

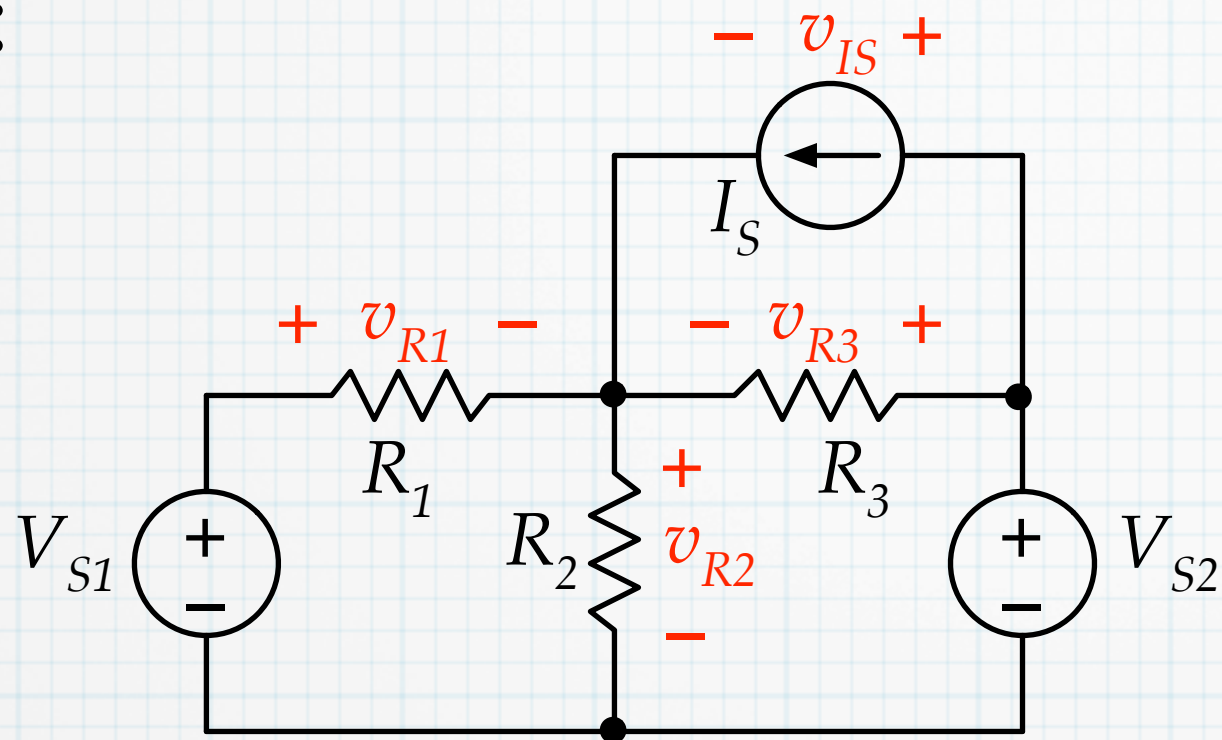
# The mesh-current method

- Equivalent resistance
- Voltage / current dividers
- Source transformations
- Node voltages
- Mesh currents
- Superposition

Mirror image of the node-voltage method.

- Define mesh currents flowing around the loops that make up a circuit.
- Then use KVL to relate the voltages around each loop.
- Convert voltage equations to mesh-current equations using Ohm's law.

## Recall:



Use KVL to relate the voltage around the three meshes (loops) that define the circuit.

$$V_{S1} - v_{R1} - v_{R2} = 0$$

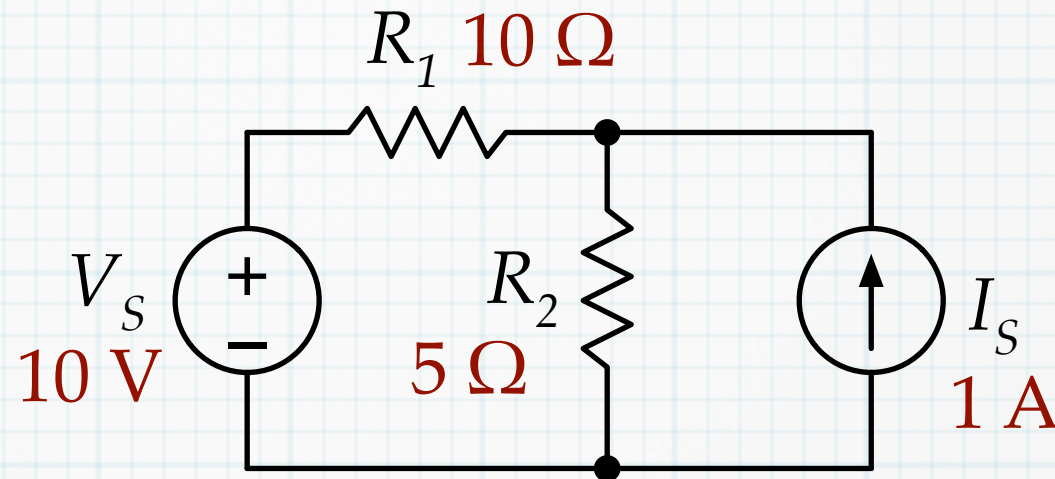
$$V_{S2} - v_{R3} - v_{R2} = 0$$

$$v_{R3} - v_{IS} = 0$$

Three equations, but four  
knowns ( $v_{R1}$ ,  $v_{R2}$ ,  $v_{R3}$ ,  $v_{IS}$ ).  
Need another equation. Or a  
better method.

