Note:- If the conclusion is of the form $r \to s$, we will take r as an additional premise and derive 's' using the given premises and 'r'.

Form of Argument:-

when a set of given statements constitute a valid argument, the argument form will be presented

Table for Rules of Inference:-

Rule in tautological form	Name of the rule
1. $(p \wedge q) \to p$ $(p \wedge q \Rightarrow p)$ $(p \wedge q) \to q$ $(p \wedge q \Rightarrow q)$	1. Simplication
2. $p \to (p \vee q)$ $q \to (p \vee q)$	2. Addition
3. $[(p) \wedge (q)] \to (p \wedge q)$	3. Conjuction
4. $[p \wedge (p \to q)] \to q$	4. Modus ponens
4. $[p \wedge (p \to q)] \to q$	4. Modus ponens.
5. $[\neg q \wedge (p \to q)] \to \neg p$	5. Modus tollens.
6. $[(p \to q) \wedge (q \to r)] \to (p \to r)$	6. Hypothetical syllogism.
7. $[(p \vee q) \wedge \neg p] \to q$	7. Disjunctive syllogism.
8. $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$	8. Resolution
9. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$	9. Dilemma.

eq of form of argument:-

(1) If it rains heavily, then travelling will be different. If the students arrive on time, then travelling was not difficult. They arrived on time. Therefore it did not rain heavily.

"p" - It rains heavily.

"q" - Travelling is difficult.

"r" - Students arrived on time.

Here the premises are $p \to q$, $r \to \neg q$ and 'r' lead to the conclusion $\neg p$.

∴ The form of argument given as follows to shows that the premises lead to the conclusion.

Sl. no	Statement	Reason
1.	$p \to q$	Rule P
2.	$\neg q \to \neg p$	T, contrapositive of 1
3.	$r \to \neg q$	Rule P
4.	$r \to \neg p$	Steps (2,3) and Rule T and Hypothetical Syllogism
5.	r	Rule P.
6.	$\neg p$	T, Steps 4 & 5 and modus Ponen.

Now, we can shows the premises leads the conclusion by three Method.

1. Direct Method.
2. Indirect Method.
3. Inconsistant Method.

1. Direct Method:-

whatever the premises is given using the given premises leads the conclusion directly is called the direct method.

2. Indirect Method:-

If we want to shows the conclusion (A) by using the set of premises.

Let us assume the conclusion A is false and take $\neg A$ is a extra consadulion premises. Now we are using all the premises then prove it's a contradiction is called a indirect Method.

3. Inconsistant Method:-

A set of premises $A_1 A_2 \ldots A_n$ is said to be inconsistant, if their conjuction implies a contradiction.

2. S.T $t \wedge s$ can be derived from the premises $p \to q,\ q \to \neg r,\ r,\ p \vee (t \wedge s)$.

Sl. no	Statement	Reason
1.	$p \to q$	Rule P.
2.	$q \to \neg r$	Rule P
3.	$p \to \neg r$	Rule T, 1, 2 & H. syllo
4.	$r \to \neg p$	Rule T, 3 and $p \to q \equiv \neg q \to \neg p$.
5.	r	Rule P
6.	$\neg p$	T, 4, 5 & M.P $p \wedge (p \to q) \to q$
7.	$p \vee (t \wedge s)$	Rule P.
8.	$t \wedge s$	Rule T, 6, 7 & Disjunctive syllogism. $\{(p \vee q) \wedge \neg p \to q\}$

3. S.T $a \vee b$ follows logically from the primises $p \vee q,\ (p \vee q) \to \neg r,\ \neg r \to (s \wedge \neg t)$ and $(s \wedge \neg t) \to (a \vee b)$.

Sl. no	Statement	Reason
1.	$(p \vee q) \to \neg r$	Rule P
2.	$\neg r \to (s \wedge \neg t)$	Rule P
3.	$(p \vee q) \to (s \wedge \neg t)$	Rele T, 2, 3 & Hyp. Sy.
4.	$p \vee q$	Rule P
5.	$s \wedge \neg t$	Rule T, 4, 5 & M.P
6.	$(s \wedge \neg t) \to (a \vee b)$	P
7.	$a \vee b$	T, 5, 6 & M.P

1. give a direct proof for the implication

$p \to (q \to s),\ (\neg r \vee p),\ q \Rightarrow (r \to s)$

Sl. no	Statement	Reason.
1.	$\neg r \vee p$	P
2.	$r \to p$	T, 1 and equivalence of 1
3.	$p \to (q \to s)$	P
4.	$r \to (q \to s)$	T, 2,3 and hyp. syllo
5.	$\neg r \vee (\neg q \vee s)$	T, 4 & equivalences of 4
6.	q	P
7.	$q \wedge (\neg r \vee \neg q \vee s)$	T, 7,8 and conjuction.
8.	$q \wedge (\neg r \vee s)$	T (7,8) and negation & Domination Laws
9.	$\therefore \wedge (\neg r \vee s)$	T, 8, Simplication
10.	$r \to s$	T, 9 and equivalences of 9

