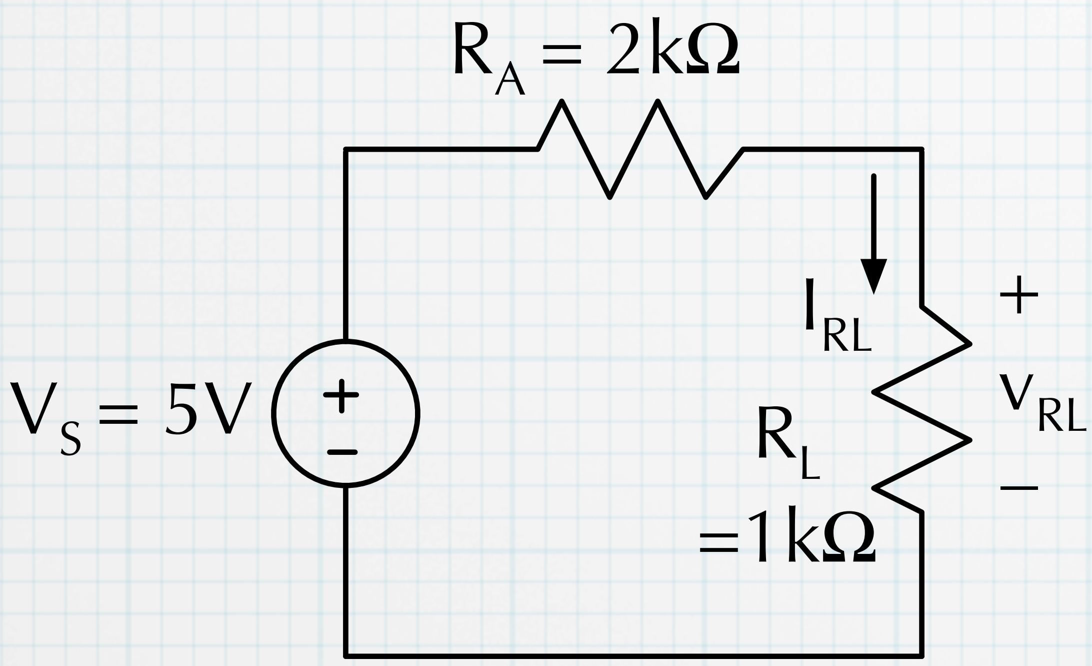
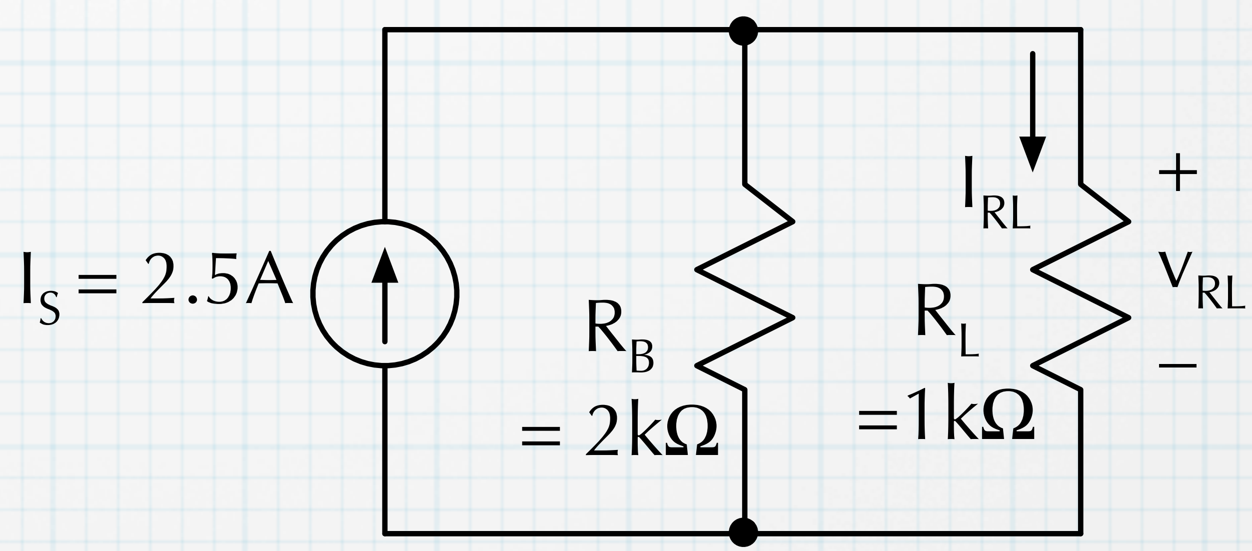
Source transformations

Consider the two circuits below. In particular, look at the current and voltage of R_L in each circuit. Using any of the techniques we seen so far, it is easy to find I_{RL} and V_{RL} for each case.