# The mesh-current method

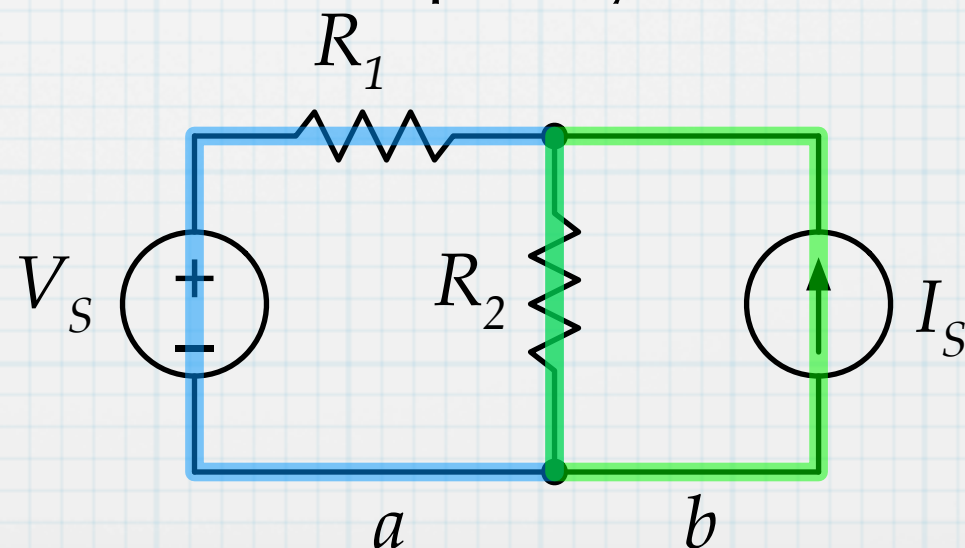
Let's illustrate the mesh-current method, using a familiar circuit (2-sources, 2-resistors).



Of course, we have solved this one previously using the source-transformation and node-voltage methods. But this makes it a good choice for looking at something new, since we already know how to solve it.

1. Identify the meshes that define the circuit. A mesh (or loop) is a closed, continuous path around some part of the circuit. Generally, we want the set of the smallest meshes that completely define the circuit.

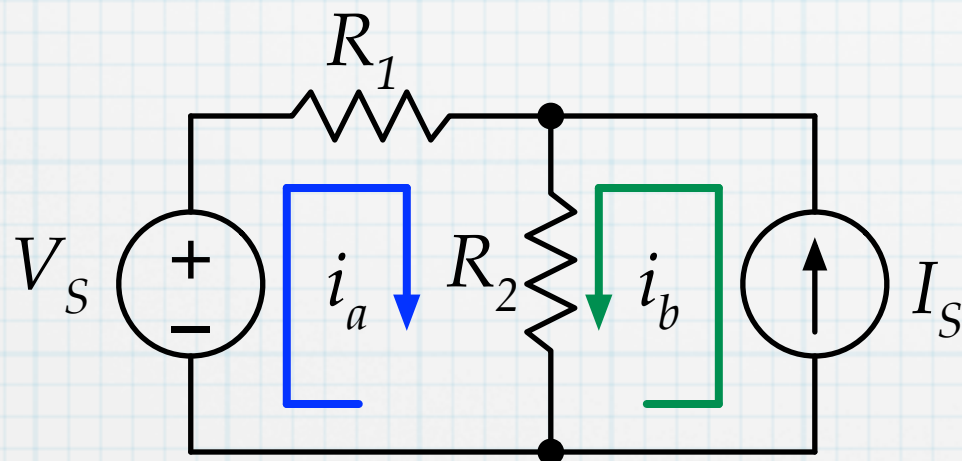
Our simple circuit has two meshes, labeled  $a$  and  $b$ .





We see (by KCL) that the current around the outside of mesh  $a$  (through  $V_S$  and  $R_1$ ) and the current through the outside part of mesh  $b$  (which is just  $I_S$ ) add together to flow through  $R_2$ , where the two meshes coincide. Therefore, we can...

2. Define mesh currents that circulate around each mesh. The current through components on the outside branches of the mesh is just the mesh current. The current through components on interior branches, where two meshes coincide, will be a combination of two mesh currents.



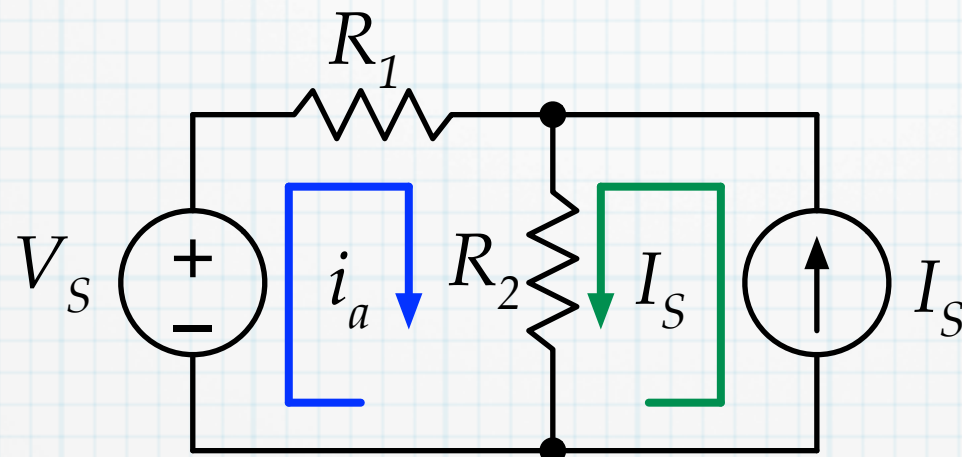
$$i_{V_S} = i_{R_1} = i_a$$

$$I_S = i_b$$

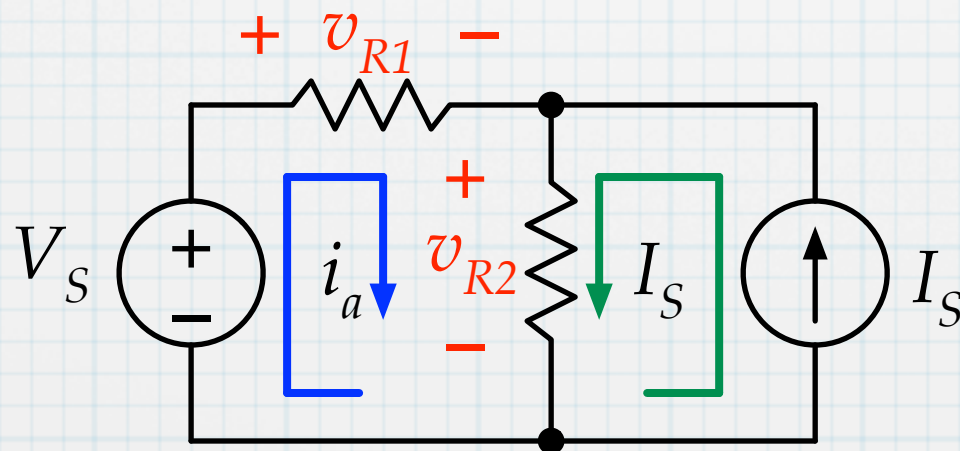
$$i_{R_2} = i_a + i_b$$

As always, you can define the mesh currents in either. At the end, when we compute the actual values for the currents, the sign of the value will tell us the actual direction.