5. $p \vee q,\ p \to r,\ q \to s \Rightarrow s \vee r.$

Steps	Statement	Reason.
1.	$p \vee q$	P
2.	$\neg p \to q$	T, conditional of 1
3.	$p \to r$	P
4.	$q \to s$	P
5.	$\therefore \neg p \to s$	Rule T, 2,4 & Hyp. syllo.
6.	$p \vee s$	Rule T, conditionals of 5.
7.	$\neg p \vee r$	Rule T, conditionals of 3.
8.	$s \vee r$	Rule T, Resolution of 6,7.

6. Prove by indirect method to S.T

$r \to \neg q, \; r \vee s, \; s \to \neg q, \; p \to q \Rightarrow \neg p.$

Solution: To use the indirect method, we will include $\neg(\neg p) \equiv p$ as an additional premise and prove a contradiction.

Indirect

Sl. no	Statement	Reason
1.	p	P (Additional)
2.	$p \to q$	P
3.	q	T, 1, 2 and M.P
4.	$r \to \neg q$	P
5.	$s \to \neg q$	P
6.	$(r \vee s) \to \neg q$	T, Equivalences of 4, 5
7.	$r \vee s$	P
8.	$\neg q$	T, 6, 7 & M.P
9.	$q \wedge \neg q$	T, 3, 8 and Conjuction.
10.	F	T, 9 and negation Law.

7. S.T $p \to (q \to s)$ using the C.P Rule (if necessary) from the premises $p \to (q \to r)$ and $q \to (r \to s)$.

C.P-rule

Sl. no	Statement	Reason
1.	p	P (Additional) [Assumed Premises]
2.	$p \to (q \to r)$	P
3.	$q \to r$	T, 1, 2 and M.P
4.	$\neg q \vee r$	T, 3 and equivalence of 3
5.	$q \to (r \to s)$	P
6.	$\neg q \vee (r \to s)$	T, 5 and equivalence of 5
7.	$\neg q \vee (r \wedge (r \to s))$	T, 4, 6 and distribution
8.	$\neg q \vee s$	T, 7 and M.P
9.	$q \to s$	T, 8 and equivalence of 8.
10.	$p \to (q \to s)$	T, 9 and CP-rule.

8. S.T $(p \to q) \wedge (r \to s)$, $(q \to t) \wedge (s \to u)$, $\neg(t \wedge u)$ and $(p \to r) \Rightarrow \neg p$.

Soln:

Sl.no	Statement	Reason
1.	$(p \to q) \wedge (r \to s)$	P
2.	$p \to q$	T, 1, Simplication
3.	$r \to s$	T, 1, Simplication
4.	$(q \to t) \wedge (s \to u)$	P
5.	$q \to t$	T, 4, Simplication
6.	$s \to u$	T, 4, Simplication
7.	$p \to t$	T, 2,5 and hyp. syllo
8.	$r \to u$	T, 3, 6 and hy. syllo
9.	$p \to r$	P.
10.	$p \to u$	T, 8,9 and hyp. syllo.
11.	$\neg t \to \neg p$	T and 7 contrapositive
12.	$\neg u \to \neg p$	T and 10 contrapositive
13.	$(\neg t \vee \neg u) \to \neg p$	T, 11, 12 and $(a \to b) \wedge (c \to b) \Rightarrow (a \vee c) \to b$
14.	$\neg(t \wedge u) \to \neg p$	T, 13 and De-Morgan's Law
15.	$\neg(t \wedge u)$	P
16.	$\neg p$	T, 14,15 and M. P

9. 8.T $(a \to b) \wedge (a \to c), \neg(b \wedge c), (d \vee a) \Rightarrow d$

Direct

Sl.no	Statement	Reason
1.	$(a \to b) \wedge (a \to c)$	P
2.	$a \to b$	T, 1 and Simplication
3.	$a \to c$	T, 1 and Simplication
4.	$\neg b \to \neg a$	T, 2 and contrapositive
5.	$\neg c \to \neg a$	T, 3 and contrapositive
6.	$(\neg b \vee \neg c) \to \neg a$	T, 4 and 5
7.	$\neg(b \wedge c) \to \neg a$	T, 6 and De-Morgan's Law
8.	$\neg(b \wedge c)$	P
9.	$\neg a$	T, 7, 8 M.P
10.	$d \vee a$	P
11.	$(d \vee a) \wedge \neg a$	T, 9, 10 and conjuction
12.	$(d \wedge \neg a) \vee (a \wedge \neg a)$	T, 11 and distributive Law
13.	$(d \wedge \neg a) \vee F$	T, 12 and Negation Law
14.	$d \wedge \neg a$	T, 13 and Identity
15.	d	T, 14 and Simplication

10. Use the Indirect method to show that,

Indirect.

