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# ***Error Detection & Correction***

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# Introduction

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*Data can be corrupted during  
transmission due to...*

*accidents, sudden increase in  
electricity and voltage / decrease in  
signal power over distance*

*For reliable communication, errors  
must be detected and corrected*

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# What is an Error ?

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- *Whenever bits flow from one point to another, they are subject to unpredictable changes because of interference*
  - *The interference can change the shape of the signal, thus the bit value either from “1” to “0” or from “0” to “1”*
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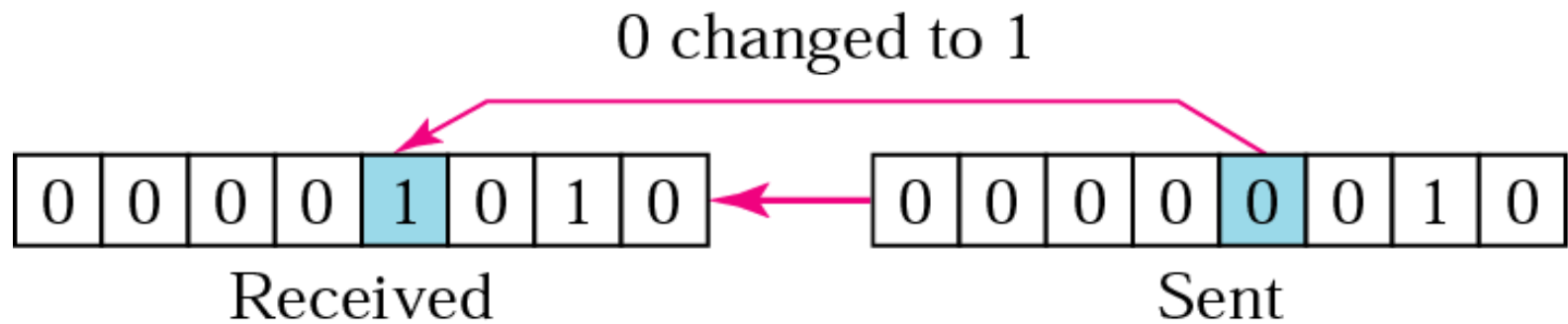
# Two Types of Errors

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- *Single-Bit Errors* : only one bit in the data unit has changed
  - *Burst Errors of length 'n'* : 2 or more bits in the data unit have changed ( 'n' is the distance between the FIRST and LAST errors in the data block )
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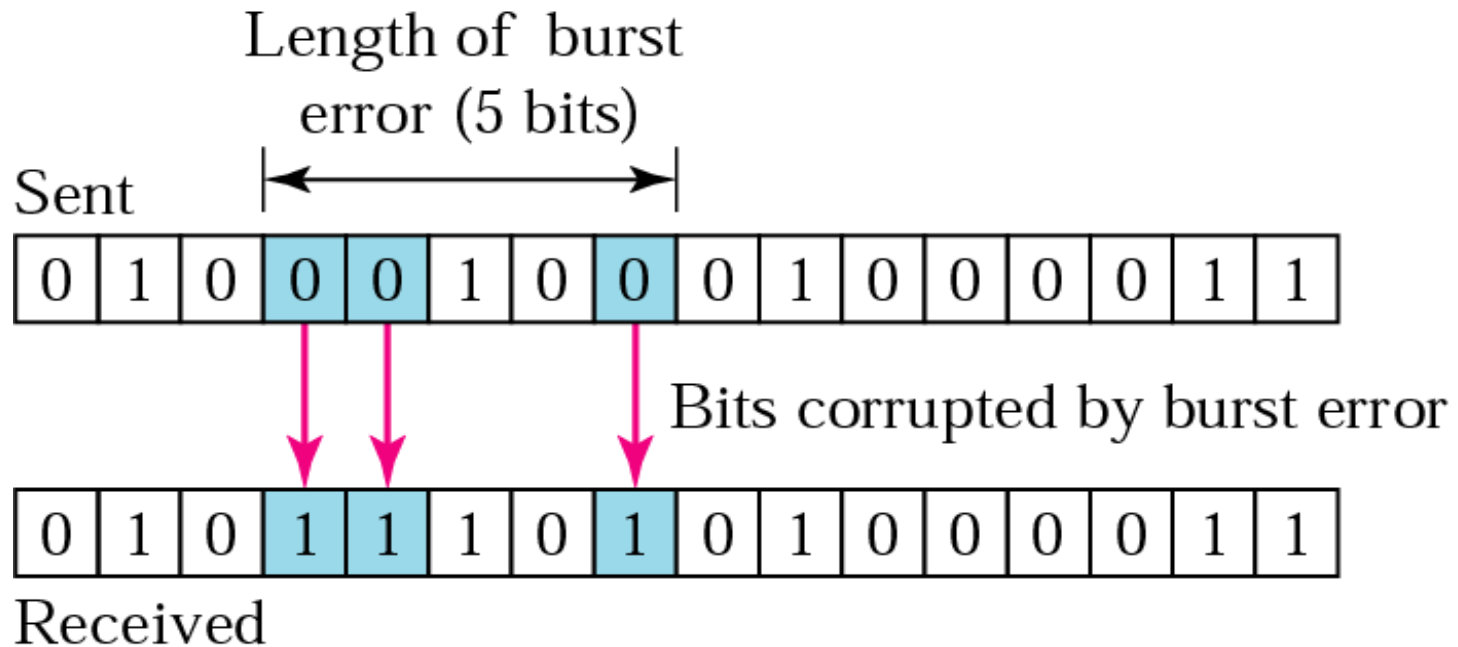
# Single-Bit Errors

