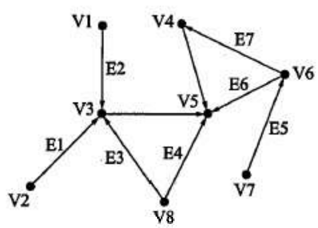
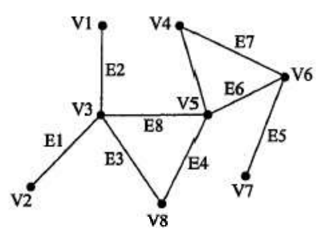
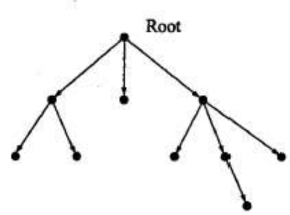
Graphs

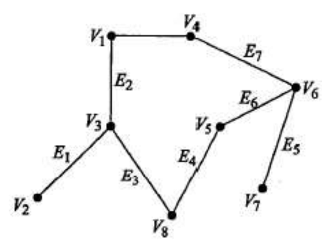


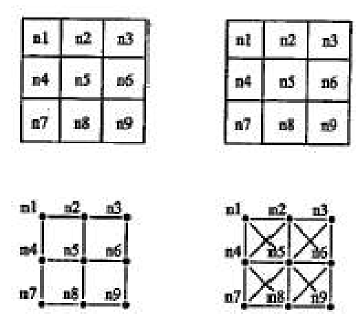


Collection of Edges and Nodes (Vertices)



A tree





Search in Path Planning

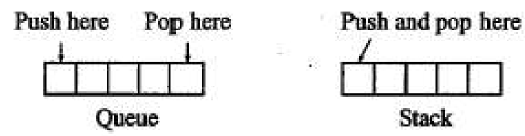
- Find a path between two locations in an unknown, partially known, or known environment
- Search Performance
  - Completeness
  - Optimality → Operating cost
  - Space Complexity
  - Time Complexity

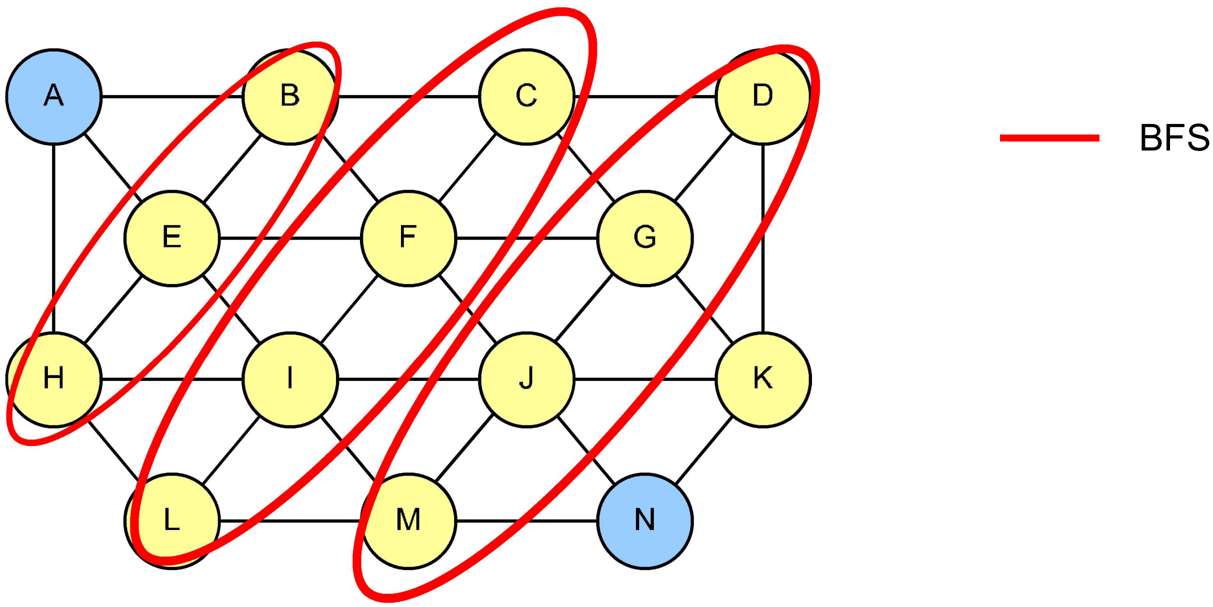
Search

- Uninformed Search
  - Use no information obtained from the environment
  - Blind Search: BFS (Wavefront), DFS
- Informed Search
  - Use evaluation function
  - More efficient
  - Heuristic Search: A*, D*, etc.

