R, L and C Circuits With Sinusoidal Excitation :-

→ R, L, C Circuit elements have diff. electrical properties.

R – opposes current
L – Opposes changes in current
C – Opposes change in voltage.

} ⇒ AC contains only pure values.

power factor :-

→ cosine angle between the voltage and current. (ϕ angle)

power factor $= \cos\phi$. $P = VI\cos\phi$.

Lagging → In Inductor Load, Leading → In capacitor Load.

Active (or) Real (or) True power :-

→ product of RMS value of voltage and current with cosine of the angle between them.

$$P = I_{rms} \cdot V_{rms} \cos\phi \Rightarrow VI\cos\phi$$ → Unit is Watts (W).

Reactive power :-

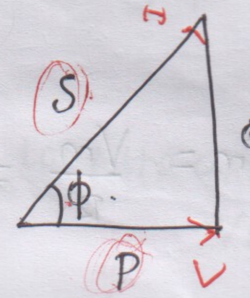
→ product of rms value of voltg & current with sine of the angle b/w them.

$$Q = V_{rms} I_{rms} \sin\phi \Rightarrow VI\sin\phi.$$ [Unit is reactive volt ampere (Var)]

Apparent power :- → Product of rms value of voltg & current is called apparent power (S). Unit (Volt Ampere) (VA).

$$S = V_{rms} I_{rms} \quad (or) \quad S = VI.$$

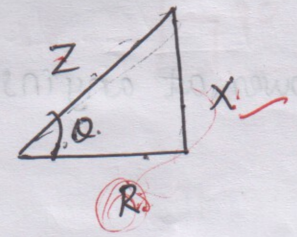
$$S = \sqrt{P^2 + Q^2}.$$



$\sin\phi = \frac{QP}{VI}$

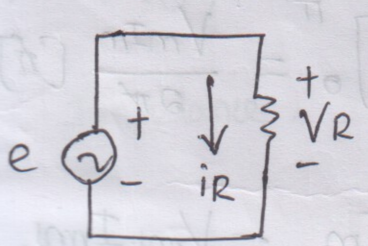
$$\text{power factor} = \frac{\text{True power}(P)}{\text{Apparent power}(S)} = \frac{VI\cos\phi}{VI}.$$

$$\text{powerfactor} = \cos\phi = \frac{R}{Z} \; \frac{(Adj)}{(hy)}.$$

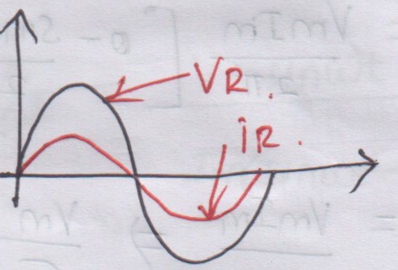


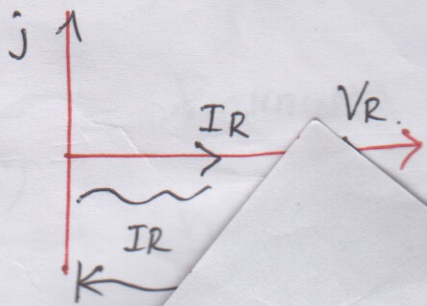
Resistance & sinusoidal AC :-

→ Resistor R is connected across an alternating voltg source. In a purely resistive circuit, current i_R & voltage V_R are in phase. Since votg, & current waveforms coincide, their phasors also coincide.



(Circuit)





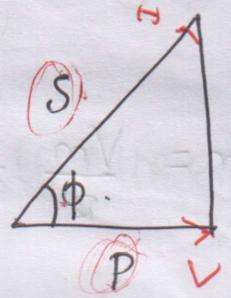
(Phasor)

let, Applied voltage is $V = V_m \sin\omega t$. —— (1)

Apparent power: → Product of rms value of voltg & current is called apparent power (S). Unit (Volt Ampere) (VA).

