UNIT - I

20/07/2023

SET THEORY

NOTATIONS

U - Universal set

All capitals letters denote set

EX : $A = \{1, 2, 3, 4, 5\}$

$Z = \{0, \pm 1, \pm 2, \cdots\}$

SET OPERATIONS

Union - $\cup$

$A \cup B = \{x / x \in A \text{ or } x \in B\}$

Intersection - $\cap$

$A \cap B = \{x / x \in A \text{ and } x \in B\}$

Subtraction :

$A - B = \{x / x \in A \text{ and } x \notin B\}$

Direct Sum :

$A \oplus B = (A - B) \cap (B - A)$

PROBLEM :

If $A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 6, 7, 8\}$

find (i) $A \cap (B - A)$

(ii) $A \oplus B$

(iii) $(A - B) \cup B$

SOL : (i) $B - A = \{6, 7, 8\}$

$A \cap (B - A) = \phi$

(ii) $A - B = \{1, 4, 5\}$

$B - A = \{6, 7, 8\}$

$A \oplus B = \phi$

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CARDINALITY OF A SET 'A'

DEFINITION :

The cardinality of a set A is the number of elements in A.

It is defined by $n(A)$.

EX :

(iv) find $n(A-B)$
(v) find $n(A \oplus B)$

(iv) $n(A-B) = 3$
(v) $n(A \oplus B) = 0$

CARTESIAN PRODUCT OF TWO SETS A & B

Let $A = \{a / a \in A\}$
$B = \{b / b \in B\}$

$A \times B = \{(a,b) / a \in A \text{ and } b \in B\}$

PROBLEM :

If $A = \{a, b, c\}$
$B = \{1, 2\}$
(i) find $A \times B$
(ii) find $n(A \times B)$

(i) $A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$
$n(A \times B) = 6$

IMPORTANT LAWS

Let A, B and C be three sets and U denots union set.

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1. ASSOCIATIVITY LAW :

(i) $A \cup (B \cup C) = (A \cup B) \cup C$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

2. DISTRIBUTIVE LAW :

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. COMPLEMENT LAW :

(i) $A \cap A^c = \phi$

(ii) $A \cup A^c = U$

4. DEMORGAN'S LAW :

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(A \cap B)^c = A^c \cup B^c$

5. COMMUTATIVE LAW :

(i) $A \cup B = B \cup A$.

(ii) $A \cap B = B \cap A$.

Prove Demorgan's law theoretically

Sol: $(A \cup B)^c = A^c \cap B^c$

$= \{x \mid x \in (A \cup B)^c\}$

$= \{x \mid x \notin (A \cup B)\}$

$= \{x \mid x \notin A \text{ or } x \notin B\}$

$= \{x \mid x \in A^c \cap B^c\} = A^c \cap B^c$

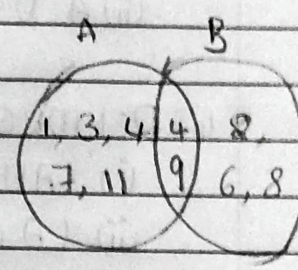
PROBLEM :

If A, B and C are 3 sets: Prove analytically that $A - (B \cap C) = (A - B) \cup (A - C)$

Sol: $A-(B\cap C) = \{x / x \in A-(B\cap C)\}$
$= \{x / x \in A \text{ and } x \notin (B\cap C)\}$
$(A-B)\cup(A-C) = \{x / x \in A \text{ and } x \notin B \text{ or } x \notin C\}$
$= \{x / x \in A \text{ and } x \notin B \text{ or } x \in A \text{ and } x \notin C\}$
$= \{x / x \in (A-B) \text{ or } x \in (A-C)\}$
$= \{x / x \in (A-B)\cup(A-C)\}$

PROBLEM:

Find the sets A and B such that
$A-B = \{1,3,7,11\}$
$B-A = \{2,6,8\}$
$A\cap B = \{4,9\}$



