that 200 items are inspected each day.

Day (i):	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defective (np):	6	6	6	5	0	0	6	14	4	0	1	8	2	4	7

35. On inspection of 20 lots each of 50 items, the following numbers of defectives were found: Construct the control chart for the fraction defectives and comment on the state of control.

    1, 4, 3, 4, 4, 3, 2, 4, 2, 3, 5, 1, 2, 4, 2, 4, 8, 3, 2, 2

36. In an integrated circuit production line, 15 samples of 100 units are checked for electrical specifications on alternate days of a month and the number of defectives found are tabulated below: Draw the p-chart and comment on the nature of control.
    24, 62, 26, 38, 33, 44, 45, 34, 30, 52, 44, 52, 36, 34 and 38.
37. On inspection of 10 samples, each of size 400, the numbers of defective articles were:
    19, 4, 9, 12, 9, 15, 26, 14, 15, 17.
    Draw the np-chart and p-chart and comment on the state of control.
38. Draw the appropriate control chart for the following data and comment on the state of control:

Day:	1	2	3	4	5	6	7	8	9	10
No. inspected:	150	184	181	196	180	174	210	210	195	210
No. of defectives:	25	10	3	14	6	15	43	28	39	25

39. Ten samples of varying size are taken from a production line and the number of defectives is found in each sample. The results are given below. Draw the appropriate control chart and comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. inspected:	155	160	156	156	164	160	161	173	148	167
No. of defectives:	8	8	8	7	8	6	5	10	7	9

40. Twenty half-litre milk bottles are selected at random from a process and the numbers of air bubbles (defects) observed from the bottles are given in the following table. Draw the appropriate control chart and comment on the nature of control.

Bottle no. (i) :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
No. of defects (c) :	4	5	7	3	3	5	6	2	4	8	3	5	4	3	4	5	7	3	6	13

41. The following data relate to the number of defects in each of 15 units drawn randomly from a production process. Draw the control chart for the number of defects and comment on the state of control.

    6, 4, 9, 10, 11, 12, 20, 10, 9, 10, 15, 10, 20, 15, 10.
42. Construct a c-chart for the number of defects from the following data which represent the number of imperfections in 20 pieces of cloth of equal length in a certain production of a mill. Is the process under control?
    No. of imperfections: 3, 3, 4, 10, 10, 3, 3, 3, 6, 5, 6, 10, 4, 7, 4, 7, 4, 8, 4 and 7.

43. The specifications for the length of a certain product are (30 ± 10) mm. 15 samples each of size 5 gave the following values for $\overline{X}$ and R in mm. Compute the tolerance limits of the production process and also find if the process will meet the specifications:

Sample no. (i) :	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15					
$\overline{X}$:	25	30	23	38	15	28	38	18	25	40
	29	39	37	29	36					
R :	20	10	20	21	11	22	22	11	20	19
	14	9	16	18	34					

44. The specification limits for a quality characteristic are 1.100 and 1.120 (in certain units). 15 samples of 5 measurements each gave the following values for $\overline{X}$ and R (Range values are multiplied by 10^3). Compute the tolerance limits of the process and check if the process will meet the specifications:

Sample no. (i) :	1	2	3	4	5	6	7	8
	9	10	11	12	13	14	15	
$\overline{X}$:	1.115	1.116	1.114	1.112	1.114	1.112	1.114	1.112
	1.113	1.111	1.113	1.114	1.111	1.113	1.111	
R :	18	17	8	6	7	5	5	7
	3	4	6	4	3	5	7	

45. The following data give the measurements of 10 samples, each of size 5, drawn from a process at regular intervals. Find the tolerance limits for the process and test if the specifications (50 ± 10) are met by the process.