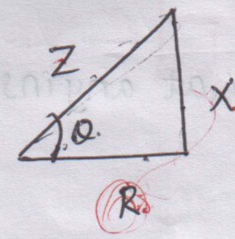
$$S = V_{rms} I_{rms} \quad (or) \quad S = VI.$$

$$S = \sqrt{P^2 + Q^2}.$$



power factor $= \dfrac{\text{True power}(P)}{\text{Apparent power}(S)} = \dfrac{VI\cos\phi}{VI}.$

$$\text{powerfactor} = \cos\phi = \frac{R}{Z} \; \frac{(Adj)}{(hy)}.$$



Resistance & sinusoidal AC:-

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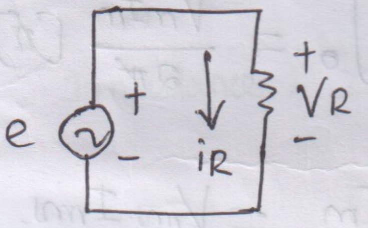

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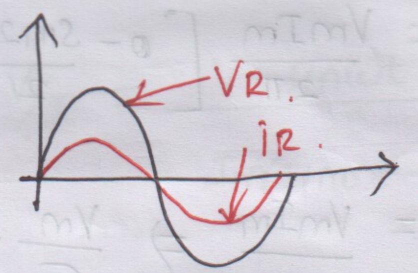
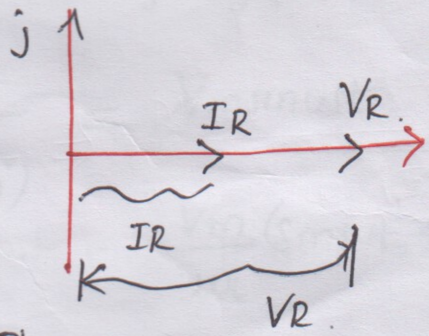




(Phasor)

let, Applied voltage is $V = V_m \sin\omega t$. ——— ①

Resulting current has an instantaneous value, $i = V/R$.

$$i = \frac{V_m \sin\omega t}{R} = \left(\frac{V_m}{R}\right)\sin\omega t = I_m \sin\omega t$$

$I_m = \frac{V_m}{R} \Rightarrow$ peak value of current.

Impedance $Z = V/i \Rightarrow \frac{V_m \sin\omega t}{I_m \sin\omega t} = \frac{V_m}{V_m/R} = R$ {V & I in each other

power at any instant, $P = vi$

$$= (V_m \sin\omega t)(I_m \sin\omega t)$$

$$P = V_m I_m \sin^2\theta \quad (\theta = \omega t)$$

Power,

Avg. power over a cycle, $P = \frac{V_m I_m}{\pi}\int_0^{\pi} \sin^2\theta \, d\theta$

$$= \frac{V_m I_m}{2\pi}\int_0^{\pi}(1-\cos 2\theta)\, d\theta$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{2\pi}[\pi]$$

$$= \frac{V_m I_m}{2} \Rightarrow \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$$

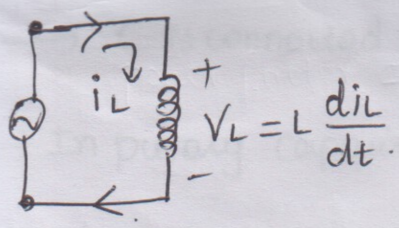
$$\boxed{P = VI \text{ Watts}}$$

Scanned by VueScan - get a free trial at www.hamrick.com power factor; → cosine angle between voltage and current.

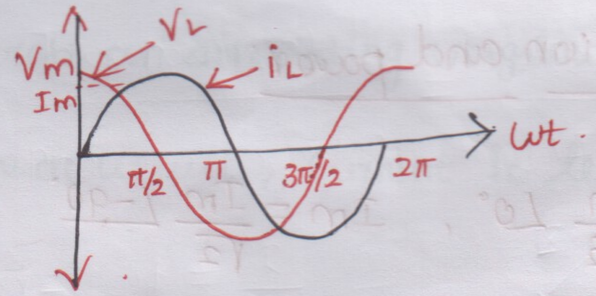
$$\cos\phi = \cos 0 = 1 \cdot \left[\phi = 0 \text{ ; bcz } V \text{ \& } I \text{ in phase}\right].$$

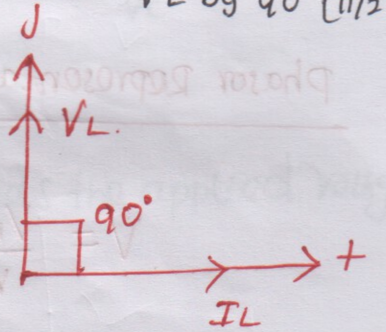
Inductance and Sinusoidal AC:-

→ current i_L lags the applied voltage V_L by 90° ($\pi/2$ r



(circuit)





Applied voltage, $V = V_m \sin\omega t$ —— ①.

current through the inductor (i), $\quad i = \dfrac{1}{L}\int V\,dt.$

$$= \frac{1}{L}\int V_m \sin\omega t \cdot dt = \frac{V_m}{\omega L}(-\cos\omega t$$

$$\boxed{i = I_m(\sin\omega t - 90^\circ)} \quad —— ②.$$

from ① & ②, current through an inductor lags the voltage by 90°.