Uninformed Search

Graph Search from A to N





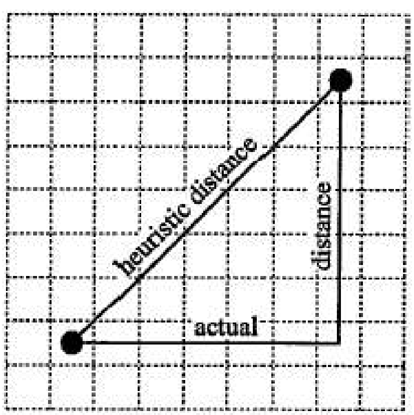
Informed Search: A*

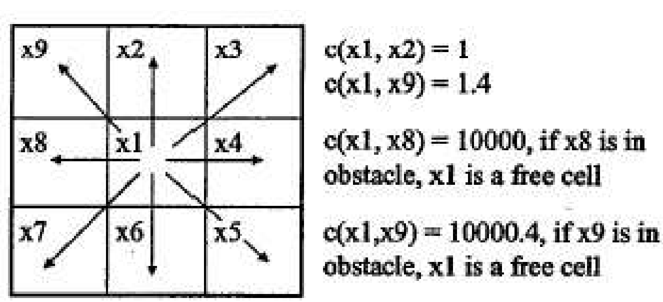
Notation

- ***n*** → node/state
- ***c(n_1,n_2)*** → the length of an edge connecting between n_1 and n_2
- ***b(n_1) = n_2*** → backpointer of a node n_1 to a node n_2.

Informed Search: A*

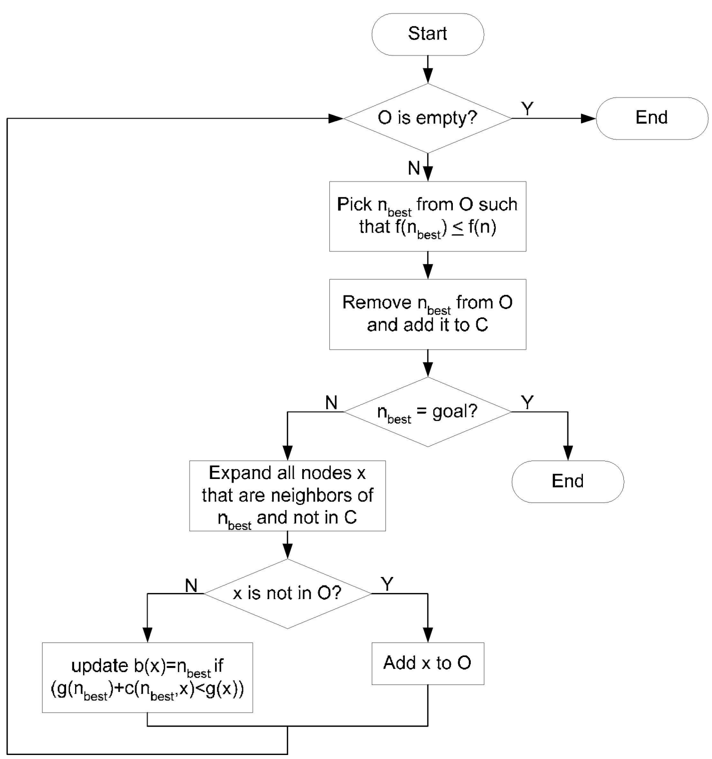
- Evaluation function, ***f(n) = g(n) + h(n)***
- Operating cost function, ***g(n)***
  - Actual operating cost having been already traversed
- Heuristic function, ***h(n)***
  - Information used to find the promising node to traverse
  - Admissible → never overestimate the actual path cost





Cost on a grid

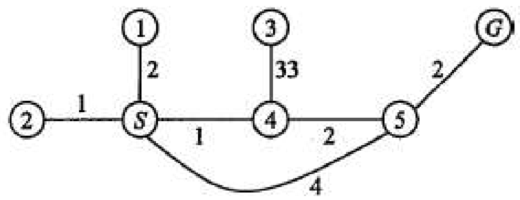
A*: Algorithm



The search requires 2 lists to store information about nodes

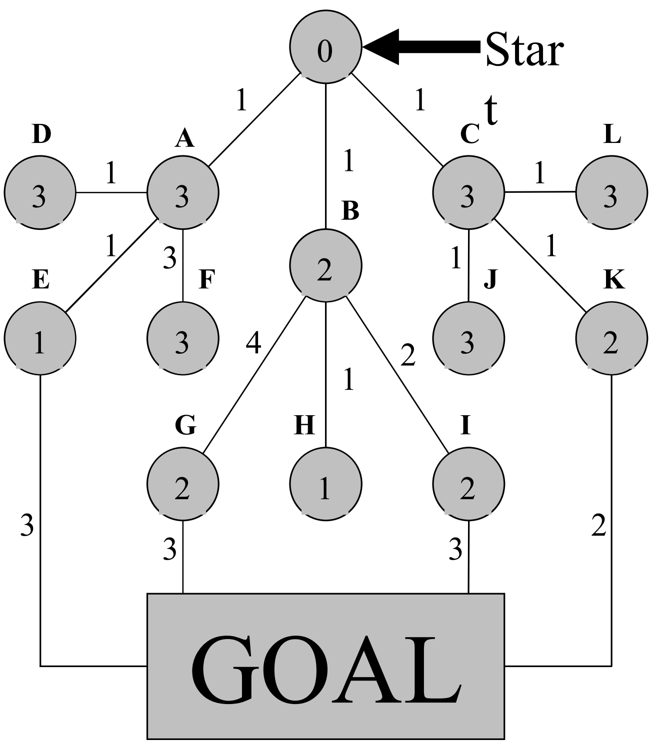
1) **Open list (O)** stores nodes for expansions
2) **Closed list (C)** stores nodes which we have explored

