UNIT - I

20/07/2023

SET THEORY

NOTATIONS

U - Universal set

All capitals letters denote set

EX : $A = \{1, 2, 3, 4, 5\}$

$Z = \{0, \pm 1, \pm 2, \cdots\}$

SET OPERATIONS

Union - $\cup$

$A \cup B = \{x / x \in A \text{ or } x \in B\}$

Intersection - $\cap$

$A \cap B = \{x / x \in A \text{ and } x \in B\}$

Subtraction :

$A - B = \{x / x \in A \text{ and } x \notin B\}$

Direct Sum :

$A \oplus B = (A - B) \cap (B - A)$

PROBLEM :

If $A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 6, 7, 8\}$

find (i) $A \cap (B - A)$

(ii) $A \oplus B$

(iii) $(A - B) \cup B$

SOL : (i) $B - A = \{6, 7, 8\}$

$A \cap (B - A) = \phi$

(ii) $A - B = \{1, 4, 5\}$

$B - A = \{6, 7, 8\}$

$A \oplus B = \phi$

CARDINALITY OF A SET 'A'

DEFINITION :

The cardinality of a set A is the number of elements in A.

It is defined by $n(A)$.

EX :

(iv) find $n(A-B)$

(v) find $n(A \oplus B)$

(iv) $n(A-B) = 3$

(v) $n(A \oplus B) = 0$

CARTESIAN PRODUCT OF TWO SETS A & B

Let $A = \{a / a \in A\}$

$B = \{b / b \in B\}$

$A \times B = \{(a,b) / a \in A \text{ and } b \in B\}$

PROBLEM :

If $A = \{a, b, c\}$

$B = \{1, 2\}$

(i) find $A \times B$

(ii) find $n(A \times B)$

(i) $A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

$n(A \times B) = 6$

IMPORTANT LAWS

Let A, B and C be three sets and U denots union set.

1. ASSOCIATIVITY LAW :

(i) $A \cup (B \cup C) = (A \cup B) \cup C$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

2. DISTRIBUTIVE LAW :

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. COMPLEMENT LAW :

(i) $A \cap A^c = \phi$

(ii) $A \cup A^c = U$

4. DEMORGAN'S LAW :

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(A \cap B)^c = A^c \cup B^c$

5. COMMUTATIVE LAW :

(i) $A \cup B = B \cup A$.

(ii) $A \cap B = B \cap A$.

Prove Demorgan's law theoretically

Sol: $(A \cup B)^c = A^c \cap B^c$

$= \{x \mid x \in (A \cup B)^c\}$

$= \{x \mid x \notin (A \cup B)\}$

$= \{x \mid x \notin A \text{ or } x \notin B\}$

$= \{x \mid x \in A^c \cap B^c\} = A^c \cap B^c$

PROBLEM :

If A, B and C are 3 sets: Prove analytically that $A - (B \cap C) = (A - B) \cup (A - C)$

Sol: $A-(B\cap C) = \{x / x \in A-(B\cap C)\}$

$= \{x / x \in A \text{ and } x \notin (B\cap C)\}$

$(A-B)\cup(A-C) = \{x / x \in A \text{ and } x \notin B \text{ or } x \notin C\}$

$= \{x / x \in A \text{ and } x \notin B \text{ or } x \in A \text{ and } x \notin C\}$

$= \{x / x \in (A-B) \text{ or } x \in (A-C)\}$

$= \{x / x \in (A-B)\cup(A-C)\}$

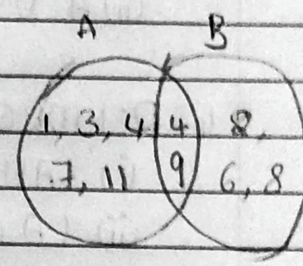
PROBLEM:

Find the sets A and B such that

$A-B = \{1,3,7,11\}$

$B-A = \{2,6,8\}$

$A\cap B = \{4,9\}$



$A = \{1,3,7,11,4,9\}$

$B = \{4,9,2,6,8\}$

Prove that $(A-C)\cap(C-B) = \phi$ analytically

sol: $(A-C)\cap(C-B) = \{x \in A \text{ and } x \in C \text{ and } x \in C \text{ and } x \notin B\}$

$= \{x \in A \text{ and } (x \notin \bar{C} \text{ and } x \in C) \text{ and } x \notin B\}$

$= \{x \in A \text{ and } (x \in \bar{C}\cap C) \text{ and } x \notin B\}$

$= \{x \in A \text{ and } x \in \phi \text{ and } x \notin B\}$

$= \{x \in A \cap \phi \cap B^c\}$

$= \{x / x = \phi\}$

$= \phi$

Prove that $A\times(B\cap C) = (A\times B)\cap(A\times C)$

sol: Now $A\times(B\cap C) = \{(x,y) / x \in A \text{ and } y \in B\cap C\}$

$= \{(x,y) / x \in A \text{ and } (y \in B \text{ and } y \in C)\}$

$= \{(x,y) / (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)\}$

$= \{(x,y) / (x,y) \in A \times B \text{ and } (x,y) \in A \times C\}$
$= (A \times B) \cap (A \times C)$

PROBLEM

Let $A = \{1, 2, 3\}$

Find all possible partitions of A.

sol:
(i) $A_1 = \{1\}$, $A_2 = \{2, 3\}$
(ii) $A_1 = \{2\}$, $A_2 = \{1, 3\}$
(iii) $A_1 = \{3\}$, $A_2 = \{1, 2\}$
(iv) $A_1 = \{1\}$, $A_2 = \{2\}$, $A_3 = \{3\}$

PARTITION OF A SET

Let A be a set

Then the following subset of A.
$A_1, A_2, A_3 \ldots\ldots A_n$ form a partition of A if

(i) $A_1 \cup A_2 \cup \ldots\ldots \cup A_n = A$
(ii) $A_i \cap A_j = \emptyset$ for $i \neq j$

PROBLEM

Let $A = \{a, b, c, d\}$

Find all possible partitions of A.

$A_1 = \{a\}$ $\quad$ $A_2 = \{b, c, d\}$
$A_1 = \{b\}$ $\quad$ $A_2 = \{a, c, d\}$
$A_1 = \{c\}$ $\quad$ $A_2 = \{a, b, d\}$
$A_1 = \{d\}$ $\quad$ $A_2 = \{a, b, c\}$
$A_1 = \{a, b\}$ $\quad$ $A_2 = \{c, d\}$
$A_1 = \{a, c\}$ $\quad$ $A_2 = \{b, d\}$
$A_1 = \{a, d\}$ $\quad$ $A_2 = \{b, c\}$
$A_1 = \emptyset$ $\quad$ $A_2 = \{a, b, c, d\}$

POWERSET OF A SET 'A'.

The power set of a set A is set of all possible subsets of A.

NOTE

Let $n(A) = m$

Then $n(P(A)) = 2^m$

Ex

Let $A = \{1, 2, 3\}$

Find $P(A)$ and also $n(P(A))$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

$2^3 = 8$

MIN SETS

DEFINITION

Let A be a set,

Let B_1, B_2 be two subsets of A.