Impedance, $Z = V/I = \dfrac{V_m \sin\omega t}{I_m(\sin\omega t - 90^\circ)} = \dfrac{V_m \sin\omega t}{\frac{V_m}{\omega L}(\sin\omega t - 90^\circ)}$

$$= \omega L \cdot \frac{\sin\omega t}{\sin(\omega t - 90^\circ)} \Rightarrow \omega L \angle 90^\circ = j\omega L.$$

$$\boxed{Z = j X_L.}$$

$Z = \omega L \rightarrow$ inductive reactance & X_L. Unit is Ohm (Ω).

$$X_L = \omega L = 2\pi f L \ , \ \omega = 2\pi f.$$

In a purely inductive circuit, the oppostion to the flow of alternating current is called inductive reactance (X_L).

Phasor Representation and power:

$$V = \frac{V_m}{\sqrt{2}} \angle 0^\circ , \quad I_m = \frac{I_m}{\sqrt{2}} \angle -90^\circ$$

Instantaneous power (or) power at any instant } $(p) = VI \Rightarrow V_m \sin\theta \cdot I_m \sin(\theta - \pi/2)$.

$$p = -V_m I_m \sin\theta \cos\theta.$$

$$\text{Avg. power} = \frac{-V_m I_m}{\pi} \int_0^{\pi} (\sin\theta\cos\theta) d\theta.$$

$$= \frac{-V_m I_m}{\pi} \int_0^{\pi} \sin 2\theta \, d\theta \Rightarrow \frac{V_m I_m}{2\pi} \left(\frac{\cos 2\theta}{2} \right)_0^{\pi}$$

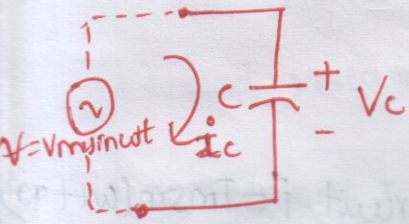
$$P = \frac{V_m I_m}{4\pi} (\cos 2\pi - \cos 0) = 0.$$

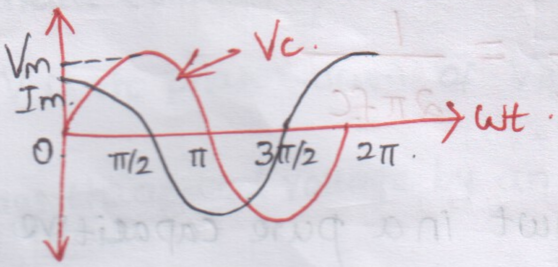
So, pure inductor does not consume any real power.

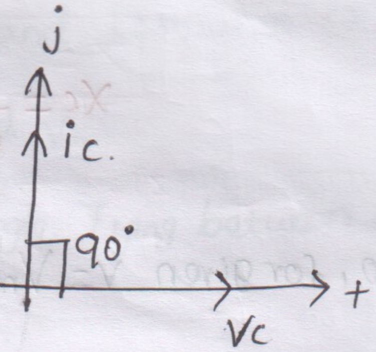
power factor $= \cos\phi \Rightarrow \cos 90 = 0$.

Power factor is zero lagging.

Capacitance and Sinusoidal AC:-







→ C is connected across an alternating voltg source.

→ In purely capacitive ac circuit, current i_C leads the applied voltg V_C by 90°. ($\pi/2$ rads).

Applied voltage $V = V_m \sin\omega t$ —①.

Voltg across the capacitor is, $V = \frac{1}{C}\int i\,dt$

current through capacitor $i = C\frac{dv}{dt}$.

$$i = C\frac{d}{dt}(V_m \sin\omega t) \Rightarrow V_m.\omega C.\cos\omega t.$$

$$= \frac{V_m}{1/\omega C}.\cos\omega t.$$

$$i = I_m \cos\omega t \Rightarrow I_m(\sin\omega t + 90°)$$

where, $I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_C}$, i.e., $X_C = 1/\omega C$. → Capacitive reactance in ohms.