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# Burst Errors





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# *Error Detection*

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# Error Detection-General

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- Sender transmits every data unit twice
  - Receiver performs bit-by-bit comparison between that two versions of data
  - Any mismatch would indicate an error, which needs error correction
  - **Advantage:** very accurate
  - **Disadvantage:** time consuming
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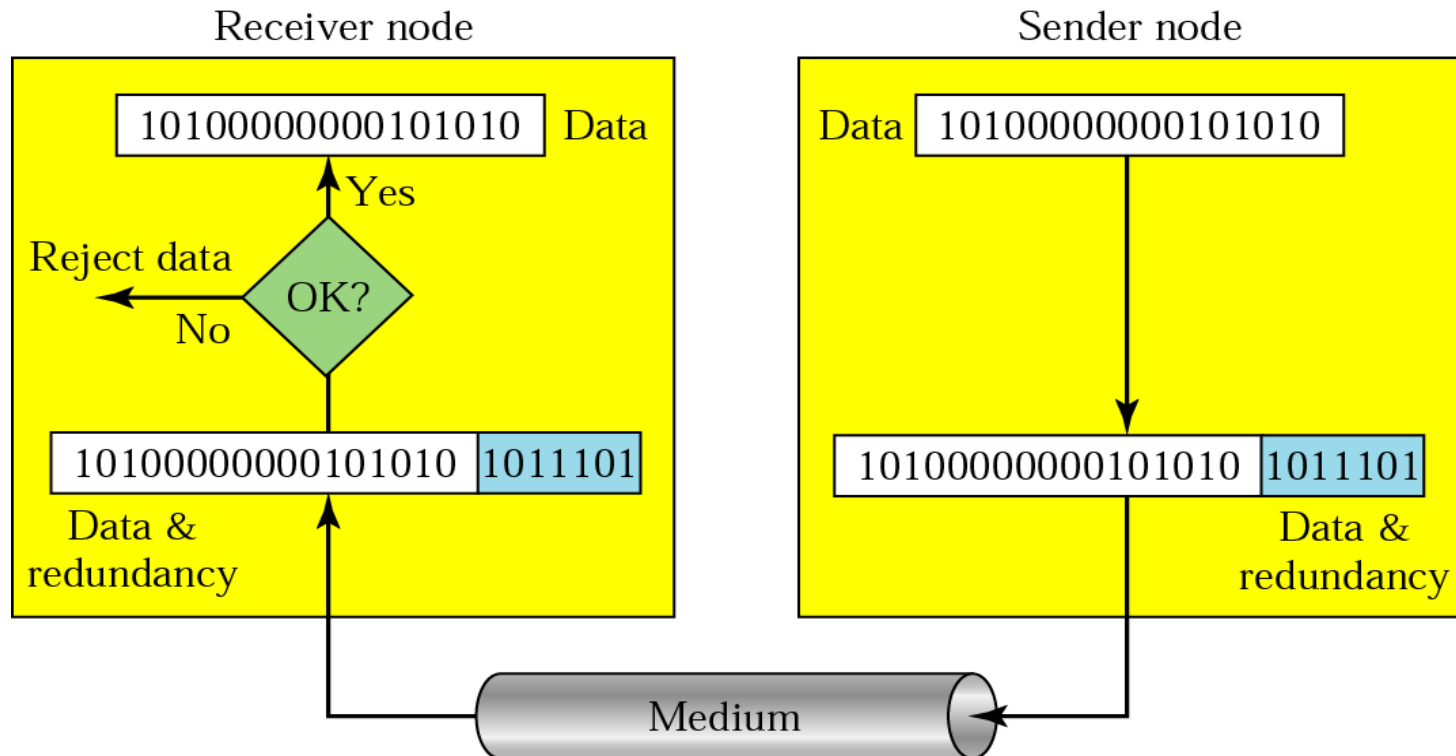
# Error Detection- Redundanc

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- Called as “redundancy” because the extra bits are redundant to the information
  - Redundant information will be discarded as soon as the accuracy of the information has been determined
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# Error Detection- Redundanc



# Error Detection-Redundanc

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## Types of Redundancy Checks