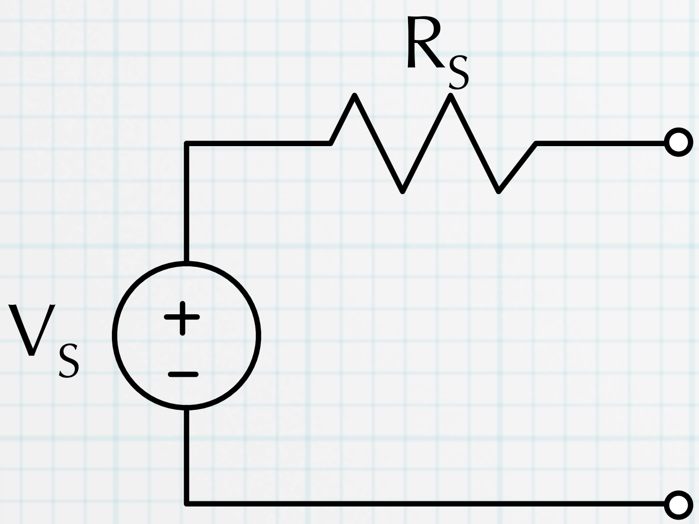


V_{RL} = 1.667 V and I_{RL} = 1.667 A

V_{RL} = 1.667 V and I_{RL} = 1.667 A

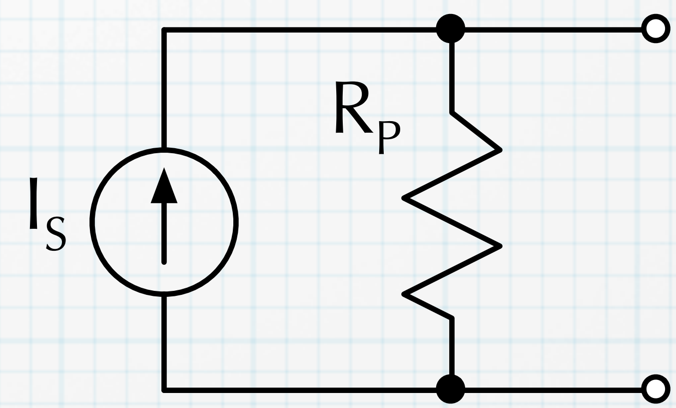
Interesting: From the point of view of the resistor R_L, the series combination of the voltage source and resistor R_A gives the exact same result as the parallel combination of current source and resistor R_B.

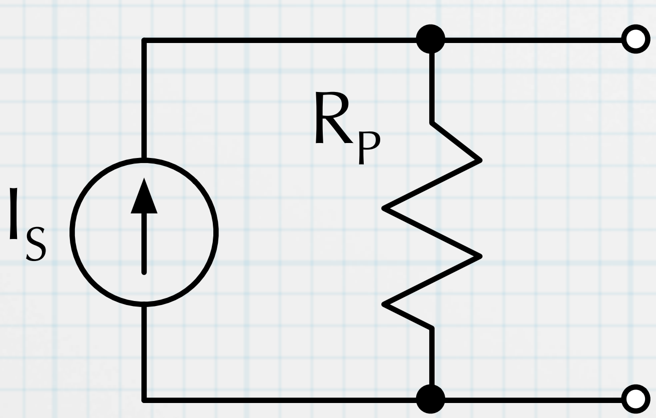
The series combination seems to behave *identically* to the parallel combination. This suggests that we may, in the right circumstances, replace one configuration for the other. Making this switch is known as a *source transformation*.