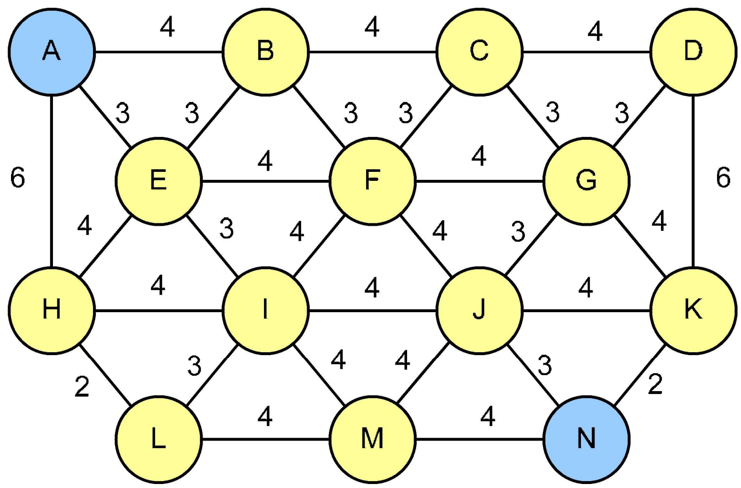
Dijkstra's Search: f(n) = g(n)



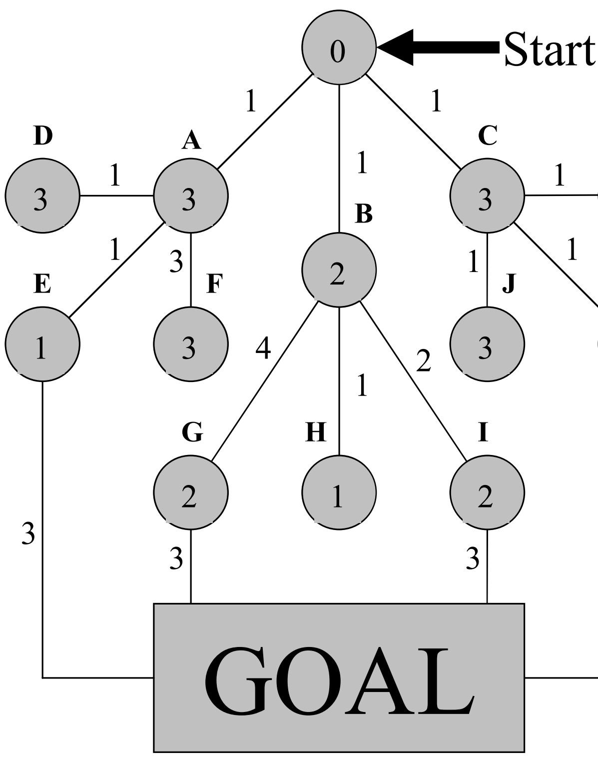
1. O = {S}
2. O = {1, 2, 4, 5}; C = {S} (1,2,4,5 all back point to S)
3. O = {1, 4, 5}; C = {S, 2} (there are no adjacent nodes not in C)
4. O = {1, 5, 3}; C = {S, 2, 4} (1, 2, 4 point to S; 5 points to 4)
5. O = {5, 3}; C = {S, 2, 4, 1}
6. O = {3, G}; C = {S, 2, 4 1} (goal points to 5 which points to 4 which points to S)

Two Examples Running A*

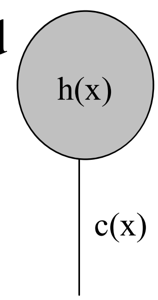




Example (1/5)



