PROBABILITY & STATISTICS

- By
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UNIT V

STATISTICAL QUALITY CONTROL

- Introduction and Process Control
- Control Charts for X and R
- Control Charts for X and S
- *p* – Chart
- *np* – Chart
- *c* – Chart

INTRODUCTION

- Quality
- Statistical Quality Control
- Quality Control Charts
- Process Control
- Product Control
- Variables
- Attributes

INTRODUCTION

- In these days of tough business competition, it has become essential to maintain the quality of the goods manufactured and market them at reasonable price.
- If the consumers feel satisfied with regard to the quality, price, etc. of the product manufactured by a certain company, it will result in goodwill for the product and in increase in sales.

INTRODUCTION (Continued…)

- If not and if proper attention is not given to the complaints of the consumers regarding quality, the manufacturer cannot push through his products in the market and ultimately he has to quit the market.
- Hence it is important to maintain and improve the quality of the manufactured products for the manufacturer to remain and flourish in his business.

INTRODUCTION (Continued…)

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Quality

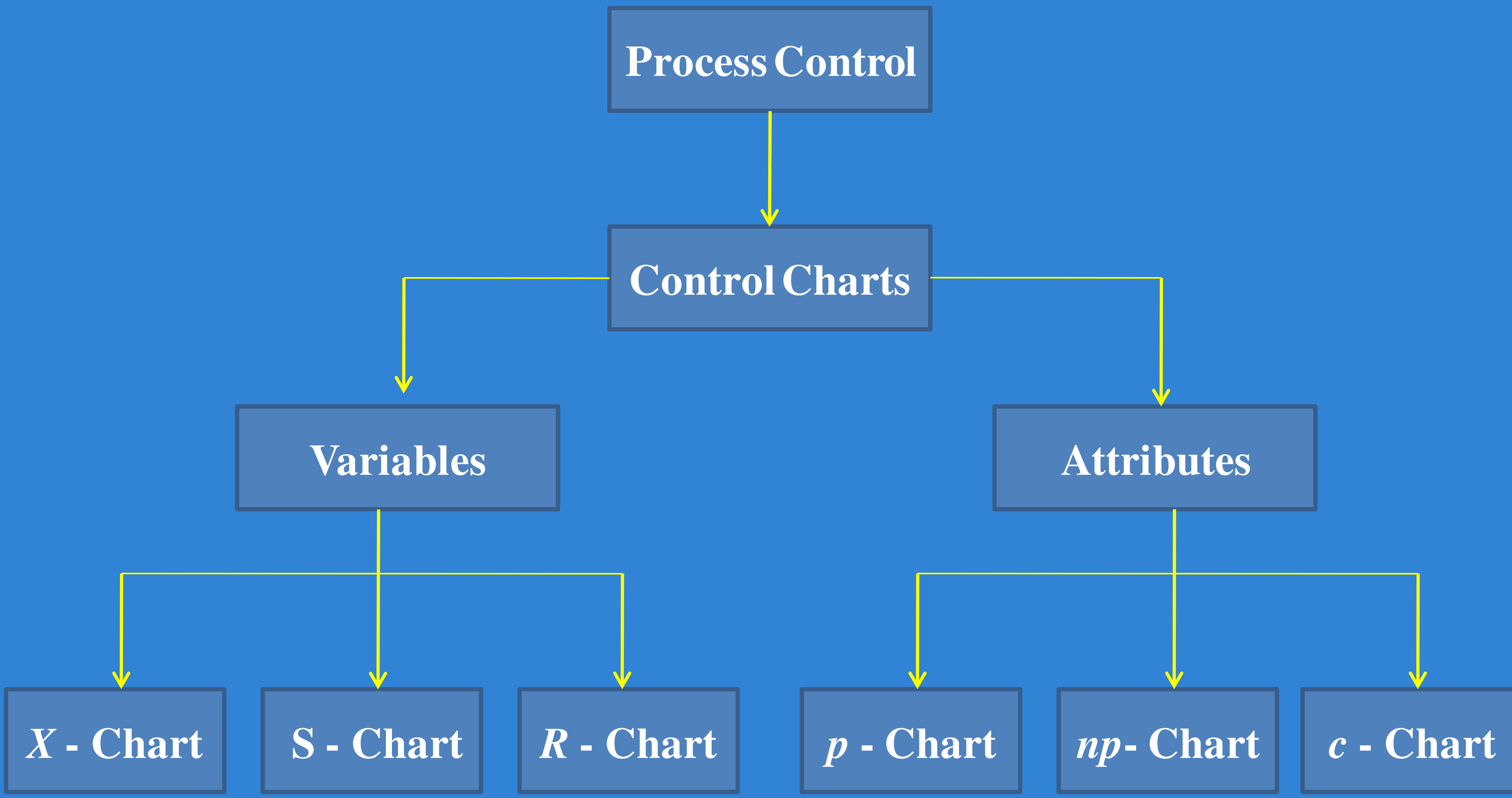
- Goods are said to be of good quality if they satisfy the consumer.
- Goods are said to be of good quality if they meet the expected functional use.
- The quality not only measures the level of satisfaction in meeting the customers needs but also is a function of time period in which it continues to meet customers needs.

INTRODUCTION (Continued…)

Statistical Quality Control (SQC)

- SQC is a statistical method for finding whether the variation in the quality of the product is due to random causes or assignable causes.
- SQC does not involve inspecting each and every item produced for quality standards, but involves inspection of samples of items produced and application of tests of significance.

VARIOUS CHARTS OF QUALITY CONTROL



Process Control

- The control of the quality of the goods while they are in the process of production.
- In process control, we could detect the defects and faults of the process and could correct the errors.

Control Chart

- Control chart is a graphical device mainly used for the study and control of the manufacturing process.
- Control chart is also called shewhart chart.

Types of Control Charts

- Control charts of variables
- Control charts of attributes

Variables

- The quality characteristics of a product that are measurable.