$I_S = V_S/R_S$

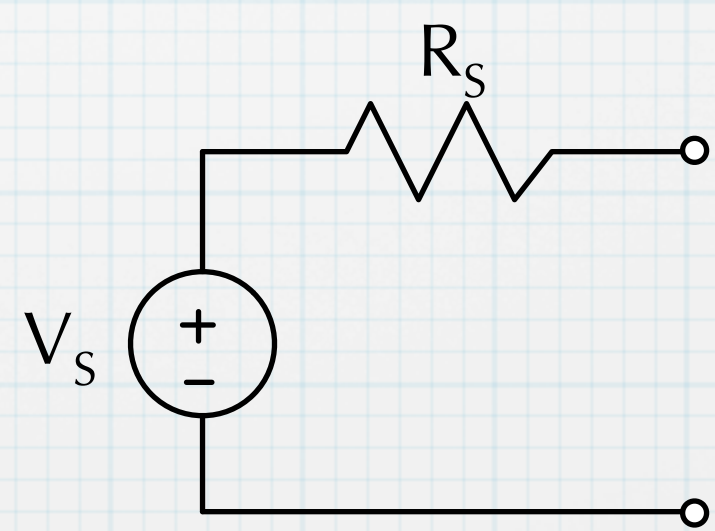
$R_P = R_S$





$V_S = I_S R_P$

$R_S = R_P$



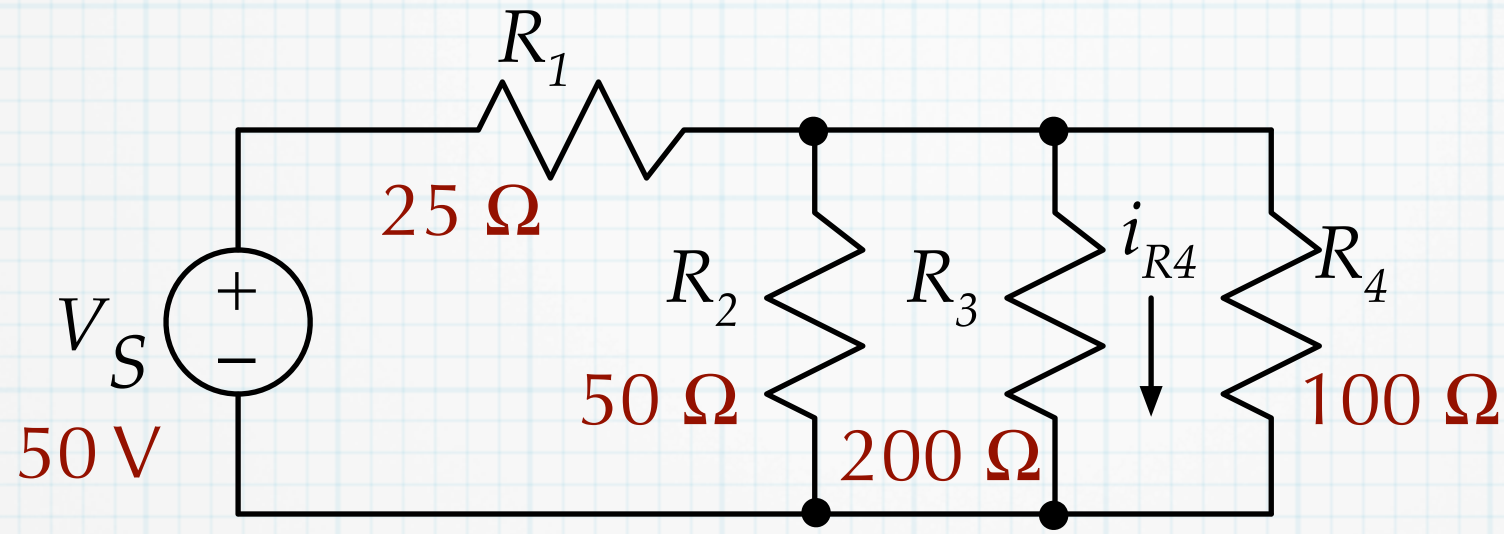
These are perfectly viable substitutions. From the point of view of whatever circuitry is attached to the two terminals, the result will be exactly the same for the two source configurations.

If you are trying calculate something about the two components (V-R_S or I-R_P), you cannot transform them. In transforming them, you lose the ability to calculate a specific property.

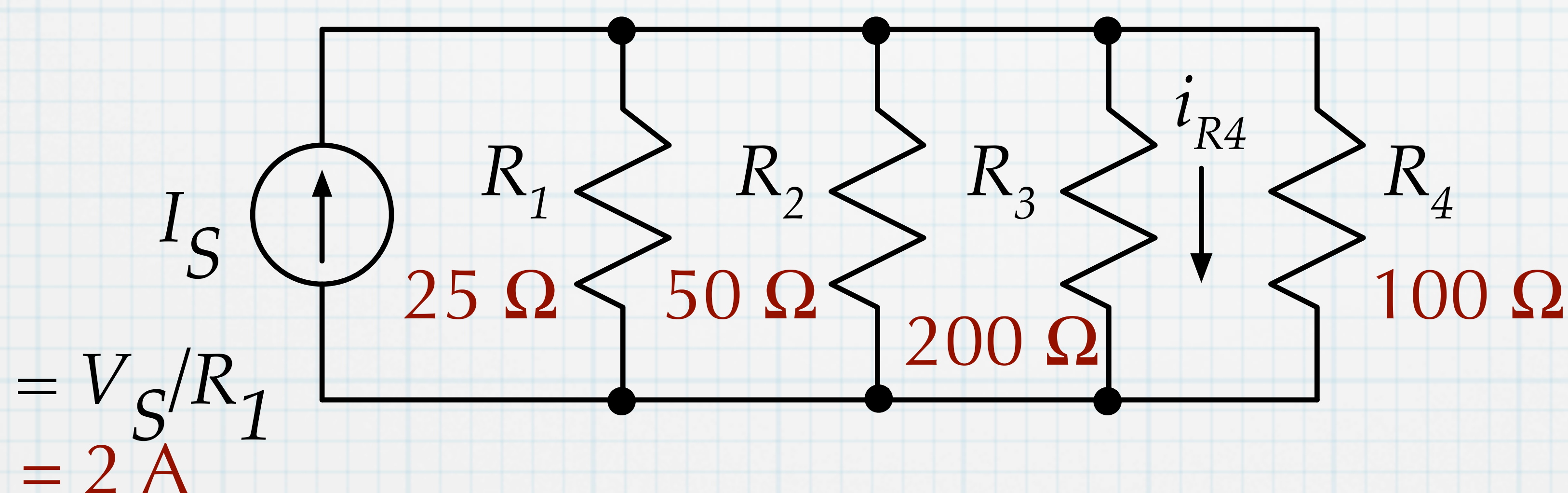
This is an example of bigger, more important idea known as the Thevenin equivalent of circuit. We will introduce this later, and make extensive use of it in discussing amplifiers, etc.

Example 1

Find i_{R4} in the circuit below



Use a source transformation to put everything in parallel.