Legend



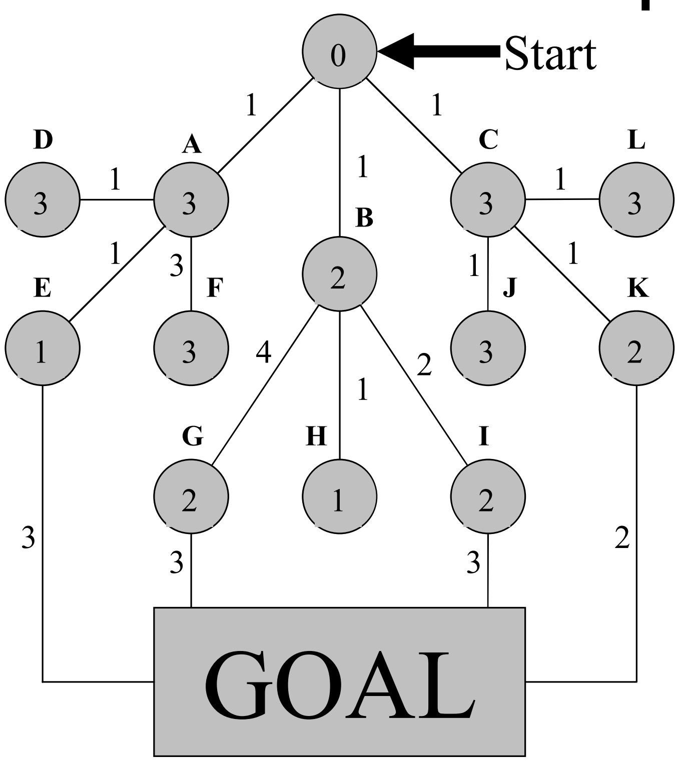
Priority = g(x) + h(x)

Note:

g(x) = sum of all previous arc costs, c(x), from start to x

Example: c(H) = 2

Example (2/5)



First expand the start node

B(3)
A(4)
C(4)

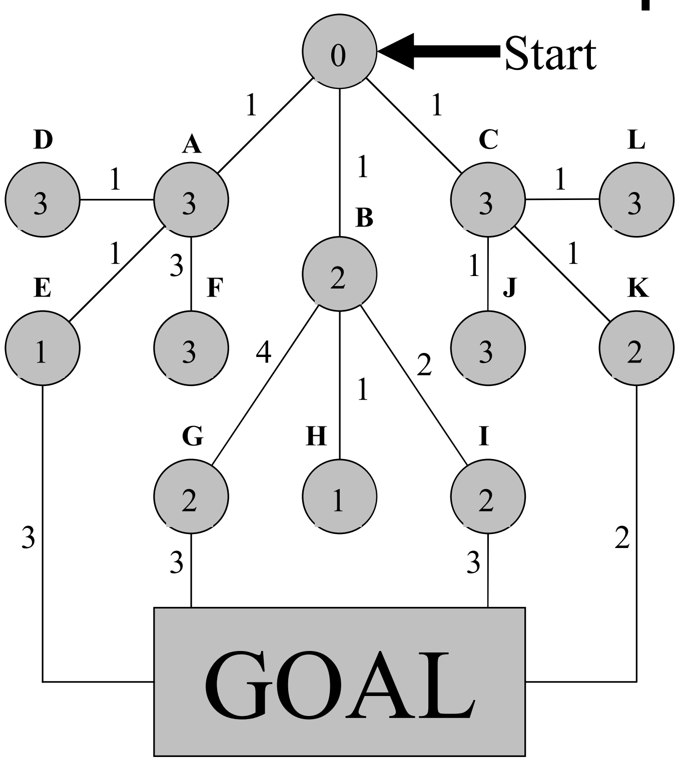
If goal not found, expand the first node in the priority queue (in this case, B)

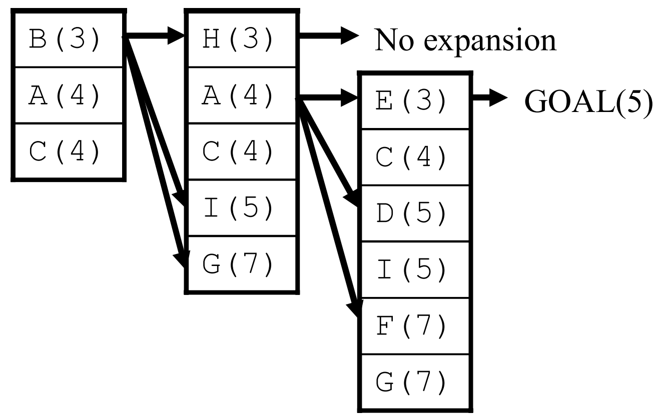
H(3)
A(4)
C(4)
I(5)
G(7)

Insert the newly expanded nodes into the priority queue and continue until the goal is found, or the priority queue is empty (in which case no path exists)

Note: for each expanded node,
you also need a pointer to its respective parent. For example, nodes A, B and C point to Start

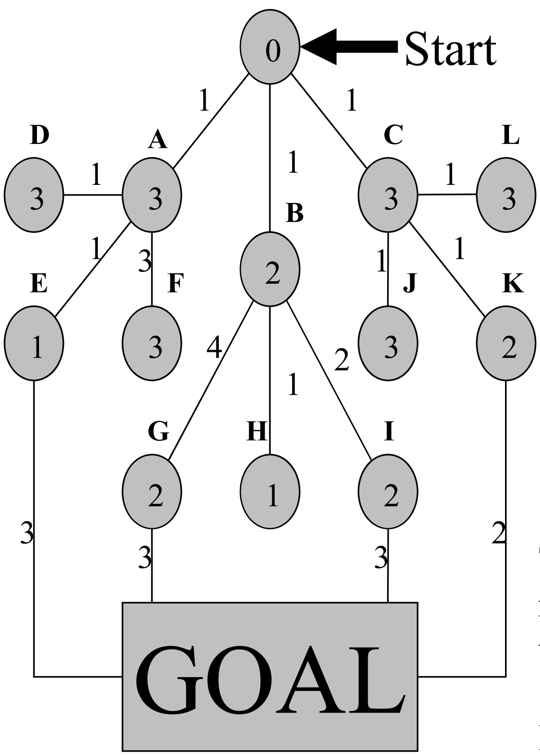
Example (3/5)