Example

- Diameter of wires
- Life of an electric bulb
- Setting time of concrete

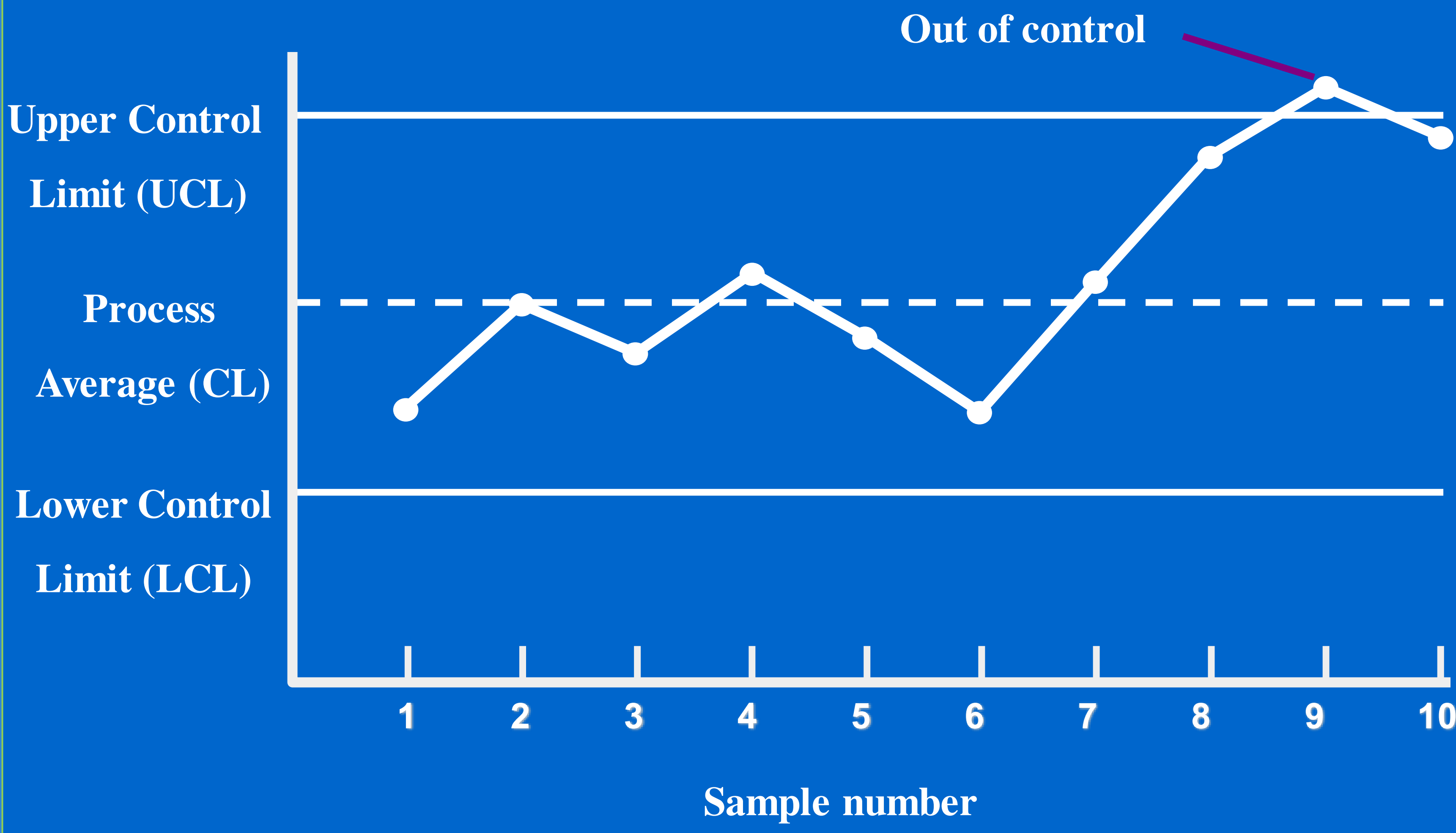
Attributes

- The quality characteristics of a product that are not measurable.

Example

- Visual defects found during inspection
- Coloured threads in white cloth
- Bubbles of air in windscreen
- Clerical errors in an invoice

CONTROL CHART



Advantages of Control Chart

- Control chart, helps us to rectify the faults and errors during the process or even after the process is over.
- By quality control methods, contain and hold the variability in the production process due to chance variation only.
- Any change of process in the production line can be tested very easily.

Advantages of Control Chart (Continued…)

- Warning can be taken when the sample points lie outside warning limits and between action or control limits.
- Since the testing is done by sampling methods based on probability, there is a lot of saving in time and cost.
- Decisions can be taken with more reliability and confidence.
- Also by use of control charts, wastage, expenditure and spoilage can be minimized.

$\bar{X}$ and $R-$ Chart

Problems:

A machine fills boxes with dry cereal . 15 samples of 4 boxes are drawn randomly. The weight of the sampled boxes are shown as follows. Draw the control charts for the sample mean and sample range and determine whether the process is in a state of control.

Sample Number	Weight of Boxes			
	1	2	3	4
1	10.00	10.20	11.30	12.40
2	10.30	10.90	10.70	11.70
3	11.50	10.70	11.40	12.40
4	11.00	11.10	10.70	11.40
5	11.30	11.60	11.90	12.10
6	10.70	11.40	10.70	11.00
7	11.30	11.40	11.10	10.30
8	12.30	12.10	12.70	10.70
9	11.00	13.10	13.10	12.40
10	11.30	12.10	10.70	11.50
11	12.50	11.90	11.80	11.30
12	11.90	12.10	11.60	11.40
13	12.10	11.10	12.10	11.70
14	11.90	12.10	13.10	12.00
15	10.60	11.90	11.70	12.10

Solution:

Sample Number	Weight of Boxes				Mean	Range
	1	2	3	4		
1	10.00	10.20	11.30	12.40	10.98	2.40
2	10.30	10.90	10.70	11.70	10.90	1.40
3	11.50	10.70	11.40	12.40	11.50	1.70
4	11.00	11.10	10.70	11.40	11.05	0.70
5	11.30	11.60	11.90	12.10	11.73	0.80
6	10.70	11.40	10.70	11.00	10.95	0.70
7	11.30	11.40	11.10	10.30	11.03	1.10
8	12.30	12.10	12.70	10.70	11.95	2.00
9	11.00	13.10	13.10	12.40	12.40	2.10
10	11.30	12.10	10.70	11.50	11.40	1.40
11	12.50	11.90	11.80	11.30	11.88	1.20
12	11.90	12.10	11.60	11.40	11.75	0.70
13	12.10	11.10	12.10	11.70	11.75	1.00
14	11.90	12.10	13.10	12.00	12.28	1.20
15	10.60	11.90	11.70	12.10	11.58	1.50
Average					11.54	1.33

$$\bar{\bar{X}} = \frac{1}{n}\sum \bar{X}_i = \frac{1}{15}(10.98 + 10.90 + \ldots + 11.58) = 11.54$$

$$\bar{R} = \frac{1}{n}\sum R_i = \frac{1}{15}(2.4 + 1.4 + \ldots + 1.5) = 1.33$$