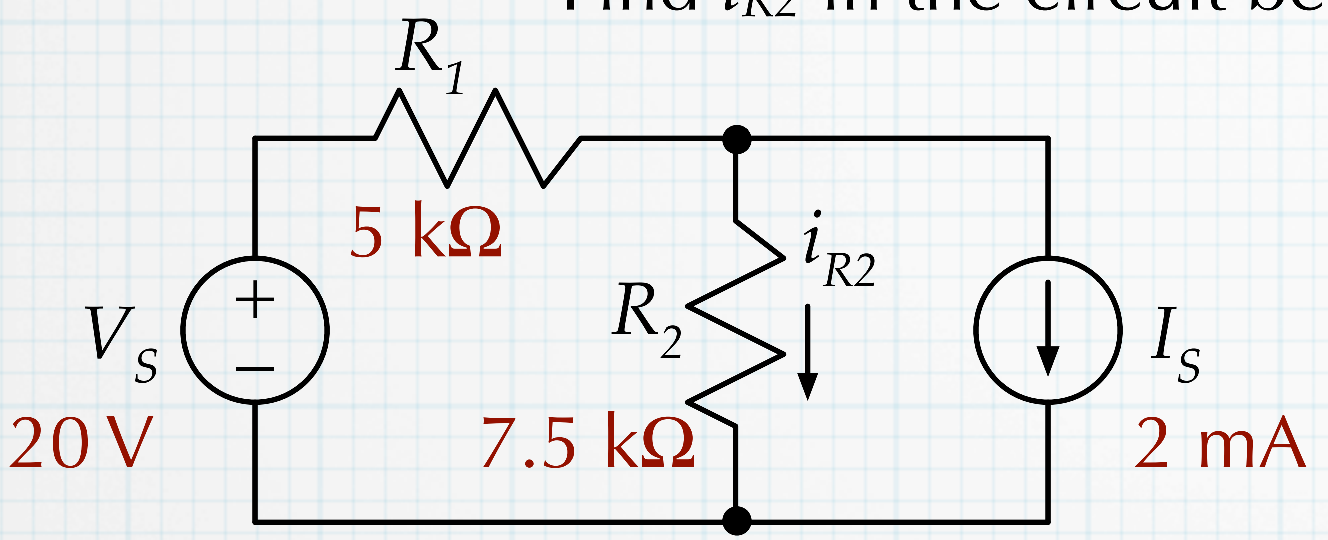


Then use a current divider:

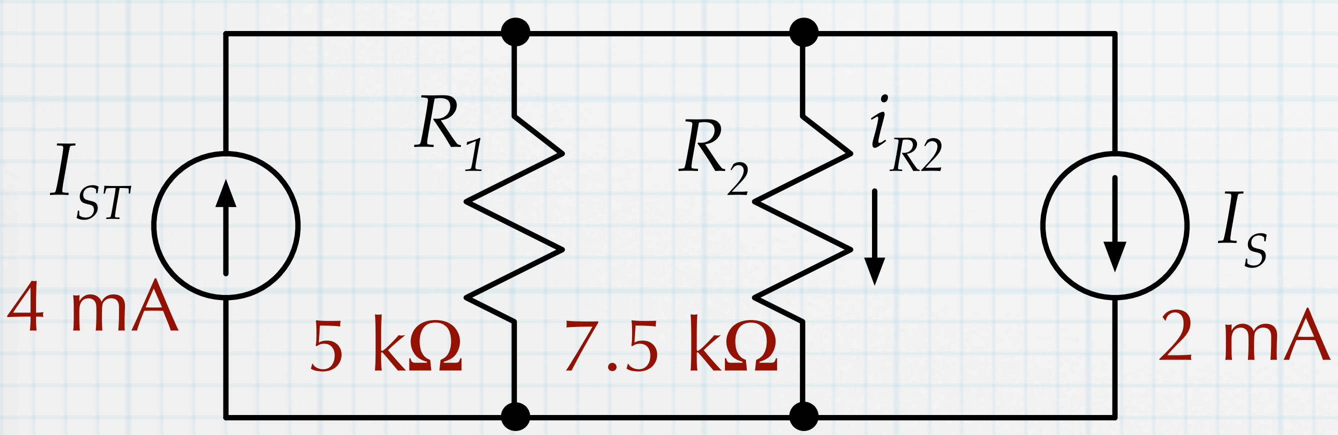
$$I_{R4} = \frac{\frac{1}{100\Omega}}{\frac{1}{25\Omega} + \frac{1}{50\Omega} + \frac{1}{200\Omega} + \frac{1}{100\Omega}} (2\text{A}) = 0.2667 \text{ A}$$

Example 2

Find i_{R2} in the circuit below.



