UNIT - II

/ /

10/08/23

PERMUTATIONS

PERMUTATIONS

An ordered arrangement of r elements of a set containing n distinct elements is called an r-permutation of n elements

It is denoted by ^nP_r (or) $P(n,r)$

$$P(n,r) = \frac{n!}{(n-r)!}$$

EX :

Out of 3 elements

2 elements can be Permuted in $\frac{3!}{(3-2)!} = \frac{6}{1} = 6$

COMBINATIONS

An unordered arrangements of r-elements of a set containing n distinct elements is called r-combination of n elements. It is denoted by $C(n,r)$ or ^nC_r

$$C(n,r) = \frac{n!}{n!(n-r)!}$$

Out of 3 elements

2 can be combined in $\frac{3!}{2!\,1!} = \frac{6}{2} = 3.$

THEOREM.

If there are n different elements, no. of permutations of n elements is $n!$

$(P(n,n) = n!)$

THEOREM

There no. of different Permutations of

/ /

n objects which include n_1 identical objects of Type 1, n_2 identical objects of Type 2 - - - n_k identical objects of Type k is

$$\frac{n!}{n_1! \times n_2! \times \cdots \times n_k!}$$

PROBLEM

How many permutations of letters A, B, C, D, E are there?

Sol. No. of Permutations = 5!

PROBLEM

How many Permutations of letters A, B, C, A, F, C are there

Sol. No. of Permutations = $\frac{6!}{2!1!2!1!} = 180$

PROBLEM

How many permutation of the letters A, B, C, D, E, F, G containing the string BCD.

Sol Treat BCD as a single letter

We have 5 elements.

BCD, A, E, F, G

No. of permutations = 5!

PROBLEM

How many positive integer n can be termed using the digits 3, 4, 4, 5, 5, 6, 7 if n has to exceed 5,00,000.

/ /

PROBLEM

Assuming that repetitions are not.

(a) Permitted how many four digit numbers can be formed from six digits 1, 2, 3, 5, 7, 8 ?

(b) How many of these numbers are less than 4000?

(c) How many of these numbers are even?

(d) How many of these numbers are odd?

Sol: (a) Forming 4 digit number using ~~six~~ six given digits. is nothing but 4-Permutation of 6 objects.

NO. of numbers $= P(6,4) = \frac{6!}{2!} = 360.$

$P(n,r) = \frac{n!}{(n-r)!}$

(b) The number should start with 3 or 2 or 1

The remaining 3 places can be filled using 1, 2, 5, 7, 8.

This can be done in $P(5,3)$

NO. of numbers $= 3 \times P(5,3)$

$= 3 \times 60.$

$= 180.$

(c) The number should end with 2 or 8

The remaining 3 places can be filled. 5 digits.

NO. of numbers $= 2 \times P(5,3) = 2 \times 60$

$= 120.$

(d) The number should end with 1, 3, 5, 7.

The remaining 3 places can be filled with 5 digits

NO. of numbers $= 4 \times P(5,3) = 4 \times 60$

$= 240.$

_/_/_

PROBLEM

How many bit strings of length 10 contain
(a) Exactly four 1's
(b) atmost four 1's
(c) atleast four 1's
(d) an equal number of 0's and 1's ?

Sol:

(a) A bit string of length 10 can be considered to have 10 positions.

No. of strings $= C(10,4) = \frac{10!}{4!\,6!} = 210.$

(b) 10 positions should be filled with no. 1 or one 1 and 9 zeros.
(or) two 1's and eight ~~zero~~ 0's (or) 3 one's and 7 zeros (or) 4 one's and 6 zero's.

No. of ways $= {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 1 + 10 + 45 + 120 + 210$

$= 3$

(c) No. of ways $= {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8$

$+ {}^{10}C_9 + {}^{10}C_{10}.$

$= 848.$

(d) No. of ways $= {}^{10}C_5 = 252$

NOTE:

Let $x_1 + x_2 + \cdots x_n = r$

then the no. of non-negative solutions is $C(n+r-1, r)$ $\quad x_i \geq 0.$

/ /

PROBLEM

Determine the number of non negative solution of the equation.

$$x_1 + x_2 + x_3 + x_4 = 32$$

$r = 32 \quad n = 4$

$$c(4+32-1, 32) = c(35, 32) = \frac{35!}{32!\,3!}$$

$$= \frac{\cancel{33}^{11} \times \cancel{34}^{17} \times 35}{\cancel{3} \times \cancel{2}}$$

$$= 6545$$

PROBLEM.

Solve $x_1 + x_2 + x_3 + x_4 = 32$ subject to the condition that $x_1, x_2 \geq 5$ and $x_3, x_4 \geq 7$

$x_1 \geq 5 \quad x_2 \geq 5 \quad x_3 \geq 7 \quad x_4 \geq 7$

$x_1 - 5 \geq 0 \quad x_2 - 5 \geq 0 \quad x_3 - 7 \geq 0 \quad x_4 - 7 \geq 0$

Let $u_1 = x_1 - 5 \quad u_2 = x_2 - 5 \quad u_3 = x_3 - 7 \quad u_4 = x_4 - 7$

$$x_1 - 5 + x_2 - 5 + x_3 - 7 + x_4 - 7 = 32 - 5 - 5 - 7 - 7$$

$$u_1 + u_2 + u_3 + u_4 = 8.$$

No. of required solutions $= (4+8-1, 8) = c(11, 8)$

$= 165.$

PROBLEM

Find the number of non-negative solution for $x_1 + x_2 + x_3 + x_5 + x_6 < 10$.