From the table of control chart constants, for the sample size n = 4,

$$A_2 = 0.729$$

$$D_3 = 0$$

$$D_4 = 2.282$$

Control Limits for $\bar{X}$ - Chart

$$CL = \bar{\bar{X}} = 11.54$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 11.54 - 0.729(1.33) = 10.59$$

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 11.54 + 0.729(1.33) = 12.53$$

Control Limits for $\bar{R}$ - Chart

$$CL = \bar{R} = 1.33$$

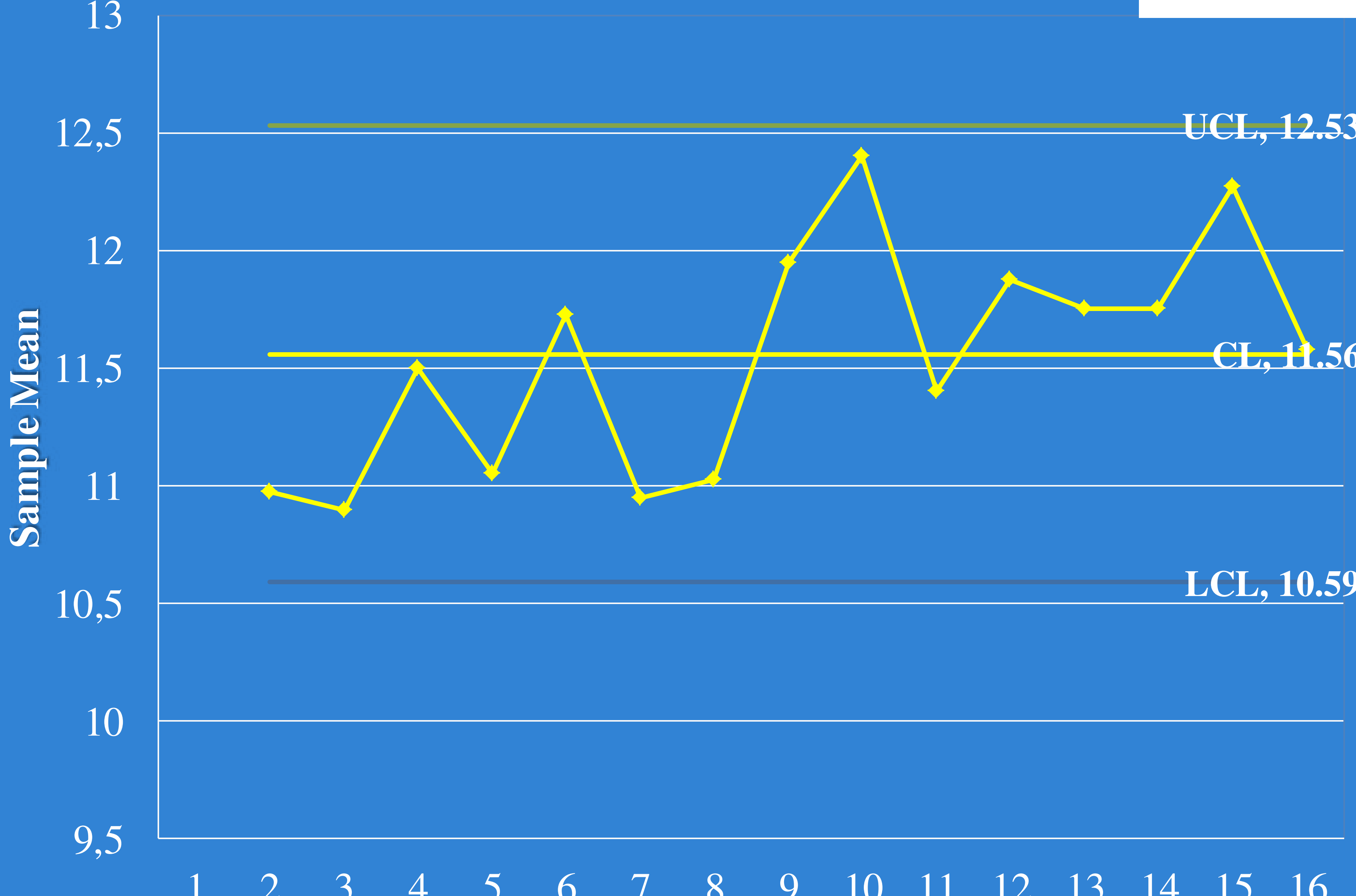
$$LCL = D_3\bar{R} = 0$$

$$UCL = D_4\bar{R} = 2.282(1.33) = 3.04$$

Conclusion

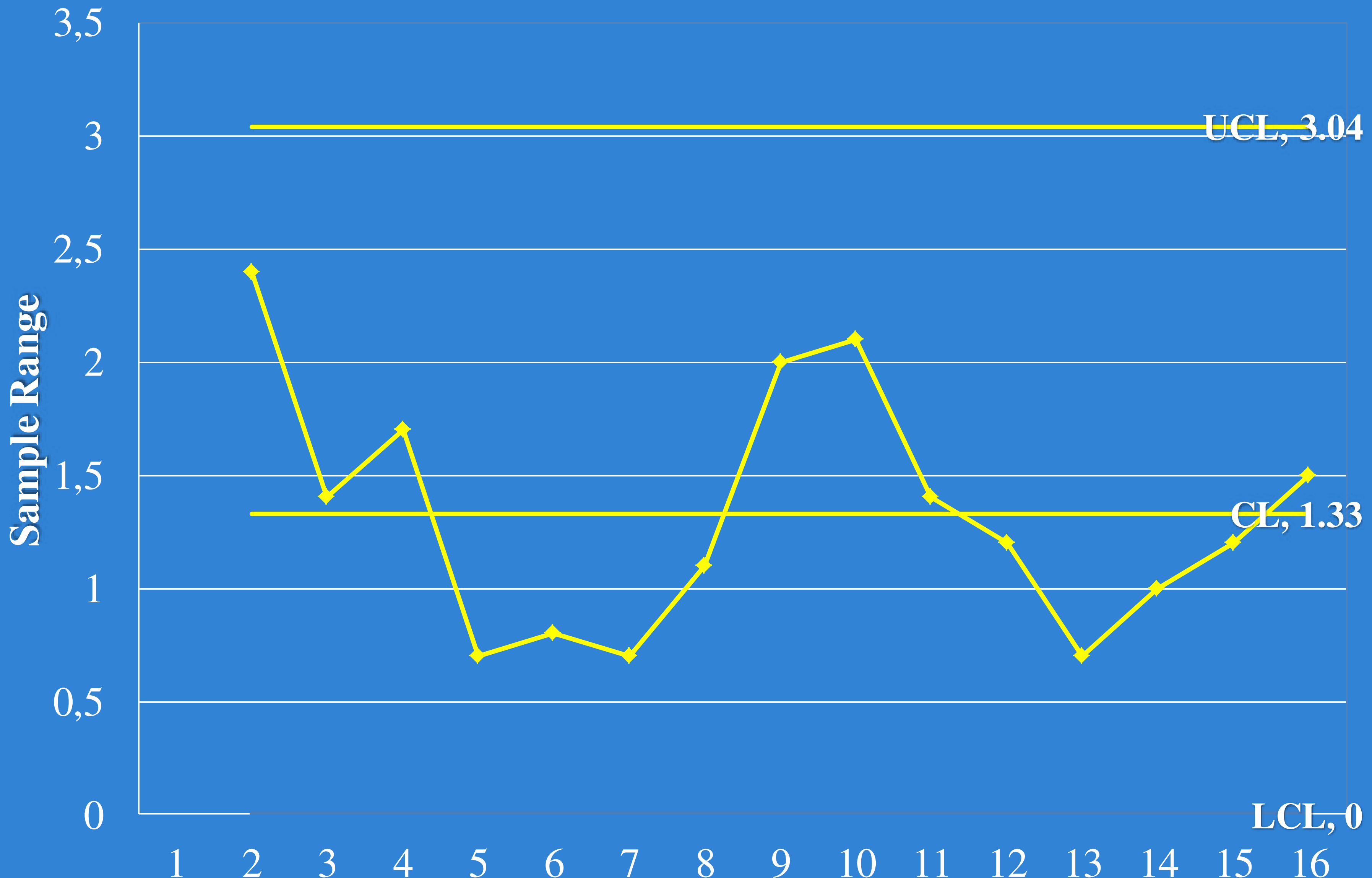
Since all the sample points lie within upper and lower line both X bar and R chart, the process in under control.

X-Bar Chart



Sample Mean
13
12,5
12
11,5
11
10,5
10
9,5
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
UCL, 12.53
CL, 11.56
LCL, 10.59

R- Chart


3,5
3
2,5
2
1,5
1
0,5
0
Sample Range
UCL, 3.04
CL, 1.33
LCL, 0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

$\bar{X}$ *and S Chart*

Problem

The following data give the coded measurements of 10 samples each of size 5, drawn from the production process at intervals 1 hour. Calculate the sample mean and S.D's and draw the control chart for $\bar{X}$ *and S Chart*.