Transform the voltage source / resistor combo



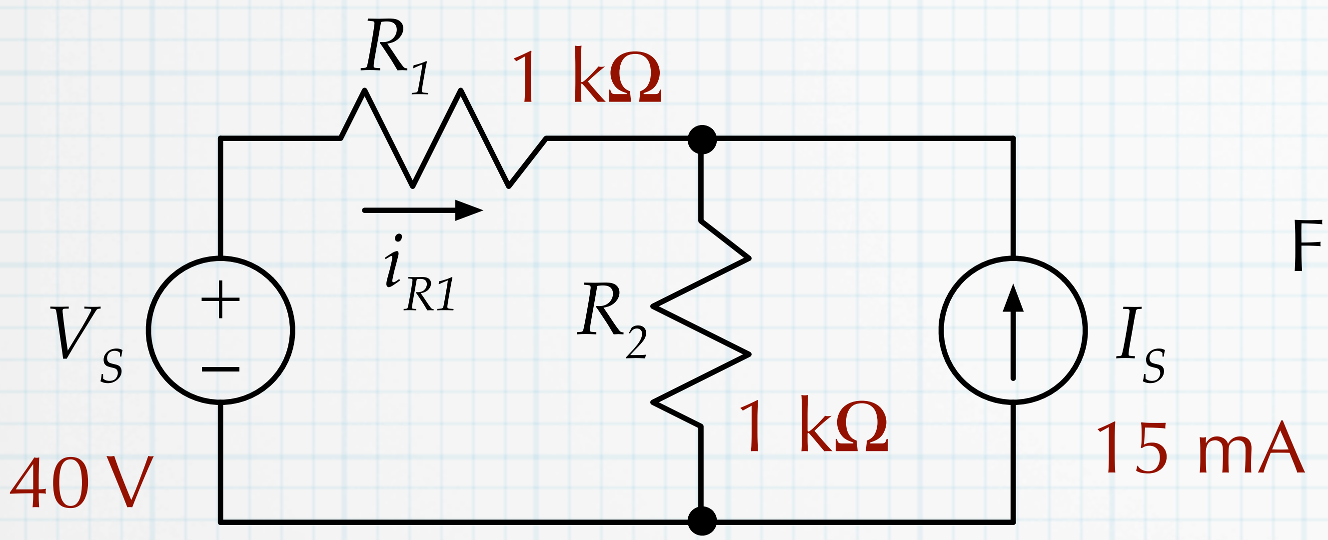
Writing a KCL equation shows that there is a net current of I_{ST} - I_S = 2 mA flowing into the parallel resistor combination.

Net current divides between the two resistors.

$$i_{R2} = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} (I_{ST} - I_S)$$

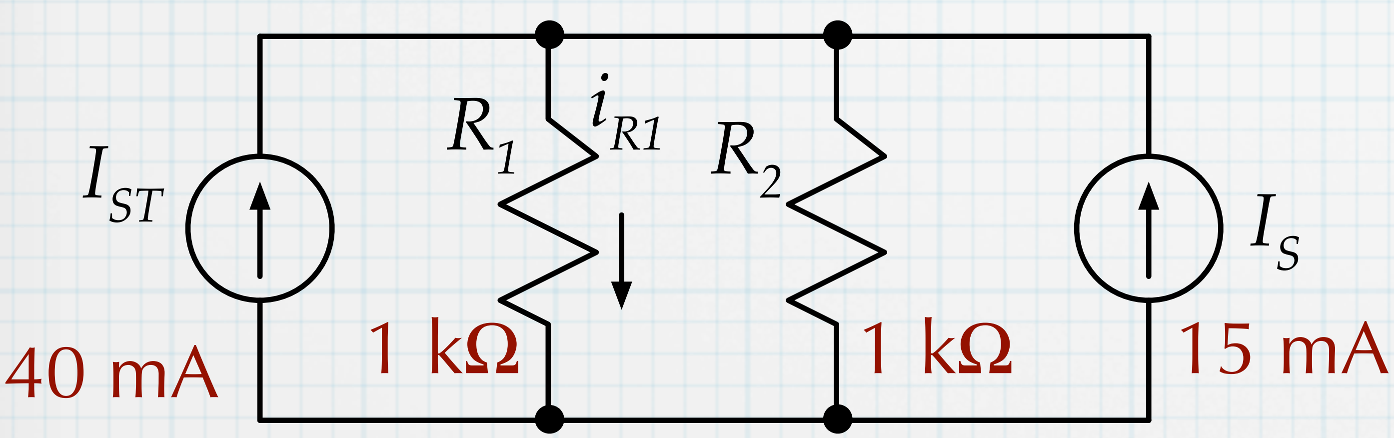
$$= \frac{\frac{1}{7.5\text{k}\Omega}}{\frac{1}{5\text{k}\Omega} + \frac{1}{7.5\text{k}\Omega}} (2\text{mA}) = 0.8\text{mA}$$

Example 3 (An incorrect application)



Find i_{R1} in the circuit at left.

The previous example worked nicely so use the same method. Transform V_S & R_1, and use current divider with total current.