→ In purely capacitive circuit the opposition to the flow of alternating current is called capacitive reactance.

$$X_C = \frac{V_C}{I_C}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

So, for given $v = V_m \sin\omega t$ in a pure capacitive circuit $i = I_m \sin(\omega t + 90°)$

the current is leading voltage by $90°$.

Phasor representation & power:

$$V = \frac{V_m}{\sqrt{2}} \angle 0° \quad \& \quad I = \frac{I_m}{\sqrt{2}} \angle 90°$$

Power factor $= \cos 90 = 0 \rightarrow$ Zero leading.

Instantaneous power $(p) = vi = V_m \sin\omega t \cdot I_m \sin(\omega t + 90°)$

$$\text{Avg. power} = \frac{V_m I_m}{\pi} \int_0^{\pi} \sin\theta \cos\theta \, d\theta, \quad (\omega t = \theta)$$

$$= \frac{V_m I_m}{2\pi} \int_0^{\pi} \sin 2\theta \, d\theta$$

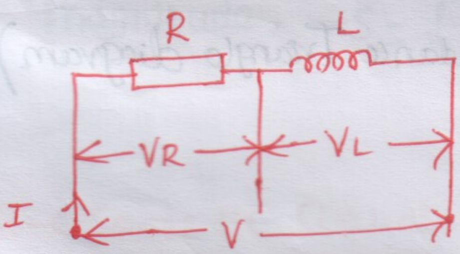
$$= \frac{-V_m I_m}{2\pi} \left[\frac{\cos 2\theta}{2} \right]_0^{\pi} = 0$$

Thus a pure capacitor does not consume any real power.

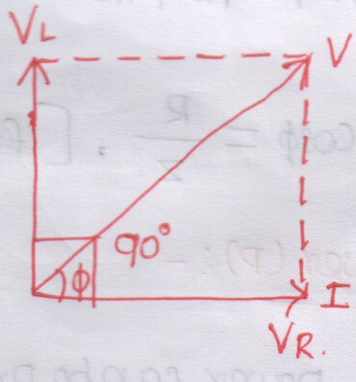
R-L-Series AC Circuit:-

→ In ac Circuit Containing inductance(L) and Resistance (R), the applied voltage V is the phasor sum of V_R and V_L.

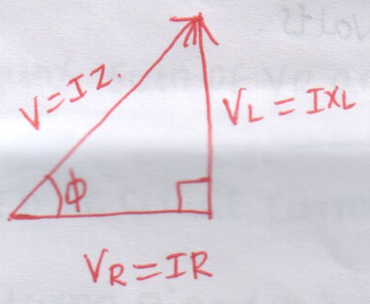
→ Current I lags the applied voltage by an angle lying between 0° and 90° shown as angle ϕ.



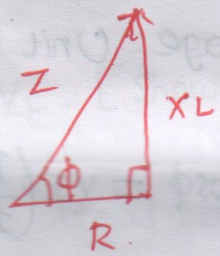
(Circuit)



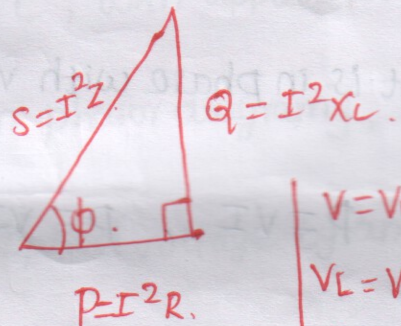
(b) Phasor diagram.



(Voltage triangle)



(Impedance Triangle).



$V = V_m \sin\omega t$ - Applied vo

V_L = Votg across inducto

V_R = Votg across Resista

→ In a Series RL circuit the current flowing through R and L is common & that taken as Reference phasor.

→ In Resistance the voltage across it V_R and current I are in phase.

→ In Inductor, voltage across it V_L leads current I by 90°.