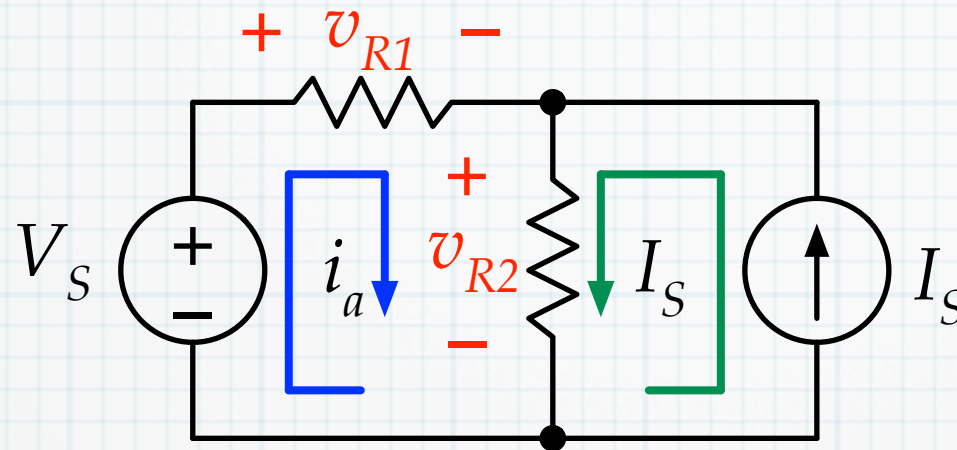
3. Simplify the problem by identifying currents that known. If there is a current source on an outside branch of mesh, then the mesh current must be equal to the source current. In this case, it is obvious that  $i_b = I_S$ . Now the only unknown current is  $i_a$ .



4. Define voltages for all of the components around the meshes where the current is not known.



5. Write KVL equations around each of the loops where mesh current is not known. There is only one in this case.



$$V_S - v_{R1} - v_{R2} = 0.$$

6. Use Ohm's law to express the resistor voltages in terms of the mesh currents.

$$v_{R1} = R_1 i_a \quad v_{R2} = R_2 (i_a + I_S)$$

7. Insert the currents into the KVL equations to form the *mesh-current equations*.

$$V_S - R_1 i_a - R_2 (i_a + I_S) = 0 \quad \text{Only one equation in this case.}$$



8. Do the math to solve the mesh equation(s) for the mesh current(s). Once all the currents are known, all the voltage and powers for the circuit components can be calculated.

$$V_S - R_1 i_a - R_2(i_a + I_S) = 0$$

$$(R_1 + R_2) i_a = V_S - R_2 I_S$$

$$\begin{aligned} i_a &= \frac{V_S - R_2 I_S}{R_1 + R_2} \\ &= \frac{10 \text{ V} - (5\Omega)(1 \text{ A})}{10\Omega + 5\Omega} = 0.333 \text{ A} \end{aligned}$$

Everything else (voltages, currents, powers) can be calculated once the mesh currents are known.

$$v_{R1} = R_1 i_a = (10 \Omega)(0.333 \text{ A}) = 3.33 \text{ V}$$

$$v_{R2} = R_2(i_a + I_S) = (5 \Omega)(0.333 \text{ A} + 1.0 \text{ A}) = 6.67 \text{ V}$$

# The mesh current method

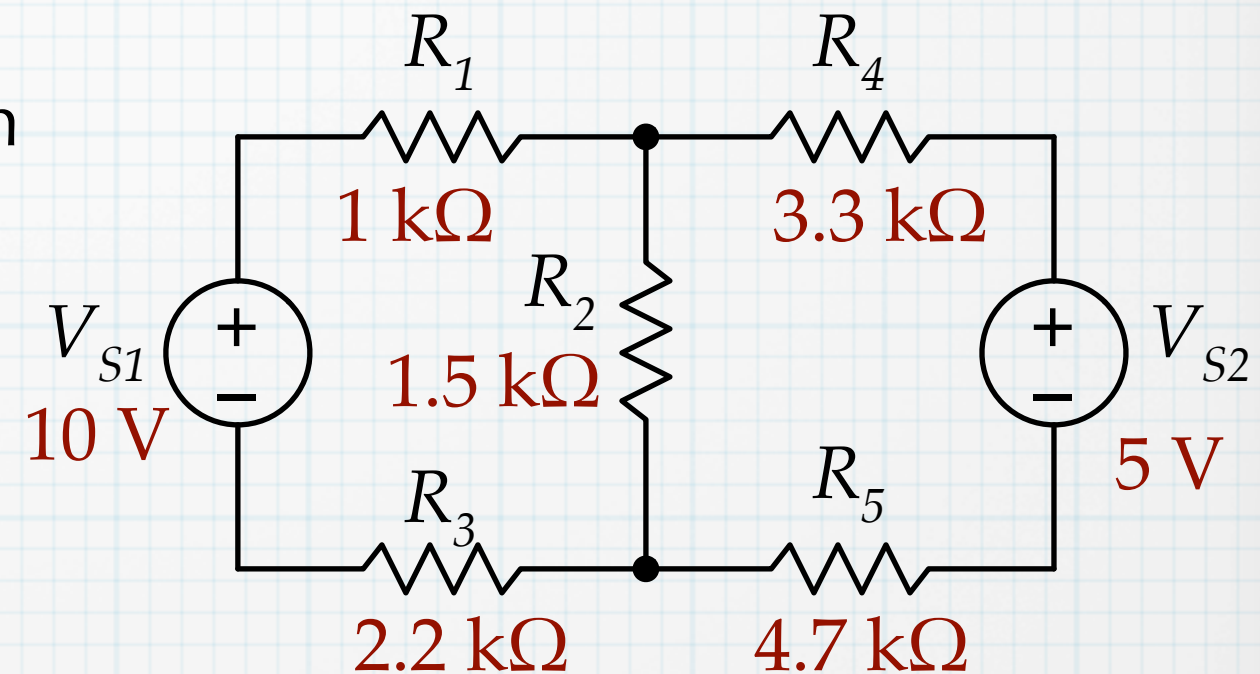
1. Identify all of the individual meshes in the circuit.
2. Assign a mesh current to each mesh.
3. Identify meshes in which the current is known because there is a current source in an outside branch of the mesh.
4. Assign voltages to all of the elements in the meshes with unknown currents.
5. Use KVL around each mesh to write loop equation.
6. Use Ohm's law to write the resistor voltages in terms of the mesh currents.
7. Insert the voltage expressions into the KVL equations to arrive at a set of mesh current equations. If there are  $n$  unknown currents, there should be  $n$  equations relating them.
8. Solve the system of equations to find the mesh currents.