$r \to \neg q,\ r \vee s,\ s \to \neg q,\ p \to q \Rightarrow \neg p.$

Soln:- To use the indirect method, we will include $\neg(\neg p) = p$ as an Additional premise and to prove it's a contradiction.

(Refer Ques.no:- 6)

11. S.T b can be derived from the premises $a\to b$, $c\to b$, $d\to(a\vee c)$, d by indirect method.

Soln:- Let us include $\neg b$ as an additional premise and prove it's a contradiction.

Sl.no	Statement	Reason.
1.	$a\to b$	P
2.	$c\to b$	P
3.	$(a\vee c)\to b$	T, 1,2 and equivalence.
4.	$d\to(a\vee c)$	P
5.	$d\to b$	T, 3,4 and hyp. Syllogism.
6.	d	P
7.	b	T, 5,6 and M. P
8.	$\neg b$	P, Additional
9.	$b\wedge\neg b$	T, 7,8 and conjection.
10.	F	T, 9 and negation Law.

12. Using the indirect method of proof, derive $p\to\neg s$ from the premises $p\to(q\vee r)$, $q\to\neg p$, $s\to\neg r$, p.

Soln:- Let us include $\neg(p\to\neg s)$ as an additional premise and prove a Contradiction.

$\because \neg(p\to\neg s)=\neg(\neg p\vee\neg s)=p\wedge s.$

$\therefore$ The additional premise to be introduced may be taken $p\wedge s$

Here, $\neg(p \to \neg s) = \neg(\neg p \vee \neg s) = p \wedge s$.

$\therefore$ the additional premise to be introduced to be introduced may be taken as $p \wedge s$.

Sl. no	Statement	Reason
1.	$p \to (q \vee r)$	P
2.	p	P
3.	$q \vee r$	T, 1,2 and M.P
4.	$p \wedge s$	P (additional)
5.	s	T, 4 and simplication.
6.	$s \to \neg r$	P
7.	$\neg r$	T, 5,6 and M.P
8.	q	T, 3,7 and D.S
9.	$q \to \neg p$	P
10.	$\neg p$	T, 8,9 and M.P
11.	$p \wedge \neg p$	T, 2,10 and conjuction.
12.	F	T, 11 and negation Law

12. Prove that the premises $p \to q$, $q \to r$, $s \to \neg r$ and $p \wedge s$ are inconsistant.

Soln:- We desire a contradiction by using the given premises, it means that they are inconsistant

Here, $\neg(p \to \neg s) = \neg(\neg p \vee \neg s) = p \wedge s$.

$\therefore$ the additional premise to be introduced to be introduced may be taken as $p \wedge s$.

Sl. no	Statement	Reason.
1.	$p \to (q \vee r)$	P
2.	p	P
3.	$q \vee r$	T, 1,2 and M.P
4.	$p \wedge s$	P (additional)
5.	s	T, 4 and simplication.
6.	$s \to \neg r$	P
7.	$\neg r$	T, 5,6 and M.P
8.	q	T, 3,7 and D.S
9.	$q \to \neg p$	P
10.	$\neg p$	T, 8,9 and M.P
11.	$p \wedge \neg p$	T, 2,10 and conjuction.
12.	F	T, 11 and negation Law

12. Prove that the premises $p \to q$, $q \to r$, $s \to \neg r$ and $q \wedge s$ are inconsistant.

<u>Soln:-</u> We derive a contradiction by using the given premises, it means that they are inconsistan

Sl. no	statement	Reason.
1.	$P \to q$	P
2.	$q \to r$	P
3.	$P \to r$	T, 1, 2 and hyp. syllogism.
4.	$s \to \neg r$	P
5.	$r \to \neg s$	T, 4 and contrapositive
6.	$q \to \neg s$	T, 2, 5 and hyp. syllogism
7.	$\neg q \vee \neg s$	T, 6 and equivalence of 6
8.	$\neg (q \wedge s)$	T, 7 and De-Margan's Law
9.	$q \wedge s$	P
10.	$(q \wedge s) \wedge \neg (q \wedge s)$	T, 8, 9 and conjuction
11.	F	T, 10 and negation Law.

13. Prove that, the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$ and $a \wedge d$ are inconsistant.

Sl. no	Statement	Reason.
1.	$a \wedge d$	P
2.	a	T, 1 and Simplication
3.	d	T, 1 and Simplication
4.	$a \to (b \to c)$	P
5.	$b \to c$	T, 2, 4 and M.P
6.	$\neg b \vee c$	T, 5 and equivalence.
7.	$d \to (b \wedge \neg c)$	P
8.	$\neg (b \wedge \neg c) \to \neg d$	T, Contrapositive of 7.
9.	$(\neg b \vee c) \to \neg d$	T, 8, and equivalence.
10.	$\neg d$	T, 6, 9 and M.P
11.	$d \wedge \neg d$	T, 3, 10 and conjuction.
12.	F	T, 11 and negation Law.