- Parity Check
    - Simple Parity Check
    - Two Dimensional Parity Check / Longitudinal Redundancy Check (LRC)
  - Cyclic Redundancy Check (CRC)
  - Check Sum
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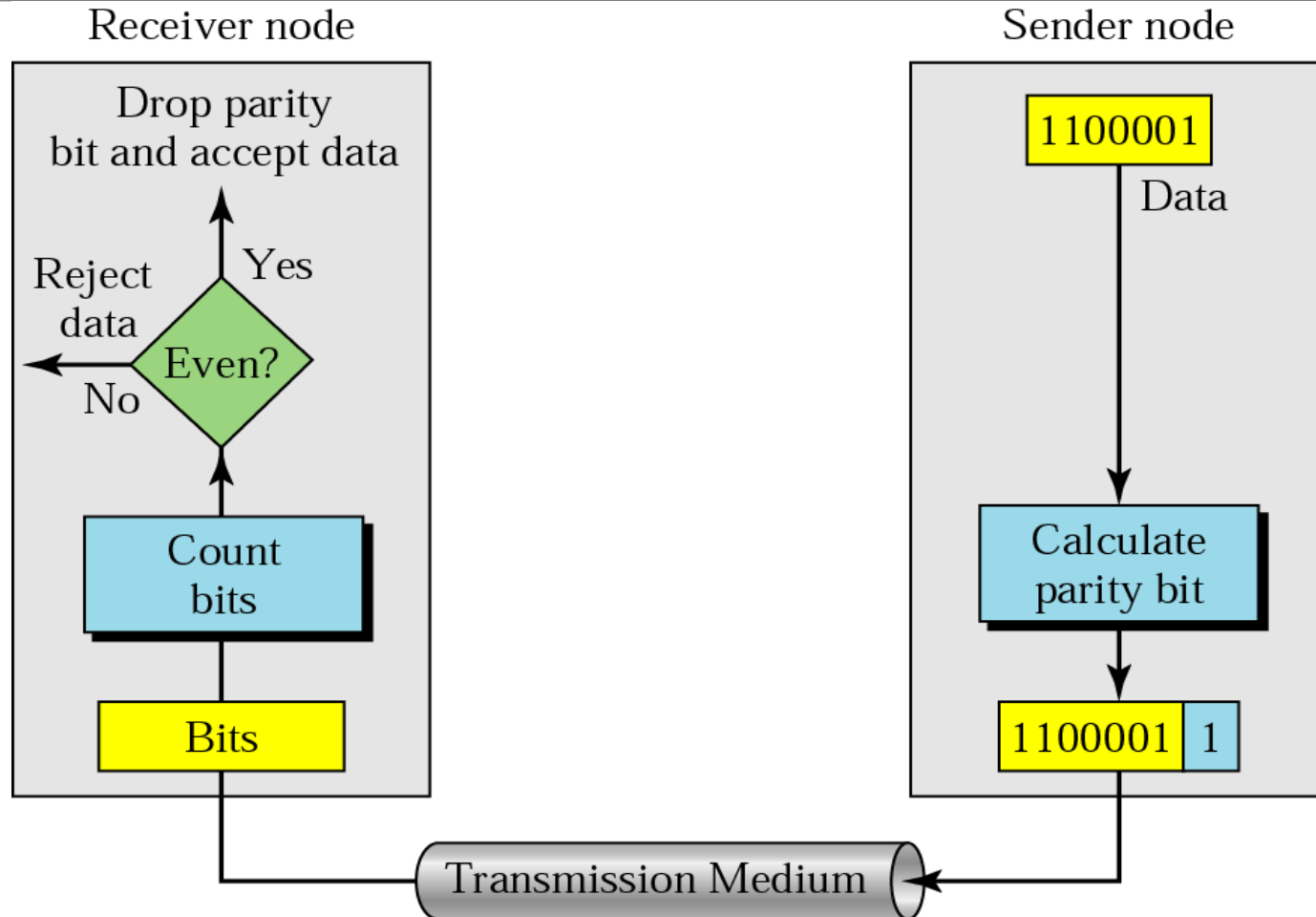
# Error Detection-Simple Parity Check

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- A redundant bit called “Parity Bit” is added to every data unit
  - Even Parity : total number of 1’s in the data unit becomes even
  - Odd Parity : total number of 1’s in the data unit becomes odd
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# Error Detection- Simple Parity Check



**Even Parity Check**

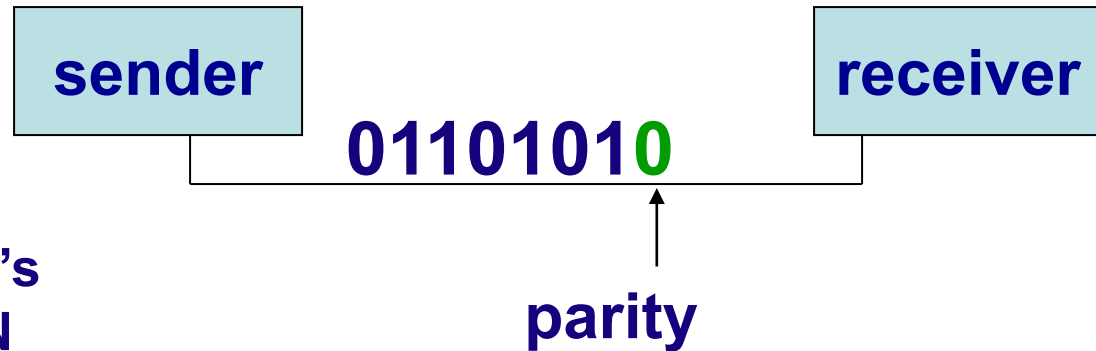
# Example of Using Parity Bits



Data to be sent: Letter **V** in 7-bit ASCII: **0110101**

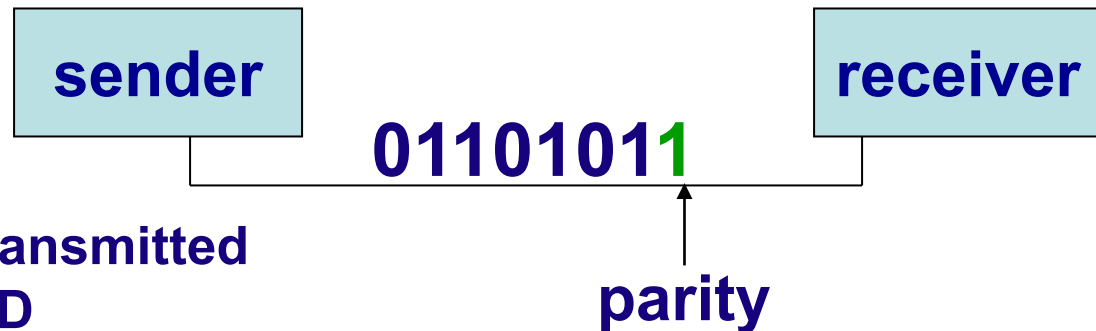
**EVEN parity**

number of all  
transmitted 1's  
remains **EVEN**



**ODD parity**

number of all transmitted  
1's remains **ODD**





# Simple Parity Check-An Example

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Data:

w                      o                      r                      l                      d