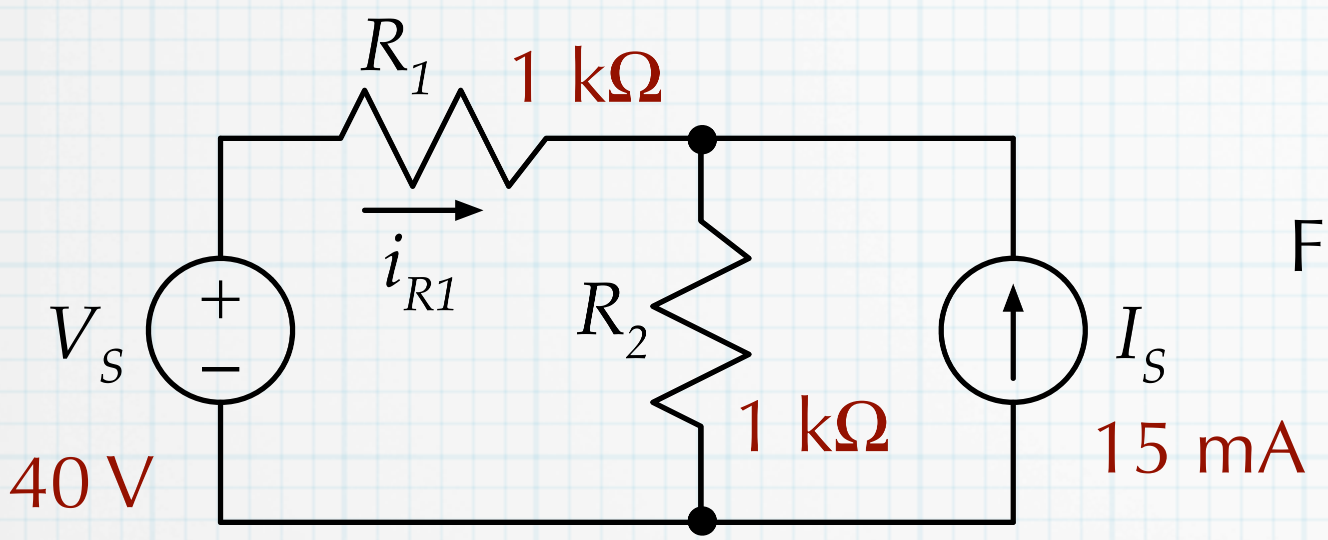


$$i_{R1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} (I_{ST} + I_S)$$

$$= \frac{\frac{1}{1\text{k}\Omega}}{\frac{1}{1\text{k}\Omega} + \frac{1}{1\text{k}\Omega}} (55\text{mA}) = 27.5\text{mA}$$

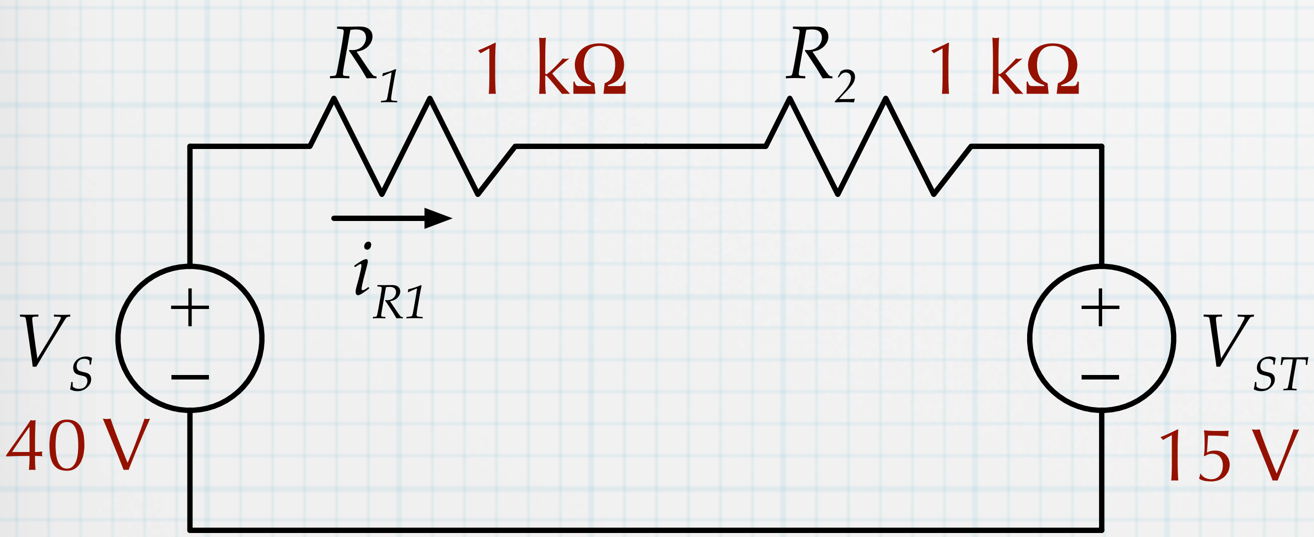
It seems nice, but it is wrong because you cannot transform the component for which you are trying to find voltage or current.

Example 3 (Redo it correctly.)



Find i_{R1} in the circuit at left.

Transform I_S & R_2.



