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# Trotting Gait Planning and Implementation for a Little Quadruped Robot

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**Abstract.** This paper presents a trotting pattern generation approach and online control strategy for a little quadruped robot. The trotting gait is scheduled based on the composite cycloid method in order to improve the stability of quadruped robot. The efficiency and performance of the proposed methods are verified by simulations and experiments by means of the little quadruped robot constructed by our laboratory. The experiment results show that the average speed of the little quadruped robot on even terrain is 0.1m/s and the proposed methods are suitable and simple in terms of controlling.

**Keywords:** Trot, Gait planning, Quadruped, Robot.

## 1 Introduction

Approximately half of the earth's land surface is inaccessible to either wheeled or tracked robots. Otherwise legged animals can move over most of earth's terrain, the quadruped robots are optimal for a multi-legged robots, because of the least complex mechanical design and a better stability of configuration [1].

The gait design and control methods for quadruped robots have been studied for several decades. The systematic planning based on mathematical analysis through forward and inverse kinematics equations and the incremental learning is two gait design methods commonly [2]. The gaits of quadruped robots are classified into static gaits and dynamical gaits. In dynamical gaits, Trot is the most energy efficient gait, which decouples pairs of legs and makes control simpler. And the research of quadruped trot will lead to understanding other dynamical gaits, such as pacing and galloping [3]. So, the focus of this paper is on trotting gait generation method and simple software control strategy of quadruped robots. As the most available locomotion pattern, the trotting pattern is generated so that the quadruped robot moves with a fast speed and stable body attitude.

The position control and force control are two main control methods for quadruped robots. The dynamical model of quadruped robots will be needed in the force control method, the dynamical model is difficult to obtain for the quadruped robot with many degrees of freedom. For decreasing the control complexity of quadruped robot, a simple control schedule is proposed through soft programming in Visual C++. The soft controlling platform controls the electronics rudder servos directly. Therefore, if

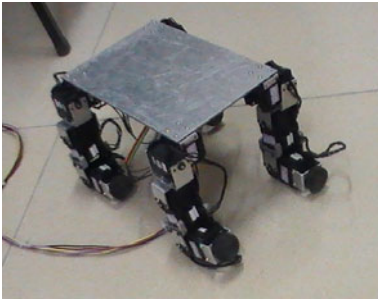
the position control is precise for the quadruped robot based on the kinematics equations, the walking speed and attitude of quadruped robot may be very well with good foot trajectory method.

First of all, this paper proposes the forward kinematics (FK) and inverse kinematics (IK) equations of the quadruped robot, Secondly, based on the composite cycloid method of gait planning [4], a trotting gait generation method on even terrain is presented. Furthermore, for verifying the simple control strategy and good foot trajectory method, a little quadruped robot is constructed by our laboratory. The other intention to design the little quadruped robot is to study and verify the solutions of our forward and inverse kinematics and online control method for the hydraulic actuated quadruped robot in the future.

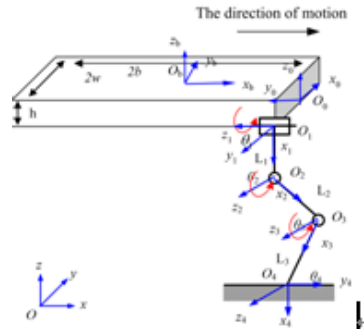
The paper is structured as follows: Section 2 describes the FK and IK equations of the quadruped robot with three degrees of freedom. Section 3 explains the reasons for choosing the trotting gait. Section 4 contains the trotting gait generation method in the swing period and support period. The simulations and experiments are proposed in Section 5. Finally, section 6 and 7 describe the conclusion and future work.

## 2 FK and IK Model of the Quadruped Robot

The quadruped robot used in this paper consists of a body frame and four legs as show in Fig. 1. Small size electronic rotary actuators are installed for the quadruped robot to carry heavy payload and move fast on even terrain. The size of this robot is 23cm x 18cm x 18cm (length, height and width, respectively). One leg module comprises one hip joint, one shoulder joint and one knee joint, so the little quadruped robot has 12 degrees of freedom, with three degrees of freedom per leg.



**Fig. 1.** The real model of quadruped robot



**Fig. 2.** The coordinates of quadruped robot

Fig. 2 shows the little quadruped robot's kinematic diagram of the right front leg [5]. Each leg consists of three degrees of freedom. The kinematic equation of the foot tips can be derived by making use of the coordinate frames established in Fig. 2.