1110111 1101111 1110010 1101100 1100100

Sent As:

11101110 11011110 11100100 11011000 11001001

Corrupted:

11111110 11011110 11101100 11011000 11001001

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# Parity Check - Performance

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- Can detect all single-bit errors
  - Can also detect burst errors if the total number of bits changed is odd (1,3,5,..)
  - Cannot detect errors where the total number of bits changed is even
  - Detects about 50% of errors
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# Error Detection- 2D/LRC

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- Adds an additional character (instead of a bit)
  - A block of bits is organized in a table
  - The Parity Bit for each data unit is calculated
  - Then Parity Bit for each column is calculated
  - Parity Bits are attached to the data unit
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# Error Detection- LRC

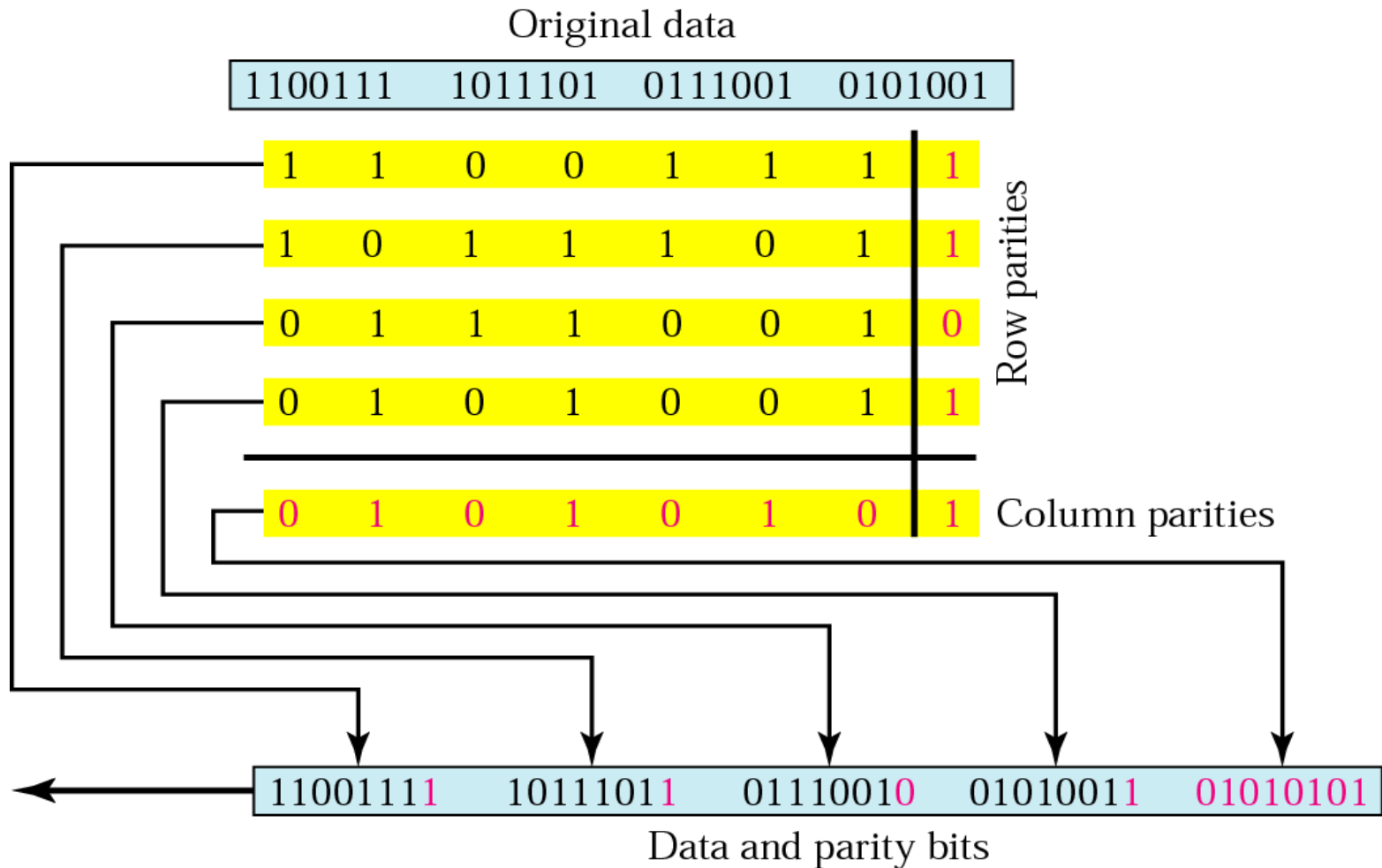
Example:

Send the message “DATA” using ODD parity and LRC

<u>Letter</u>	<u>ASCII</u>	<u>Parity bit</u>
D	1 0 0 0 1 0 0	1
A	1 0 0 0 0 0 1	1
T	1 0 1 0 1 0 0	0
A	1 0 0 0 0 0 1	1
BCC	1 1 0 1 1 1 1	1

Note that the BCC's parity bit is also determined by parity

# Error Detection- LRC



# LRC - Performance

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- Detects all burst errors up to length  $n$  (number of columns)
  - If two bits in one data unit are damaged and two bits in exactly same positions in another data unit are also damaged, the checker will not detect an error
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# Error Detection- CRC