For the R-L circuit, $|V| = \sqrt{V_R^2 + V_L^2}$.

$$\tan\phi = \frac{V_L}{V_R}, \quad \phi = \tan^{-1}\left(V_L/V_R\right)$$

In an a.c circuit, the ratio applied voltage V to current I.

$$\boxed{Z = V/I.}$$

Impedance(Z), $Z = R + jX_L$.

$$|Z| = \sqrt{R^2 + X_L^2}, \quad \tan\phi = \frac{X_L}{R}, \quad \sin\phi = \frac{X_L}{Z},$$

$$\cos\phi = \frac{R}{Z}.$$ [from Impedance Triangle diagram).

Active (or) Real power (P) :-

→ Only the Real power can be produced at the time of current component is in phase with voltage. Unit is Watts.

$$P = VI, \quad P = VI\cos\phi = VI\left(\frac{R}{Z}\right)$$

$$P = \frac{V}{Z} \times I \times R.$$

$$\boxed{P = I^2 R.}$$

Reactive power (Q) :-

→ Reactive power consumption in any circuit when a current component is in quadrature with voltage. Unit is Volt Ampere Reactive.

$$Q = V \times \text{quadrature component of current}$$

$$Q = VI\sin\phi \Rightarrow VI\frac{X_L}{Z}, \quad Q = \frac{V}{Z} * I * V_L = I^2 X_L.$$

Apparent power (S) :-

→ It is calculated as the product of voltage and current . Unit Volt Amper

$$S = V * I \Rightarrow I^2 Z \Rightarrow V^2/Z$$

$$S = P + jQ$$

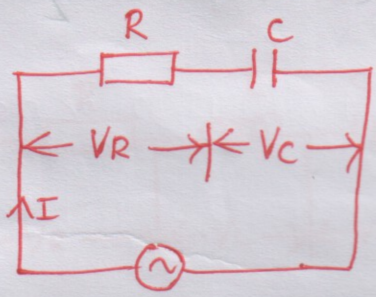
magnitude, $S = \sqrt{P^2 + Q^2}$

R-C Series AC Circuit :-

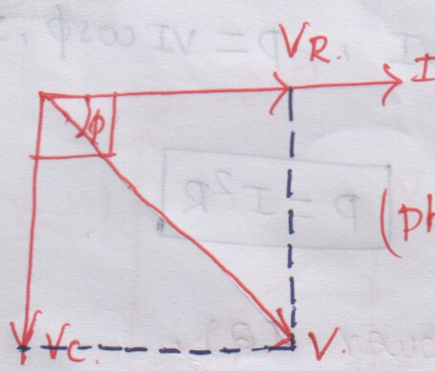
→ In an a.c circuit, which has R and C , with applied voltage V is the phasor sum of V_R and V_C. [Shown in phasor diagram].

→ In RC Circuit Current I leads the applied voltage by an angle lying between 0° and 90°.

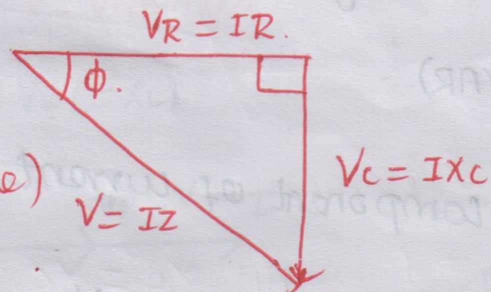
(Circuit)



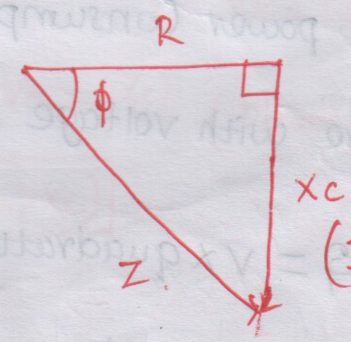
$V = V_m \sin \omega t$



(Phasor Diagram)



(Voltage triangle)



(Impedance Triangle)

In R-C circuit, $V = \sqrt{V_R^2 + V_C^2}$.

$$\tan\phi = \frac{V_C}{V_R}, \quad \phi = \tan^{-1}(V_C/V_R)$$

Then, Impedance (Z) is, $Z = \frac{V}{I}$.

from Impedance triangle ~~table~~ diagram, $Z = \sqrt{R^2 + X_C^2}$

$$\tan\phi = \frac{X_C}{R}; \quad \sin\phi = \frac{X_C}{Z}; \quad \cos\phi = \frac{R}{Z}.$$

Active (or) Real power, (P),

→ Only the power can be consumed at the time of current is in phase with voltage. (W).