Sample Number	Coded Measurements(X)				
	1	2	3	4	5
1	9	15	14	9	13
2	10	11	13	6	10
3	10	13	8	12	7
4	8	13	11	10	13
5	7	9	10	4	5
6	12	15	7	16	10
7	9	9	9	13	5
8	15	15	10	13	17
9	10	13	14	7	11
10	16	14	12	14	14

Solution

We first compute the mean and S.D for each sample

Sample Number	Coded Measurements(X)					Mean	S.D
	1	2	3	4	5		
1	9	15	14	9	13	12	2.5
2	10	11	13	6	10	10	2.3
3	10	13	8	12	7	10	2.3
4	8	13	11	10	13	11	1.9
5	7	9	10	4	5	7	2.3
6	12	15	7	16	10	12	3.3
7	9	9	9	13	5	9	2.5
8	15	15	10	13	17	14	2.4
9	10	13	14	7	11	11	2.4
10	16	14	12	14	14	14	1.3
Average						11	2.32

Now

$$\bar{\bar{X}} = \frac{1}{n}\sum \bar{X}_i = \frac{1}{10}(12+10+10+\ldots+14) = 11$$

$$\bar{s} = \frac{1}{n}\sum s_i = \frac{1}{10}(2.5+2.3+2.3+\ldots+1.3) = 2.32$$

From the table of control chart constants, for the sample size n=4, we have

$$A_1 = 1.596$$

$$B_3 = 0$$

$$B_4 = 2.089$$

Control Limits for $\bar{X}$ - Chart

$$CL = \bar{\bar{X}} = 11$$

$$LCL = \bar{\bar{X}} - A_1\sqrt{\frac{n}{n-1}}\,\bar{s} = 11 - (1.596)\sqrt{\frac{5}{4}}(2.59) = 6.86$$

