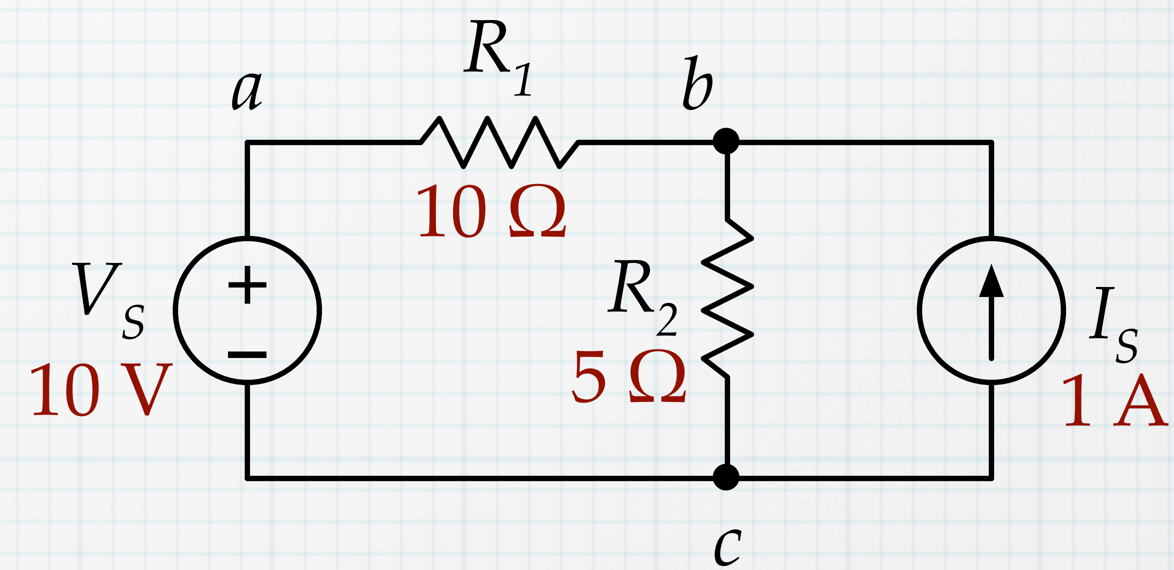
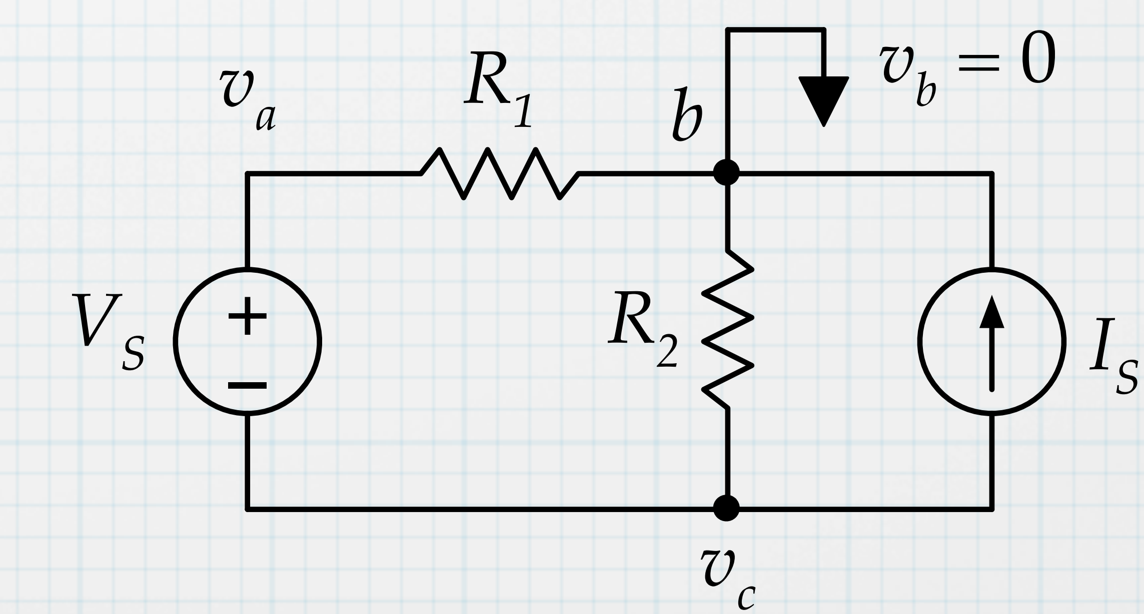
A tricky node-voltage situation