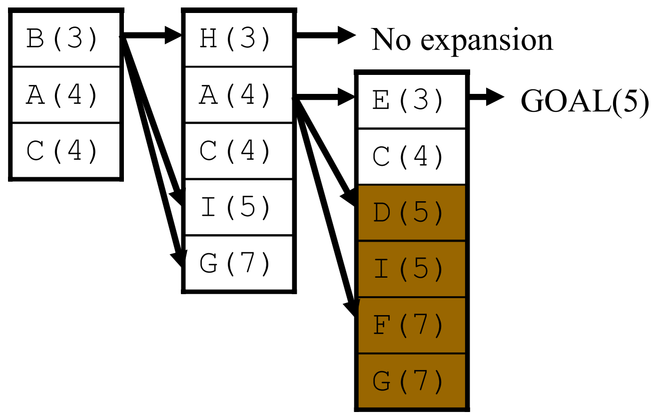


We've found a path to the goal:
Start => A => E => Goal
(*from the pointers*)

Are we done?



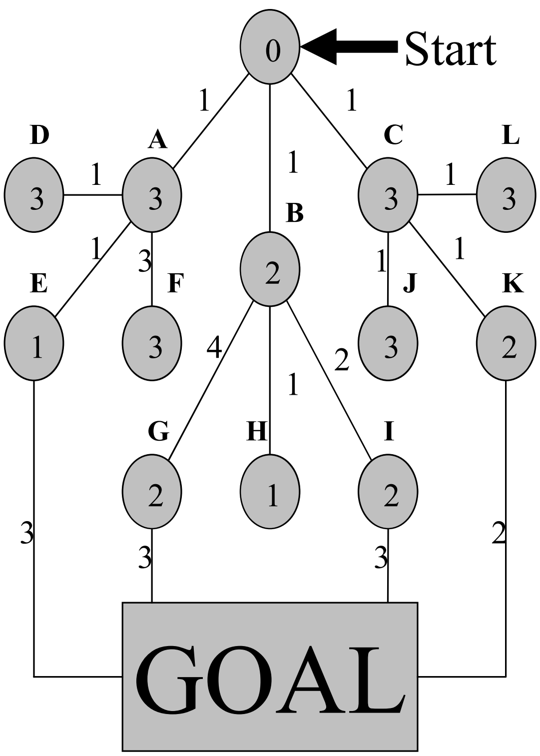
Example (4/5)



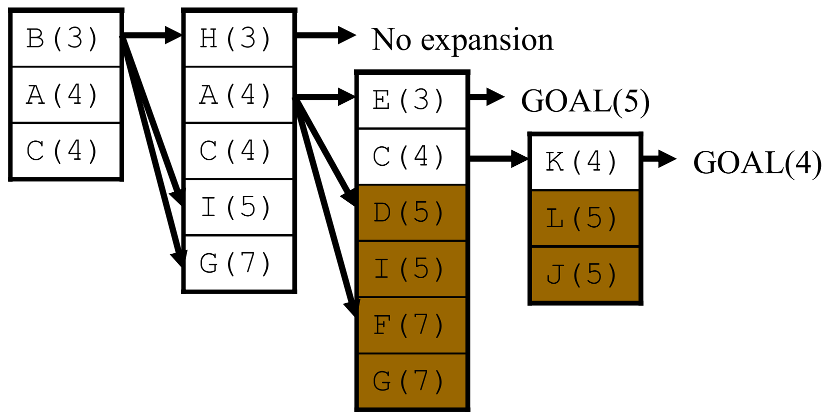
There might be a shorter path, but assuming non-negative arc costs, nodes with a lower priority than the goal cannot yield a better path.

In this example, nodes with a priority greater than or equal to 5 can be pruned.

Why don't we expand nodes with an equivalent priority? (why not expand nodes D and I?)