$A = \{1,3,7,11,4,9\}$
$B = \{4,9,2,6,8\}$

Prove that $(A-C)\cap(C-B) = \phi$ analytically

sol: $(A-C)\cap(C-B) = \{x \in A \text{ and } x \in C \text{ and } x \in C \text{ and } x \notin B\}$
$= \{x \in A \text{ and } (x \notin \bar{C} \text{ and } x \in C) \text{ and } x \notin B\}$
$= \{x \in A \text{ and } (x \in \bar{C} \cap C) \text{ and } x \notin B\}$
$= \{x \in A \text{ and } x \in \phi \text{ and } x \notin B\}$
$= \{x \in A \cap \phi \cap B^c\}$
$= \{x / x = \phi\}$
$= \phi$

Prove that $A\times(B\cap C) = (A\times B)\cap(A\times C)$

sol: Now $A\times(B\cap C) = \{(x,y) / x \in A \text{ and } y \in B\cap C\}$
$= \{(x,y) / x \in A \text{ and } (y \in B \text{ and } y \in C)\}$
$= \{(x,y) / (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)\}$

$= \{(x, y) / (x, y) \in A \times B \text{ and } (x, y) \in A \times C\}$
$= (A \times B) \cap (A \times C)$

PROBLEM

Let $A = \{1, 2, 3\}$

Find all possible partitions of A.

sol:
(i) $A_1 = \{1\}$, $A_2 = \{2, 3\}$
(ii) $A_1 = \{2\}$, $A_2 = \{1, 3\}$
(iii) $A_1 = \{3\}$, $A_2 = \{1, 2\}$
(iv) $A_1 = \{1\}$, $A_2 = \{2\}$, $A_3 = \{3\}$

PARTITION OF A SET

Let A be a set

Then the following subset of A.

$A_1, A_2, A_3 \cdots\cdots A_n$ form a.

partition of A if

(i) $A_1 \cup A_2 \cup \cdots\cdots \cup A_n = A$
(ii) $A_i \cap A_j = \emptyset$ for $i \neq j$

PROBLEM

Let $A = \{a, b, c, d\}$

Find all possible partitions of A.

$A_1 = \{a\}$ $A_2 = \{b, c, d\}$
$A_1 = \{b\}$ $A_2 = \{a, c, d\}$
$A_1 = \{c\}$ $A_2 = \{a, b, d\}$
$A_1 = \{d\}$ $A_2 = \{a, b, c\}$
$A_1 = \{a, b\}$ $A_2 = \{c, d\}$
$A_1 = \{a, c\}$ $A_2 = \{b, d\}$
$A_1 = \{a, d\}$ $A_2 = \{b, c\}$
$A_1 = \emptyset$ $A_2 = \{a, b, c, d\}$

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POWERSET OF A SET 'A'.

The power set of a set A is set of all possible subsets of A.

NOTE

Let $n(A) = m$

Then $n(P(A)) = 2^m$

Ex

Let $A = \{1, 2, 3\}$

Find $P(A)$ and also $n(P(A))$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$2^3 = 8$

MIN SETS

DEFINITION

Let A be a set,

Let B_1, B_2 be two subsets of A.

Then min sets generated by B_1 and B_2 are given by

$A_1 = B_1 \cap B_2^c$

$A_2 = B_1 \cap B_2$

$A_3 = B_2 \cap B_1^c$

$A_4 = B_2^c \cap B_1^c$

PROBLEM

Let $A = \{1, 2, 3\}$

$B_1 = \{1, 2\}$ $B_2 = \{2, 3\}$

Find all min sets generated by B_1 and B_2

Sol: $A_1 = B_1 \cap B_2^c = \{1, 2\} \cap \{1\} = \{1\}$.

$A_2 = B_1^c \cap B_2 = \{3\} \cap \{2, 3\} = \{3\}$

$A_3 = B_1 \cap B_2 = \{1, 2\} \cap \{2, 3\} = \{2\}$

$A_4 = B_1^c \cap B_2^c = \{3\} \cap \{1\} = \phi$

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PROBLEM

Let $A = \{1,2,3,4,5,6,7,8,9\}$

Let $B_1 = \{5,6,7\}$

$B_2 = \{2,4,5,9\}$

$B_3 = \{3,4,5,6,8,9\}$

Find all min sets generated by B_1, B_2 and B_3.

sol:

$A_1 = B_1 \cap B_2 \cap B_3$

$A_2 = B_1^c \cap B_2 \cap B_3$

$A_3 = B_1 \cap B_2^c \cap B_3$

$A_4 = B_1 \cap B_2 \cap B_3^c$

$A_5 = B_1^c \cap B_2^c \cap B_3$

$A_6 = B_1^c \cap B_2 \cap B_3^c$

$A_7 = B_1 \cap B_2^c \cap B_3^c$

$A_8 = B_1^c \cap B_2^c \cap B_3^c$

$B_1^c = \{1,2,3,4,8,9\}$

$B_2^c = \{1,3,6,7,8\}$

$B_3^c = \{1,2,7\}$

$A_1 = \{5,6,7\} \cap \{2,4,5,9\} \cap \{3,4,5,6,8,9\}$
$= \{5\}$

$A_2 = \{1,2,3,4,8,9\} \cap \{2,4,5,9\} \cap \{3,4,5,6,8,9\}$
$= \{4,9\}$

$A_3 = \{5,6,7\} \cap \{1,3,6,7,8\} \cap \{3,4,5,6,8,9\}$
$= \{6\}$

$A_4 = \{5,6,7\} \cap \{2,4,5,9\} \cap \{1,2,7\}$
$= \emptyset$

$A_5 = \{1,2,3,4,8,9\} \cap \{1,3,6,7,8\} \cap \{3,4,5,6,8,9\}$
$= \{3,8\}$

$A_6 = \{1,2,3,4,8,9\} \cap \{2,4,5,9\} \cap \{1,2,7\}$
$= \{2\}$

$A_7 = \{5,6,7\} \cap \{1,3,6,7,8\} \cap \{1,2,7\}$
$= \{7\}$

$A_8 = \{1,2,3,4,8,9\} \cap \{1,3,6,7,8\} \cap \{1,2,7\}$
$= \{1\}$

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RELATION

DEFINITION

Let A and B be two sets. A subset R of cartesian product A X B is called a relation satisfying some property.

EXAMPLE :

Let $A = \{1, 2, 3\}$

$B = \{2, 4, 6\}$

R is a relation defined by (a,b) is an element of R such that a+b is always even

$R = \{(2,2), (2,4), (2,6)\}$

PROPERTIES OF RELATION.

1. REFLEXIVE

Let R be a relation defined on A or from A to A.

A relation R is said to be reflexive if $(a,a) \in R$ for all $a \in A$.

NOTE : $(a,b) \in R$

a is related to b.

2. SYMMETRIC

A relation R is said to be symmetric on A if $(a,b) \in R$ implies $(b,a) \in R$

3. TRANSITIVE

A relation R is said to be transitive if $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

DEFINITION:

EQUIVALENCE RELATION

A relation R defined on A is said to be equivalent relation if it satisfies

(i) Reflexive

(ii) Symmetric

(iii) Transitive

PROBLEM:

A relation R is defined on a set of parallel lines.

Prove that R is an equivalence relation. if $(l, m) \in R$ such that l is parallel to m.

sol. Let L = set of all parallel lines.

Define R such that

$(l, m) \in R$ such that l is parallel to m.

REFLEXIVE

Let $l, m \in L$

and let $(l, m) \in R$

then l is parallel to m.

$\Rightarrow$ m is parallel to l

$\Rightarrow (m, l) \in R$.

R is symmetric

REFLEXIVE.

Let $l, m \in L$.

such $(l, m) \in R$ (i.e.,) l is parallel to m.

Now $(l, l) \in R$

$\Rightarrow$ R is reflexive

TRANSIVE

Let $l, m, n \in L$

such that $(l, m) \in R$ and $(m, n) \in R$

l is parallel to m and m is parallel to n

l is parallel to n.

$(l, n) \in R$

R is an equivalence relation.

PROBLEM:

A relation R is defined on $A = \{2, 4, 6, 8\}$ such that $(a, b) \in R$ if $a+b$ is always even. Prove that R is an equivalence relation.

Sol. $R = \{(2,2), (2,4), (2,6), (2,8), (4,2), (4,4), (4,6), (4,8), (6,2), (6,4), (6,6), (6,8), (8,2), (8,4), (8,6), (8,8)\}$

REFLEXIVE

let $a \in A$.