The node-method will always work – you can always generate enough equations to determine all of the node voltages. The prescribed method quite well, but there is one situation where things can be a bit sticky. Whenever there is a voltage source that does *not* have one of its terminals connected to ground, there will be an unknown current that gets added into the mix.

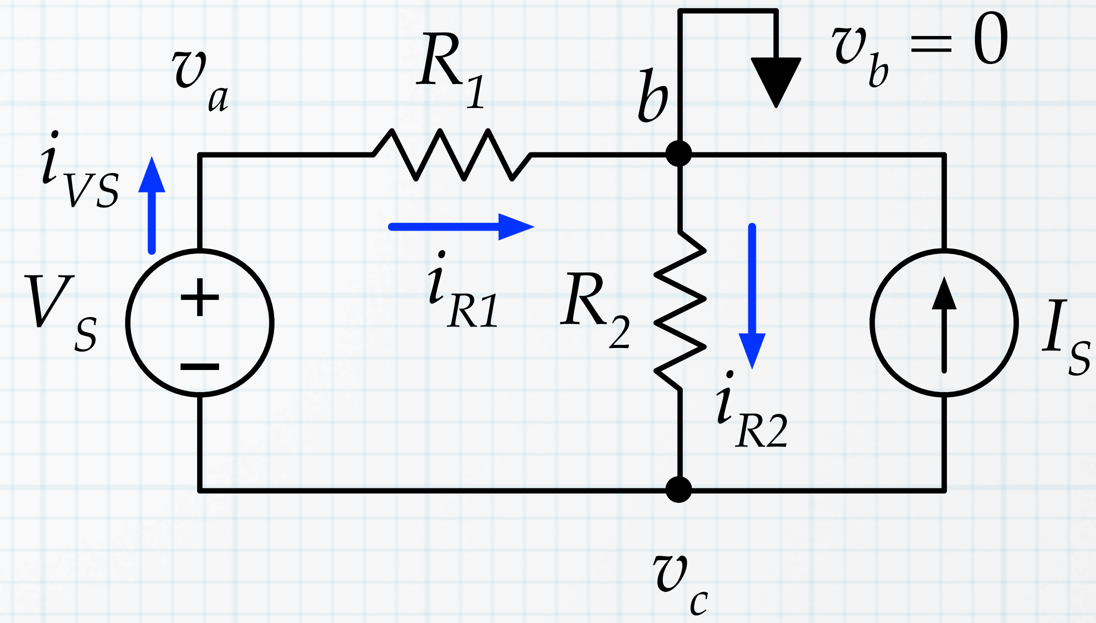
To see the source of the problem, let's consider the 2-source, 2-resistor circuit one more time.



As rookies, we might think that it a good idea to choose node b as the reference. That leaves nodes a and c to be determined with respect to b.