We convert the inequality into equality by adding some variable $x_7 > 0$

(or)

$x_7 \geq 1 \quad x_7 - 1 \geq 0$

$x_1+x_2+x_3+x_4+x_5+x_6+x_7=10$ $x_i \geq 0 \ \forall \ i=1$ to 6

$x_1+x_2+x_3+x_4+x_5+x_6+x_7-1=9$

Let $x_i=u_i \ \forall \ i=1$ to 6

$x_7-1=u_7$

$\therefore u_1+u_2+u_3+u_4+u_5+u_6+u_7=9$

Here $u_i \geq 0 \ \forall \ i=1$ to 7

No. of Solutions $= C(7+9-1,9)$

$= C(15,9)$

$= 5005$.

PROBLEM

How many positive integers less than 1000000 have the sum of their digits equal to 19

(or)

Solve for integer solution.

$x_1+x_2+x_3+x_4+x_5+x_6=19$.

$n=6 \quad r=19$

$C(6+19-1,19) = C(24,19) = \frac{24!}{19!\,5!}$

$= \frac{24 \times 23 \times 22 \times 21 \times 20}{5 \times 4 \times 3 \times 2}$

$= 42,504$

PIGEON PRINCIPLE

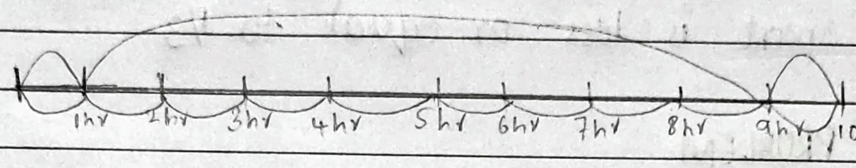
If n Pigeons are accomodated in m Pigeonholes and $n>m$ then one of the pigeoholes must contain atleast $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ pigeons.

PROBLEM

A man covered a total distance of 45 km

__/__/__

for 10 hrs. It is known that he covered 6km in the first hour and only 3 km in the last thour show that he must have covered atleast 9km within a certain period of 2 consecutive hours.



He covered 6+3=9 KM in thal first and last hour.

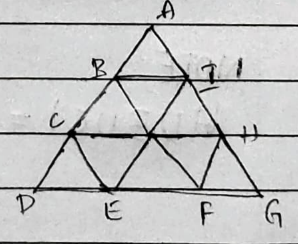
He must have covered the remaining 36km during the period from 2^{nd} to nineth hour. the The possible consecutive 2 hours are 2 and 3, 4 and 5, 6 and 7 and 8 and 9.

No. of pigeons = 36km = n

No. of Pigeons holes = 4 = m.

PROBLEM

If we select 10 points in the interior of an equilateral triangle of side 1 (unit) show that there must be atleast two points whose distance apart is less than or equal to 1/3.



Let ADG be the given equilateral Δ^{le}

The pair of points (B,C), (E,F) and (I,H) are the points of trisection AD, DG and AG respectively

Then we have 9 equilateral Δ^{les} with side 1/3 unit.

No. of pigeons = no. of points = 10 = n.

No. of Pigeonhole = no. of subtriangles = 9 = m

$n > m$

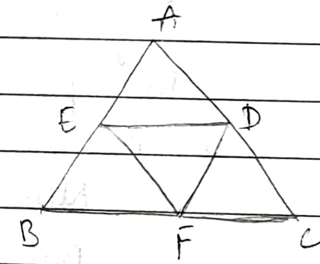
$$\left\lceil \frac{n-1}{m} \right\rceil + 1 = \left\lceil \frac{10-1}{9} \right\rceil + 1$$

$$= 1 + 1 = 2$$

There must be atleast two points whose apart is less or equal to 1/3.

PROBLEM

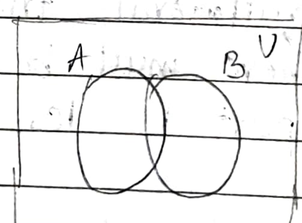
Of any 5 points chosen within an equilateral Δ^{le} whose sides are of pegion, 1unit. Prove that atleast two points are within a distance of 1/2 and of each other



PRINCPLE OF INCLUSION AND EXCLUSION

If A and B are finite subsets of a finite universal set U then

$|A \cup B| = |A| + |B| - |A \cap B|$



NOTE:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

PROBLEM

There are 250 students in an Engineering collage of these 188 have taken a course in Fortran, 100 have taken a course in C and 35 have taken a course in Java. Further

88 have taken courses in both in a and Java.
Fortran and C. 23 have taken courses in both in C and Java. 29 have taken courses in both Fortran and Java. If 19 of those students have taken all 3 courses. How many of these 250 students have not taken a course in any of these 3 courses.

Let A be the set of students who have taken Fotran.

Let B be the set of students who have taken C.

Let C be the set of students who have taken Java.

$|A| = 188$ $\quad |A \cap B| = 88$ $\quad |A \cap B \cap C| = 19.$

$|B| = 100$ $\quad |A \cap C| = 23$

$|C| = 35$ $\quad |A \cap C| = 29$

Now

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 188 + 100 + 35 - 88 - 23 - 29 + 19$$

$$= 202$$

No. of students who have not taken any course $= 250 - 202 = 48.$