Then min sets generated by B_1 and B_2 are given by

$A_1 = B_1 \cap B_2^c$

$A_2 = B_1 \cap B_2$

$A_3 = B_2 \cap B_1^c$

$A_4 = B_2^c \cap B_1^c$

PROBLEM

Let $A = \{1, 2, 3\}$

$B_1 = \{1, 2\}$ $B_2 = \{2, 3\}$

Find all min sets generated by B_1 and B_2

Sol:

$A_1 = B_1 \cap B_2^c = \{1, 2\} \cap \{1\} = \{1\}$

$A_2 = B_1^c \cap B_2 = \{3\} \cap \{2, 3\} = \{3\}$

$A_3 = B_1 \cap B_2 = \{1, 2\} \cap \{2, 3\} = \{2\}$

$A_4 = B_1^c \cap B_2^c = \{3\} \cap \{1\} = \phi$

PROBLEM

Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Let $B_1 = \{5, 6, 7\}$

$B_2 = \{2, 4, 5, 9\}$

$B_3 = \{3, 4, 5, 6, 8, 9\}$

Find all min sets generated by B_1, B_2 and B_3.

sol:

$A_1 = B_1 \cap B_2 \cap B_3$

$A_2 = B_1^c \cap B_2 \cap B_3$

$A_3 = B_1 \cap B_2^c \cap B_3$

$A_4 = B_1 \cap B_2 \cap B_3^c$

$A_5 = B_1^c \cap B_2^c \cap B_3$

$A_6 = B_1^c \cap B_2 \cap B_3^c$

$A_7 = B_1 \cap B_2^c \cap B_3^c$

$A_8 = B_1^c \cap B_2^c \cap B_3^c$

$B_1^c = \{1, 2, 3, 4, 8, 9\}$

$B_2^c = \{1, 3, 6, 7, 8\}$

$B_3^c = \{1, 2, 7\}$

$A_1 = \{5, 6, 7\} \cap \{2, 4, 5, 9\} \cap \{3, 4, 5, 6, 8, 9\}$
$= \{5\}$

$A_2 = \{1, 2, 3, 4, 8, 9\} \cap \{2, 4, 5, 9\} \cap \{3, 4, 5, 6, 8, 9\}$
$= \{4, 9\}$

$A_3 = \{5, 6, 7\} \cap \{1, 3, 6, 7, 8\} \cap \{3, 4, 5, 6, 8, 9\}$
$= \{6\}$

$A_4 = \{5, 6, 7\} \cap \{2, 4, 5, 9\} \cap \{1, 2, 7\}$
$= \emptyset$

$A_5 = \{1, 2, 3, 4, 8, 9\} \cap \{1, 3, 6, 7, 8\} \cap \{3, 4, 5, 6, 8, 9\}$
$= \{3, 8\}$

$A_6 = \{1, 2, 3, 4, 8, 9\} \cap \{2, 4, 5, 9\} \cap \{1, 2, 7\}$
$= \{2\}$

$A_7 = \{5, 6, 7\} \cap \{1, 3, 6, 7, 8\} \cap \{1, 2, 7\}$
$= \{7\}$

$A_8 = \{1, 2, 3, 4, 8, 9\} \cap \{1, 3, 6, 7, 8\} \cap \{1, 2, 7\}$
$= \{1\}$

__/__/__

RELATION

DEFINITION

Let A and B be two sets. A subset R of cartesian product $A \times B$ is called a relation satisfying some property.

EXAMPLE :

Let $A = \{1, 2, 3\}$

$B = \{2, 4, 6\}$

R is a relation defined by (a,b) is an element of R such that $a+b$ is always even

$R = \{(2,2), (2,4), (2,6)\}$

PROPERTIES OF RELATION.

1. REFLEXIVE

Let R be a relation defined on A or from A to A.

A relation R is said to be reflexive if $(a,a) \in R$ for all $a \in A$.

NOTE : $(a,b) \in R$

a is related to b.

2. SYMMETRIC

A relation R is said to be symmetric on A if $(a,b) \in R$ implies $(b,a) \in R$

3. TRANSITIVE

A relation R is said to be transitive if $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

DEFINITION:

EQUIVALENCE RELATION

A relation R defined on A is said to be equivalent relation if it satisfies

(i) Reflexive

(ii) Symmetric

(iii) Transitive

PROBLEM:

A relation R is defined on a set of parallel lines.

Prove that R is an equivalence relation. if $(l, m) \in R$ such that l is parallel to m.

sol. Let L = set of all parallel lines.

Define R such that

$(l, m) \in R$ such that l is parallel to m.

REFLEXIVE

Let $l, m \in L$

and let $(l, m) \in R$.

then l is parallel to m.

$\Rightarrow$ m is parallel to l

$\Rightarrow (m, l) \in R$.

R is symmetric.

REFLEXIVE.

Let $l, m \in L$.

such $(l, m) \in R$ (i.e.,) l is parallel to m.

Now $(l, l) \in R$

$\Rightarrow$ R is reflexive

TRANSIVE

Let $l, m, n \in L$

such that $(l, m) \in R$ and $(m, n) \in R$

l is parallel to m and m is parallel to n

l is parallel to n.

$(l, n) \in R$

R is an equivalence relation.

PROBLEM :

A relation R is defined on $A = \{2, 4, 6, 8\}$ such that $(a, b) \in R$ if $a+b$ is always even. Prove that R is an equivalence relation.

Sol. $R = \{(2,2), (2,4), (2,6), (2,8), (4,2), (4,4), (4,6),$
$(4,8), (6,2), (6,4), (6,6), (6,8), (8,2), (8,4),$
$(8,6), (8,8)\}$

REFLEXIVE

let $a \in A$.

Then $(a, a) \in R$ for all $a \in A$.

SYMMETRIC

Let $(a, b) \in R$

Then, $a+b$ is even

$\Rightarrow b+a$ is also even.

$(b, a) \in R$

TRANSITIVE

Let $(a, b) \in R$

$(b, c) \in R$

$a+b$ is even, $b+c$ is even

$a+b = 2m$ — (1) $\quad b+c = 2n$ — (2)

(1) + (2) gives,

$a + 2b + c = 2(m+n)$

$a + c = 2(m+n) - 2b$

$a + c = 2(m+n-b)$

$a+c$ is even

$(a, c) \in R$

$(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

$\therefore$ R is an equivalence Relation.

NOTE : congruency

$a \equiv b \pmod{m}$

$a \equiv b \pmod{m}$

$a-b$ is divisible by m.

$5 \equiv 1 \pmod{2}$

PROBLEM :

If R is a relation on set of integers. such that prove that $(a,b) \in R$ if $a \equiv b \pmod{m}$ then prove that R is an equivalence relation,

sol: REFLEXIVE :

$a \equiv a \pmod{m}$, since 0 is divisible by m

$(a,a) \in R$

SYMMETRIC

Let $(a,b) \in R$

Then $a \equiv b \pmod{m}$

$\Rightarrow a-b$ is divisible by m.

