

PART-C

Q.2
(i)

$$(D^2 + 1)y = \tan x$$

eg. auxiliary Equation is $m^2 + 1 = 0$

$$m = \pm i \quad \alpha = 0, \beta = 1$$

~~Complementary function~~ $C.F. = e^{\alpha x} (C_1 \cos \beta x + C_2 \sin \beta x)$

$$\begin{aligned} f_1(x) &= \cos x & f_1'(x) &= -\sin x & F(x) &= \tan x \\ f_2(x) &= \sin x & f_2'(x) &= \cos x \end{aligned}$$

$$\omega = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$\begin{aligned} &= \cos^2 x - (-\sin^2 x) \\ &= \cos^2 x + \sin^2 x \\ &= 1 \end{aligned}$$

$$P = \int \frac{f_2}{\omega} F(x) dx$$

$$= \int \frac{\sin x}{\cos x} dx \quad \begin{aligned} \cos x &= t \\ -\sin x dx &= dt \end{aligned}$$

$$= \int -t^2 dt = -\frac{t^3}{3} = -\frac{\sin^3 x}{3}$$

PONNURI ANIRUPDHAN

RA2112704010015

$$\begin{aligned} Q &= \int \frac{\cos x \times \sin x}{\cos x} dx \\ &= \int \sin x dx \\ &= -\cos x \end{aligned}$$

$$\begin{aligned} P.I &= P.f_1 + Q.f_2 \\ &= \left[\frac{\sin^3 x \cos x}{3} + \cos x \sin x \right] \end{aligned}$$

$$y = C_1 \cos x + C_2 \sin x - \frac{\sin^3 x \cos x}{3} - \cos x \sin x$$

Part - B

Q26

$$u = x^2 + y^2$$

$$x = at^2$$

$$y = 2at$$

$$u = a^2 t^4 + 4a^2 t^2$$

$$\frac{du}{dt} = \frac{d}{dt} (a^2 t^4 + 4a^2 t^2)$$

$$= 4a^2 t^3 + 8a^2 t$$

$$= 4a^2 t [t^2 + 2]$$

Q25

$$(x^2 D^2 - xD + 1) y = 0$$

$$\log(x) = z$$

$$D' = \frac{d}{dz}$$

$$xD = D'$$
$$x^2 D^2 = D'(D'-1)$$

$$((D')^2 - D' - D' + 1) y = 0$$

$$((D')^2 - 2D' + 1) y = 0$$

The required auxiliary eqn is $m^2 - 2m + 1 = 0$

$$(m-1)^2 = 0$$

$$m = 1$$

$$C.F. = (C_1 + C_2 x) e^x$$

Q24

$$(D^2 - 4D + 4) y = \cos 2x$$

The required auxiliary eq is $m^2 - 4m + 4 = 0$

$$(m-2)^2 = 0$$

$$m = 2$$

$$C.F. = (C_1 + C_2 x) e^x$$

$$P.I = \frac{\cos 2x}{D^2 - 4D + 4}$$

$$a = 2$$

$$a^2 = 4$$

$$-D^2 = a^2$$

$$D^2 = -4$$

$$= \frac{\cos 2x}{-4 + 4D + 4}$$

$$= \frac{\cos 2x}{-4D}$$

$$= \frac{D(\cos 2x)}{-4D^2}$$

$$D^2 = -4$$

$$= \frac{-2 \sin 2x}{16}$$

$$= \frac{-\sin 2x}{8}$$

$$y = (C_1 + C_2 x) e^x - \frac{\sin 2x}{8}$$

Q23

$$(D^2 + 6D + 9)y = e^{-3x} x$$

auxiliary Equation is $m^2 + 6m + 9 = 0$

$$(m+3)^2 = 0$$

$$m = -3$$

$$C.P = (C_1 + C_2 x) e^{-3x}$$

PONNULI ANURODHANA
RA 241204010015

Page No.	
Date	

$$u = 2xy$$

$$\frac{\partial u}{\partial x} = 2y \quad \frac{\partial u}{\partial y} = 2x$$

$$v = x^2 - y^2$$

$$\frac{\partial u}{\partial x} = 2x \quad \frac{\partial u}{\partial y} = -2y$$

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix}$$

$$= -4y^2 - 4x^2$$

$$= -4(x^2 + y^2)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$= -4(r^2 \cos^2 \theta + r^2 \sin^2 \theta)$$

$$= -4r^2$$