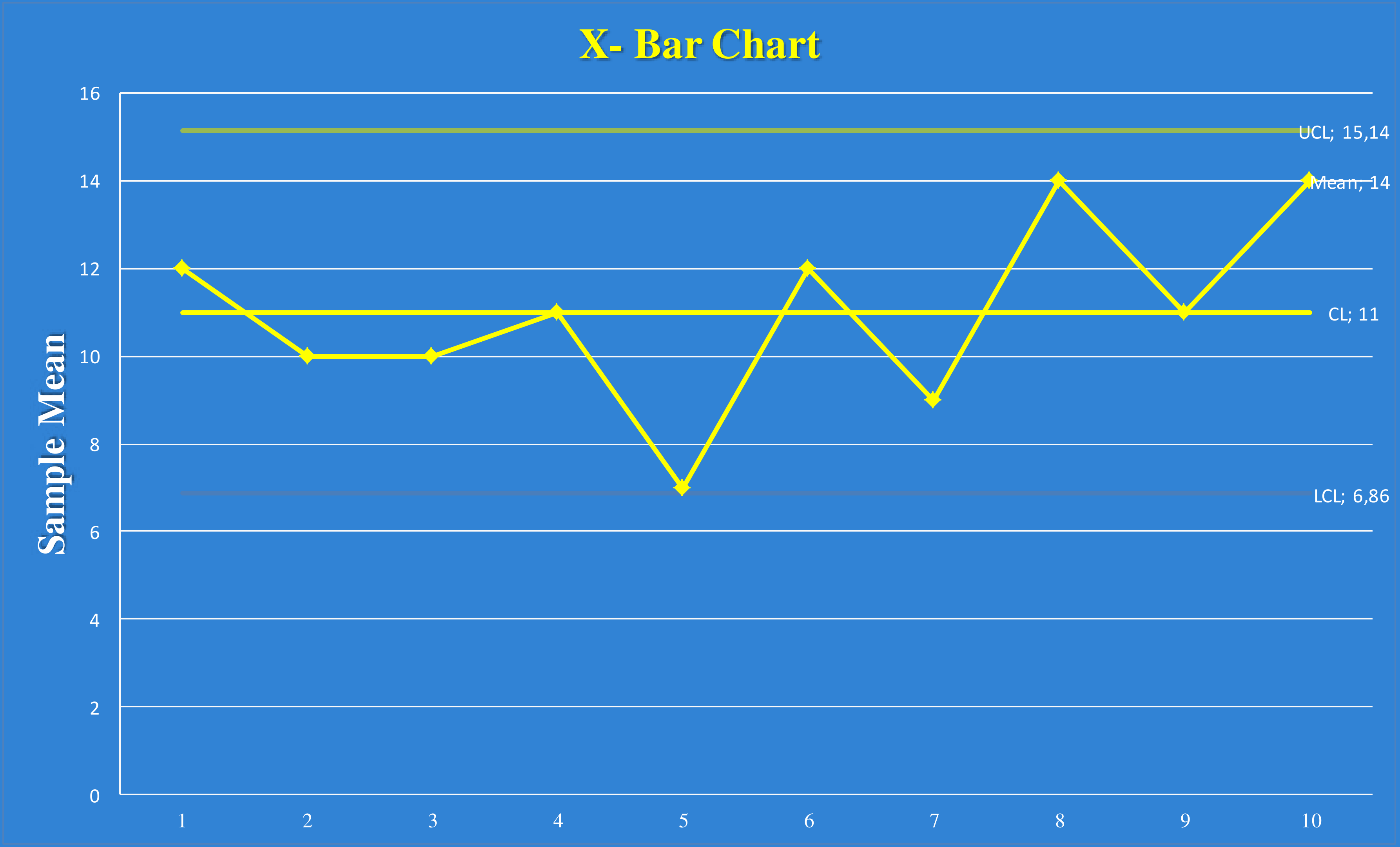
$$UCL = \bar{\bar{X}} + A_1\sqrt{\frac{n}{n-1}}\,\bar{s} = 11 + (1.596)\sqrt{\frac{5}{4}}(2.32) = 15.14$$

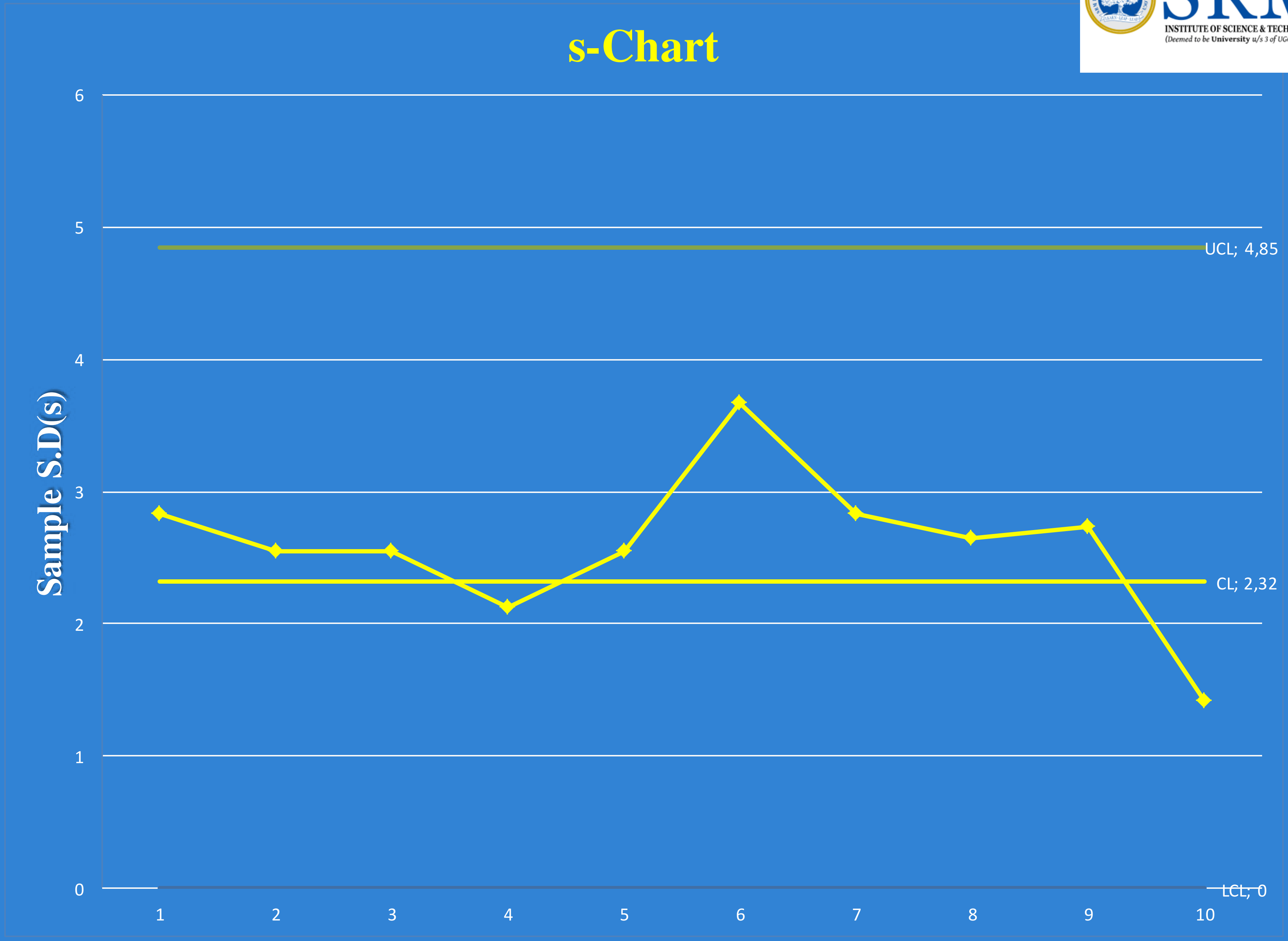
Control Limits for s - Chart

$$CL = \bar{s} = 2.32$$

$$LCL = B_3\bar{s} = 0$$

$$UCL = B_4\bar{s} = 2.089(2.32) = 4.85$$


X- Bar Chart
Sample Mean
16
14
12
10
8
6
4
2
0
UCL; 15,14
Mean; 14
CL; 11
LCL; 6,86
1
2
3
4
5
6
7
8
9
10


s-Chart
Sample S.D(s)
6
5
4
3
2
1
0
UCL; 4,85
CL; 2,32
LCL; 0
1
2
3
4
5
6
7
8
9
10

Conclusion

The given mean values lies between 6.86 and 15.14 and the given S.D values lies between 0 and 4.85. Hence the process is under control with respect to average and variability.