$b-a$ is also divisible by m

$b \equiv a \pmod{m}$

R is symmetric

TRANSITIVITY :

Let $(a,b) \in R$ and $(b, c) \in R$

Then $a \equiv b \pmod{m}$ $b \equiv c \pmod{m}$

$a-b = k_1 m$ $\quad\quad$ $b_1 - c = k_2 m$

$a-b+b-c = (k_1+k_2)m$

$a-c = (k_1+k_2)m$

$\Rightarrow a \equiv c \pmod{m}$

R is an equivalence relation.

PROBLEM:

If R is a relation defined on N×N where $N = \{1, 2, 3 \ldots\}$ and $((a,b),(c,d)) \in R$ if $ad = bc$.

Prove that R is an equivalence relation.

Sol. REFLEXIVE

Let $(a,b) \in N \times N$

Now we can say $((a,b),(a,b)) \in R$

since $ab = ba$

R is reflexive

SYMMETRIC

Let $((a,b),(c,d)) \in R$

$\Rightarrow ad = bc$

$\Rightarrow bc = ad$

$\Rightarrow ((c,d),(a,b)) \in R$

R is symmetric.

TRANSITIVITY:

Let $((a,b),(c,d)) \in R$

and $((c,d),(e,f)) \in R$

Then $ad = bc \rightarrow (1)$

$cf = de \rightarrow (2)$

$de = cf \rightarrow (2)$

$\frac{(2)}{(1)} = \frac{e}{a} = \frac{f}{b} \Rightarrow af = be$

$((a,b),(e,f)) \in R$

R is ~~reflt~~ transitive

R is an equivalence relation.

PROBLEM :

If R is a relation defined on set of integers such that $(a,b) \in R$ if $3a+4b = 7n$ then prove that R is an equivalence relation.

sol: REFLEXIVE

$3a+4a = 7a$

$\Rightarrow (a,a) \in R$

R is reflexive

SYMMETRIC

Let $(a,b) \in R$ $\quad 3a+4b = 7n$ —— (i)

$7b-3b+7a-4a = 7n$

$7(b+a-n) = 3b+4a$

$3b+4a = 7(k)$

$(b,a) \in R$

TRANSITIVITY

Let $(a,b) \in R$ and $(b,c) \in R$

$3a+4b = 7k_1$ —— (i)

$3b+4c = 7k_2$ —— (ii)

Adding (i) and (ii)

$3a+4c+7b = 7(k_1+k_2)$

$3a+4c = 7(k_1+k_2-b)$

$(a,c) \in R$

$\therefore$ R is transitive

$\therefore$ R is an equivalence relation.

DEFINITION.

Domain and Range

Let R be a relation from A to B

$R = \{(a,b)\} \; a \in A$ and $b \in B\}$

Domain $= \{a \in A\}$

Range =

PROBLEM:

Let R be a relation defined on $A = \{1,2,3\}$ and $R = \{(1,2),(2,3)\}$

Domain $= \{1,2\}$

Range $= \{2,3\}$

DEFINITION.

INVERSE RELATION R^{-1}

Let $R = \{(a,b) / a \in A$ and $b \in B\}$

$R^{-1} = \{(b,a) / (a,b) \in R\}$

PROBLEM

Let R be a relation defined on $A = \{1,2,3\}$ and $R = \{(1,2),(2,3)\}$. Find R^{-1}.

Sol. $R^{-1} = \{(2,1),(3,2)\}$

COMPOSITION OF TWO RELATIONS.

Let R be a relation from A to B.

Let S be a relation from B to C.

Then composition of R and S is given by

$R \circ S = \{(a,c) / \exists$ some $b \in B$ for $(a,b) \in R$ and $(b,c) \in S\}$

PROBLEM:

Let R and S be two relations defined

_/_/_

on $A = \{1, 2, 3, 4\}$

Let $R = \{(1,1), (1,3), (3,2), (3,4), (4,2)\}$

Let $S = \{(2,1), (3,3), (3,4), (4,1)\}$

Find (i) $R \circ S$ (ii) $S \circ R$ (iii) $R \circ R$

Sol: $R \circ S = \{(1,3), (1,4)\ (3,1), (4,1)\}$

$S \circ R = \{(2,1)\ (2,3)\ (3,2)\ (3,4)\ (4,1)\ (4,3)\}$

$R \circ R = \{(1,1)\ (1,3), (1,2)\ (1,4)\ (3,2)\}$.

PROBLEM

Let R and S be two relations defined on $A = \{1, 2, 3, 4\}$

Such that (i) $(a,b) \in R$ if $a+b$ is even

(ii) $(a,b) \in S$ if $a+b$ is odd.

Then find (i) Domain of R

(ii) Domain of S

(iii) $R \circ S$

(iv) Range of R

(v) Range of S.

$R = \{(1,1), (1,3), (2,2)\ (2,4), (3,1), (3,3), (4,2), (4,4)\}$

$S = \{(1,2), (1,4), (2,1)\ (2,3)\ (3,2), (3,4)\ (4,1), (4,3)\}$

Domain of $R = D(R) = A$

Domain of $S = D(S) = A$

Range of $R = A$

Range of $S = A$.

$S \circ R = \{(2,1)\ (2,3)\ (3,2)\ (3,4)\ (4,1)\ (4,3)\}$

$R \circ R = \{(1,1)\ (1,3)\ (1,2)\ (1,4)\ (3,2)\}$

MATRIX REPRESENTATION OF A RELATION.

Let R be a relation defined from A to B

Then R can be represented by a matrix

$M_R = (m_{ij})$

__/__/__

where $m_{ij} = \begin{cases} 1 & \text{if } (a,b) \in R \\ 0 & \text{if } (a,b) \notin R \end{cases}$

PROBLEM

Let $A = \{0,1,2,3\}$

$B = \{2,6,7\}$

Define R by $(a,b) \in R$ if $a+b$ is Prime

Sol:

$$M_R = \begin{array}{c|c} & \begin{matrix} 2 & 6 & 7 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{(0,2),(0,7),(1,2),(1,6),(3,2)\}$

NOTE:

$M_{R^{-1}}$ - Matrix representation of R^{-1}

$$\boxed{M_{R^{-1}} = M_R^T}$$

OPERATIONS DEFINED ON M_R

V - OPERATION (OR OPERATION)

Let R and S be two relations defined on A. Then the matrix representation ~~RO~~ $R \cup S$ is the join of M_R and M_S obtained by Putting "1" in the positions where either M_R or M_S has a "1" and it is denoted by

$M_{R \cup S} = M_R \vee M_S$.

$\wedge$ - OPERATION (AND Operation)

The matrix representing $R \cap S$ is the meet of M_R and M_S obtained by putting 1 in

_/_/_

the positions where both M_R and M_S have a "1" and is denoted by

$$\boxed{M_{R\cap S} = M_R \wedge M_S}$$

CLOSURES

REFLEXIVE CLOSURE

Let R be a relation on A.