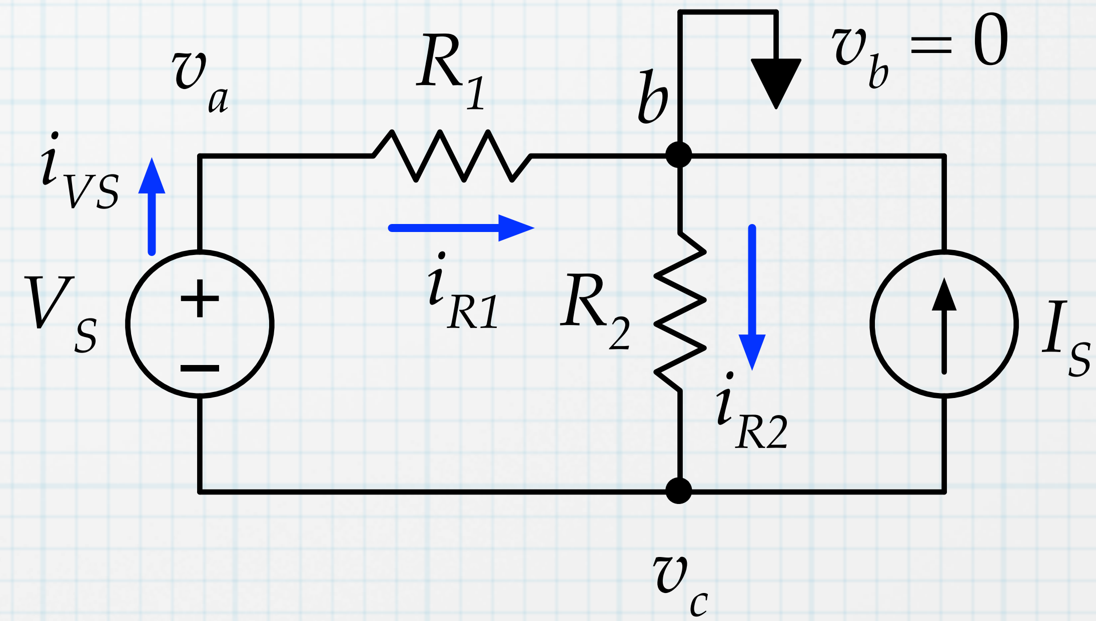
We proceed in the usual fashion, defining the currents and setting up KCL equations. We note that since we don't know v_a directly, we need to write a KCL there, and that this will involve i_{VS}.



Writing the KCL equations:

a: $i_{VS} = i_{R1}$

c: $i_{R2} = i_{VS} + I_S$

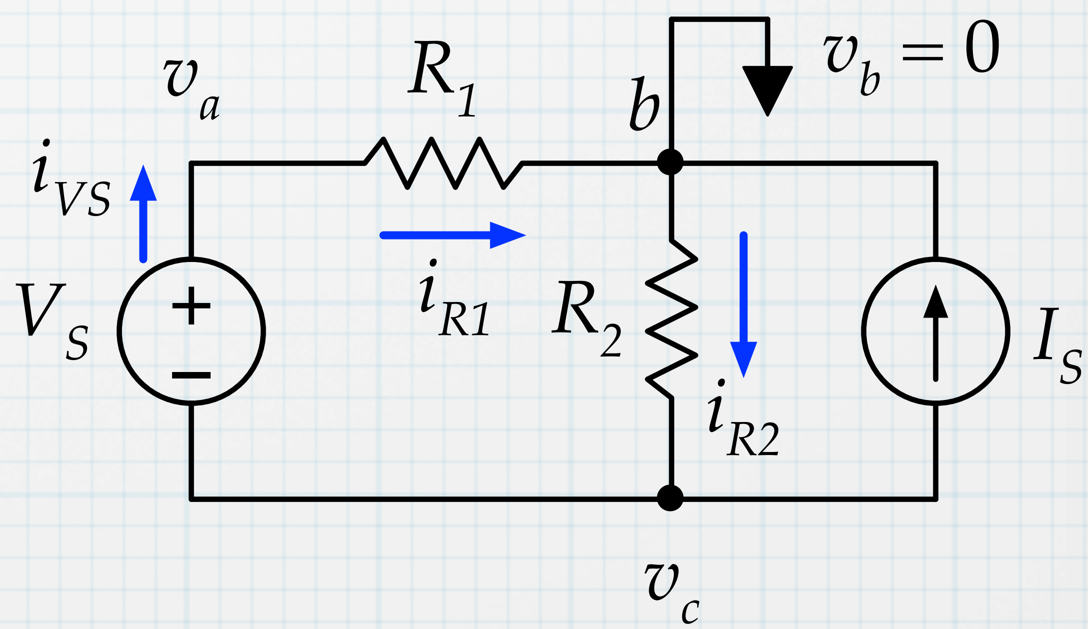


Using Ohm's law to turn these into node-voltage equations:

$$i_{VS} = \frac{v_a}{R_1} \qquad \frac{-v_c}{R_2} = i_{VS} + I_S$$

And now we see the essence of the difficulty here – in using Ohm's law to convert the KCL equations to node-voltage equation, we can't do anything with the voltage-source current. We don't know its value and we can't do anything with it because we can't apply Ohm's law to it. We are left with three unknowns (v_a, v_c and i_{VS}), but only two equations.