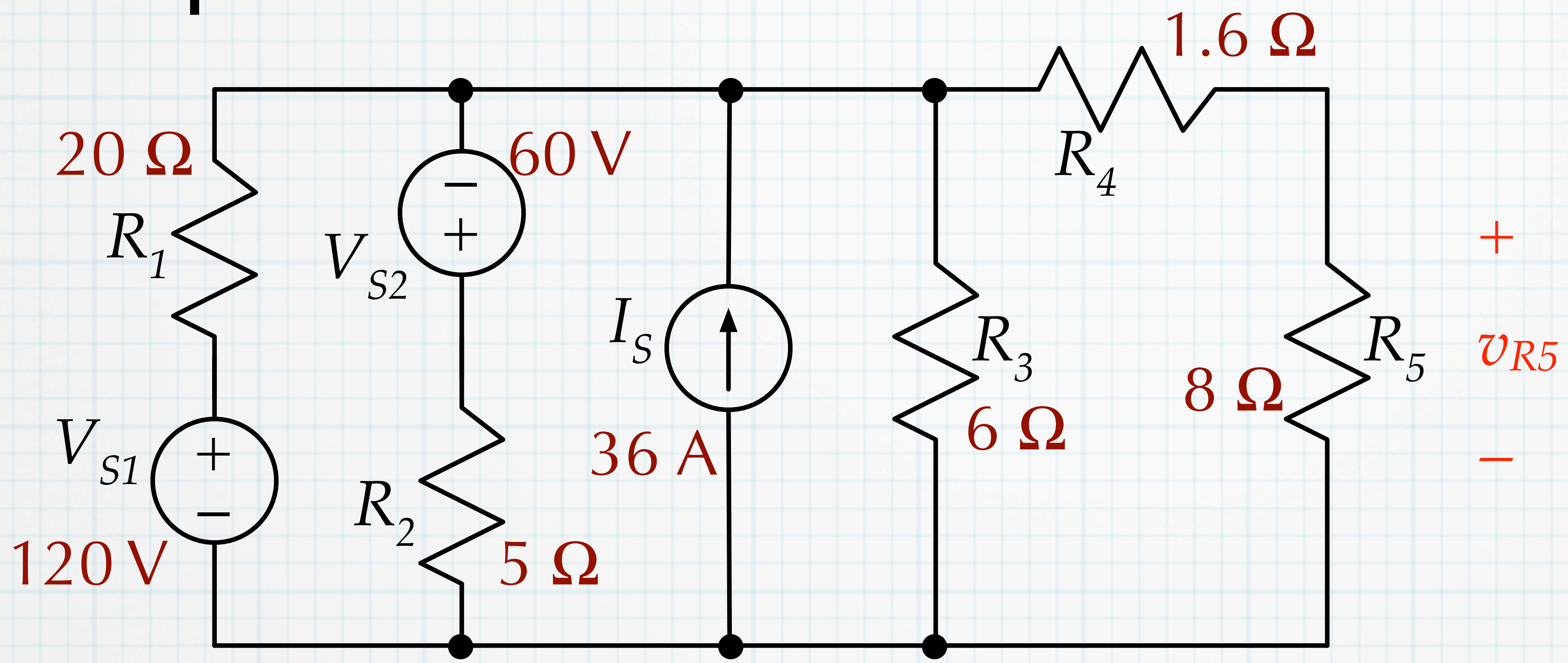
Writing a KVL loop equation and solving for i_{R1} gives

$$i_{R1} = \frac{V_S - V_{ST}}{R_1 + R_2}$$

$$= \frac{40\text{V} - 15\text{V}}{1\text{k}\Omega + 1\text{k}\Omega} = 12.5\text{mA}$$

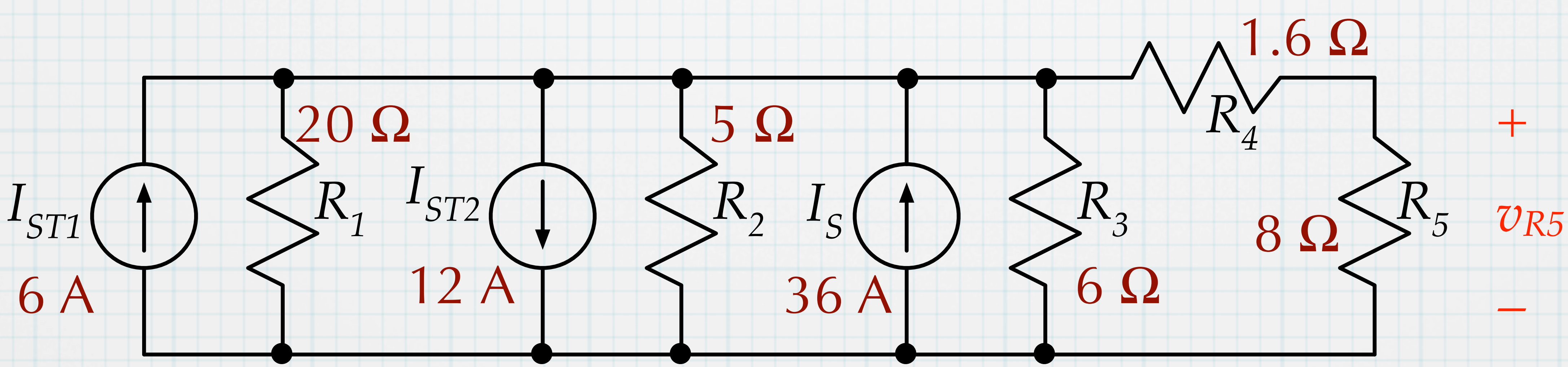
This is the correct answer.