$$P = VI, \quad P = VI\cos\phi, = VI(R/Z), \quad P = \frac{V}{Z} * I * R$$

$$\boxed{P = I^2 R}$$

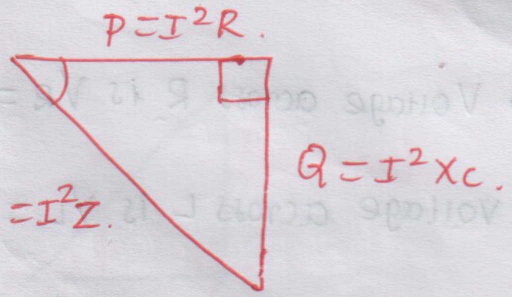
Reactive power, (Q),

→ Reactive power Consumption in any ckt when a current is in quadrature with voltage. (VAR).

$$Q = V \times \text{quadrature component of current}$$

$$Q = VI\sin\phi, \quad Q = VI\left(\frac{X_C}{Z}\right), \quad Q = \frac{V}{Z} * I * X_C$$

$$\boxed{Q = I^2 X_C}$$



Apparent power (S),

→ Product of voltage and current. $S = VI. = I^2 Z \Rightarrow V^2/Z.$

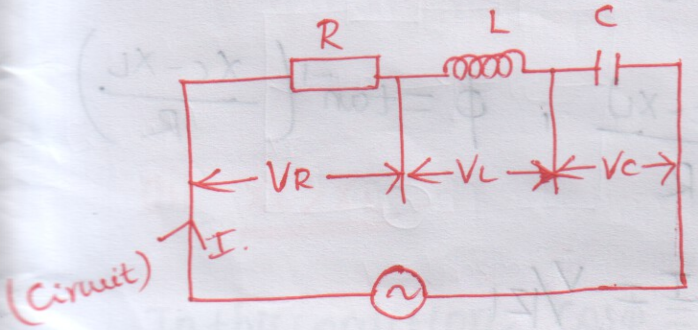
$$S = P + jQ.$$

$$\boxed{S = \sqrt{P^2 + Q^2}.}$$

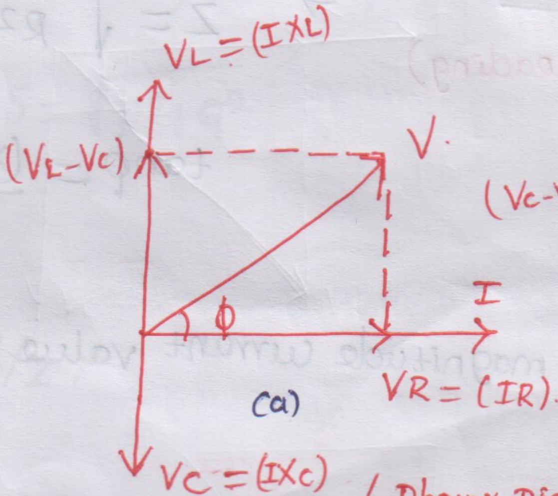
R-L-C Series AC Circuit :-

→ In a.c Circuit containing resistance R, inductance L, & capacitance the applied voltg V is the phasor sum of V_R. V_L and V_C. [shown in Phasor diagram].

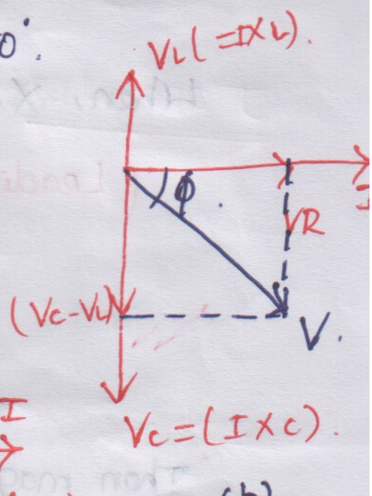
→ V_C and V_L are anti-phase. ie, displaced by 180°.



$V = V_m \sin \omega t$



(a)

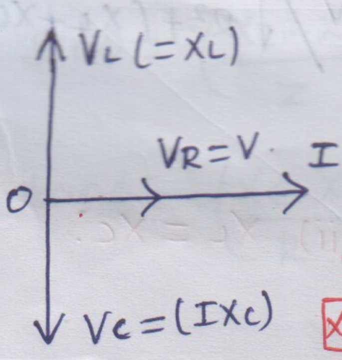


(b).