Then $(a, a) \in R$ for all $a \in A$.

SYMMETRIC

Let $(a, b) \in R$

Then, $a+b$ is even

$\Rightarrow b+a$ is also even.

$(b, a) \in R$

TRANSITIVE

Let $(a, b) \in R$

$(b, c) \in R$

$a+b$ is even, $b+c$ is even

$a+b = 2m$ — (1) $\quad b+c = 2n$ — (2)

(1) + (2) gives,

$a + 2b + c = 2(m+n)$

$a + c = 2(m+n) - 2b$

$a + c = 2(m+n-b)$

$a+c$ is even

$(a,c) \in R$

$(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

$\therefore$ R is an equivalence Relation.

NOTE : congruency

$a \equiv b \pmod{m}$

$a \equiv b \pmod{m}$

$a-b$ is divisible by m.

$5 \equiv 1 \pmod{2}$

PROBLEM :

If R is a relation on set of integers. such that prove that $(a,b) \in R$ if $a \equiv b \pmod{m}$ then prove that R is an equivalence relation,

sol: REFLEXIVE :

$a \equiv a \pmod{m}$, since 0 is divisible by m

$(a,a) \in R$

SYMMETRIC

Let $(a,b) \in R$

Then $a \equiv b \pmod{m}$

$\Rightarrow a-b$ is divisible by m.

$b-a$ is also divisible by m

$b \equiv a \pmod{m}$

R is symmetric

TRANSITIVITY :

Let $(a,b) \in R$ and $(b,c) \in R$

Then $a \equiv b \pmod{m}$ $b \equiv c \pmod{m}$

$a-b = k_1 m$ $\qquad b_1 - c = k_2 m$

$a-b+b-c = (k_1+k_2)m$

$a-c = (k_1+k_2)m$

$\Rightarrow a \equiv c \pmod{m}$

R is an equivalence relation.

PROBLEM:

If R is a relation defined on NXN where $N = \{1, 2, 3 \ldots\}$ and $((a,b),(c,d)) \in R$ if $ad = bc$.

Prove that R is an equivalence relation.

Sol. REFLEXIVE

Let $(a,b) \in N \times N$

Now we can say $((a,b),(a,b)) \in R$

since $ab = ba$

R is reflexive

SYMMETRIC

Let $((a,b),(c,d)) \in R$

$\Rightarrow ad = bc$

$\Rightarrow bc = ad$

$\Rightarrow ((c,d),(a,b)) \in R$

R is symmetric.

TRANSITIVITY:

Let $((a,b),(c,d)) \in R$

and $((c,d),(e,f)) \in R$

Then $ad = bc \rightarrow (1)$

$cf = de \rightarrow (2)$

$de = cf \rightarrow (2)$

$\frac{(2)}{(1)} = \frac{e}{a} = \frac{f}{b} \Rightarrow af = be$

$((a,b),(e,f)) \in R$

R is ~~reflex~~ transitive

R is an equivalence relation.

PROBLEM :

If R is a relation defined on set of integers such that $(a,b) \in R$ if $3a+4b=7n$ then prove that R is an equivalence relation.

sol: REFLEXIVE

$3a+4a=7a$

$\Rightarrow (a,a) \in R$

R is reflexive

SYMMETRIC

Let $(a,b) \in R$ $\quad 3a+4b=7n$ —(i)

$7b-3b+7a-4a=7n$

$7(b+a-n)=3b+4a$

$3b+4a=7(k)$

$(b,a) \in R$

TRANSITIVITY

Let $(a,b) \in R$ and $(b,c) \in R$

$3a+4b=7k_1$ —(i)

$3b+4c=7k_2$ —(ii)

Adding (i) and (ii)

$3a+4c+7b=7(k_1+k_2)$

$3a+4c=7(k_1+k_2-b)$

$(a,c) \in R$

$\therefore$ R is transitive

$\therefore$ R is an equivalence relation.

DEFINITION.