Example 4



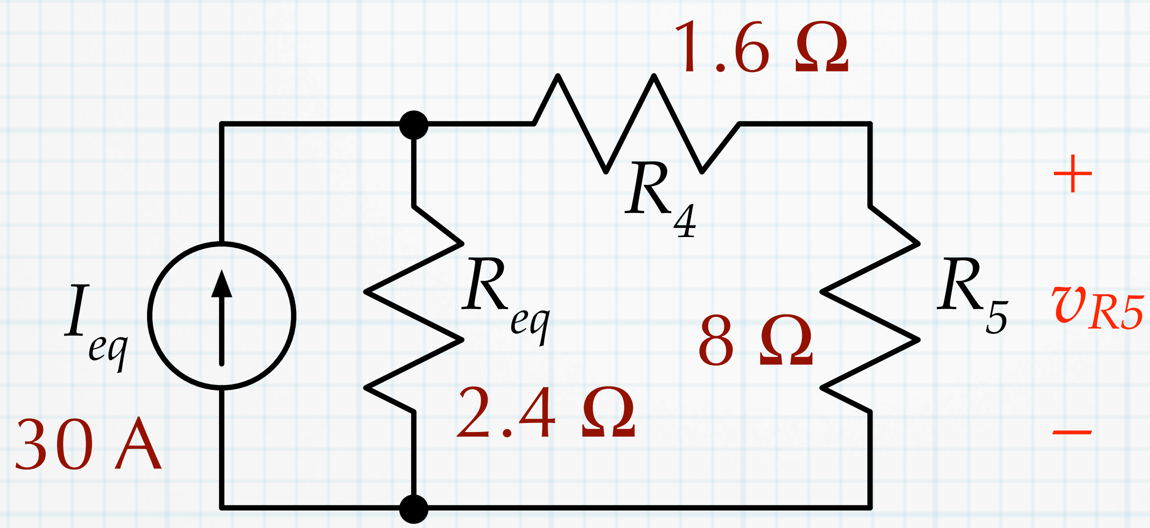
Find v_{R5} in the circuit at left.

Transform two voltage sources to current sources. (Pay attention to polarity.)



Example 4 (cont.)

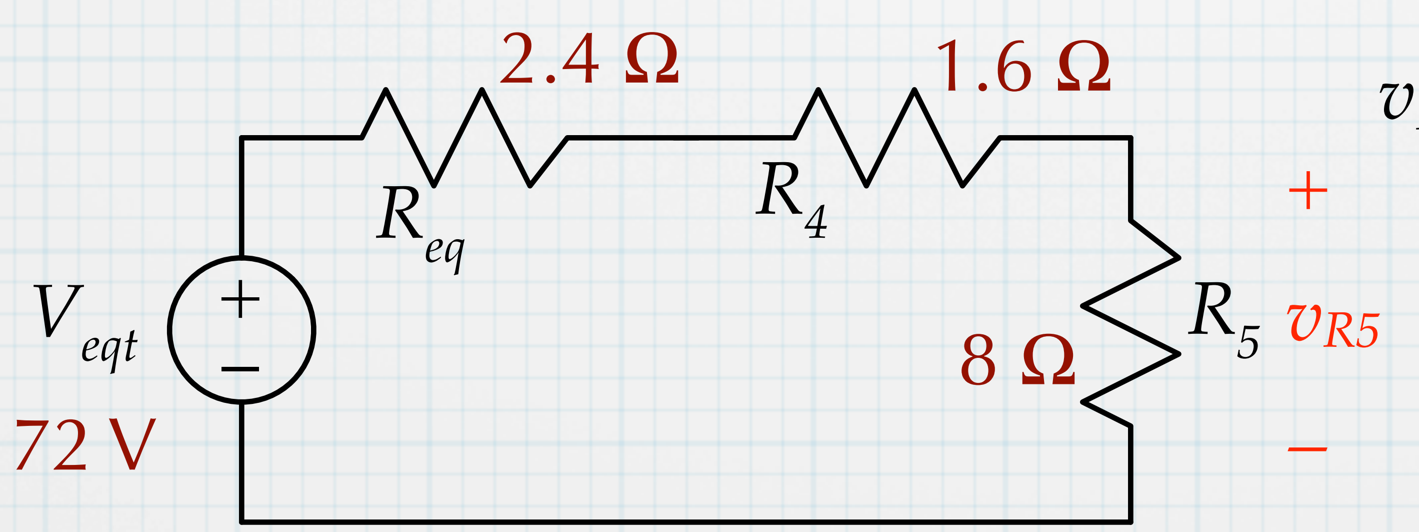
Add the parallel current sources into one. Combine the parallel resistors into one.



I_{eq} = 6A – 12 A + 36 A = 30 A.

R_{eq} = 20 Ω || 5 Ω || 6 Ω = 2.4 Ω.

Transform I_{eq} & R_{eq}:



Use voltage divider:

$$v_{R5} = \frac{R_5}{R_{eq} + R_4 + R_5} V_{eqt}$$

$$= \frac{8\Omega}{2.4\Omega + 1.6\Omega + 8\Omega} (72\text{V})$$

$$= 48\text{V}$$