Let $\Delta_R = \{(a,a) / a \in A\}$

The smallest relation R' that contains Δ_R is called reflexive closure.

SYMMETRIC CLOSURE

Let R be a relation on A

Let $\Delta_S = \{(a,b) / (b,a) \in R\}$

The smallest relation R' that contains Δ_S is called symmetric closure

TRANSITIVE CLOSURE

Let $\Delta_T = \{(a,c) / (a,b), (b,c) \in R\}$

The smallest relation R' that contains Δ_T is called the transitive closure

Let R be a relation defined on $A = \{a,b,c\}$

PROBLEM

Let $R = \{(a,a), (a,b), (b,c)\}$ be a relation. Find its reflexive closure R'.

Sol: $R' = \{(a,a), (a,b), (b,c)\} \cup \{(b,b), (c,c)\}$

PROBLEM

Let R be a relation on $A = \{a,b,c\}$.

Let $R = \{(a,a), (a,b), (b,c)\}$. Find symmetric clouser R'.

__/__/__

Sol $R' = \{(a,a), (a,b), (b,c)\} \cup \{(b,a), (c,b)\}$

Find the transitive closure R'

$R' = \{(a,a), (a,b), (b,c)\} \cup \{(a,c)\}$

PROBLEM

Let R be a relation on $A = \{a,b,c\}$

Let $R = \{(a,b) (b,c), (c,a)\}$. Find the transitive closure

Sol: $R' = \{(a,b), (b,c), (c,a)\}$

$\cup \{(a,c), (b,a), (c,b)\}$

WARSHALL'S ALGORITHM FOR FINDING TRANSITIVE CLOSURE

If $|A| = n$, then the matrix representation of transitive closure is W_n.

Let $\boxed{W_0 = M_R}$

Step 1: First transfer to W_k all 1's in W_{k-1}

Step 2: List the locations P_1, P_2 in column k of W_{k-1} when the entry is "1" and locations q_1, q_2 ---- in row k of W_{k-1} when entry is "1".

PROBLEM :

Using warshall's algorithm for finding transitive closure of $R = \{(a,b) (b,c), (c,a)\}$ which is defined on $A = \{a,b,c\}$

$$M_R = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

__/__/__

$$W_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

	In W_{k-1}		Positions of 1's in W_K	W_K
K	positions of 1's in K^{th} column	Positions of 1's in k^{th} row		
1	3	2	(3, 2)	$W_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
2	1, 3	3	(1,3) (3,3)	$W_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
3	1, 2, 3	1, 2, 3	(1,1) (1,2) (1,3) (2,1) (2,2) (2,3) (3,1) (3,2) (3,3)	$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

PROBLEM:

Let $R = \{(a,a), (a,b) (b,c)\}$ be a relation on $A = \{a, b, c\}$. Find the transitive clousre of R

$$M_R = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad W_0 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

__/__/__

K	In W_{k-1} Positions of 1's in k^{th} column	Positions of 1's in k^{th} row.	Positions of 1's in W_k	W_k
1	1	1, 2	(1, 1) (1, 2)	$W_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
2	1	3	(1, 3)	$W_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
3.	1, 2	-	-	$W_3 = W_2$

$R' = \{(a,a), (a,b), (a,c), (b,c)\}$

PROBLEM

Let R be a relation defined on $A = \{1, 2, 3\}$ such that $(a,b) \in R$ if $a+b$ is even. Find the transitive closure of R. using warshall's algorithm.

sol:

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \qquad W_0 = M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

K	In W_{k-1} Positions of 1's in k^{th} column	Positions of 1's in k^{th} row	Positions of 1's in W_k	W_k
1.	1, 3	1, 3	(1, 1) (1, 3) (3, 1) (3, 3)	$W_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

_/_/_

2	2	-	(2, 2)	$W_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
3	1, 3	1, 3	(1,1), (1,3), (3,1), (3,3)	$W_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$\boxed{R' = R}$$

$R' = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

PARTIALLY ORDERED SET (Poset)

Let R be a relation defined on A Then R is said to be a poset

if (i) R is reflexive

(ii) R is antisymmetric (not symmetric)

(iii) R is transitive

PROBLEM

Let R be a relation defined on $A = \{0, 1, 2, 3\}$ such that ~~(a,b) ∈ R if a≤b~~. $(S_1, S_2) \in R$ if $S_1 \subset S_2$ where S_i's are subsets of A and P(A) ($S_i \in P(A)$)

Prove that R is a Poset.

sol. $S_i \subseteq S_i \ \forall i$

$(S_i, S_i) \in R$

R is reflexing

Let $(S_i, S_j) \in R$

Then $S_i \subset S_j$

$\Rightarrow S_j \not\subset S_i$

R is not symmetric

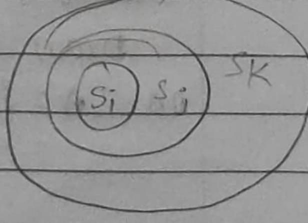
Let $(S_i, S_j) \in R$

$(S_j, S_k) \in R$

$S_i \subset S_j,\ S_j \subset S_k$

$\Rightarrow S_i \subset S_k$

$\Rightarrow (S_i, S_k) \in R.$



GRAPHICAL REPRESENTATION OF A RELATION:

Let R be a relation defined on A

To represent R graphically,

each element of A can be considered as a point when an element a is related to b. an arc is drawn from a to b. with an arrow mark. This graph is called a digraph

HASSE DIAGRAM.

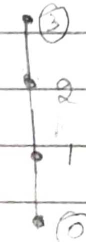
Hasse diagram representing a partial ordering can be obtained from the diagraph by removing all loops that are present due to transitivity and drawing each edge without arrow so that in initial vertex is below its end vertex.

PROBLEM

Draw a diagraph for the following relation R. defined on $A = \{0,1,2,3\}$ $(a,b) \in R$ if $a<b$

$$M_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

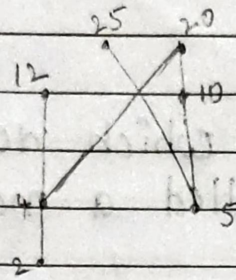
HASSE DIAGRAM

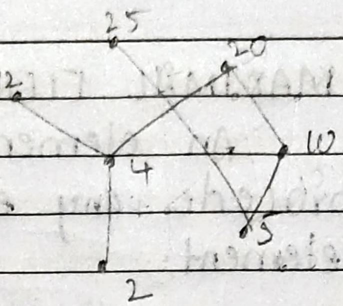


Let $R = \{(0,1),(0,2),(0,3),(1,2),(1,3),(2,3)\}$

PROBLEM

Draw a Hasse diagram for a relation R defined on $A = \{2,4,5,10,12,20,25\}$ $(a,b) \in R$ if a divides b