Sample no.	1	2	3	4	5	6	7	8	9	10
Measurement (X)	49	50	50	48	47	52	49	55	53	54
	55	51	53	53	49	55	49	55	50	54
	54	53	48	51	50	47	49	50	54	52
	49	46	52	50	44	56	53	53	47	54
	53	50	47	53	45	50	45	57	51	56

Table 2.1 Table of Control Chart Constants

Sample size	Factors for p-chart			Factors for s-chart					Factors for R-chart				
n	A	A_1	A_2	c_2	B_1	B_2	B_3	B_4	d_2	D_1	D_2	D_3	D_4
2	2.121	3.760	1.880	0.5642	0	1.843	0	3.267	1.128	0	3.686	0	3.262
3	1.732	2.394	1.023	0.7236	0	1.858	0	2.568	1.693	0	4.358	0	2.575
4	1.500	1.880	0.729	0.7979	0	1.808	0	2.266	2.059	0	4.698	0	2.282
5	1.342	1.596	0.577	0.8407	0	1.756	0	2.089	2.326	0	4.918	0	2.115
6	1.225	1.410	0.483	0.8686	0.026	1.711	0.030	1.970	2.534	0	5.078	0	2.004
7	1.134	1.277	0.419	0.8882	0.105	1.672	0.118	1.882	2.704	0.205	5.203	0.076	1.924
8	1.061	1.175	0.373	0.9027	1.167	1.638	0.185	1.815	2.847	0.387	5.307	5.136	0.864
9	1.000	1.094	0.337	0.9139	0.219	0.609	0.239	1.760	2.970	0.546	5.394	0.184	1.816
10	0.949	1.028	0.308	0.9227	0.262	1.584	0.284	1.716	3.078	0.687	5.469	0.223	1.777
11	0.905	0.973	0.285	0.9300	0.299	1.561	0.321	1.679	3.173	0.812	5.534	0.256	0.744
12	0.866	0.925	0.266	0.9359	0.331	1.541	0.354	1.646	3.258	0.924	5.592	0.284	1.716
13	0.832	0.884	0.249	0.9410	0.359	1.523	0.382	1.618	3.336	1.026	5.646	0.308	1.692
14	0.802	0.848	0.235	0.9453	0.384	1.507	0.406	1.594	3.407	1.121	5.693	0.329	1.671
15	0.775	0.816	0.223	0.9490	0.406	1.492	0.428	1.572	3.472	1.207	5.737	0.348	1.652
16	0.750	0.788	0.212	0.9523	0.427	1.478	0.448	1.552	3.532	1.285	5.779	0.364	1.636
17	0.728	0.762	0.203	0.9551	0.445	1.465	0.466	1.534	3.588	1.359	5.817	0.379	1.621
18	0.707	0.738	0.194	0.9576	0.461	0.459	0.482	1.518	3.640	1.426	5.854	0.392	1.608
19	0.688	0.717	0.184	0.9599	0.477	0.443	0.497	1.503	3.689	1.490	5.888	0.404	1.596
20	0.671	0.697	0.110	0.9619	0.491	1.433	0.510	1.490	3.735	1.544	5.922	0.418	1.586

Guide for using appropriate control chart constants

Quality Controlled	*Central Line*	*LCL*	*UCL*	*Sample size (n)*	*Remarks*
$\overline{X}$ (Mean)	μ	$\mu - A\sigma$	$\mu + A\sigma$	Any value of n	When μ, σ are given
$\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{X}} - A_2\overline{R}$	$\overline{\overline{X}} + A_2\overline{R}$	$n \le 10$	When μ, σ are not known,
$\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{X}} - A_1\sqrt{\frac{n-1}{n}}\,\overline{s}$	$\overline{\overline{X}} + A_1\sqrt{\frac{n-1}{n}}$	$n \le 25$ (n constant)	When μ, s are not known, but $\overline{\overline{X}}\,\overline{s}$, are known
$\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{x}} - \frac{3\overline{s}}{\sqrt{n}}$	$\overline{\overline{X}} + \frac{3\overline{s}}{\sqrt{n}}$	n >25 (n varying slightly)	-Do-
R (Range)	$d_2\sigma$	$D_1\sigma$	$D_2\sigma$	$n \le 10$	When σ is given
R	$\overline{R}$	$D_3\overline{R}$	$D_4\overline{R}$	$n \le 10$	When σ is not known
σ(S.D.)	$c_2\sqrt{\frac{n}{n-1}}\sigma$	$B_1\sqrt{\frac{n}{n-1}}\sigma$	$B_2\sqrt{\frac{n}{n-1}}\sigma$	Any value of n	When σ is given
s	s	$B_3\,\overline{s}$	$B_4\,\overline{s}$	$n \le 25$ (n constant)	When σ is not known, but $\overline{s}$ is known
s	$\overline{s}$	$\overline{s} - \frac{3\overline{s}}{\sqrt{2n}}$	$\overline{s} + \frac{3\overline{s}}{\sqrt{2n}}$	$n > 25$ (n varying slightly)	-Do-