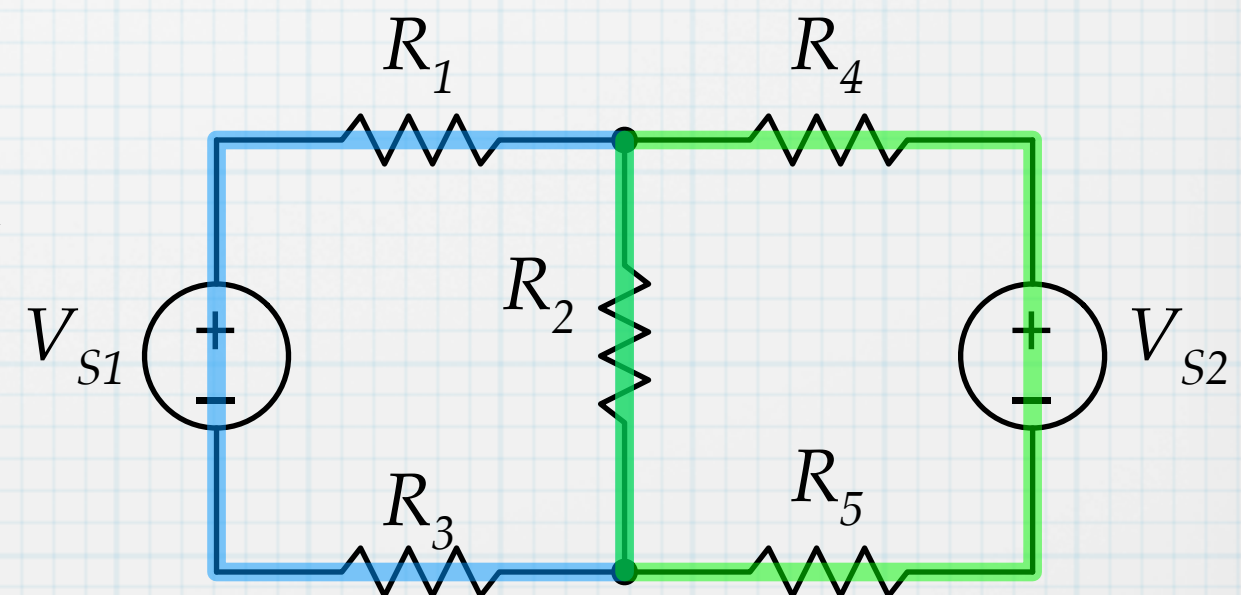
# Another example

The 2-source, 2-resistor circuit is pretty simple, and we have already seen two ways to solve it previously. Why do we need yet another method?

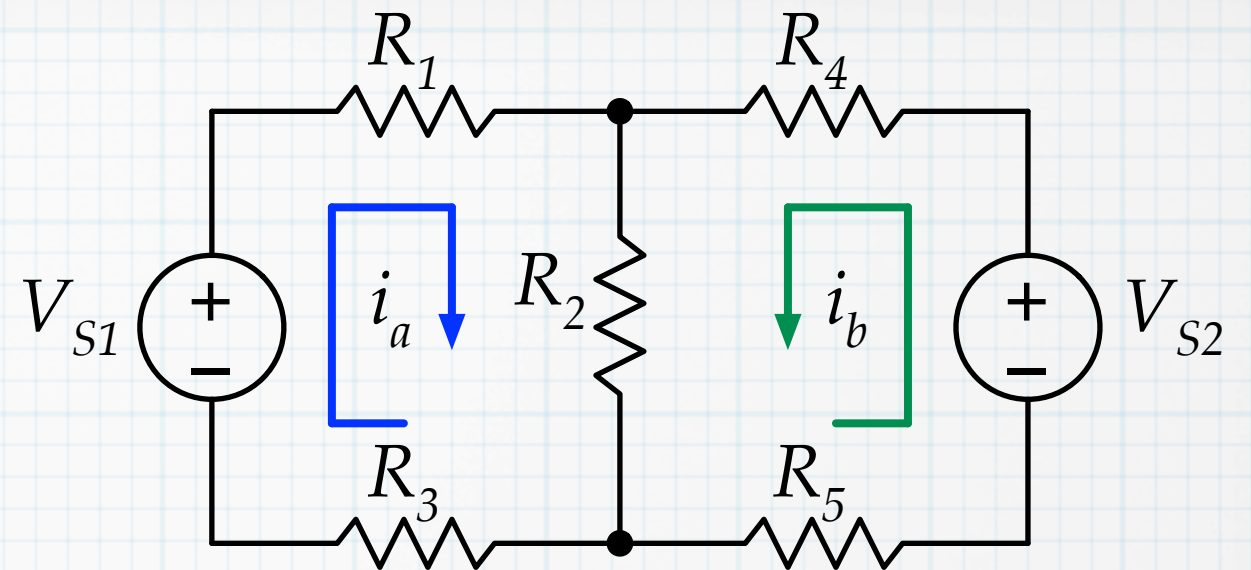
To see how the mesh-current approach can be advantageous, consider the circuit at right. There are six nodes (count them). So the node-voltage method will have at least 4 equations and 4 unknowns.



