

DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

Module-I, Lecture-14

**Fermi-Dirac Distribution Function
 $f(E)$ and Influence of donors in
semiconductors**



The free electron gas in a solid obeys Fermi-Dirac statistics.

Suppose in an assemblage of fermions, there are $M(E)$ allowed quantum states in an energy range between E and $E+dE$ and $N(E)$ is the number of particles in the same range.

The Fermi-Dirac distribution function is defined as,

$$\frac{N(E)}{M(E)} = \frac{1}{1 + \exp(E - E_F)/kT}$$

$N(E) / M(E)$ is the fraction of the possible quantum states which are occupied.

The distribution of electrons among the levels is described by function $f(E)$, *probability of an electron occupying an energy level 'E'.*

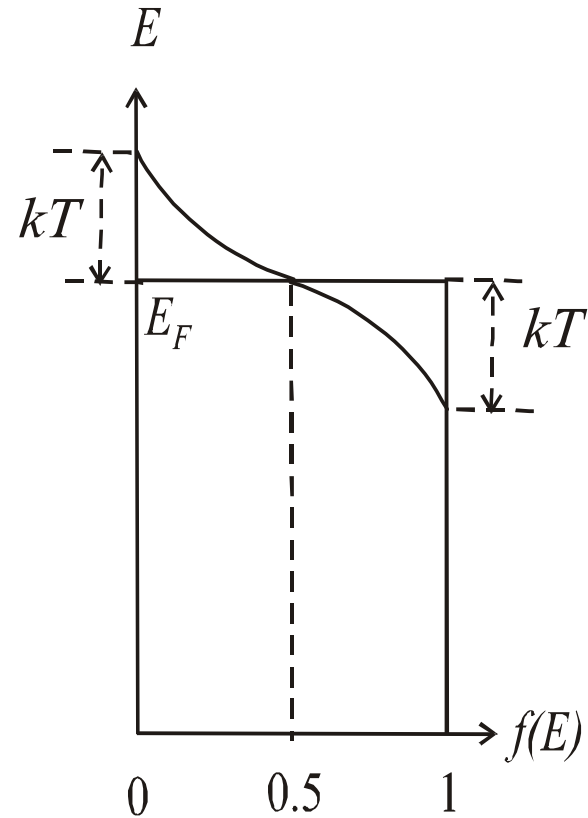
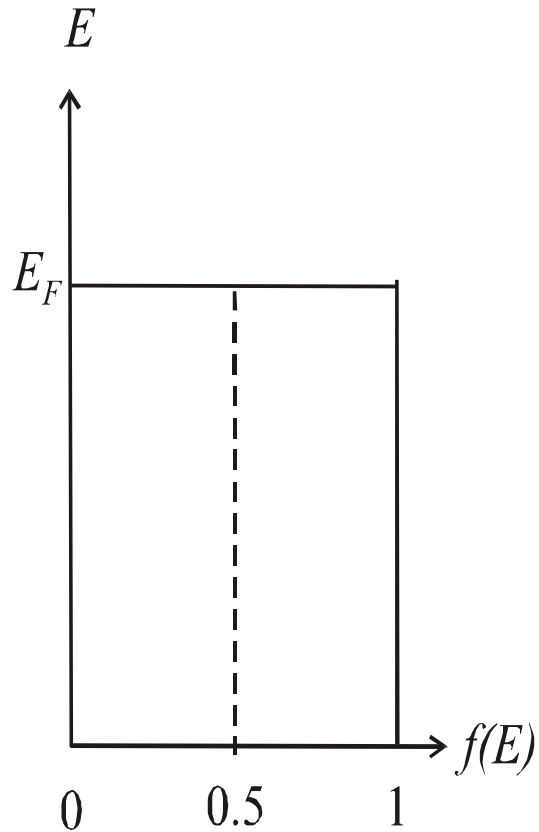
If the level is certainly empty, then $f(E) = 0$. Generally the $f(E)$ has a value in between zero and unity.

When $E < E_F$ (i.e.,) for energy levels lying below E_F , $(E - E_F)$ is a negative quantity and hence,

$$f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1$$

That means all the levels below E_F are occupied by the electrons.

Fermi-Dirac Distribution Function $f(E)$



Fermi Dirac distribution function at different temperatures



When $E > E_F$ (i.e.) for energy levels lying above E_F , $(E - E_F)$ is a positive quantity

$$f(E) = \frac{1}{1 + e^{\infty}} = \frac{1}{1 + \infty} = 0$$

This equation indicates all the levels above E_F are vacant.

At absolute zero, all levels below E_F are completely filled and all levels above E_F are completely empty.