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- Powerful error detection scheme
  - Rather than addition, binary division is used
  - A sequence of redundant bits, called “CRC” or “CRC remainder” is appended to the data unit, so that the resulting data unit becomes divisible by a predetermined binary number
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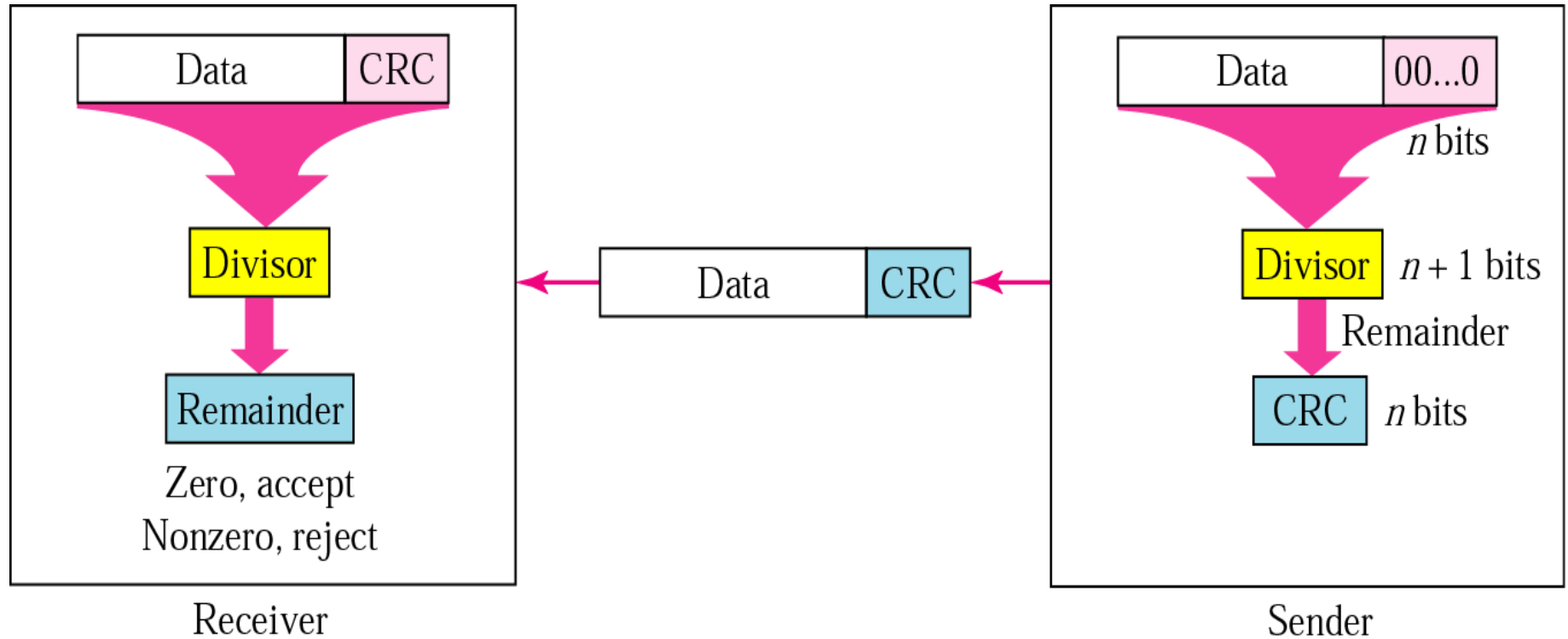
# Error Detection- CRC

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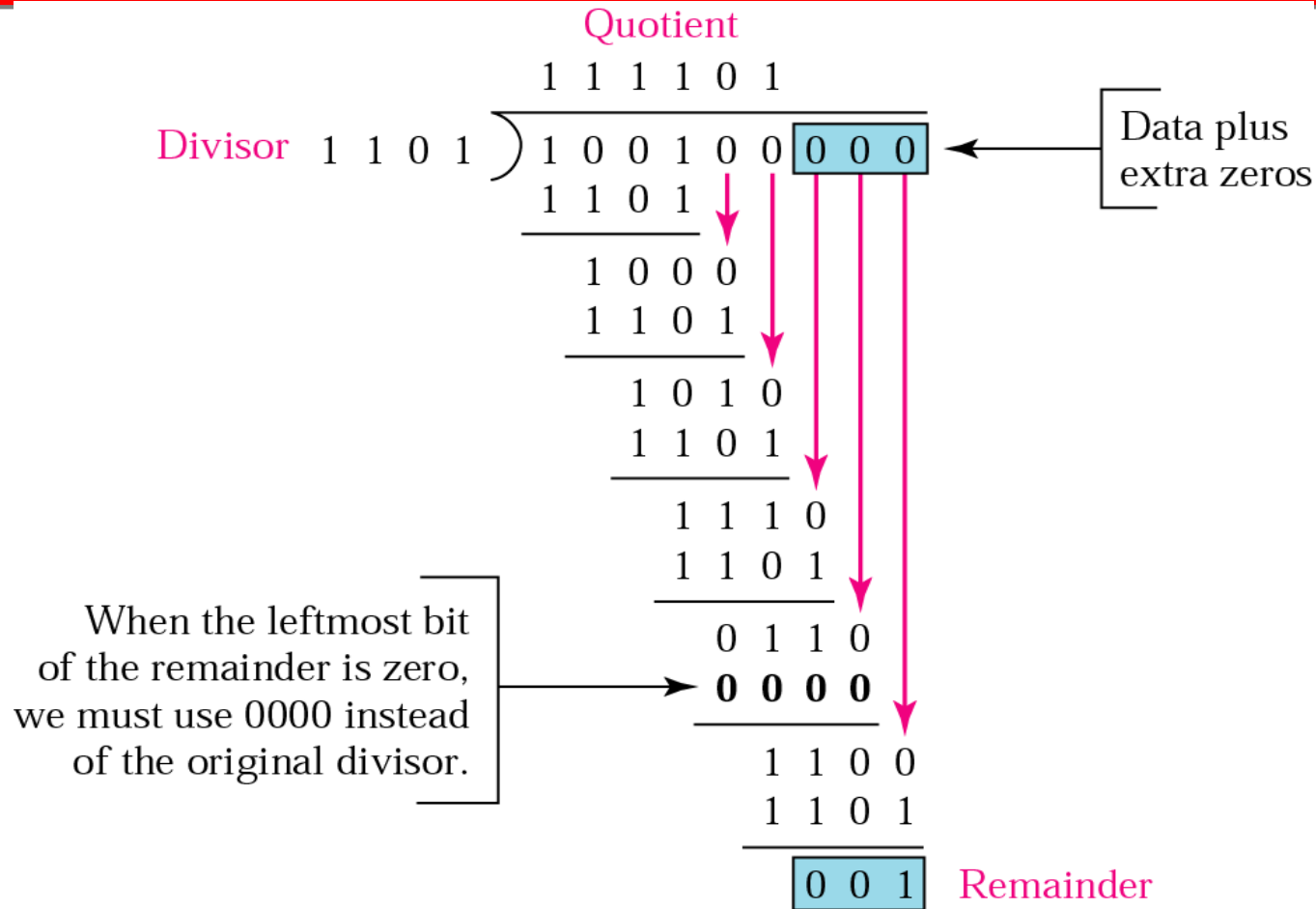
- At the receiver side, the incoming data unit is divided by the same predetermined number.
  - If there is no remainder, the data unit is accepted
  - If there is a remainder, the receiver indicates that the data unit has been damaged during transmission
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# Error Detection- CRC