1. However, we see that there are only two meshes and so there will be two mesh currents. This should be easier.



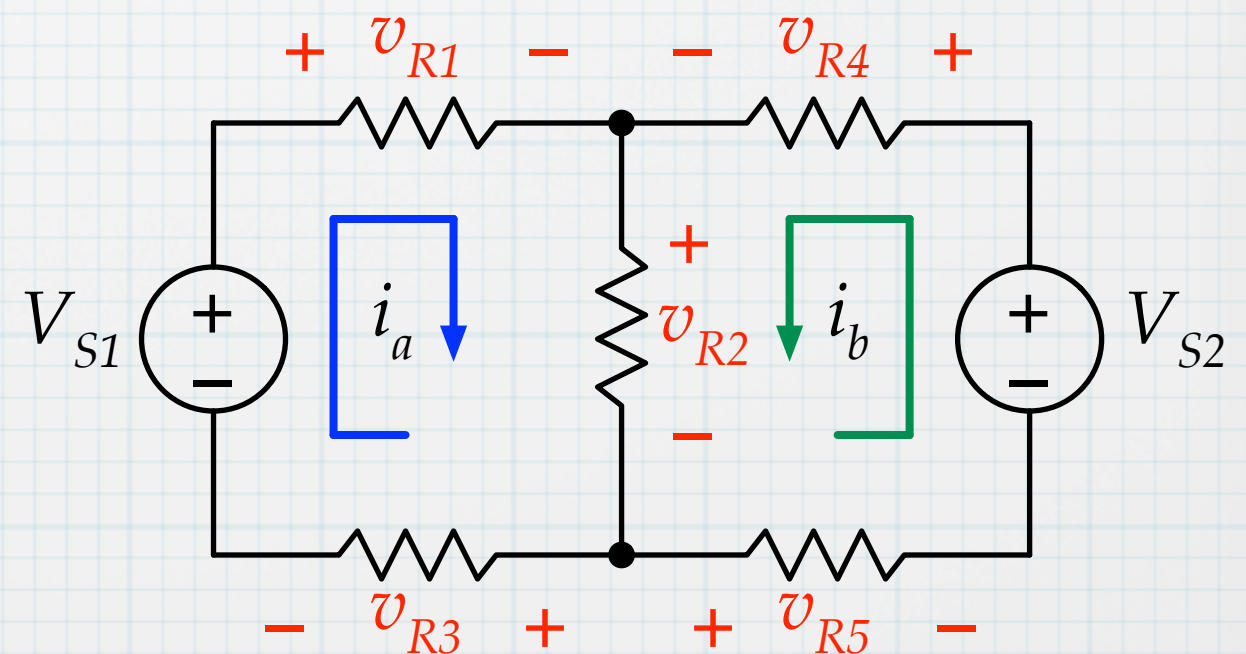
2. Define the two mesh currents.



According to the way that we have defined the currents,  $i_a$  flows in  $V_{S1}$ ,  $R_1$ , and  $R_4$ , and  $i_b$  flows in  $V_{S2}$ ,  $R_4$ , and  $R_5$ . The current in  $R_2$  is the sum of  $i_a$  and  $i_b$ .

3. There are no current sources, so no simplifications are possible. This will be a “two equations, two unknowns” problem.

4. Assign voltages around each loop.



5. Use KVL to relate the voltages around each mesh.

a:  $V_{S1} - v_{R1} - v_{R2} - v_{R3} = 0.$

(As always with KVL,  
starting point is irrelevant.)

b:  $V_{S2} - v_{R4} - v_{R2} - v_{R5} = 0.$

6. Use Ohm's law to write the resistor voltages in terms of the mesh currents.

$$v_{R1} = R_1 i_a \quad v_{R2} = R_2 (i_a + i_b) \quad v_{R3} = R_3 i_a \quad v_{R4} = R_4 i_b \quad v_{R5} = R_5 i_b$$

7. Insert the voltage expressions into the KVL equations to form mesh equations.

a:  $V_{S1} - R_1 i_a - R_2 (i_a + i_b) - R_3 i_a = 0.$

b:  $V_{S2} - R_4 i_b - R_2 (i_a + i_b) - R_5 i_b = 0.$



8. Do the math.

$$(R_1 + R_2 + R_3) i_a + R_2 i_b = V_{S1}.$$

$$R_2 i_a + (R_4 + R_2 + R_5) i_b = V_{S2}.$$

$$(1 \text{ k}\Omega + 1.5 \text{ k}\Omega + 2.2 \text{ k}\Omega) i_a + (1.5 \text{ k}\Omega) i_b = 10 \text{ V}.$$

$$(1.5 \text{ k}\Omega) i_a + (3.3 \text{ k}\Omega + 1.5 \text{ k}\Omega + 4.7 \text{ k}\Omega) i_b = 5 \text{ V}.$$