The FK and IK equations are deduced based on the D-H method. The solution of forward kinematics for each leg is described as:

$$P_{RF} = \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{Bmatrix} b + L_2 s_2 + L_3 (c_2 s_3 + s_2 c_3) \\ -w - (L_1 s_1 + L_2 s_1 c_2 + L_3 (s_1 c_2 c_3 - s_1 s_2 s_3)) \\ -L_1 c_1 - L_2 c_1 c_2 + L_3 (c_1 s_2 s_3 - c_1 c_2 c_3) - h \end{Bmatrix} \quad (1)$$

$$P_{RH} = \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{Bmatrix} -b + L_2 s_2 + L_3 (c_2 s_3 + s_2 c_3) \\ -w - (L_1 s_1 + L_2 s_1 c_2 + L_3 (s_1 c_2 c_3 - s_1 s_2 s_3)) \\ -L_1 c_1 - L_2 c_1 c_2 + L_3 (c_1 s_2 s_3 - c_1 c_2 c_3) - h \end{Bmatrix} \quad (2)$$

$$P_{LH} = \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{Bmatrix} -b + L_2 s_2 + L_3 (c_2 s_3 + s_2 c_3) \\ w + (L_1 s_1 + L_2 s_1 c_2 + L_3 (s_1 c_2 c_3 - s_1 s_2 s_3)) \\ -L_1 c_1 - L_2 c_1 c_2 + L_3 (c_1 s_2 s_3 - c_1 c_2 c_3) - h \end{Bmatrix} \quad (3)$$

$$P_{LF} = \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = \begin{Bmatrix} b + L_2 s_2 + L_3 (c_2 s_3 + s_2 c_3) \\ w + (L_1 s_1 + L_2 s_1 c_2 + L_3 (s_1 c_2 c_3 - s_1 s_2 s_3)) \\ -L_1 c_1 - L_2 c_1 c_2 + L_3 (c_1 s_2 s_3 - c_1 c_2 c_3) - h \end{Bmatrix} \quad (4)$$

where LF, RF, RH and LH is left front, right front, right hind and left hind leg respectively,  $\{p_x, p_y, p_z\}$  are the foot tips of the leg.

The inverse kinematics of the legs are also analyzed based the D-H method. For simplicity, the RF inverse kinematics is only as follows:

$$\theta_1 = \tan^{-1} \left( \frac{w + p_y}{p_z + h} \right) \quad (5)$$

$$\theta_2 = \tan^{-1} \frac{E - D \frac{\sqrt{D^2 + E^2 - K^2}}{K}}{D + E \frac{\sqrt{D^2 + E^2 - K^2}}{K}} \quad (6)$$

$$\theta_3 = \cos^{-1} \left( \frac{D^2 + E^2 - L_2^2 - L_3^2}{2L_2 L_3} \right) \quad (7)$$

where  $D = -s_1 p_y - c_1 p_z - w s_1 - h c_1 - L_1$ ,  $E = p_x - b$ ,  $L_2 + L_3 c_3 = K$ . By inputting the foot-tips value, the respective angles,  $\theta_1, \theta_2, \theta_3$  will be solved by the aid of IK equation. In reverse, by formulating the FK equations and knowing the foot tips position of the quadruped robot, we will be able find the angles of the quadruped robot. After the three angles have been found, it will be able to construct the pattern of gait planning for trotting.

### 3 Trotting Gait Planning

Gait means a pattern of discrete foot placements performed in a given sequence. The quadruped robots have two different kinds of gaits. Static gaits (e.g., crawl, wave) occur when the vertical projection of center of gravity remains always inside the polygon formed by the supporting legs. Dynamic gaits (e.g., trot, pace and gallop)

mean that the vertical projection of center of gravity is not necessary to remain inside the polygon formed by the supporting legs with the dynamic balance to be maintained.

The trotting gait was chosen for the little quadruped robot in this paper because of its observed energy efficiency over a wide range of running speed and its wide use in nature [3], [6]. The trot, as shown in Fig. 3, is a symmetric gait during which the diagonal forelimb and hind limb move in unison, ideally contacting and leaving the ground at the same time. Fig. 3 shows the complete trotting stride with two transfer phases and two support phases.