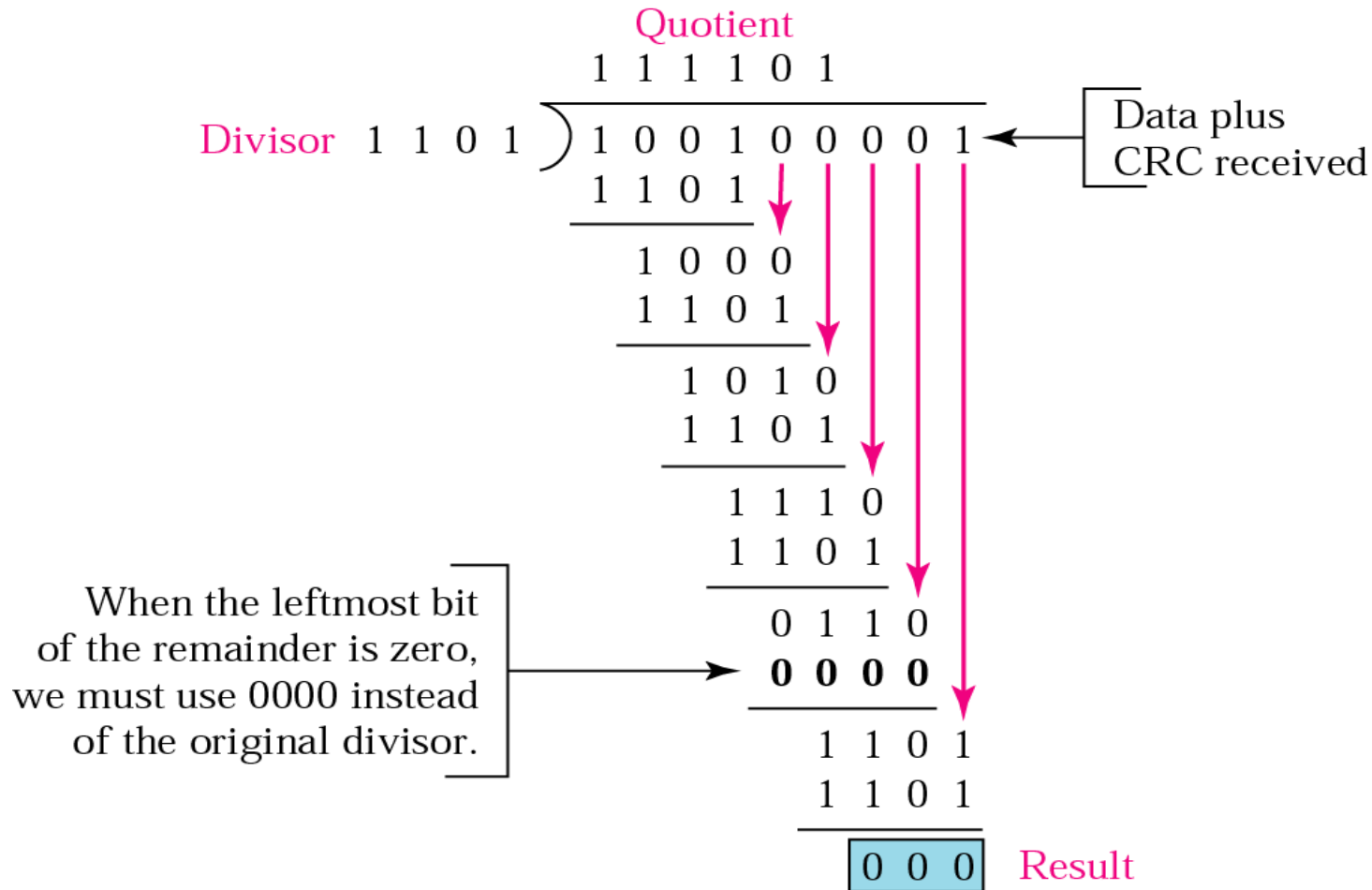


# Error Detection- CRC Generator





# Error Detection- CRC Checker



# Error Detection- CRC Polynomial

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- The divisor in the CRC generator is most often represented as an algebraic *Polynomial*.
  - Reasons:
    - It is short
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# Error Detection- CRC Polynomial



$$x^7 + x^5 + x^2 + x + 1$$

Polynomial

$$x^7 + x^5 + x^2 + x + 1$$

$x^6$

$x^4$

$x^3$

1 0 1 0 0 1 1 1

Divisor

# Error Detection- CRC Polynomials



Name	Polynomial	Application
CRC-8	$x^8 + x^2 + x + 1$	ATM header
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^2 + 1$	ATM AAL
ITU-16	$x^{16} + x^{12} + x^5 + 1$	HDLC
ITU-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$	LANs