$$(4.7 \text{ k}\Omega) i_a + (1.5 \text{ k}\Omega) i_b = 10 \text{ V}.$$

$$(1.5 \text{ k}\Omega) i_a + (9.5 \text{ k}\Omega) i_b = 5 \text{ V}.$$

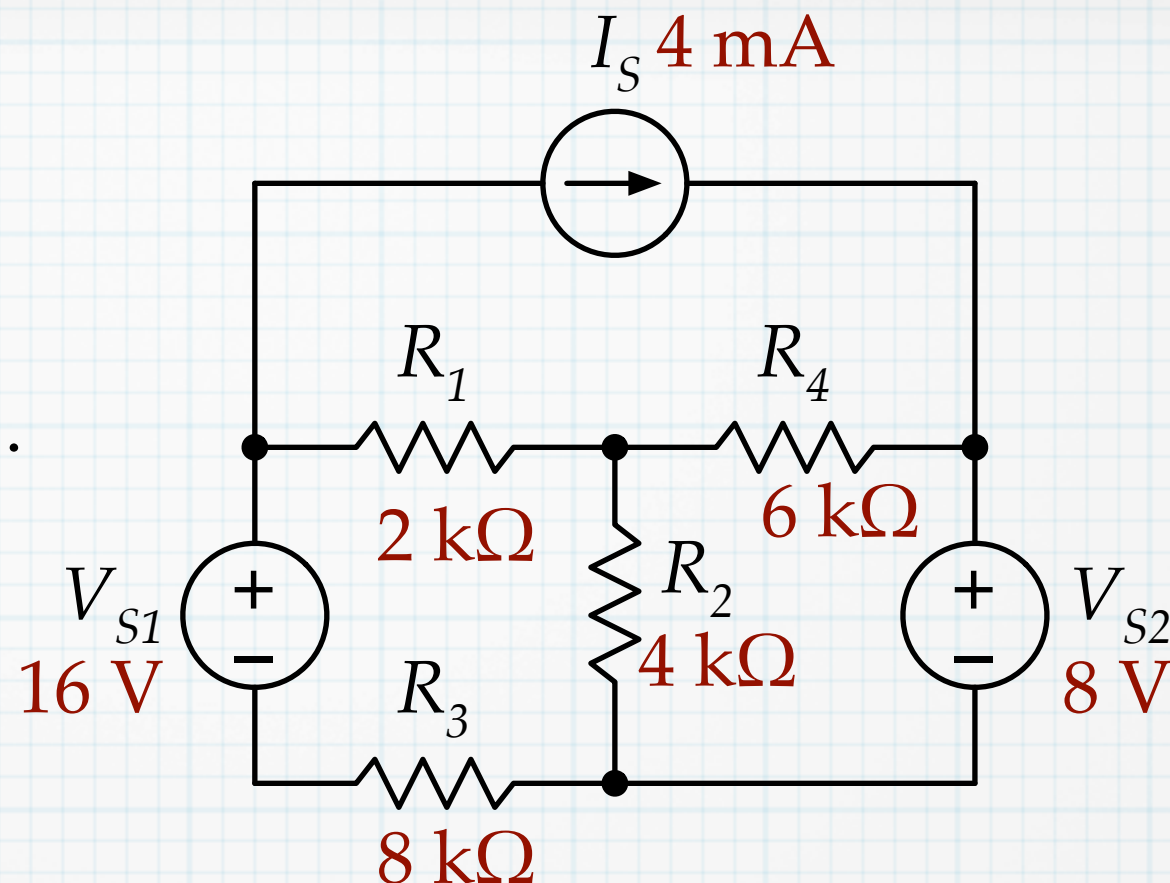
$$i_a = 2.06 \text{ mA}.$$

$$i_b = 0.21 \text{ mA}.$$

Now you can find all the voltages and powers, if needed.

## Example 3

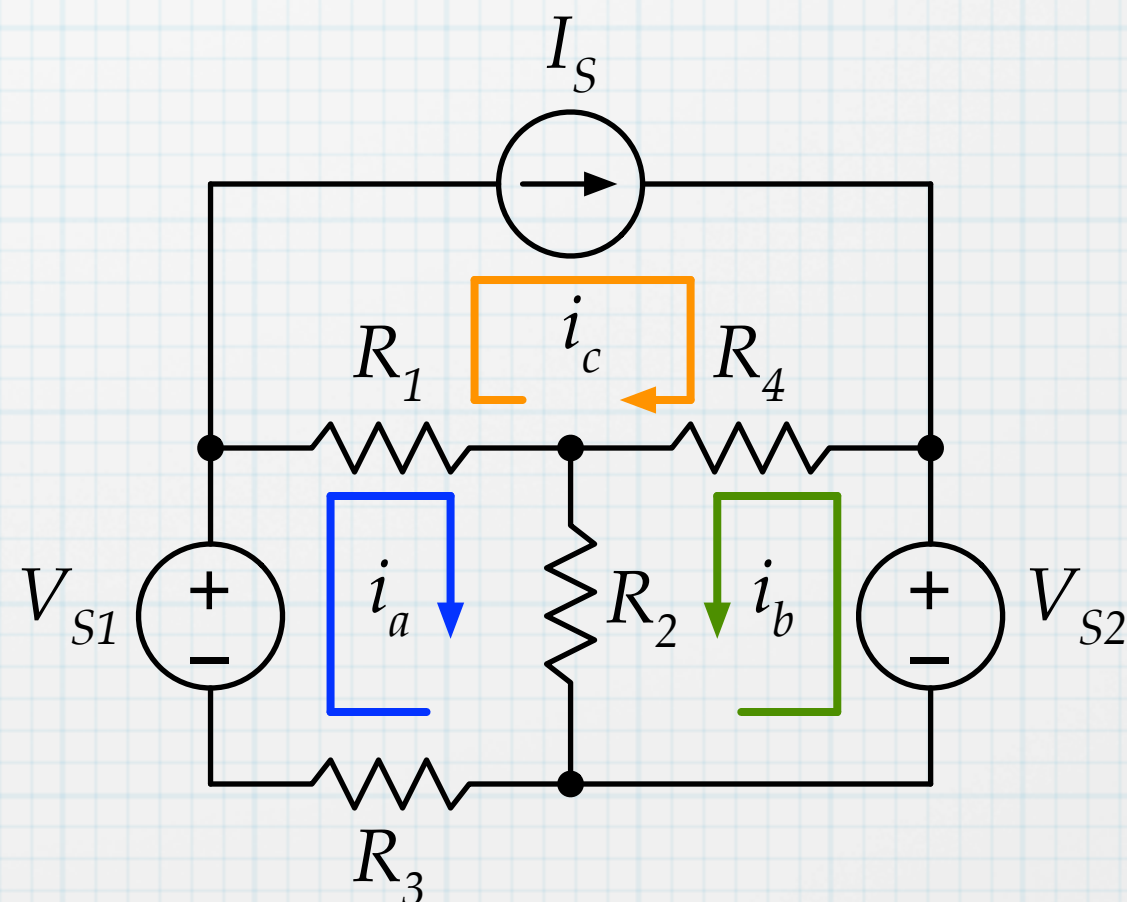
Determine the power being delivered (or absorbed) by  $V_{S1}$ .



