SRM INSTITUTE OF SCIENCE & TECHNOLOGY (Deemed to be University u/s 3 of UGC Act 1956)

SRM Institute of Science and Technology

College of Engineering and Technology

DEPARTMENT OF MATHEMATICS

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

Academic Year: 2022 -2023 (EVEN)

SLOT – A2

Test	: **CLAT-2**		**Date**	: **31.03.23**
Course Code & Title	: **21MAB301T – PROBABILITY AND STATISTICS**		**Duration**	: **2 Periods**
Year & Semester	: **II & IV**		**Max. Marks**	: **50**

Note:

(i) **Part-A** should be answered in Question paper within **20** minutes and the same should be handed over to hall invigilator at the end of **20th** minute.

(ii) Only **A/B/C/D** have to be mentioned as answer for MCQ in the space provided in the Question paper.

(iii) Any Striking or overwriting in the answer **(A/B/C/D)** under **Part-A** will not be accepted.

(iv) **Part-B** should be answered in answer booklet.

Course Articulation Matrix

S.N.	Course Outcome		PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
1	CO1	Implement the concept of probability and random variable in engineering problems.	3	3	-	-	-	-	-	-	-	-	-	-
2	CO2	Identify random variable and model them using various distribution.	3	3	-	-	-	-	-	-	-	-	-	-
3	CO3	Infer results by using hypothesis testing on large and small samples.	3	3	-	-	-	-	-	-	-	-	-	-
4	CO4	Utilize quality control techniques to solve wide variety of real-world problems in industry.	3	3	-	-	-	-	-	-	-	-	-	-
5	CO5	Apply the probability techniques and statistics in science and engineering.	3	3	-	-	-	-	-	-	-	-	-	-

Part - A

(11 x 1 = 11 Marks)

Q. No	Answer	Marks	BL	CO	PO
1.	(d) $\frac{1}{p}$	1	1	2	1
2.	(a) $P(X > t)$	1	2	2	1
3.	(b) $\frac{(b-a)^2}{12}$	1	1	2	1
4.	(c) $\frac{1}{3}$	1	2	2	2
5.	(a) λ	1	2	2	1
6.	(a) 1.645	1	1	3	1
7.	(b) $v = n_1 + n_2 - 2$	1	1	3	1
8.	(b) Type II error	1	1	3	2
9.	(d) $H_1: \mu \neq 0$	1	1	3	2
10.	(c) the level of significance	1	2	3	1
11	(a) Sample, population	1	2	3	1

Q. No	Questions	Marks	BL	CO	PO
12	It is known that the probability of an item produced by a certain machine will be defective is 5%. If the produced items are sent to the market in packets of 20, find the number of packets containing (i) At least 2 defective items (ii) Exactly 2 defective items (iii) At most 2 defective items using Binomial distribution. $n = 20,\ p = 0.05,\ \ q = 0.95,\ P(X = x) = 20C_x(0.05)^x(0.95)^{20-x}$ (i) $P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)] = 0.2641$ (ii) $P(X = 2) = 20C_2(0.05)^2(0.95)^{20-2} = 0.189$ (iii) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.9246$	8	3	2	2
13	(a) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. Calculate the probability that he will finally pass the test (a) On the fourth trial (b) In less than 4 trials? **Solution :** Let X denote the number of trials required to achieve the first success. Then X follows a geometric distribution given by $P(X = r) = q^{r-1}p; r = 1,2,3,\ldots$ Here $p = 0.8$ and $q = 0.2$ (a) $P(X = 4) = 0.8 \times (0.2)^{4-1} = 0.0064$ (b) $P(X < 4) = \sum_{r=1}^{3} (0.8) \times (0.2)^{r-1} = 0.9984$.	4	3	2	2
	(b) The time (in hours) required to repairs a machine is exponential, distributed with parameter $\lambda = \frac{1}{2}$. (i) What is the probability that the repair time exceeds 2 hours? (ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours? $f(x) = \lambda e^{-\lambda x},\ x > 0$ (i) $P(X > 2) = \int_2^\infty \frac{1}{2} e^{-\frac{x}{2}} dx = \frac{1}{2}\left[\frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\right]_2^\infty = e^{-1} = 0.3679$ (ii) $P(X \geq 10/X > 9) = P(X > 1) = \int_1^\infty \frac{1}{2} e^{-\frac{x}{2}} dx = \frac{1}{2}\left[\frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\right]_1^\infty = e^{-\frac{1}{2}} = 0.6065$	4	3	2	2
14	(a) The heights of college students in a city are normally distributed with S.D. 6 cm. A sample of 100 students has mean height 158 cm. Test the hypothesis that the mean height of college students in the city is 160 cm . $n = 100, \mu = 160, \bar{x} = 158, \sigma = 6, H_0: \mu = 160,\ H_1: \mu \neq 160$ (two tailed), $z_\alpha = 1.96$ at 5% $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{158-160}{6/\sqrt{100}} = 3.333$ $\lvert z \rvert > z_\alpha$, H_0 is rejected	4	4	3	2