Domain and Range

Let R be a relation from A to B

$R = \{ (a,b) \} \ a \in A$ and $b \in B \}$

Domain $= \{ a \in A |$

Range $=$

PROBLEM :

Let R be a relation defined on $A = \{1, 2, 3\}$ and $R = \{ (1,2), (2,3) \}$

Domain $= \{1, 2\}$

Range $= \{2, 3\}$

DEFINITION.

INVERSE RELATION R^{-1}

Let $R = \{ (a,b) / a \in A$ and $b \in B \}$

$R^{-1} = \{ (b,a) / (a,b) \in R \}$

PROBLEM

Let R be a relation defined on $A = \{1, 2, 3\}$ and $R = \{ (1,2), (2,3) \}$. Find R^{-1}.

Sol. $R^{-1} = \{ (2,1), (3,2) \}$

COMPOSITION OF TWO RELATIONS.

Let R be a relation from A to B.

Let S be a relation from B to C.

Then composition of R and S is given by

$R \circ S = \{ (a,c) | \exists$ some $b \in B$ for $(a,b) \in R$ and $(b,c) \in S \}$

PROBLEM :

Let R and S be two relations defined

on $A = \{1, 2, 3, 4\}$

Let $R = \{(1,1), (1,3), (3,2), (3,4), (4,2)\}$

Let $S = \{(2,1), (3,3), (3,4), (4,1)\}$

Find (i) $R \circ S$ (ii) $S \circ R$ (iii) $R \circ R$

Sol: $R \circ S = \{(1,3), (1,4)\ (3,1), (4,1)\}$

$S \circ R = \{(2,1)\ (2,3)\ (3,2)\ (3,4)\ (4,1)\ (4,3)\}$

$R \circ R = \{(1,1)\ (1,3), (1,2)\ (1,4)\ (3,2)\}$.

PROBLEM

Let R and S be two relations defined on $A = \{1, 2, 3, 4\}$

Such that (i) $(a,b) \in R$ if $a+b$ is even

(ii) $(a,b) \in S$ if $a+b$ is odd.

Then find (i) Domain of R

(ii) Domain of S

(iii) $R \circ S$

(iv) Range of R

(v) Range of S.

$R = \{(1,1), (1,3), (2,2)\ (2,4), (3,1), (3,3), (4,2), (4,4)\}$

$S = \{(1,2), (1,4), (2,1)\ (2,3)\ (3,2), (3,4)\ (4,1), (4,3)\}$

Domain of $R = D(R) = A$

Domain of $S = D(S) = A$

Range of $R = A$

Range of $S = A$.

$S \circ R = \{(2,1)\ (2,3)\ (3,2)\ (3,4)\ (4,1)\ (4,3)\}$

$R \circ R = \{(1,1)\ (1,3)\ (1,2)\ (1,4)\ (3,2)\}$

MATRIX REPRESENTATION OF A RELATION.

Let R be a relation defined from A to B

Then R can be represented by a matrix

$M_R = (m_{ij})$

$$m_{ij} = \begin{cases} 1 & \text{if } (a,b) \in R \\ 0 & \text{if } (a,b) \notin R \end{cases}$$

where

PROBLEM

Let $A = \{0, 1, 2, 3\}$

$B = \{2, 6, 7\}$

Define R by $(a,b) \in R$ if $a+b$ is Prime

Sol:

$$M_R = \begin{array}{c|ccc} & 2 & 6 & 7 \\ \hline 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \end{array}$$

$R = \{(0,2), (0,7), (1,2), (1,6), (3,2)\}$

NOTE:

$M_{R^{-1}}$ - Matrix representation of R^{-1}

$$\boxed{M_{R^{-1}} = M_R^T}$$

OPERATIONS DEFINED ON M_R

V - OPERATION (OR OPERATION)

Let R and S be two relations defined on A. Then the matrix representation $R \cup S$ is the join of M_R and M_S obtained by putting "1" in the positions where either M_R or M_S has a "1" and it is denoted by

$M_{R \cup S} = M_R \vee M_S$.

$\wedge$ - OPERATION (AND Operation)

The matrix representing $R \cap S$ is the meet of M_R and M_S obtained by putting 1 in the positions where both M_R and M_S have a "1" and is denoted by

$$\boxed{M_{R \cap S} = M_R \wedge M_S}$$

CLOSURES

REFLEXIVE CLOSURE

Let R be a relation on A.