$\boxed{(X_C > X_L)}$

(Phasor Diagram)

$\boxed{(X_L > X_C)}$.



(c).

$\boxed{X_L = X_L}$.

→ Voltage across R is $V_R = IR$ → in phase with current I.

→ voltage across L is $V_L = IX_L \angle 90^\circ$ → current ~~leads~~ lags voltage by 90°
(jIX_L) → votg leads current by 90°

→ voltage across C is $V_C = IX_C \angle 90^\circ$ → current leads voltage by 90°.
($-jIX_L$) → votg lags current by 90°

Applied voltage $V = V_m \sin\omega t$.

$$V = V_R + (V_L - V_C)$$

$$\boxed{V = \sqrt{V_R^2 + (V_L - V_C)^2}}$$

Impedance (z) is, $Z = V/I$.

When, $X_L > X_C \Rightarrow$ $Z = \sqrt{R^2 + (X_L - X_C)^2}$
(Lagging)

$$\tan\phi = \frac{(X_L - X_C)}{R}, \quad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right).$$

When, $X_C > X_L \Rightarrow$ $Z = \sqrt{R^2 + (X_C - X_L)^2}$
(Leading)

$$\tan\phi = \frac{(X_C - X_L)}{R}, \quad \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Then magnitude current value. $I = V/|Z|$

$$I = V\Big/\sqrt{R^2 + (X_L - X_C)^2}$$

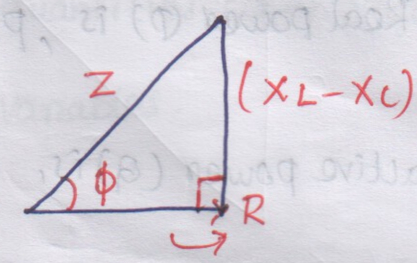
Two conditions,

(i) $X_L > X_C \Rightarrow$ Lagging p.f

(ii) $X_C > X_L \Rightarrow$ Leading p.f

(iii) $X_L = X_C$.

(i) $X_L > X_C$:-



In this condition, $\cos\phi = \frac{R}{Z}$.

$\sin\phi = \left(\frac{X_L - X_C}{Z}\right)$

$\tan\phi = \left(\frac{X_L - X_C}{R}\right)$

So, Real power(p) is, $P = VI$, $P = VI\cos\phi \Rightarrow VI\left(R/Z\right)$

$$\boxed{P = I^2R}$$

Reactive power (Q) is, $Q = VI\sin\phi$

$$Q = V \cdot I \cdot \left(\frac{X_L - X_C}{Z}\right)$$

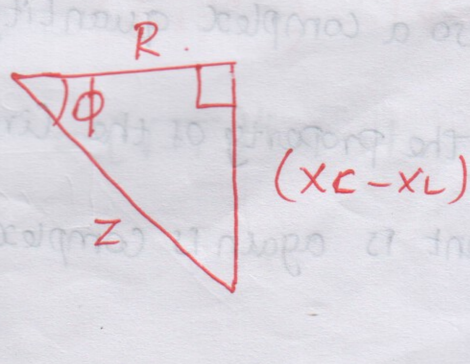
$$\boxed{Q = I^2(X_L - X_C)}$$

Apparent power (S) is, $S = VI \Rightarrow I^2 Z \Rightarrow V^2/Z$.

$$S = \sqrt{P^2 + Q^2}.$$

(ii) $X_C > X_L$:-

In this condition, $\cos\phi = R/Z$,



$\sin\phi = \left(\frac{X_C - X_L}{Z}\right)$

$\tan\phi = \left(\frac{X_C - X_L}{R}\right)$

Real power (P) is, $P = VI\cos\phi$ $\boxed{P = I^2R.}$

Reactive power (Q) is, $Q = VI\sin\phi$.

$$Q = VI.\left(\frac{X_C - X_L}{Z}\right)$$

$$\boxed{Q = I^2(X_C - X_L)}$$

Apparant power (S) is, $S = VI.$, $S = I^2Z$, $S = V^2/Z$.

$$S = \sqrt{P^2 + Q^2}.$$

(iii) $X_L = X_C$.

→ When $X_C = X_L$, the applied voltage V and the current are in phase.

This effect is called series resonance.