$p-Chart$

Problems

15 samples of 200 items each were drawn from the output of a process. The number of defective items in the samples are given below, prepare a control chart for the fraction defective and comment on the state of control

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defective	12	15	10	8	19	15	17	11	13	20	10	8	9	5	8

Solution:

We first compute the fraction defectives

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defective (np)	12	15	10	8	19	15	17	11	13	20	10	8	9	5	8
Fraction defectives	0.060	0.075	0.050	0.040	0.095	0.075	0.085	0.055	0.065	0.100	0.050	0.040	0.045	0.025	0.040

Now

$$\sum np = 12+15+10+\ldots+8) = 180$$

$$n\overline{p} = \frac{1}{n}\sum np = \frac{180}{15} = 12$$

$$\overline{p} = \frac{12}{200} \ (\because \text{each sample contains 200 items})$$

$$= 0.06$$

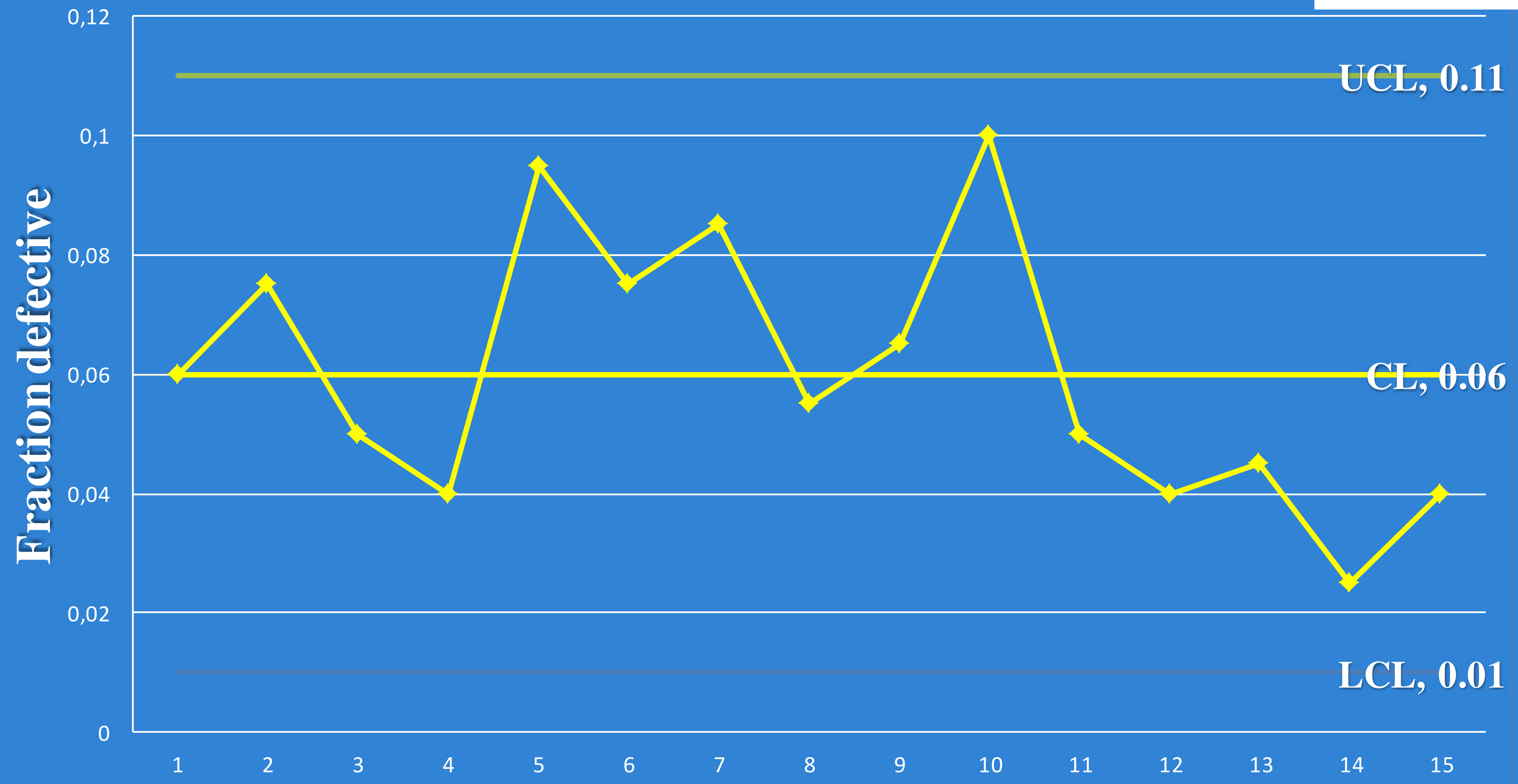
For the p - Chart

$$CL = \overline{p} = 0.06$$

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.06 - (3)\sqrt{\frac{0.06\times0.945}{200}} = 0.01$$

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.06 + (3)\sqrt{\frac{0.06\times0.945}{200}} = 0.11$$

p-Chart



Conclusion

Since all the sample points lies between the LCL and UCL lines, the process is under control.

$np - Chart$

Problems

In a factory producing spark plugs, the number of defectives found in the inspection of 15 lots of 100 each is given below. Draw the control chart for the number of defectives and comment on the state of control.