Let $\Delta_R = \{(a,a) / a \in A\}$

The smallest relation R' that contains Δ_R is called reflexive closure.

SYMMETRIC CLOSURE

Let R be a relation on A

Let $\Delta_S = \{(a,b) / (b,a) \in R\}$

The smallest relation R' that contains Δ_S is called symmetric closure

TRANSITIVE CLOSURE

Let $\Delta_T = \{(a,c) / (a,b), (b,c) \in R\}$

The smallest relation R' that contains Δ_T is called the transitive closure

Let R be a relation defined on $A = \{a,b,c\}$

PROBLEM

Let $R = \{(a,a), (a,b), (b,c)\}$ be a relation

Find its reflexive closure R'.

Sol: $R' = \{(a,a), (a,b), (b,c)\} \cup \{(b,b), (c,c)\}$

PROBLEM

Let R be a relation on $A = \{a,b,c\}$.

Let $R = \{(a,a), (a,b), (b,c)\}$. Find symmetric clouser R'.

Sol $R' = \{(a,a), (a,b), (b,c)\} \cup \{(b,a), (c,b)\}$

Find the transitive closure R'

$R' = \{(a,a), (a,b), (b,c)\} \cup \{(a,c)\}$

PROBLEM

Let R be a relation on $A = \{a,b,c\}$

Let $R = \{(a,b) (b,c), (c,a)\}$. Find the transitive closure

Sol: $R' = \{(a,b), (b,c), (c,a)\}$

$\cup \{(a,c), (b,a), (c,b)\}$

WARSHALL'S ALGORITHM FOR FINDING TRANSITIVE CLOSURE

If $|A| = n$, then the matrix representation of transitive closure is W_n.

Let $\boxed{W_0 = M_R}$

Step 1: First transfer to W_k all 1's in W_{k-1}

Step 2: List the locations P_1, P_2 in column k of W_{k-1} when the entry is "1" and locations q_1, q_2 ---- in row k of W_{k-1} when entry is "1".

PROBLEM :

Using warshall's algorithm for finding transitive closure of $R = \{(a,b) (b,c), (c,a)\}$ which is defined on $A = \{a,b,c\}$

$$M_R = \begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 1 & 0 \\ b & 0 & 0 & 1 \\ c & 1 & 0 & 0 \end{array}$$

$$W_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

	In W_{k-1}		Positions of 1's in W_K	W_K
K	positions of 1's in K^{th} column	Positions of 1's in K^{th} row		
1	3	2	(3, 2)	$W_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
2	1, 3	3	(1,3) (3, 3)	$W_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
3	1, 2, 3	1, 2, 3	(1,1) (1,2) (1,3) (2,1) (2, 2) (2, 3) (3, 1) (3,2) (3,3)	$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

PROBLEM:

Let $R = \{(a,a), (a,b), (b,c)\}$ be a relation on $A = \{a, b, c\}$. Find the transitive clousre of R

$$M_R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad W_0 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

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k	In W_{k-1}: Positions of 1's in k^{th} column	In W_{k-1}: Positions of 1's in k^{th} row.	Positions of 1's in W_k	W_k
1	1	1, 2	(1, 1) (1, 2)	$W_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
2	1	3	(1, 3)	$W_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
3.	1, 2	–	–	$W_3 = W_2$

$$R' = \{(a,a), (a,b), (a,c), (b,c)\}$$

PROBLEM

Let R be a relation defined on $A = \{1, 2, 3\}$ such that $(a,b) \in R$ if $a+b$ is even. Find the transitive closure of R. using warshall's algorithm.

sol:

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

$$W_0 = M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

k	In W_{k-1}: Positions of 1's in k^{th} column	In W_{k-1}: Positions of 1's in k^{th} row	Positions of 1's in W_k	W_k
1.	1, 3	1, 3	(1,1) (1,3) (3,1) (3,3)	$W_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

2	2	-	(2, 2)	$W_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
3	1, 3	1, 3	(1,1),(1,3),(3,1), (3,3)	$W_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$\boxed{R' = R}$$

$$R' = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$$

PARTIALLY ORDERED SET (Poset)

Let R be a relation defined on A Then R is said to be a poset

if (i) R is reflexive

(ii) R is antisymmetric (not symmetric)

(iii) R is transitive

PROBLEM

Let R be a relation defined on $A = \{0,1,2,3\}$ such that ~~(a, b) ∈ R if a≤b~~. $(S_1, S_2) \in R$ if $S_1 \subset S_2$ where S_i's are subsets of A and P(A) ($S_i \in P(A)$)

Prove that R is a Poset.