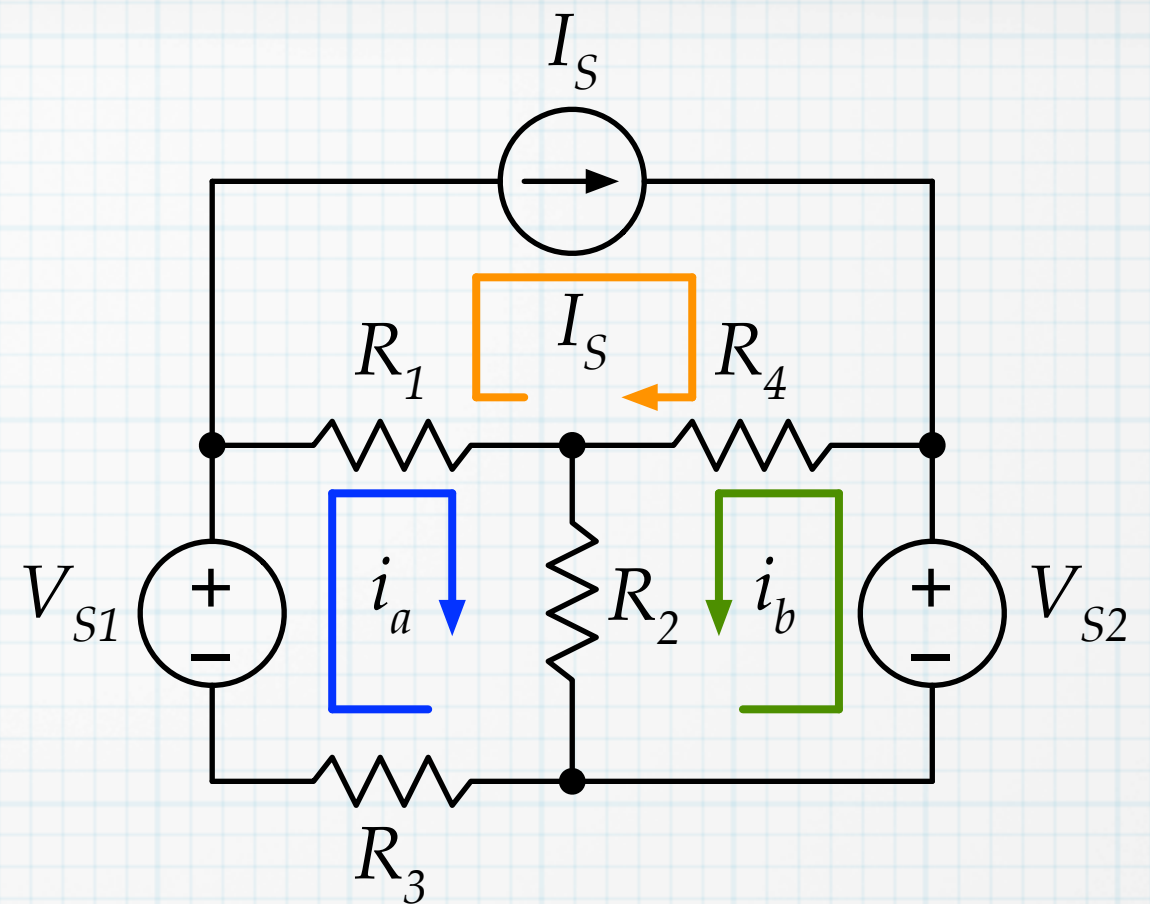
We will need to find  $i_{V_{S1}} = I_S + i_{R1}$ . The resistor  $R_3$  on the bottom would make the use of node voltages a bit unwieldy, so try mesh currents.

1&2. The circuit has three meshes, so define three mesh currents.

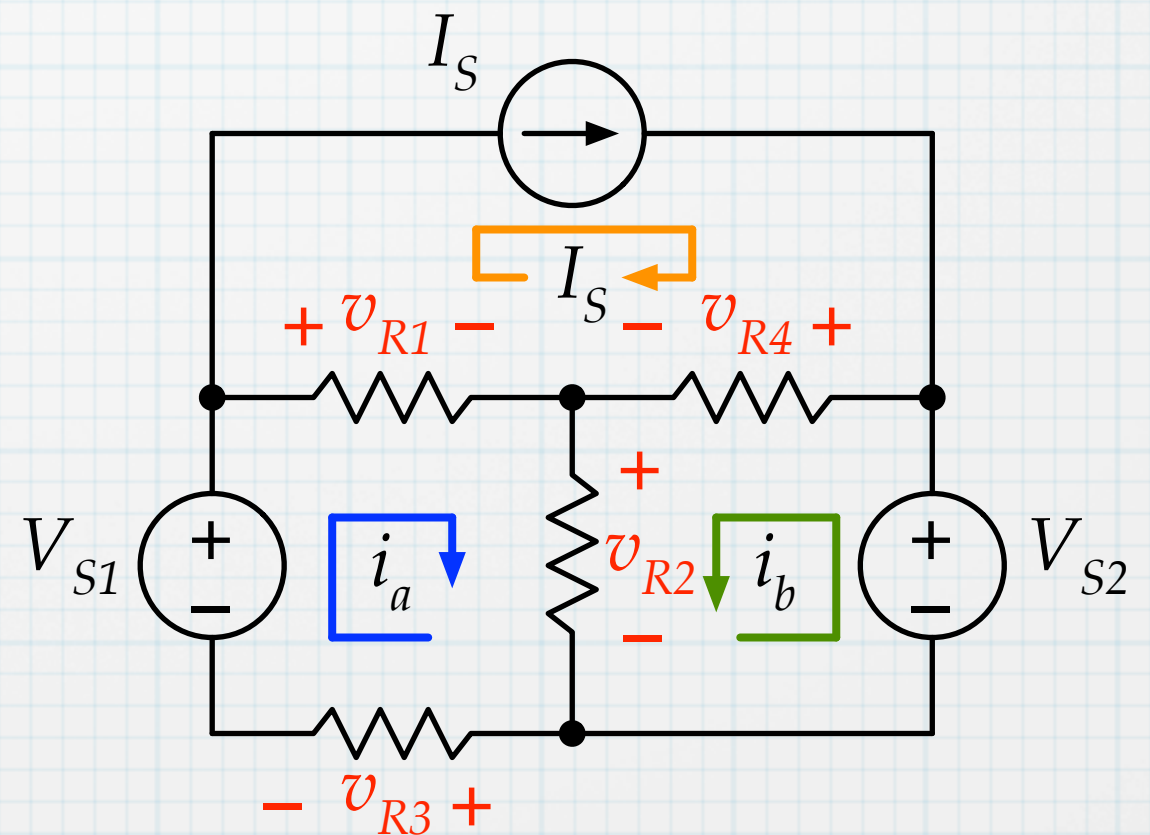
Now note that  $i_{V_{S1}} = i_a$ .