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defective	5	10	12	8	6	4	6	3	4	5	4	7	9	3	4

Now

$$\sum np = 5+10+12+\ldots+3+4) = 90$$

$$n\overline{p} = \frac{1}{N}\sum np = \frac{90}{15} = 6$$

$$\overline{p} = \frac{6}{100} \; (\because \text{each sample contains } 100 \text{ items})$$

$$= 0.06$$

For the np - Chart

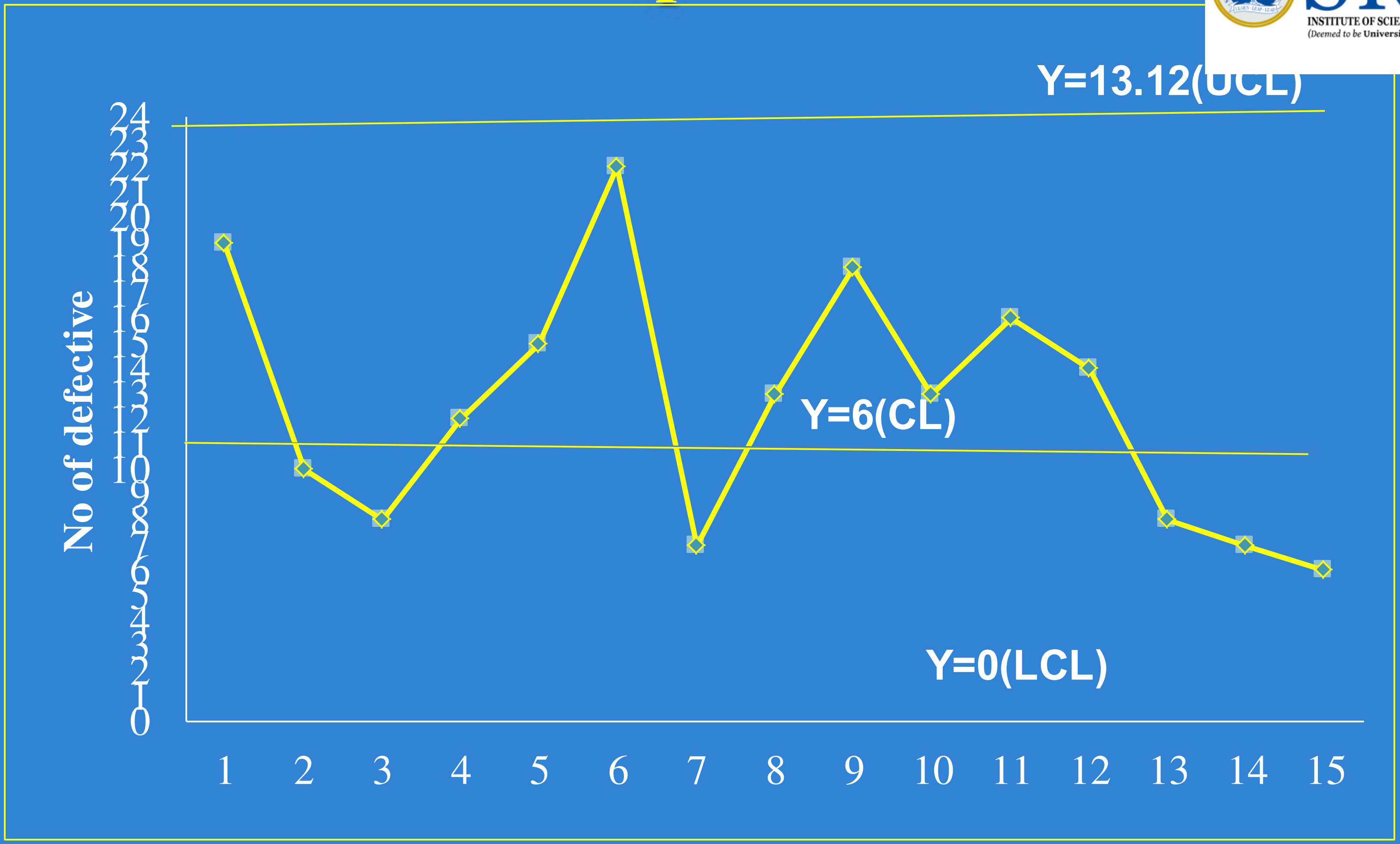
$$CL = n\overline{p} = 6$$

$$LCL = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} = 6 - (3)\sqrt{6\times 0.945} = -1.12$$

$LCL = 0$ (cannot be nagative)

$$UCL = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} = 6 + (3)\sqrt{6\times 0.945} = 13.12$$