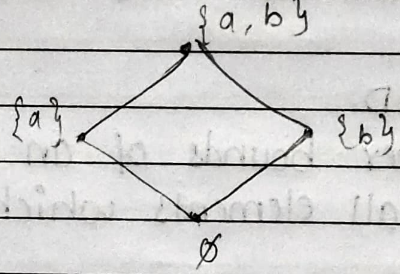


PROBLEM

Draw a Hasse Diagram for a relation R defined on P(A), where $A = \{a, b\}$

$(S_i, S_j) \in R$ if $S_i \subset S_j$ where $S_i \in P(A)$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

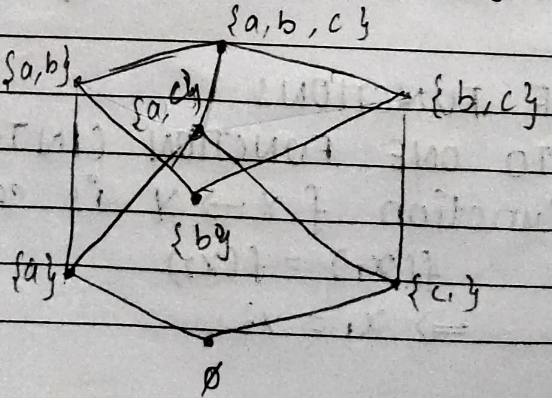


4 PROBLEM

Draw a Hasse Diagram for a relation R defined on P(A) where $A = \{a, b, c\}$

$(S_i, S_j) \in R$ if $S_i \subset S_j$, where $S_i \in P(A)$

$\Rightarrow$ $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$.



_/_/_

MAXIMAL ELEMENT

An element of a set which does not procede any element is called a maximal element.

MINIMAL ELEMENT

An element of a set which does not succeed any element is called a minimal element.

UPPER BOUND

The upper bounds of an element "a" is set of all elements which precedes "a".

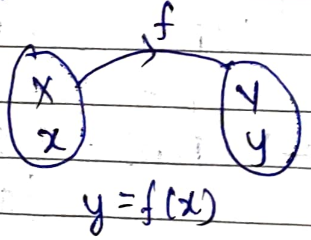
LOWER BOUND.

The lower bounds of an element "a" is the set of all elements which succeeds "a"

FUNCTIONS

A relation f from X to Y is called a function. If for every $x \in X$, there exists a $y \in Y$ such that $y = f(x)$

$f : X \longrightarrow Y$



$y = f(x)$

TYPES OF FUNCTIONS

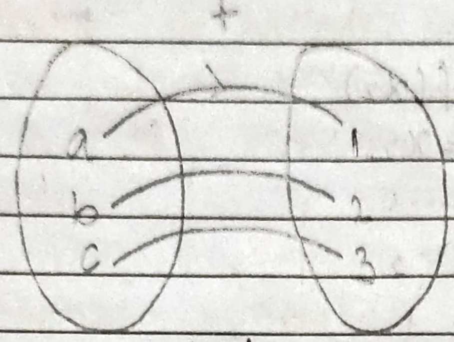
1. ONE TO ONE FUNCTION (INJECTIVE)

A function $f \; X \to Y$ is called 1-1 if,

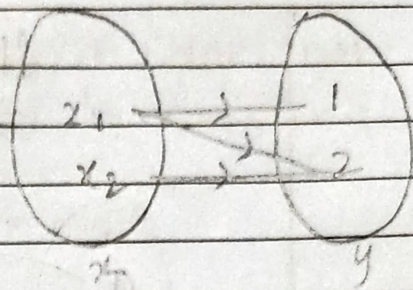
$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

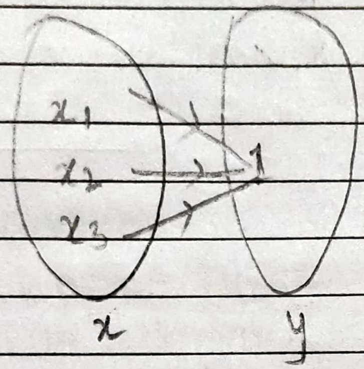
___/___/___



1-1



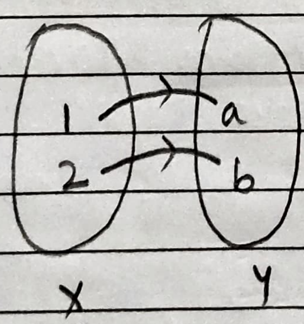
not 1-1



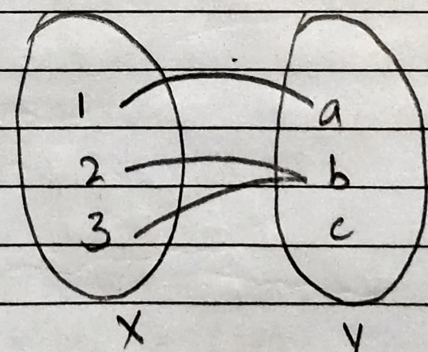
not 1-1

ONTO FUNCTION (surjective)

A function of $X \longrightarrow Y$ is called onto for every $y \in Y$ there exists $x \in X$ such that $y = f(x)$



onto



not onto

DEFINITION

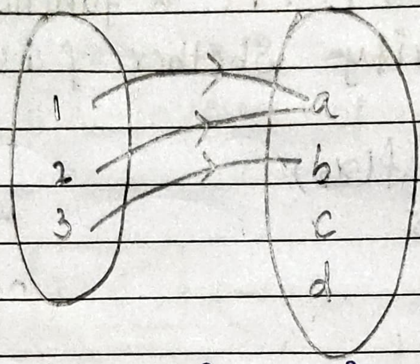
DOMAIN AND RANGE OF A FUNCTION

Let $f: X \longrightarrow Y$ be a function.

Domain of f is X

_/_/_

Range of f = $\{ y / y = f(x) \in Y \}$



Domain = $\{1, 2, 3\}$
Range = $\{a, b\}$

PROBLEM.

Let $f: Z^+ \longrightarrow Z^+$ be a function defined by $f(x) = 3x$ verify whether it is bijection.

sol: Let $f(x_1) = f(x_2)$

$3x_1 = 3x_2$

$3(x_1 - x_2) = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

f is 1-1.

Let $y \in Z^+$

Then $y = 3x$

$x = y/3$

For $y \in Z^+$ we can not find $x \in Z^+$ such that $y = 3x$.

[If $y = 2$, $x = \frac{2}{3} \notin Z^+$]

f is not onto.

It is not a bijection.

__/__/__

PROBLEM:

If $f: \mathbb{Z} \to \mathbb{Z}$ is a function given by $f(x) = x + 5$. verify whether f is bijection.

Let $f(x_1) = f(x_2)$

$x_1 + 5 = x_2 + 5$

$\boxed{x_1 = x_2}$

f is 1-1

Let $y \in \mathbb{Z}$

Then $y = x + 5$

$x = y - 5 \in \mathbb{Z}$

$\boxed{f \text{ is onto}}$

f is a bijection.

IDENTITY FUNCTION

Let $f: A \to A$ be a function Then $f(a) = a \; \forall a$ is called the identity function.