3. With the current source in the outside branch of mesh c, we know directly that  $i_c = I_S$ . That leaves two unknown mesh currents.



4. Define voltages for each component around each of the unknown meshes.

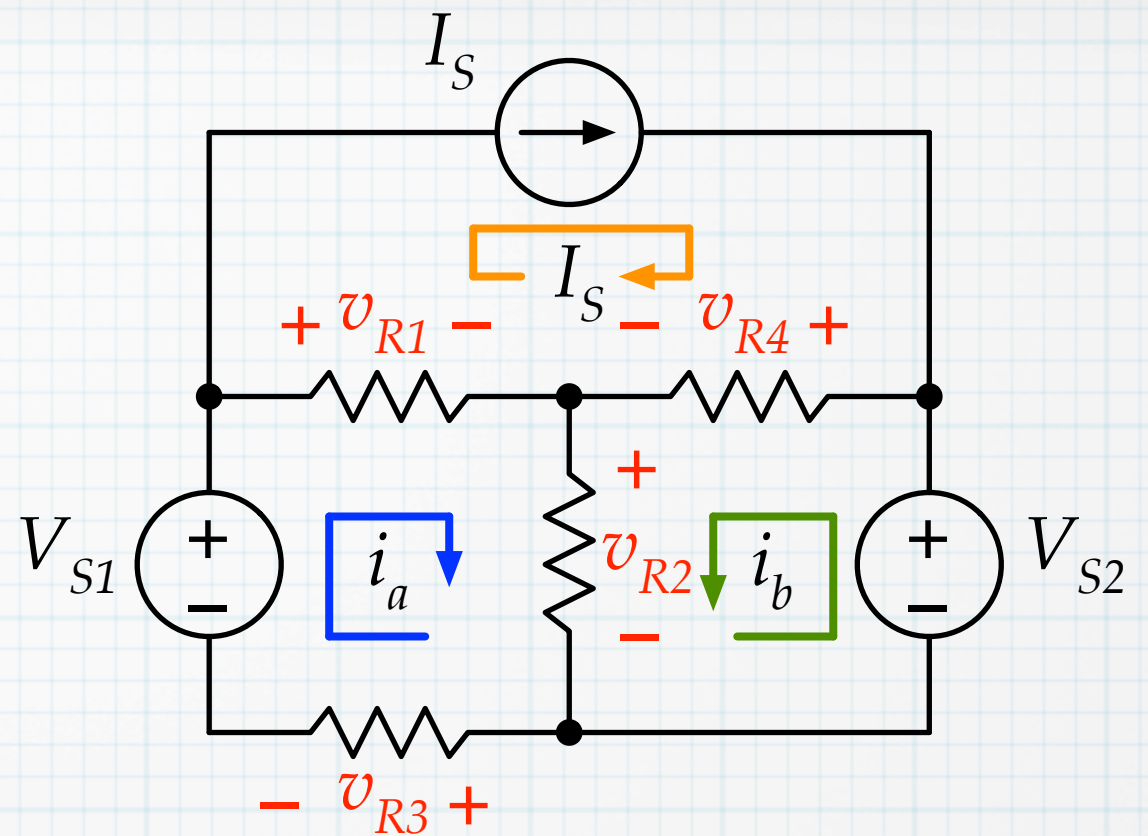




5. Use KVL to relate the voltages around each mesh.

a:  $V_{S1} - v_{R1} - v_{R2} - v_{R3} = 0.$

b:  $V_{S2} - v_{R4} - v_{R2} = 0.$



6. Use Ohm's law to write the resistor voltages in terms of the mesh currents. It is important to pay attention to current directions and voltage polarities.

$$v_{R1} = R_1 (i_a - I_S) \quad v_{R2} = R_2 (i_a + i_b) \quad v_{R3} = R_3 i_a \quad v_{R4} = R_4 (i_b + I_S)$$

7. Insert the voltage expressions into the KVL equations to form mesh equations.

a:  $V_{S1} - R_1 (i_a - I_S) - R_2 (i_a + i_b) - R_3 i_a = 0.$

b:  $V_{S2} - R_4 (i_b + I_S) - R_2 (i_a + i_b) = 0.$

8. Do the math.

$$(R_1 + R_2 + R_3) i_a + R_2 i_b = V_{S1} + R_1 I_S$$

$$R_2 i_a + (R_4 + R_2) i_b = V_{S2} - R_4 I_S$$

$$(2 \text{ k}\Omega + 4 \text{ k}\Omega + 8 \text{ k}\Omega) i_a + (4 \text{ k}\Omega) i_b = 16 \text{ V} + (2 \text{ k}\Omega)(4 \text{ mA})$$

$$(4 \text{ k}\Omega) i_a + (6 \text{ k}\Omega + 4 \text{ k}\Omega) i_b = 8 \text{ V} - (6 \text{ k}\Omega)(4 \text{ mA})$$

$$(14 \text{ k}\Omega) i_a + (4 \text{ k}\Omega) i_b = 24 \text{ V}.$$

$$(4 \text{ k}\Omega) i_a + (10 \text{ k}\Omega) i_b = -16 \text{ V}.$$

$$i_a = 2.45 \text{ mA}.$$

$$i_b = -2.58 \text{ mA}.$$

$$P_{VS1} = V_{S1} i_{VS1}$$

$$= V_{S1} i_a$$

$$= (16 \text{ V})(2.45 \text{ mA}) = 39.2 \text{ mW}$$

# Node-voltage or mesh-current?

Deciding which approach to take in a particular circuit usually boils down to determining which method leads to easier math – fewest number of simultaneous equations.

node number (N)

1. Count number of nodes in the circuit.
2. Subtract 1 for ground.
3. Subtract 1 for each voltage source which has a connection ( + or – ) to ground.
4. Add 1 for each voltage source which has no connection to ground. (More on this later.)

mesh number (M)

1. Count number of meshes in the circuit.
2. Subtract 1 for each current source which is located in an outside branch of a mesh.
3. Add 1 for each current source which is located in an interior branch (shared between two meshes). (More on this later.)

If  $N < M$ , the node-voltage method should have less math.

If  $M < N$ , the mesh-current method should have less math.