Example (5/5)



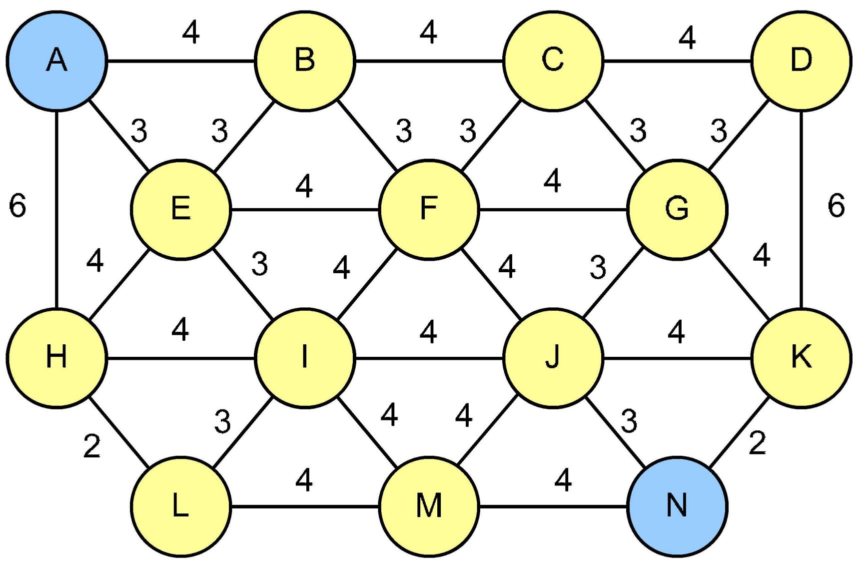
We can continue to throw away nodes with priority levels lower than the lowest goal found.

As we can see from this example, there was a shorter path through node K. To find the path, simply follow the back pointers.

Therefore the path would be:
Start => C => K => Goal

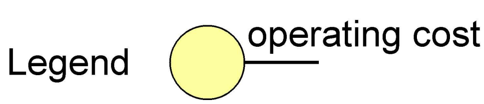
If the priority queue still wasn't empty, we would continue expanding while throwing away nodes with priority lower than 4.
(remember, lower numbers = higher priority)

A*: Example (1/6)

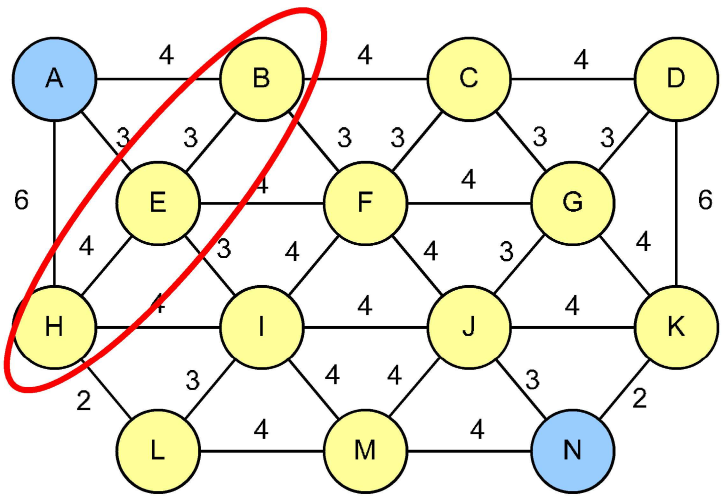


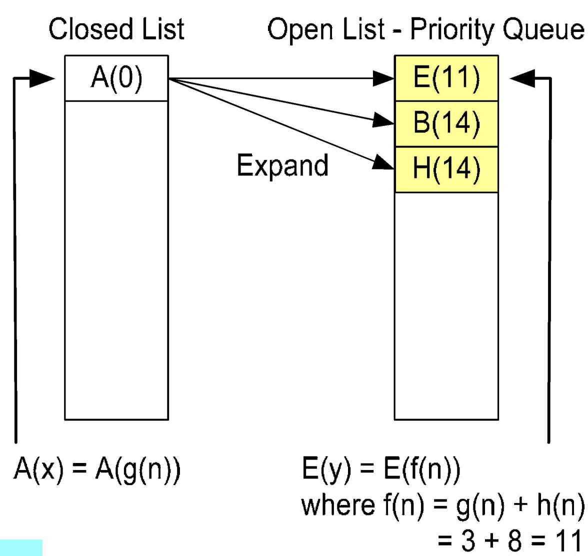
Heuristics

A = 14	H = 8
B = 10	I = 5
C = 8	J = 2
D = 6	K = 2
E = 8	L = 6
F = 7	M = 2
G = 6	N = 0



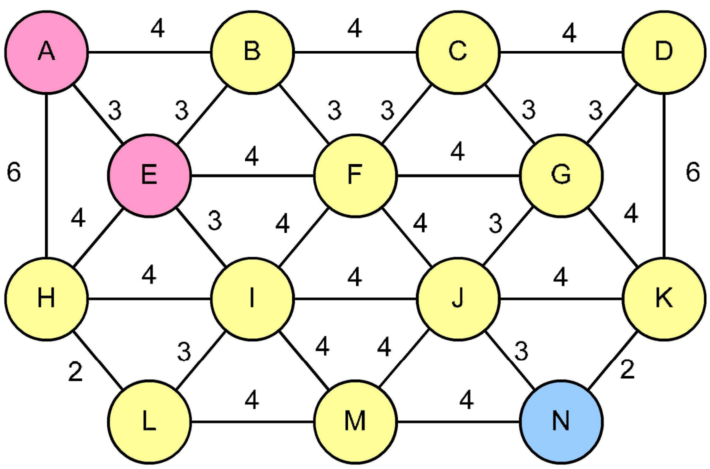
A*: Example (2/6)