Usually it is denoted by I_A.

$\boxed{I_A(a) = a}$

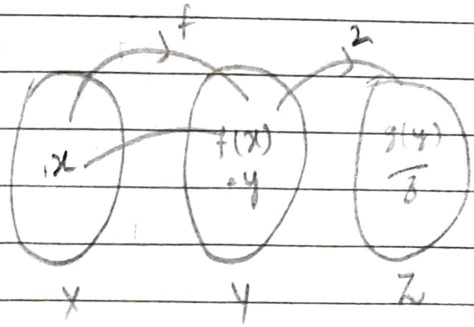
COMPOSITION OF TWO FUNCTIONS.

Let $f: X \to Y$ be a function.

and $g: Y \to Z$ be another function.

Then $(g \circ f)(x)$ is called the composition of f and g

$(g \circ f)(x) = g(f(x))$



Ex: $\sin(\log x)$

_ / _ / _

INVERSE OF A FUNCTION

If $f: A \longrightarrow B$ and $g: B \longrightarrow A$ then g is called the inverse of f if $gof = I_A$

$fog = I_B$

NOTE : $\boxed{g = f^{-1}}$

PROPERTIES OF FUNCTION

① Composition is associative

(or)

Let $f: A \rightarrow B$ and $g: B \rightarrow C$, $h: C \rightarrow D$

Then $ho(gof) = (hog)of$.

Sol: Given. $f: A \rightarrow B$

$\exists b$ in B such that $b = f(a)$

$g: B \rightarrow C$

$\exists c$ in C such that $c = g(b)$

$h: C \rightarrow D$

$\exists d$ in D such that $h(c) = d$.

$(gof)(a) = g(f(a))$

$= g(b) = c$

$(ho(gof))(a)$

$= h((gof)(a))$

$= h(c)$

$= d$.

$(ho(gof))(a) = d$ — ①

NOW

$((hog)of)(a)$

$= ((hog)(f(a)))$

$= h(g(h))$

$= h(c)$

$(h \circ (g \circ f))(a) = d$ —— ②

PROPERTY 2

If $f: A \rightarrow B$ and $g: B \rightarrow C$ are bijective then $g \circ f: A \rightarrow C$ is also bijective

Sol

As $f: A \rightarrow B$ is 1-1

$f(a_1) = f(a_2)$

$\Rightarrow a_1 = a_2$

As $g: B \rightarrow C$ is 1-1

$g(b_1) = g(b_2)$

$\Rightarrow b_1 = b_2$.

To Prove $g \circ f: A \rightarrow C$ is 1-1

Let $(g \circ f)(a_1) = (g \circ f)(a_2)$

$g(f(a_1)) = g(f(a_2))$.

$\Rightarrow f(a_1) = f(a_2)$ ($\because g$ is 1-1)

$\Rightarrow \boxed{a_1 = a_2}$ ($\because f$ is 1-1

$g \circ f$ is 1-1

As $g: B \rightarrow C$ is onto. then $c \in C$ such that $\boxed{g(b) = c}$

As $f: A \rightarrow B$ is onto.,

$\exists$ $b \in B$ such that $b = f(a)$

Now For $c \in C$,

$g(b) = c$

$g(f(a)) = c$

$(g \circ f)(a) = c$

$\Rightarrow g \circ f$ is onto

$g \circ f$ is bijective.

/ /

PROPERTY 3 :

The necessary and sufficient condition for the function $f : A \to B$ to be invertible is that f is bijective

PROOF

Assume that

$f : A \to B$ is invertible

By definition of invertible, function there existing $g : B \to A$ such that

$gof = I_A$ and $fog = I_B$

Let $a_1, a_2 \in A$ such that

$f(a_1) = f(a_2)$

As $g : B \to A$ is a function.

$g(f(a_1)) = g(f(a_2))$

$(gof)(a_1) = (gof)(a_2)$

$I_A(a_1) = I_A(a_2)$

$a_1 = a_2 \Rightarrow \boxed{f \text{ is } 1\text{-}1}$

Let $b \in B$ then $g(b) \in A$

Now $b = I_B(b)$

$b = (fog)(b) = f(g(b)) \Rightarrow \boxed{f \text{ is onto}}$

$\therefore$ f is a bijection.

THEOREM

If $f : A \to B$ and $g : B \to C$ are invertible functions, then $gof : A \to C$ is also invertible and $(gof)^{-1} = f^{-1} o g^{-1}$

Sol :- As $f : A \to B$ is invertible.

f is both 1-1 and onto

As $g : B \to C$ is invertible

g is both 1-1 and onto

$\Rightarrow gof$ is both 1-1 and onto
$\Rightarrow gof$ is invertible
$gof : A \to C$ is invertible

Now

$g^{-1} : C \to B$, $f^{-1} : B \to A$
$f^{-1} o g^{-1} : C \to A$.
$(gof)^{-1}$ and $f^{-1} o g^{-1}$ are functions
from $C \to A$.
for any $a \in A$, $b = f(a)$
for any $c \in C$, $c = g(b)$
$(gof)(a) = g(f(a))$
$= g(b)$
$= c$.

$(gof)(a) = c$
$(gof)^{-1}(c) = a$ — ①
$(f^{-1} o g^{-1})(c) = f^{-1}(g^{-1}(c))$
$= f^{-1}(b)$
$= a$

$(f^{-1} o g^{-1})(c) = a$ — ②
from ① and ②
$\boxed{(gof)^{-1} = f^{-1} o g^{-1}}$

PROBLEM.

Determine whether each of the following function is an injection and/or surjection or both

(a) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x^3 + x$
(b) $f : Z^+ \to Z^+$ defined by $f(x) = x^2 + 2$.

Sol: (a) Let $f(x_1) = f(x_2)$

$3x_1^3 + x_1 = 3x_2^3 + x_2$

$3(x_1^3 - x_2^3) + (x_1 - x_2) = 0$

$(x_1 - x_2)[3(x_1^2 + x_1x_2 + x_2^2) + 1] = 0$

$\Rightarrow 3(x_1^2 + x_1x_2 + x_2^2) + 1 \neq 0$

$x_1 - x_2 = 0$

$\boxed{x_1 = x_2}$

f is injective

Let $y \in R$

$y = 3x^3 + x$

$\Rightarrow$ one of the root of the above equation is a real $x \in R$

For $y \in R$, $\exists x \in R$ such

$y = 3x^3 + x$

f is subjective.

(b) Let $f(x_1) = f(x_2)$

$x_1^2 + 2 = x_2^2 + 2$

$x_1^2 \pm x_2^2$

$(x_1^2 - x_2^2) = 0$

$(x_1 - x_2)(x_1 + x_2) = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

f is injective

Let $y \in Z^+$

$y = x^2 + 2$

$x^2 = y - 2$

$x = \sqrt{y-2}$

For $y = 5 \in Z^+$

$x = \sqrt{5-2} = \sqrt{3} \notin Z^+$

For $y \in Z^+$, $x \in Z^+$ such that

$\boxed{y = x^2 + 2}$

$f(x)$ is not subjective

TO COMPUTE NO. OF FUNCTIONS.