14. Conduct an argument to show that the following premises imply the conclusion "it rained".

"If it does not rain (or) if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on", "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded".

p :- It rains

q :- There is traffic dislocation

r :- Sports day will be held.

s :- Cultural programme will go on.

t :- The trophy will be awarded.

To prove,

$\neg p \vee \neg q \rightarrow r \wedge s$, $r \rightarrow t$, $\neg t \Rightarrow p$

Sl. no	Statement	Reason.
1.	$\neg p \vee \neg q \rightarrow r \wedge s$	P
2.	$(\neg p \rightarrow (r \wedge s)) \wedge (\neg q \rightarrow (r \wedge s))$	T, 1, and equivalence $(a \vee b) \rightarrow c \equiv (a \rightarrow c) \wedge (b \rightarrow c)$
3.	$\neg(r \wedge s) \rightarrow p$	T, 2 and contrapositive.
4.	$r \rightarrow t$	P
5.	$\neg t \rightarrow \neg r$	T, 4 and contrapositive.
6.	$\neg t$	P
7.	$\neg r$	T, 5, 6 and M.P
8.	$\neg r \vee \neg s$	T, 7 and addition.
9.	$\neg(r \wedge s)$	T, 8 and De-Morgan's law.
10.	p	T, 3, 9 and M.P

14. Construct an argument to show that the following premises imply the conclusion "it rained".

"If it does not rain (or) if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on", "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded"

p :- It rains

q :- There is traffic dislocation

r :- Sports day will be held.

s :- Cultural programme will go on.

t :- The trophy will be awarded.

To prove,

$$\neg p \vee \neg q \to r \wedge s,\ r \to t,\ \neg t \Rightarrow p$$

Sl. no	Statement	Reason.
1.	$\neg p \vee \neg q \to r \wedge s$	P
2.	$(\neg p \to (r \wedge s)) \wedge (\neg q \to (r \wedge s))$	T, 1, and equivalence $(a \vee b) \to c \equiv (a \to c) \wedge (b \to c)$
3.	$\neg(r \wedge s) \to p$	T, 2 and contrapositive.
4.	$r \to t$	P
5.	$\neg t \to \neg r$	T, 4 and contrapositive.
6.	$\neg t$	P
7.	$\neg r$	T, 5, 6 and M.P
8.	$\neg r \vee \neg s$	T, 7 and addition.
9.	$\neg(r \wedge s)$	T, 8 and De-Morgan's Law.
10.	p	T, 3, 9 and M.P

15. S.T the following set of premises is inconsistent
If Rama get his degree, he will go for a job.
If he goes for a job, he will get married soon.
If he goes for higher study, he will not get married.
Rama gets his degree and goes for higher study.
∴ The statements are,

p :- Rama gets his degree
q :- He will go for a job
r :- He will get married soon.
s :- He goes for higher study.

To prove,
$p \to q$, $q \to r$, $s \to \neg r$ and $p \wedge s$ are inconsistant

Sl.no	Statement	Reason.
1.	$p \to q$	P
2.	$q \to r$	P
3.	$p \to r$	T, 1,2 and H.S
4.	$p \wedge s$	P
5.	p	T, 4 and simplication
6.	s	T, 4 and simplication.
7.	$s \to \neg r$	P
8.	$\neg r$	T, 6,7 and ~~hyp. sylto~~ M.P
9.	r	T, 3,5 and M.P
10.	$r \wedge \neg r$	T, 8,9 and conjuction
11.	F	T, 10 and negation Law.

18. S.T the following set of premises is inconsistent

If Rama get his degree, he will go for a job.

If he goes for a job, he will get married soon.

If he goes for higher study, he will not get married.

Rama gets his degree and goes for higher study.

∴ The statements are,

p :- Rama gets his degree

q :- He will go for a job

r :- He will get married soon.

s :- He goes for higher study.

To prove,

$p \to q$, $q \to r$, $s \to \neg r$ and $p \wedge s$ are inconsistent

Sl.no	Statement	Reason
1.	$p \to q$	P
2.	$q \to r$	P
3.	$p \to r$	T, 1,2 and H.S
4.	$p \wedge s$	P
5.	p	T, 4 and Simplication
6.	s	T, 4 and Simplication.
7.	$s \to \neg r$	P
8.	$\neg r$	T, 6,7 and ~~hyp. sy~~ M.P
9.	r	T, 3,5 and M. P
10.	$r \wedge \neg r$	T, 8,9 and Conjuction
11.	F	T, 10 and negation Law.