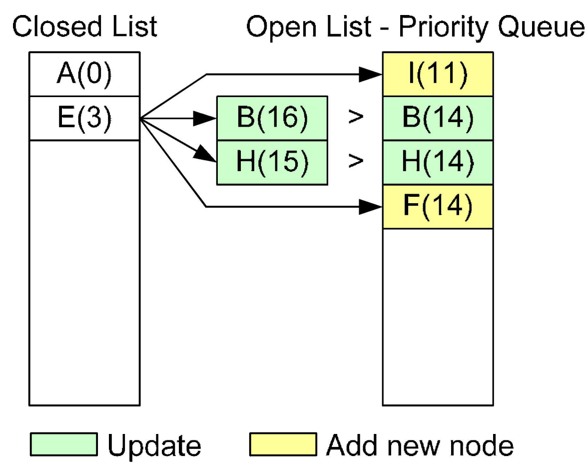




Heuristics
A = 14, B = 10, C = 8, D = 6, E = 8, F = 7, G = 6
H = 8, I = 5, J = 2, K = 2, L = 6, M = 2, N = 0

A*: Example (3/6)

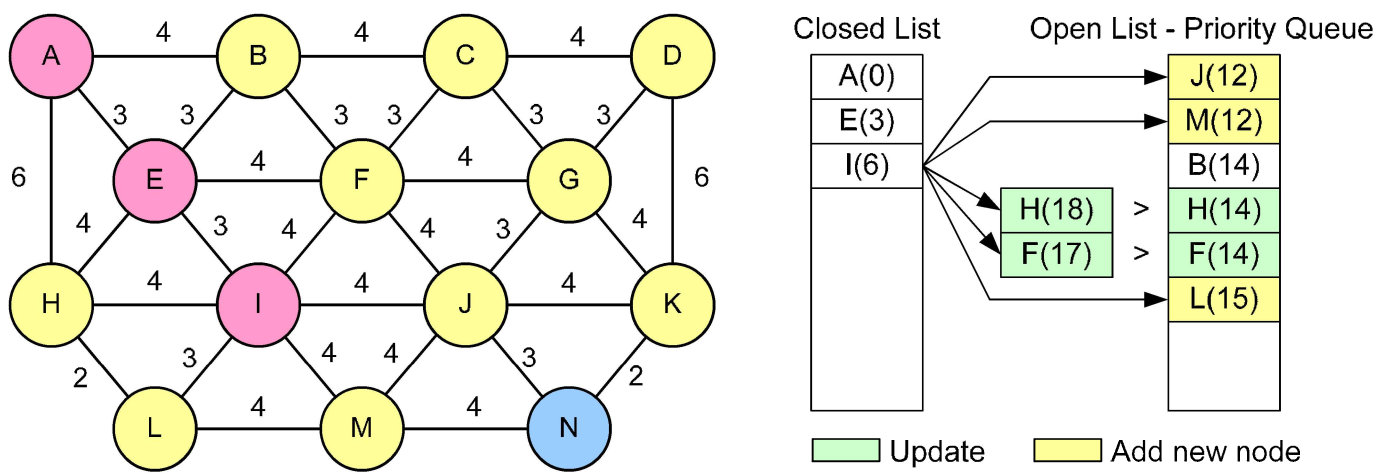




Heuristics
A = 14, B = 10, C = 8, D = 6, E = 8, F = 7, G = 6
H = 8, I = 5, J = 2, K = 2, L = 6, M = 2, N = 0

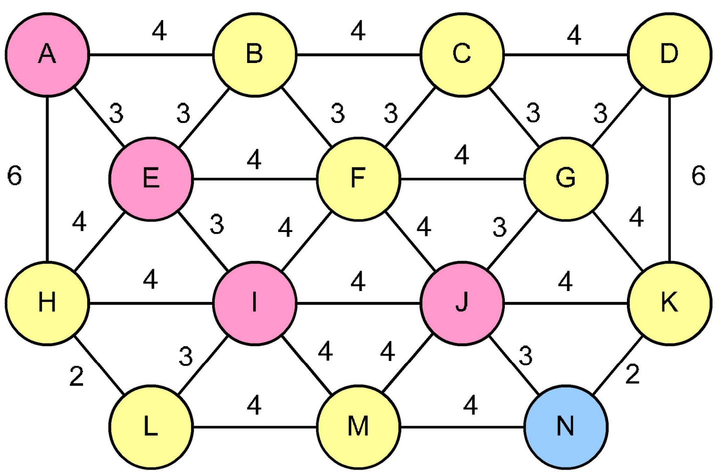
Since A → B is smaller than A → E → B, the f-cost value of B in an open list needs not be updated