This level, which divides the filled and vacant states, is known as the Fermi energy level.



When $E = E_F$,

$$f(E) = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2} \quad \text{at all temperatures}$$

The probability of finding an electron with energy equal to the Fermi energy in a metal is $\frac{1}{2}$ at any temperature.

At $T = 0$ K all the energy level upto E_F are occupied and all the energy levels above E_F are empty .

When $T > 0$ K, some levels above E_F are partially filled while some levels below E_F are partially empty.

The group V element has five valence electrons. Four of these will contribute to the covalent bonding with the silicon atoms, leaving the fifth more loosely bound to the phosphorus atom. the fifth valence electron as a donor electron.

The phosphorus atom without the donor electron is positively charged.

At very low temperatures, the donor electron is bound to the phosphorus atom.

However By intuition, it should seem clear that the energy required to elevate the donor electron into the conduction band is considerably less than that for the electrons involved in the covalent bonding.

If a small amount of energy, such as thermal energy, is added to the donor electron, it can be elevated into the conduction band, leaving behind a positively charged phosphorus ion.

The electron in the conduction band can now move through the crystal generating a current, while the positively charged ion is fixed in the crystal.

This type of impurity atom donates an electron to the conduction band and so is called a *donor impurity atom*.

The donor impurity atoms add electrons to the conduction band without creating holes in the valence band. The resulting material is referred to as an n -type semiconductor (n for the negatively charged electron)

Influence of donors in semiconductors

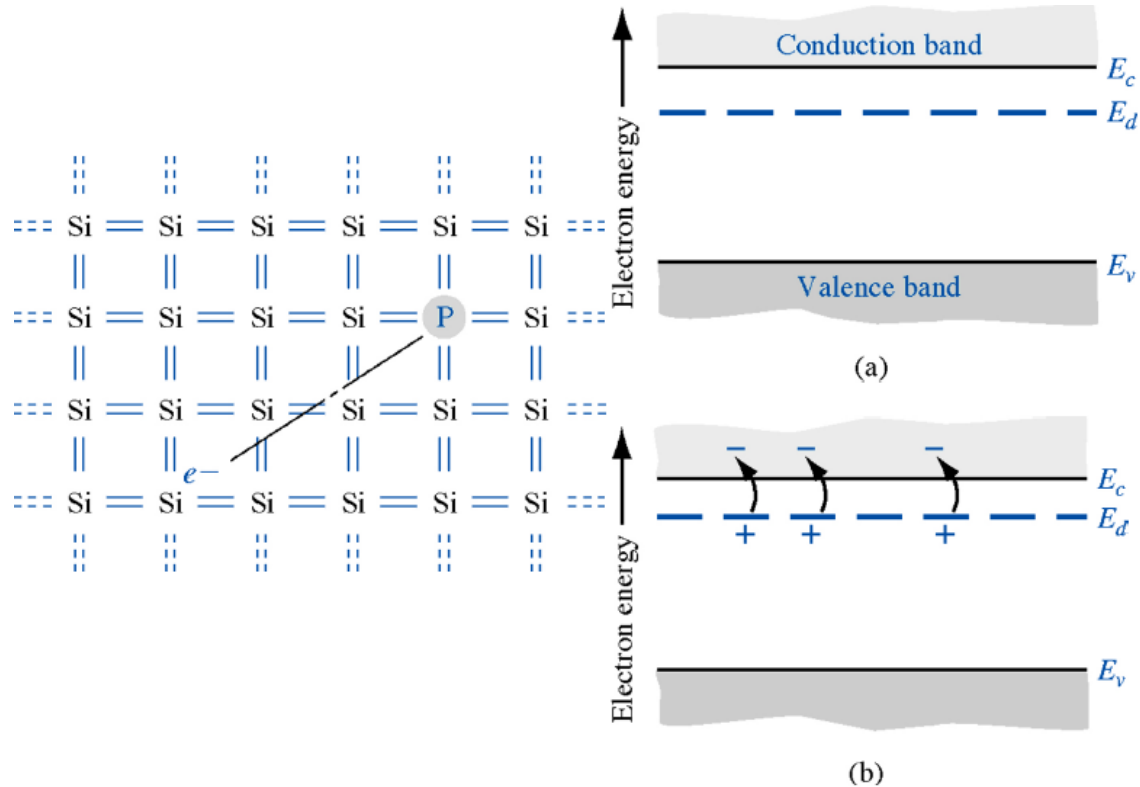


Figure shows the energy-band diagram that we would expect. The energy level, E_d , is the energy state of the donor electron.