Sol. $S_i \subseteq S_i \ \forall i$

$(S_i, S_i) \in R$

R is reflexing

Let $(S_i, S_j) \in R$

Then $S_i \subset S_j$

$\Rightarrow S_j \not\subset S_i$

R is not symmetric

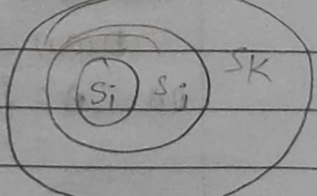
Let $(S_i, S_j) \in R$

$(S_j, S_k) \in R$

$S_i \subset S_j$, $S_j \subset S_k$

$\Rightarrow S_i \subset S_k$

$\Rightarrow (S_i, S_k) \in R$.



GRAPHICAL REPRESENTATION OF A RELATION :-

Let R be a relation defined on A.

To represent R graphically,

each element of A can be considered as a point when an element a is related to b. an arc is drawn from a to b. with an arrow mark. This graph is called a digraph

HASSE DIAGRAM.

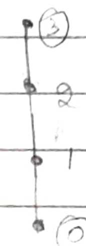
Hasse diagram representing a partial ordering can be obtained from the diagraph by removing all loops that are present due to transitivity and drawing each edge without arrow so that in initial vertex is below its end vertex.

PROBLEM

Draw a diagraph for the following relation R. defined on $A = \{0,1,2,3\}$ $(a,b) \in R$ if $a<b$

$$M_R = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 0 \end{array}$$

HASSE DIAGRAM



Let $R = \{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$

PROBLEM

Draw a Hasse diagram for a relation R defined on $A = \{2,4,5,10,12,20,25\}$ $(a,b) \in R$ if a divides b.

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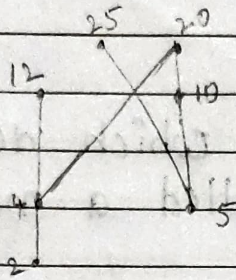
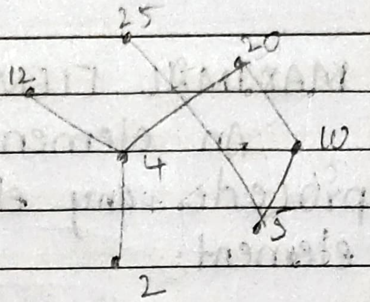
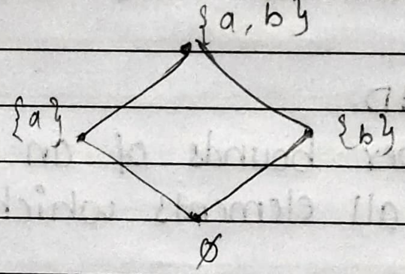




PROBLEM

Draw a Hasse Diagram for a relation R defined on P(A), where A = {a, b}
$(S_i, S_j) \in R$ if $S_i \subset S_j$ where $S_i \in P(A)$

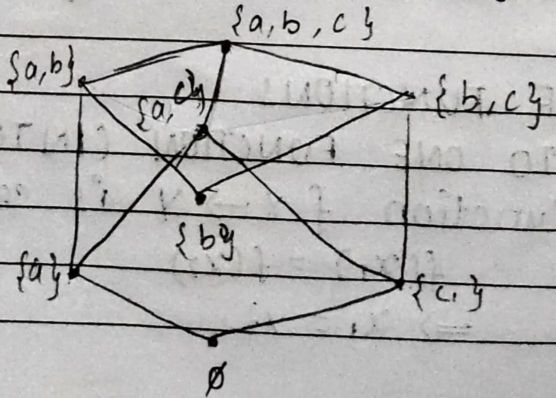
P(A) = { ∅, {a}, {b}, {a, b} }



4 PROBLEM

Draw a Hasse Diagram for a relation R defined on P(A) where A = {a, b, c}
$(S_i, S_j) \in R$ if $S_i \subset S_j$, where $S_i \in P(A)$

=> P(A) = { ∅, {a}, {b}, {c}, {a, b}, {b, c}
{a, c}, {a, b, c} }.