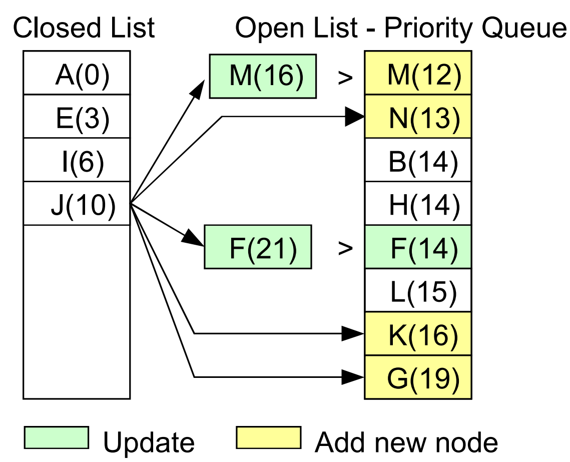
A*: Example (4/6)



Heuristics
A = 14, B = 10, C = 8, D = 6, E = 8, F = 7, G = 6
H = 8, I = 5, J = 2, K = 2, L = 6, M = 2, N = 0

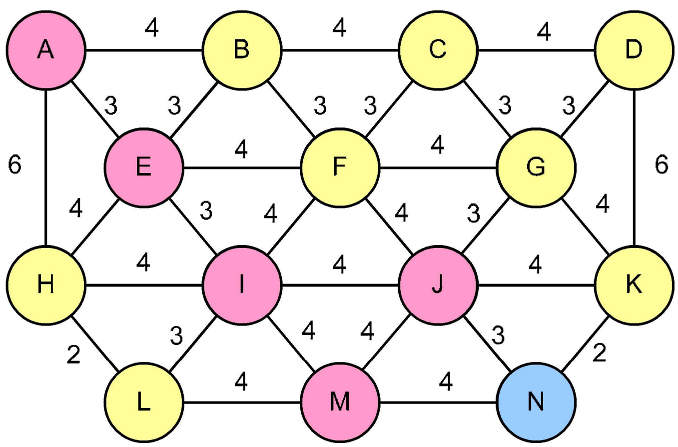
A*: Example (5/6)

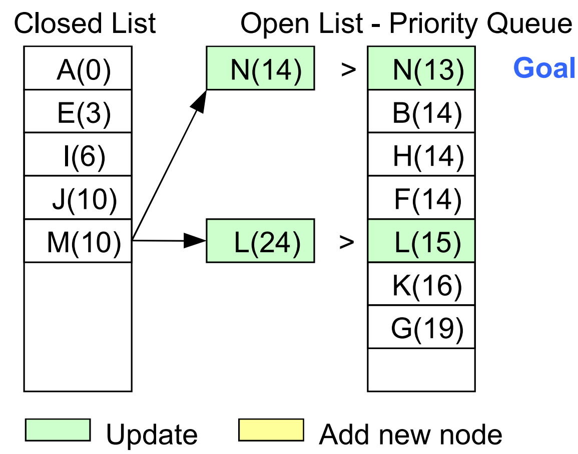




Heuristics
A = 14, B = 10, C = 8, D = 6, E = 8, F = 7, G = 6
H = 8, I = 5, J = 2, K = 2, L = 6, M = 2, N = 0

A*: Example (6/6)

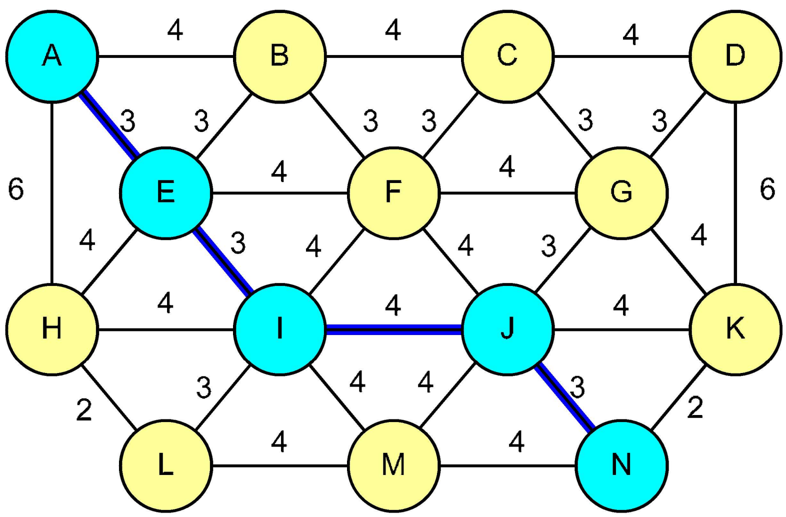




Heuristics
A = 14, B = 10, C = 8, D = 6, E = 8, F = 7, G = 6
H = 8, I = 5, J = 2, K = 2, L = 6, M = 2, N = 0

Since the path to N from M is greater than that from J, **the optimal path to N is the one traversed from J**

A*: Example Result



Generate the path from the goal node back to the start node through the back-pointer attribute