# CRC-Performance

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- CRC can detect all burst errors that affect an odd number of bits
  - CRC can detect all burst errors of length less than or equal to the degree of the polynomial
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# Error Detection- Check Sum

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- ***The sender follows these steps:***
    - The data unit is divided into “k” sections, each of “n” bits
    - All sections are added using one’s complement to get the sum
    - The sum is complemented and becomes the checksum.
    - The checksum is appended and sent with the data.
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# Error Detection- Check Sum

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- ***The receiver follows these steps:***
    - The unit is divided into “k” sections, each of “n” bits
    - All sections are added using one’s complement to get the sum.
    - The sum is complemented.
    - If the result is zero, the data are accepted; otherwise, rejected
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# Error Detection- Check Sum



Receiver

Section 1  $n$  bits  
Section 2  $n$  bits  
.....  
Checksum  $n$  bits  
.....  
Section  $k$   $n$  bits

Sum  $n$  bits

Complement

Result

If the result is 0, keep;  
otherwise, discard.

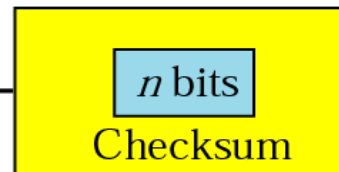
Sender

Section 1  $n$  bits  
Section 2  $n$  bits  
.....  
Checksum All 0s  
.....  
Section  $k$   $n$  bits

Sum  $n$  bits

Complement

Checksum







# Check Sum-An Example

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Data:

10101001 00111001

Computing Checksum:

10101001

00111001

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Sum 11100010

Checksum 00011101

Data Sent :

10101001 00111001 00011101

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# Check Sum-An Example

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Receiver Side:

10101001

00111001

00011101

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Sum 11111111

Complement 00000000

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# *Error Correction*

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# Error Correction Technique

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- Retransmission
  - Forward Error Correction
  - Burst Error Correction
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# Error Correction-Retransmissi

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*When an error is discovered,  
the receiver can ask the sender  
to retransmit the entire data unit*

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# Error Correction-Forward Error Correction

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- A receiver can use an error-correcting code, which automatically corrects certain errors
  - Single-bit errors:
    - Can be detected by the addition of parity bit which helps to find “error” or “no error” which is sufficient to detect errors
    - To correct errors the receiver can simply invert 0 to 1 or 1 to 0, but the problem is “locating” the position of error
    - To do so requires enough redundancy bits
    - Condition:  $2^r \geq m + r + 1$
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# Error Correction

Number of data bits $m$	Number of redundancy bits $r$	Total bits $m + r$
1	2	3
2	3	5
3	3	6
4	3	7
5	4	9
6	4	10
7	4	11

**Relationship b/w data and redundancy bits**

# Error Correction-Hamming Code

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- Hamming Code can be applied to data units of any length and uses the relationship between data and redundancy bits
  - For example: a 7-bit ASCII code requires 4 redundancy bits that can be added to the end of the data unit or mixed with the original data bits, which are placed in positions 1, 2, 4 and 8 i.e  $x^0, x^1, x^2, x^3$  and so on.
-



# Error Correction-Hamming Code



11	10	9	8	7	6	5	4	3	2	1
d	d	d	$r_8$	d	d	d	$r_4$	d	$r_2$	$r_1$

**Positions of Redundancy bits in Hamming Code**

# Error Correction-Hamming Code

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- In the Hamming Code, each “r” bit for one combination of data bits as below:

r1: bits 1, 3, 5, 7, 9, 11  
r2: bits 2, 3, 6, 7, 10, 11  
r3: bits 4, 5, 6, 7  
r4: bits 8, 9, 10, 11

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# Error Correction-Hamming Code



$r_1$  will take care of these bits.

11		9		7		5		3		1
d	d	d	$r_8$	d	d	d	$r_4$	d	$r_2$	$r_1$

$r_2$  will take care of these bits.

11	10			7	6			3	2	
d	d	d	$r_8$	d	d	d	$r_4$	d	$r_2$	$r_1$

$r_4$  will take care of these bits.

				7	6	5	4			
d	d	d	$r_8$	d	d	d	$r_4$	d	$r_2$	$r_1$

$r_8$  will take care of these bits.