MAXIMAL ELEMENT

An element of a set which does not procede any element is called a maximal element.

MINIMAL ELEMENT

An element of a set which does not succeed any element is called a minimal element.

UPPER BOUND

The upper bounds of an element "a" is set of all elements which precedes "a".

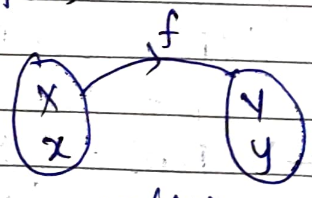
LOWER BOUND.

The lower bounds of an element "a" is the set of all elements which succeeds "a"

FUNCTIONS

A relation f from X to Y is called a function. If for every $x \in X$, there exists a $y \in Y$ such that $y = f(x)$

$f: X \longrightarrow Y$



$y = f(x)$

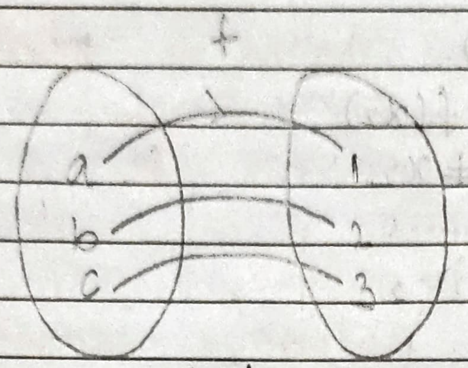
TYPES OF FUNCTIONS

1. ONE TO ONE FUNCTION (INJECTIVE)

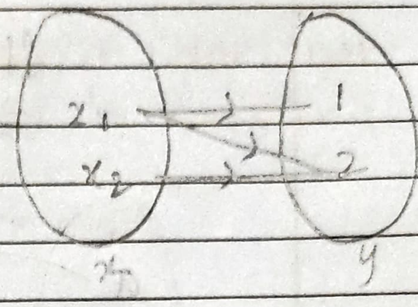
A function $f\ X \to Y$ is called 1-1 if,

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$



1-1



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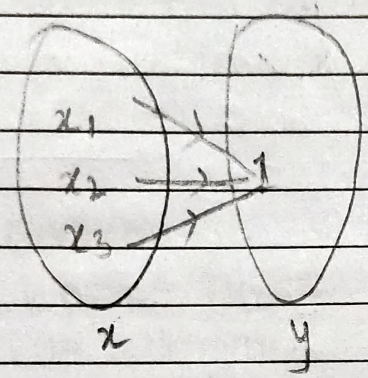
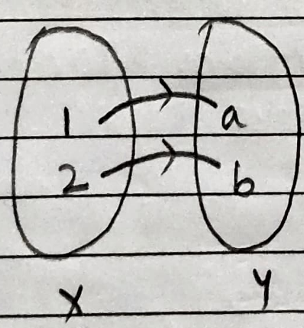


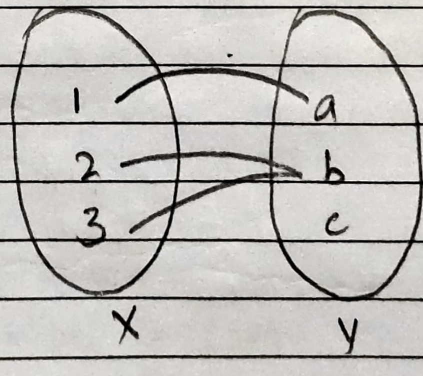
not 1-1

ONTO FUNCTION (surjective)

A function of $X \longrightarrow Y$ is called onto for every $y \in Y$ there exists $x \in X$ such that $y = f(x)$



onto



not onto

DEFINITION

DOMAIN AND RANGE OF A FUNCTION

Let $f: X \longrightarrow Y$ be a function.

Domain of f is X