**Fig. 3.** The complete trotting stride of horse [11]

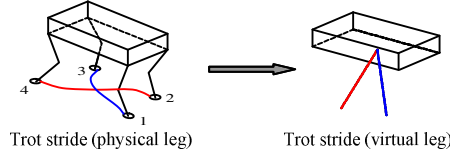
In the description of gaits, the transfer phase of a leg is the period in which the foot is not on the ground. The support phase of a leg is the period in which the foot is on the ground and the cycle time  $T$  is the time for a complete cycle of leg locomotion of a periodic gait.

Compared with other dynamic walking gaits, the trotting gait is a more practical way. The reasons are as follows:

- 1) The trotting gait has a high rate of energy efficiency and greater range of adaptation of speed.
- 2) Even if the two support diagonal legs overturned, with the help of touching the ground quickly, the other two transfer diagonal legs also can prevent the robot overturning.
- 3) The trotting gait is adaptation to the static gait of crawling, and the gait transition from trot to crawl is easy and vice versa. The trotting gait has good adaptability to complex terrain.
- 4) The diagonal legs have the same motion phase. In theory, they strike the ground at the same time. They leave the ground at the same time, and they swing forward at the same time. So, the symmetry trotting gait can implement the symmetry motion of the robot, and can keep the self-stability and reduce the complexity of attitude control.

In trotting gait, the diagonal legs form pairs. The members of a pair strike the ground in unison and they leave the ground in unison. While one pair of legs provides support, the other pair of legs swings forward in preparation for the next step [7]. In order to simplify the complexity of gait planning, the virtual legs introduced by Raibert are used, one virtual leg symbolizes two legs with simultaneous motion, as shown in Fig. 4. The dimension of each virtual leg is the same as those of the real legs. The use of virtual legs implies that the behavior of the two real legs in simultaneous motion is identical [8], [9], [10]. Thus, the method of gait planning is summarized as follows:

- 1) According to the working tasks ( e.g., motion control, stability control ) of quadruped robot is completed and the terrain information (uneven terrain, terrain slope, etc.) , the gait of virtual legs is planned.
- 2) The gait of the virtual legs is decomposed to the physical legs of the diagonal anterior and posterior, and the gait coordination and planning task of the physical legs is done.



**Fig. 4.** In the trot diagonal pairs of legs act in unison [7]

## 4 Gait Generation of Transfer and Support Period

The motion of the quadruped robot is determined by two factors, the trajectory generation of transfer legs and the angle computation of supporting legs. Based on the FK and IK equations of the quadruped robot, the trotting gait is constructed in terms of the foot trajectory of the transfer legs and the posture of the supporting legs. The foot trajectory is obtained by the stride and the height of leg. The stride of one step is determined by the desired body velocity of forward direction. And the height of foot trajectory is obtained by the height of obstacle on the ground or the terrain environment during walking.

### 4.1 Transfer Gait Generation

Quadruped robots need to have the ability to move on the uneven or slope terrains. In the dynamic trotting walking case, the relative speed between the foot and the ground causes loss of body balance. For reducing the impact between the foot and the ground and consideration is given to a trajectory where the foot velocity and acceleration can be continuous both at the time of departure and landing [4]. Therefore, this paper presents a pattern generation strategy based on the composite cycloid method for the trotting gait. The formula of the composite cycloid is as follows:

$$\begin{aligned}
 X(t) &= a\left(\frac{t}{t_0} - \frac{1}{2\pi}\sin(2\pi\frac{t}{t_0})\right) & 0 \leq t \leq t_0 \\
 Z(t) &= 2h\left(\frac{t}{t_0} - \frac{1}{4\pi}\sin(4\pi\frac{t}{t_0})\right) & 0 \leq t \leq \frac{t_0}{2}
 \end{aligned} \tag{8}$$

where  $a$  is the stride length,  $h$  is the maximum foot height of the quadruped robot,  $t$  is the sampling time of the gait trajectory.  $t_0$  is the transfer time. The  $x$  axis is taken on a linear line connecting the departing point and landing point and the  $z$  axis is on the vertical line. So  $X(t)$  is the horizontal trajectory and  $Z(t)$  is the vertical trajectory of transfer leg of the quadruped robot.