11	10	9	8							
d	d	d	$r_8$	d	d	d	$r_4$	d	$r_2$	$r_1$

**Redundancy bit Calculation**

# Error Correction-Hamming Code



Data:  
1 0 0 1 1 0 1

1	0	0		1	1	0		1		
11	10	9	8	7	6	5	4	3	2	1

Adding  $r_1$

<div></div>										
1	0	0		1	1	0		1		1
11	10	9	8	7	6	5	4	3	2	1

Adding  $r_2$

<div></div>										
1	0	0		1	1	0		1	0	1
11	10	9	8	7	6	5	4	3	2	1

Adding  $r_4$

<div></div>										
1	0	0		1	1	0	0	1	0	1
11	10	9	8	7	6	5	4	3	2	1

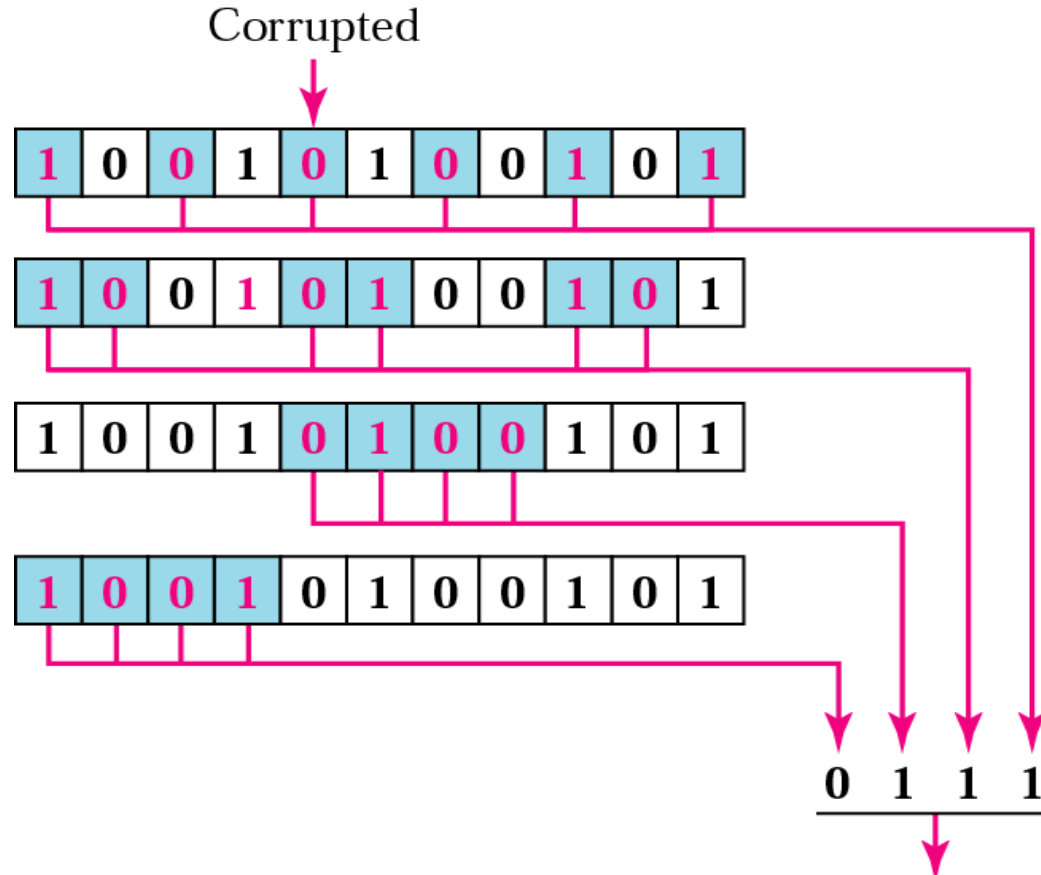
Adding  $r_8$

<div></div>										
1	0	0	1	1	1	0	0	1	0	1
11	10	9	8	7	6	5	4	3	2	1

Code:  
1 0 0 1 1 1 0 0 1 0 1

Example of Redundancy bit Calculation

# Error Detection-Using Hamming Code



The bit in position 7 is in error. 7

# Error Correction-Burst Error Correction

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- Instead of sending all the bits in a data unit together, we can organize “N” units in a column and then send the first bit of each , followed by the second bit of each and so on
  - In this way, if a burst error of M bits occurs ( $M < N$ ), then the error does not corrupt M bits of one single unit; it corrupts only 1 bit of a unit.
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# Burst Error Correction - Example

Error → 1111?000011

Error → 1010?011111

11111001100

Error → 011?1011001

Error → 011?1010110

Error → 011?1001111

Received data

11111000011

10101011111

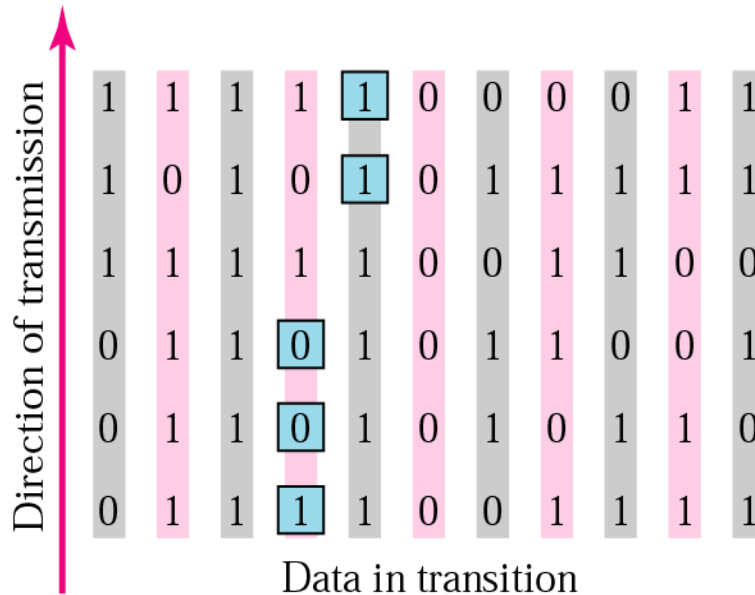
11111001100

01101011001

01101010110

01111001111

Data before being sent





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***Thank You***

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