Answers

Exercise

8. For $\overline{X}$-chart, LCL = 10.46, UCL = 11.14;
   For R-chart, LCL = 0, UCL = 1.05.
10. For $\overline{X}$-chart, LCL = 11.43, UCL = 18.57;

For s-chart, LCL = 0, UCL = 5.22.

13. For p-chart, LCL = 0.0013, UCL = 0.1687;
    For np-Chart, LCL = 0.134, UCL = 16.867.
19. 1.1056, 1.1196
21. For $\overline{X}$-chart, LCL = 112.08, UCL = 161.42;
    For R-chart, LCL = 0, UCL = 90.42
    Process under control.
22. For $\overline{X}$-chart, LCL = 61.61, UCL = 69.11;
    For R-chart, LCL = 0, UCL = 13.75.
    Process under control.
23. LCL for $\overline{X}$ = 5.46, UCL for $\overline{X}$ = 20.84;
    LCL for R = 0, UCL for R = 24.84; under control
24. LCL for $\overline{X}$ = 1.488, UCL for $\overline{X}$ = 2.148;
    LCL for R = 0, UCL for R = 6.662; under control
25. LCL for $\overline{X}$ = 9.74, UCL for $\overline{X}$ = 11.58;
    LCL for R = 0, UCL for R = 3.35; Out of control
26. LCL for $\overline{X}$ = 47.25, UCL for $\overline{X}$ = 54.75;
    LCL for R = 0, UCL for R = 13.75; out of control
27. LCL for $\overline{X}$ = 0.67, UCL for $\overline{X}$ = 2.93;
    LCL for s = 0, UCL for s = 1.18; out of control
28. LCL for $\overline{X}$ = 45.5, UCL for $\overline{X}$ = 56.5;
    LCL for s = 0, UCL for s = 6.39; under control
29. LCL for $\overline{X}$ = 6.09, UCL for $\overline{X}$ – 15.81;
    LCL for s = 0, UCL for s = 5.69; under control
30. LCL for $\overline{X}$ = – 1.55, UCL for $\overline{X}$ = 2.07;
    LCL for s = 0, UCL for s = 2.12; under control
31. LCL for $\overline{X}$ = 5.62, UCL for $\overline{X}$ = 16.18;
    LCL for s = 0, UCL for s = 5.51; under control
32. LCL = 0, UCL = 7.18; under control
33. LCL = 0, UCL = 13.12; under control
34. LCL = 0, UCL = 10.96; out of control
35. LCL = 0, UCL = 0.166; under control
36. LCL = 0.2481, UCL = 0.5413; out of control
37. LCL for np = 2.97, UCL for np = 25.03;
    LCL for p = 0.028, UCL for p = 0.063; out of control.
38. LCL = 0.042, UCL = 0.178; out of control.
39. LCL = 0, UCL = 0.0979; under control.
40. LCL = 0, UCL = 11.708; out of control.
41. LCL = 1.27, UCL = 21.13; under control.
42. LCL = 0, UCL = 12.62; under control.
43. (7.77,56.39); does not meet the specifications.
44. (1.1056, 1.1196); meets the specifications.
45. (42.98, 59.90); meets the specifications.