Let $f: A \to B$ be a function such that $|A| = m$ and $|B| = n$

The total number of functions $= n^m$

NO OF 1-1 FUNCTIONS

If $m > n$, no. 1-1 function is possible

If $m \leq n$ no. of 1-1 functions

$= n(n-1)(n-2)(n-3) \cdots (n-(n-1))$

NO OF onto FUNCTIONS.

If $m < n$, no. onto function is possible.

If $m \geq n$, no onto functions

$$= \sum_{r=0}^{n-1} (-1)^r \, {}^nC_{n-r} (n-r)^m$$

m = cardinality of domain

n = cardinality of co-domain

PROBLEM

If $f: A \to B$ be a function where $|A| = 4$ and $|B| = 3$

(1) Find no. of functions

(2) Find no. of onto functions

Sol. $m = 4$ $n = 3$

No. of functions $= 3^4 = 81$

No. of onto functions.

$$= \sum_{r=0}^{2} (-1)^r \, {}^3C_{3-r} (3-r)^4$$

_/_/_

$= {}^3C_3 \times 3^4 + (-1)\, {}^3C_2\, 2^4 + (-1)^2\, {}^3C_1 (1)^4$

$= 81 - 3 \times 16 + 3$

$= 36.$

PROBLEM

If $f: Z \to N$ is defined by

$$f(x) = \begin{cases} 2x-1 & \text{if } x > 0 \\ -2x & \text{if } x \leq 0. \end{cases}$$

(i) prove that f is a bijection

(ii) find f^{-1}

Sol: Let $x_1, x_2 \in Z$ and let $f(x_1) = f(x_2)$

Case (i)

$f(x_1)$ and $f(x_2)$ are odd

$f(x_1) = f(x_2)$

$2x_1 - 1 = 2x_2 - 1$

f is 1-1

$x_1 = x_2$

Case ii

$f(x_1)$ and $f(x_2)$ are even

$f(x_1) = f(x_2)$

$-2x_1 = -2x_2$

$x_1 = x_2$

f is 1-1

To prove f is onto

Let $y \in N$ and y is odd

$2x - 1 = y$

$x = \dfrac{y+1}{2} \in Z$

Let $y \in N$ and y is even

$$y = -2x$$
$$x = -\frac{y}{2} \in \mathbb{Z}$$

For $y \in N$ $\exists\, x \in \mathbb{Z}$

$$y = \begin{cases} 2x-1 & \text{if } x > 0 \\ -2x & \text{if } x = 0 \end{cases}$$

f is onto.

(iii) $$y = \begin{cases} 2x-1 & \text{if } x > 0 \\ -2x & \text{if } x \le 0 \end{cases}$$

$2x - 1 = y$ for $x > 0$

$2x = y + 1$

$x = \frac{y+1}{2}$

$-2x = y$ for $x \le 0$

$\boxed{x = -\frac{y}{2}}$

$\cancel{-2x = y}$ ~~for~~

UNIT - II

10/08/23

PERMUTATIONS

PERMUTATIONS

An ordered arrangement of r elements of a set containing n distinct elements is called an r-permutation of n elements.

It is denoted by ^nP_r (or) $P(n,r)$

$$P(n,r) = \frac{n!}{(n-r)!}$$

EX:

Out of 3 elements 2 elements can be Permuted in $\frac{3!}{(3-2)!} = \frac{6}{1} = 6$

COMBINATIONS

An unordered arrangements of r-elements of a set containing n distinct elements is called r-combination of n elements. It is denoted by $C(n,r)$ or ^nC_r

$$C(n,r) = \frac{n!}{n!(n-r)!}$$

Out of 3 elements 2 can be combined in $\frac{3!}{2!1!} = \frac{6}{2} = 3.$

THEOREM.

If there are n different elements, no. of permutations of n elements is $n!$.

$(P(n,n) = n!)$

THEOREM

There no. of different Permutations of

/ /

n objects which include n_1 identical objects of Type 1, n_2 identical objects of Type 2 ... n_k identical objects of Type k is

$$\frac{n!}{n_1! \times n_2! \times \cdots \times n_k!}$$

PROBLEM

How many permutations of letters A, B, C, D, E are there?

Sol. No. of Permutations = 5!

PROBLEM

How many Permutations of letters A, B, C, A, F, C are there

Sol No. of Permutations = $\frac{6!}{2!1!2!1!} = 180$

PROBLEM

How many permutation of the letters A, B, C, D, E, F, G containing the string BCD.

Sol Treat BCD as a single letter

We have 5 elements.

BCD, A, E, F, G

No. of permutations = 5!

PROBLEM

How many positive integer n can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if n has to exceed 5,00,000.

PROBLEM

Assuming that repetitions are not.

(a) Permitted how many four digit numbers can be formed from six digits 1, 2, 3, 5, 7, 8 ?

(b) How many of these numbers are less than 4000?

(c) How many of these numbers are even?

(d) How many of these numbers are odd?

Sol: (a) Forming 4 digit number using ~~sig~~ six given digits. is nothing but 4-Permutation of 6 objects.

No. of numbers $= P(6,4) = \frac{6!}{2!} = 360.$

$P(n,r) = \frac{n!}{(n-r)!}$

(b) The number should start with 3 or 2 or 1

The remaining 3 places can be filled using 1, 2, 5, 7, 8.

This can be done in $P(5,3)$

No. of numbers $= 3 \times P(5,3)$

$= 3 \times 60.$

$= 180.$

(c) The number should end with 2 or 8

The remaining 3 places can be filled. 5 digits.

No. of numbers $= 2 \times P(5,3) = 2 \times 60$

$= 120.$

(d) The number should end with 1, 3, 5, 7.

The remaining 3 places can be filled with 5 digits

No. of numbers $= 4 \times P(5,3) = 4 \times 60$

$= 240.$

PROBLEM

How many bit strings of length 10 contain

(a) Exactly four 1's

(b) atmost four 1's

(c) atleast four 1's

(d) an equal number of 0's and 1's ?

Sol:

(a) A bit string of length 10 can be considered to have 10 positions.

No. of strings $= C(10,4) = \frac{10!}{4!\,6!} = 210.$

(b) 10 positions should be filled with no. 1 or one 1 and 9 zeros.

(or) two 1's and eight zero's (or) 3 one's and 7 zeros (or) 4 one's and 6 zero's.

No. of ways $= {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 1 + 10 + 45 + 120 + 210$

$= 3$

(c) No. of ways $= {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8$

$+ {}^{10}C_9 + {}^{10}C_{10}$

$= 848.$

(d) No. of ways $= {}^{10}C_5 = 252$

NOTE:

Let $x_1 + x_2 + \cdots x_n = r$

then the no. of non-negative solutions is $C(n+r-1, r)$ $x_i \geq 0$.