## 4.2 Support Gait Generation

Suppose the locomotion speed of the quadruped robot  $v$  is 0 in the  $z$  direction for keeping the stability of the body, as is shown in Fig. 5. The control objective of support legs for the quadruped robot is to maintain the constant body height and the desired body velocity. The desired body motion is obtained from the motion of the support legs. Therefore, when the initiate and end states of the support period are determined, the trajectories of each joint of support legs can be obtained in terms of the IK equation.

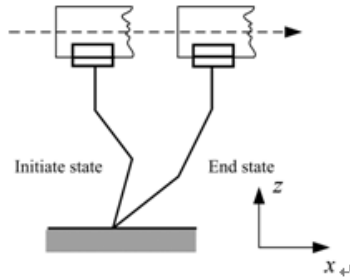


Fig. 5. Gait planning of the support legs

## 5 Simulation and Experiments

To test the trotting gait generation method and simple software controlling strategy proposed in this paper experimentally, a relatively simple forward trotting gait on even terrain was chosen. The cycle period is 1 s and the stride of one leg is set to 0.1 m. Software controlling platform of the quadruped robot is shown in Fig. 6.

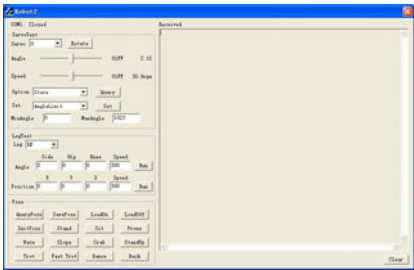
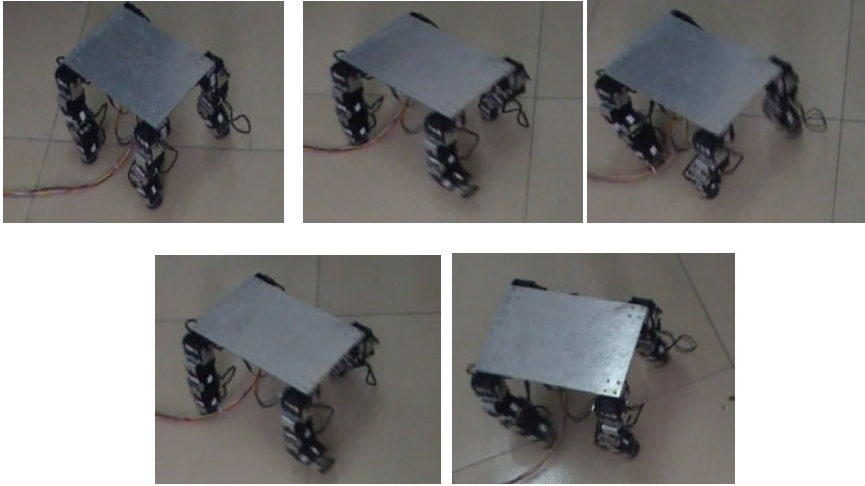


Fig. 6. The controlling platform of the little quadruped robot

The gait of the quadruped robot is controlled by the steps as follows:

- 1) The switching sequence and time of legs are scheduled based the trotting gait method;
- 2) The stride of the quadruped gait is obtained in terms of the landing position of legs;

- 3) The period of stride of the quadruped robot is computed by the desired motion velocity;
- 4) The motion trajectories and angle values of legs are planned based on the FK and IK equations;
- 5) The data frames are sent to the electrical actuated rudder servos with a delay time, the delay time is computed by the walking period divided by the key frames.



**Fig. 7.** The snapshots of a trotting gait period for the quadruped robot

Some successful experiments videos are obtained and Fig. 7 is the snapshots of a trotting gait from one video. Fig. 7 shows the trotting gait of the little quadruped robot in two walking periods. The little quadruped can walk with trotting pattern in a constant velocity 0.1m/s.

## 6 Conclusion

A little quadruped robot model is developed based on the forward and inverse kinematics and electronic rudder servo controller in this paper. The trotting motion is scheduled based on the composite cycloid method. Trotting gait in the even terrain is simulated by our real little quadruped robot and a simple online algorithm with position control method successfully compels the robot to walk continuously over a flat terrain.

## 7 Future Work

The future advancement can be carried out in the project by going for embedded processor that can process and transmit the control signal faster to the actuators.



Remote control through wireless Ethernet mode can also be considered. Only forward experimental trotting walking on even terrain is presented in this paper, the different gaits and walking on different types of terrain will be constructed in our future work.

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