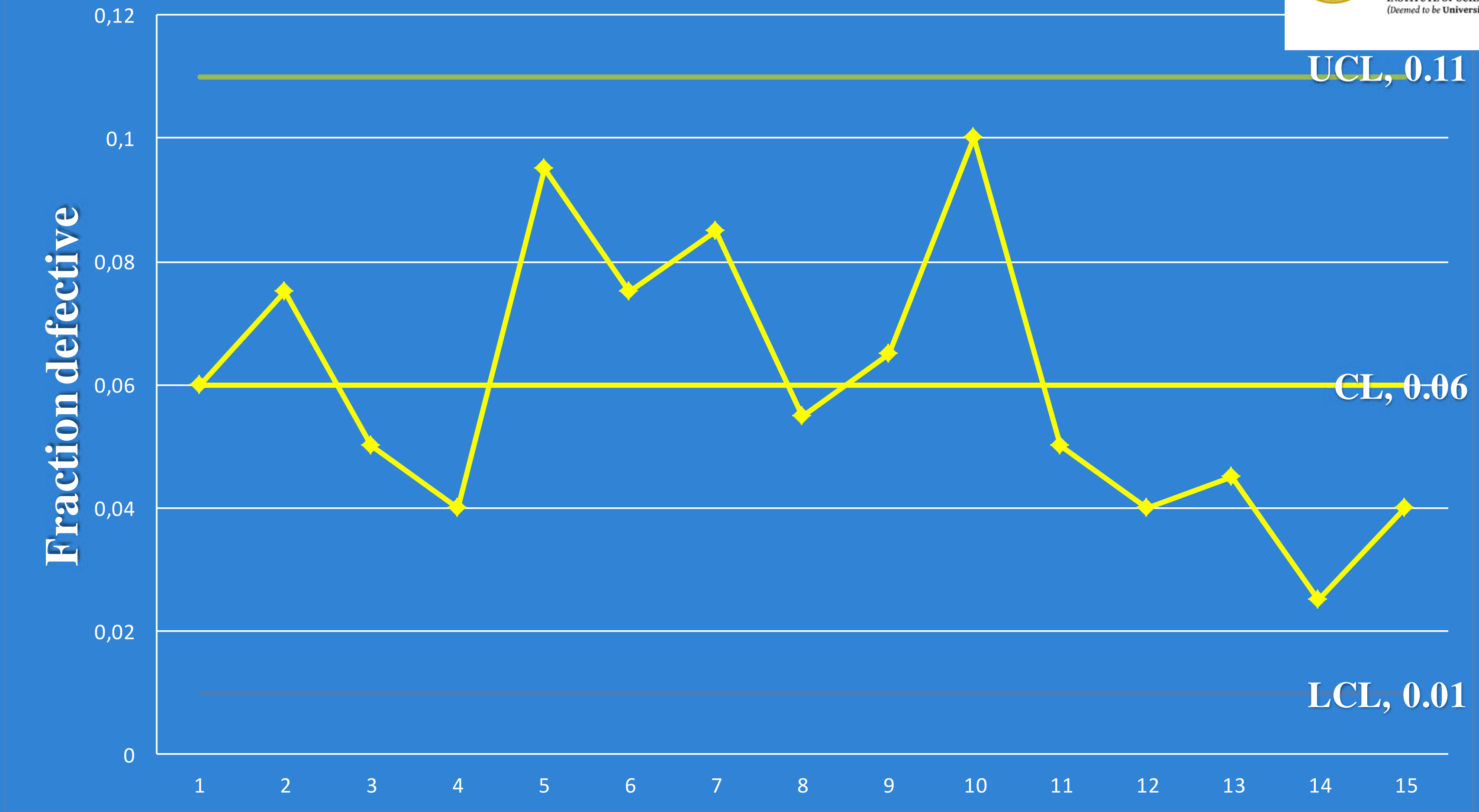
np-Chart



Conclusion

Since all the sample points lies between the LCL and UCL lines, the process is under control.

np-Chart



Conclusion

Since all the sample points lies between the LCL and UCL lines, the process is under control.

$c - Chart$

Problems

A plant produces paper for newsprint and rolls of paper are inspected for defects. The results of inspection of 20 rolls of papers are given below. Draw the c chart and comment on the state of control.

Roll No.	1	2	3	4	5	6	7	8	9	10
No. of defective	19	10	8	12	15	22	7	13	18	13
Roll No.	11	12	13	14	15	16	17	18	19	20
No. of defective	16	14	8	7	6	4	5	6	8	9

Now

$$\sum c_i = 220$$

$$\bar{c} = \frac{1}{N}\sum c_i = \frac{220}{20} = 11$$

For the c - Chart

$$CL = \bar{c} = 11$$

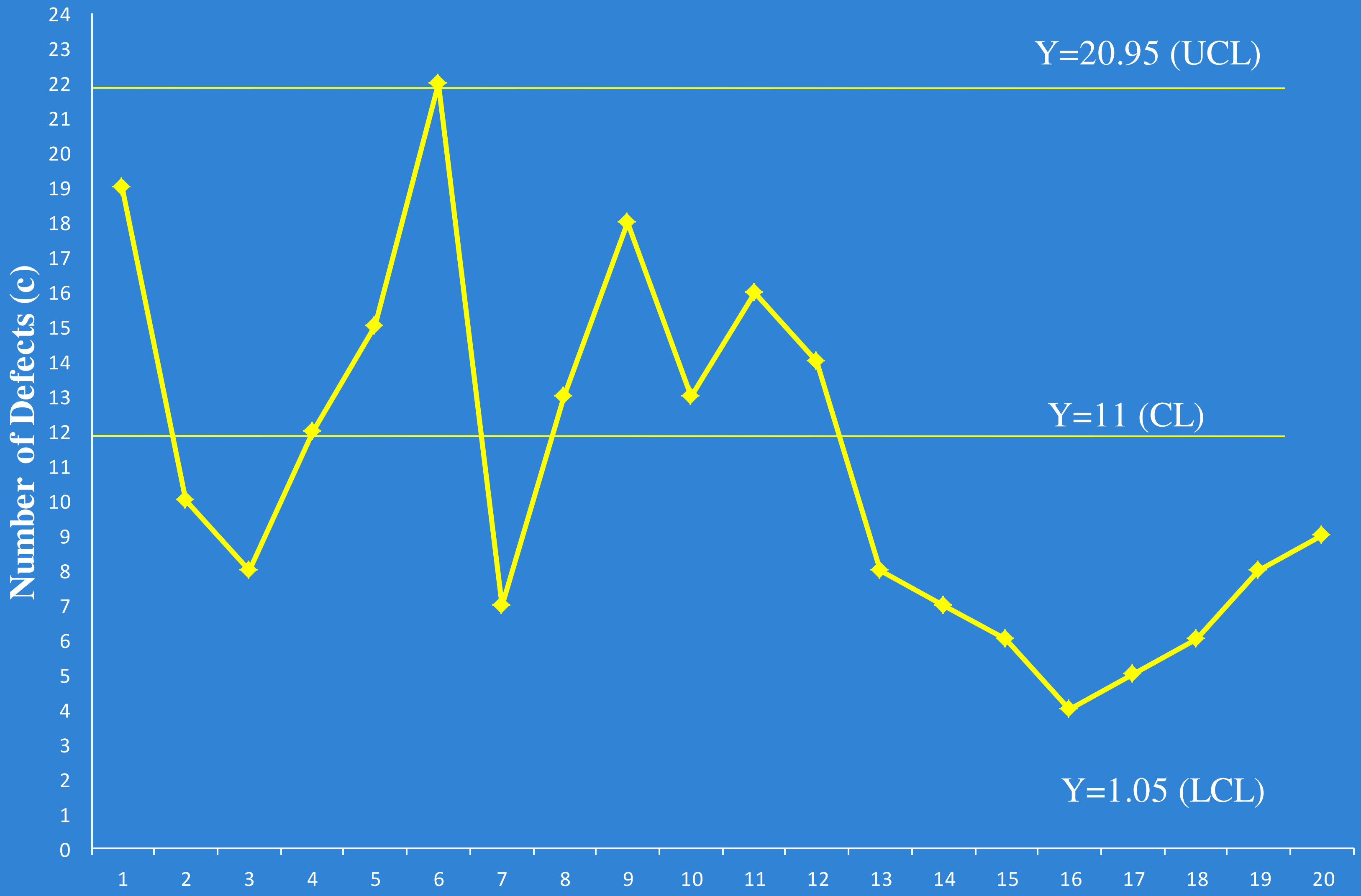
$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 11 - (3)\sqrt{11} = 1.05$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 11 + (3)\sqrt{11} = 20.95$$

Conclusion

Since one point falls outside the control lines the process is out of control.

c chart



Y=20.95 (UCL)
Y=11 (CL)
Y=1.05 (LCL)
Number of Defects (c)
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Thank you