	(b) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? Solution $p_1 = 0.2$, $p_2 = 0.185$, $n_1 = 900$ and $n_2 = 1600$. H_0: $P_1 = P_2$ H_1: $P_1 \neq P_2$ Two-tailed test is to be used. Let LOS be 5%. Therefore, $z_\alpha = 1.96$. $$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad (1)$$ Since P, the population proportion, is not given, we estimate it as $$\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{180 + 296}{900 + 1600} = 0.1904.$$ Using in (1), we have $$z = \frac{0.2 - 0.185}{\sqrt{0.1904 \times 0.8096 \times \left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.92$$ $\therefore \quad \|z\| \leq z_\alpha$ Therefore The difference between p_1 and p_2 is not significant at 5% level.	4	3	3	2
15	If 40, 42, 50, 60, 45, 40, 55, 58, 62, 60 are the sample observations drawn from a normal population. Test if the population mean can be equal to 50. *x: 40 42 50 60 45 40 55 58 62* *x^2: 1600 1764 2500 3600 2025 1600 3055 3364 3844* $\sum x = 512; \sum x^2 = 29622; n = 10$ $\bar{x} = \frac{\sum x}{n} = \frac{512}{10} = 51.2$ $s^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{29622}{10} - (51.2)^2 = 70.76$ $\|t\| = \left\|\frac{\bar{x} - \mu_0}{\sqrt{s^2 / n - 1}}\right\| = \left\|\frac{51.2 - 50}{\sqrt{70.76/10 - 1}}\right\| = 0.427$ *The 5% t distribution table value is 2.262.* ***Inference:*** ***Since the calculated value is less than the t distribution table value at L.S with n-1=10-1=9 d.f. So we accept the Ho. It may be conclude that the population mean is equal to 50.***	8	3	3	2

Part – C (1x15 = 15 Marks)

<table>
<tr><th>Q. No</th><th>Questions</th><th>Marks</th><th>BL</th><th>CO</th><th>PO</th></tr>
<tr><td>16 (a)</td><td>In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class.

$P(X < 45) = 0.10 \quad and \quad P(X > 75) = 0.05$
$P\left(-\infty < Z < \frac{45-\mu}{\sigma}\right) = 0.1 \quad and \quad P\left(\frac{75-\mu}{\sigma} < Z < \infty\right) = 0.05$
$P\left(0 < Z < \frac{\mu-45}{\sigma}\right) = 0.4 \quad and \quad P\left(0 < Z < \frac{75-\mu}{\sigma}\right) = 0.45$
From the table, $\frac{\mu-45}{\sigma} = 1.28$ and $\frac{75-\mu}{\sigma} = 1.64$
$\mu - 1.28\sigma = 45$ (1) and $\mu + 1.64\sigma = 75$ (2) Solve (1),(2) $\mu = 58.15,\ \sigma = 10.28$
$P(\text{1st class}) = P(60 < X < 75) = P(0.18 < Z < 1.64)$
$= P(0 < Z < 1.64) - P(0 < Z < 0.18) = 0.4495 - 0.0714 = 0.3781$, % first class = 38
% of second class = 100 – (failed, 1st class, got distinction) = 100 – (10 + 38 + 5) = 47.</td><td>15</td><td>3</td><td>2</td><td>2</td></tr>
<tr><td colspan="6" align="center">(OR)</td></tr>
<tr><td>(b)</td><td>The nicotine contents in two random sample of tobacco is given below:
<table>
<tr><td>Sample I</td><td>21</td><td>24</td><td>25</td><td>26</td><td>27</td><td>-</td></tr>
<tr><td>Sample II</td><td>22</td><td>27</td><td>28</td><td>30</td><td>31</td><td>36</td></tr>
</table>
Can you say that the two samples came from the same normal population?
$n_1 = 5,\ n_2 = 6,\ \bar{x} = 24.6, \bar{y} = 29;\ H_0{:}\mu_1 = \mu_2,\ H_1{:}\mu_1 \neq \mu_2 (two\ tailed)$
d.f.= $n_1 + n_2 - 2 = 9$ at 5% = 2.26;
$\Sigma(x-\bar{x})^2 = 21.2;\ \Sigma(y-\bar{y})^2 = 108;\ s^2 = \frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1+n_2-2} = 14.35$

$t = \frac{\bar{x}-\bar{y}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = -1.92 \Rightarrow |t| = 1.92$, Calculate value t < Tabulated t. H_0 is accepted.

$\hat{\sigma}_1^2 = \frac{5}{4} \times 4.24 = 5.30$ and $v = 4$; $\hat{\sigma}_2^2 = \frac{6}{5} \times 18.0 = 21.60$ and $v = 5$
$H_0{:}\ \hat{\sigma}_1^2 = \hat{\sigma}_2^2;\quad H_1{:}\ \hat{\sigma}_1^2 \neq \hat{\sigma}_2^2$
$F = \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} = \frac{21{\cdot}60}{5{\cdot}30} = 4.07$
$F_{0.05}(5, 4) = 6.26.$
Since $F < F_{0.05}$, H_0 is accepted. Therefore, the variances of the two populations can be regarded as equal.</td><td>15</td><td>4</td><td>2</td><td>2</td></tr>
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