/ /

PROBLEM

Determine the number of non negative solution of the equation.

$$x_1 + x_2 + x_3 + x_4 = 32$$

$$r = 32 \quad n = 4$$

$$C(4+32-1, 32) = C(35, 32) = \frac{35!}{32!\,3!}$$

$$= \frac{\cancel{33}^{11} \times \cancel{34}^{17} \times 35}{\cancel{3} \times \cancel{2}}$$

$$= 6545$$

PROBLEM.

Solve $x_1 + x_2 + x_3 + x_4 = 32$ subject to the condition that $x_1, x_2 \geq 5$ and $x_3, x_4 \geq 7$

$$x_1 \geq 5 \quad x_2 \geq 5 \quad x_3 \geq 7 \quad x_4 \geq 7$$

$$x_1 - 5 \geq 0 \quad x_2 - 5 \geq 0 \quad x_3 - 7 \geq 0 \quad x_4 - 7 \geq 0$$

Let $u_1 = x_1 - 5 \quad u_2 = x_2 - 5 \quad u_3 = x_3 - 7 \quad u_4 = x_4 - 7$

$$x_1 - 5 + x_2 - 5 + x_3 - 7 + x_4 - 7 = 32 - 5 - 5 - 7 - 7$$

$$u_1 + u_2 + u_3 + u_4 = 8.$$

No. of required solutions $= (4+8-1, 8) = C(11, 8)$

$$= 165.$$

PROBLEM

Find the number of non-negative solution for $x_1 + x_2 + x_3 + x_5 + x_6 < 10$.

We convert the inequality into equality by adding some variable $x_7 > 0$

(or)

$$x_7 \geq 1 \quad x_7 - 1 \geq 0$$

$x_1+x_2+x_3+x_4+x_5+x_6+x_7=10$ $\quad x_i \geq 0 \ \forall\ i=1$ to 6

$x_1+x_2+x_3+x_4+x_5+x_6+x_7-1=9$

Let $x_i=u_i \ \forall\ i=1$ to 6

$\cancel{x_7}$. $x_7-1=u_7$

$\therefore u_1+u_2+u_3+u_4+u_5+u_6+u_7=9$

Here $u_i \geq 0 \ \forall\ i=1$ to 7

No. of solutions $= C(7+9-1,9)$

$= C(15,9)$

$= 5005$.

PROBLEM

How many positive integers less than 1000000 have the sum of their digits equal to 19.

(or)

Solve for integer solution.

$x_1+x_2+x_3+x_4+x_5+x_6=19$.

$n=6 \quad r=19$

$C(6+19-1,19) = C(24,19) = \dfrac{24!}{19!\,5!}$

$= \dfrac{24 \times 23 \times 22 \times 21 \times 20}{5 \times 4 \times 3 \times 2}$

$= 42,504$

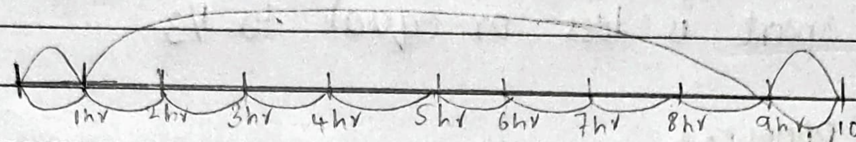
PIGEON PRINCIPLE

If n Pigeons are accomodated in m Pigeonholes and $n>m$ then one of the pigeoholes must contain atleast $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ pigeons.

PROBLEM

A man covered a total distance of 45 km

for 10 hrs. It is known that he covered 6km in the first hour and only 3 km in the last thour show that he must have covered atleast 9km within a certain period of 2 consecutive hours.



He covered $6+3=9$ KM in the first and last hour.

He must have covered the remaining 36km during the period from 2^{nd} to nineth hour. The

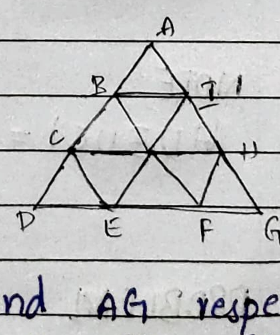
The possible consecutive 2 hours are 2 and 3, 4 and 5, 6 and 7 and 8 and 9.

No. of pigeons $= 36km = n$

No. of Pigeons holes $= 4 = m$.

PROBLEM

If we select 10 points in the interior of an equilateral triangle of side 1 (unit) show that there must be atleast two points whose distance apart is less than or equal to $1/3$.



Let ADG be the given equilateral Δ^{le}

The pair of points (B, C), (E, F) and (I, H) are the points of trisection AD, DG and AG respectively

Then we have 9 equilateral Δ^{les} with side $1/3$ unit.

No. of pigeons = no. of points $= 10 = n$.

No. of Pigeonhole = no. of subtriangles $= 9 = m$

$n > m$

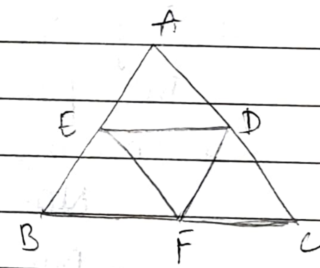
$$\left\lceil \frac{n-1}{m} \right\rceil + 1 = \left\lceil \frac{10-1}{9} \right\rceil + 1$$

$$= 1 + 1 = 2$$

There must be atleast two points whose apart is less or equal to 1/3.

PROBLEM

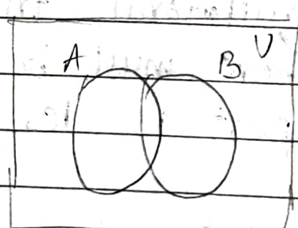
Of any 5 points chosen within an equilateral Δ^{le} whose sides are of pegion, 1unit. Prove that atleast two points are within a distance of 1/2 and of each other



PRINCPLE OF INCLUSION AND EXCLUSION

If A and B are finite subsets of a finite universal set U. then

$|A \cup B| = |A| + |B| - |A \cap B|$



NOTE:

$$|A \cup B \cup C| = |A| + |B| + |C|$$
$$- |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

PROBLEM

There are 250 students in an Engineering collage of these 188 have taken a course in Fortran, 100 have taken a course in C and 35 have taken a course in Java. Further

88 have taken courses in both in ~~C~~ and ~~Java~~. Fortran and C. 23 have taken courses in both in C and Java. 29 have taken courses in both Fortran and Java. If 19 of those students have taken all 3 courses. How many of these 250 students have not taken a course in any of these 3 courses.

Let A be the set of Students who have taken Fotran.

Let B be the set of students who have taken C.

Let C be the set of students who have taken Java.

$|A| = 188$ $\quad |A \cap B| = 88$ $\quad |A \cap B \cap C| = 19.$

$|B| = 100$ $\quad |A \cap C| = 23$

$|C| = 35$ $\quad |A \cap C| = 29$

Now

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$= 188 + 100 + 35 - 88 - 23 - 29 + 19$$
$$= 202$$

No. of students who have not taken any course $= 250 - 202 = 48.$