Two equations, three unknowns – clearly we need another equation. The "ungrounded" voltage source, which is causing the mathematical difficulty here, also gives us the way out. The source relates the voltages at nodes *a* and *c*, giving us a third equation.



$$v_a = v_c + V_S$$

$$i_{VS} = \frac{v_a}{R_1} \qquad \frac{-v_c}{R_2} = i_{VS} + I_S \qquad v_a = v_c + V_S$$

Once again, we have reached a point where we have extracted everything we need in find all the properties of the circuit – the rest is just math.

As an exercise in algebra, we can proceed in any number of ways. Once approach would be to substitute the left equation for i_{VS} into the middle equation, giving

$$\frac{-v_c}{R_2} = \frac{v_a}{R_1} + I_S$$

Then substitute in for v_a using the right-hand equation at the top, giving a single equation that can be solved for v_c.

$$\frac{-v_c}{R_2} = \frac{v_c + V_S}{R_1} + I_S$$

Writing out the remaining algebra,

$$-v_c \frac{R_1}{R_2} = v_c + V_S + R_1 I_S$$

$$-v_c \left(1 + \frac{R_1}{R_2}\right) = V_S + R_1 I_S$$

$$v_c = -\frac{V_S + R_1 I_S}{1 + \frac{R_1}{R_2}}$$

Plug in the numbers: $$v_c = -\frac{10\text{ V} + (10\Omega)\,(1\text{ A})}{1 + \frac{10\Omega}{5\Omega}} = -6.67\text{ V}$$

$$v_a = v_c + V_S = -6.67\text{ V} + 10\text{ V} = 3.33\text{ V}.$$

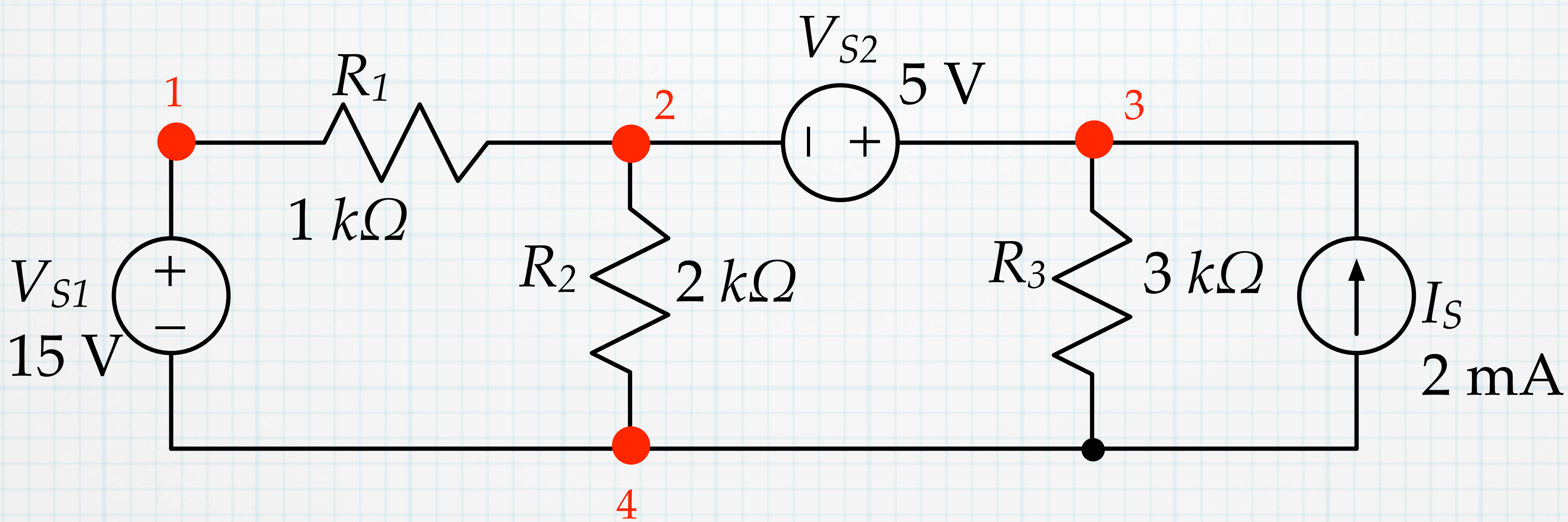
And we have the exact same results as seen twice before:

$v_{R1} = v_a - v_b = 3.33\text{ V} - 0 = 3.33\text{ V}.$ $v_{R1} = v_b - v_c = 0\text{ V} - (-6.67\text{ V}) = +6.67\text{ V}.$

$$i_{R1} = \frac{v_{R1}}{R_1} = \frac{3.33\text{ V}}{10\Omega} = 0.333\text{ A}$$ $$i_{R2} = \frac{v_{R2}}{R_2} = \frac{6.67\text{ V}}{5\Omega} = 1.33\text{ A}$$

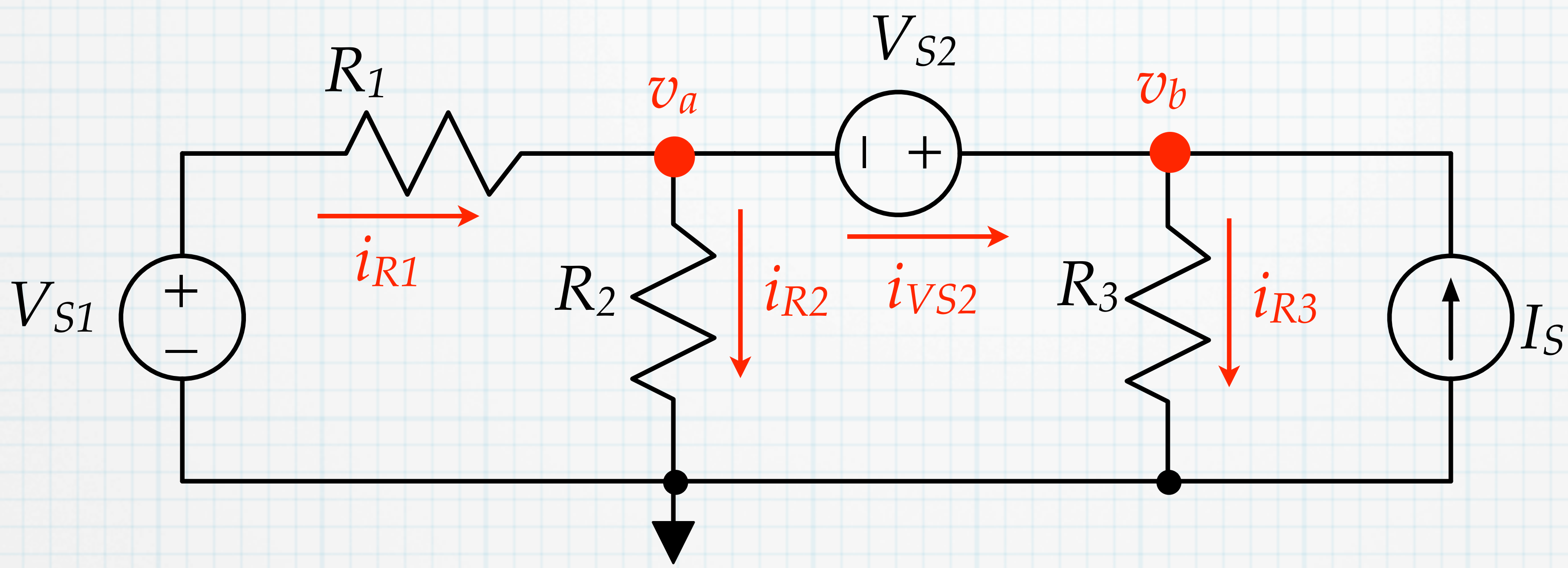
Another example

Consider the circuit below.

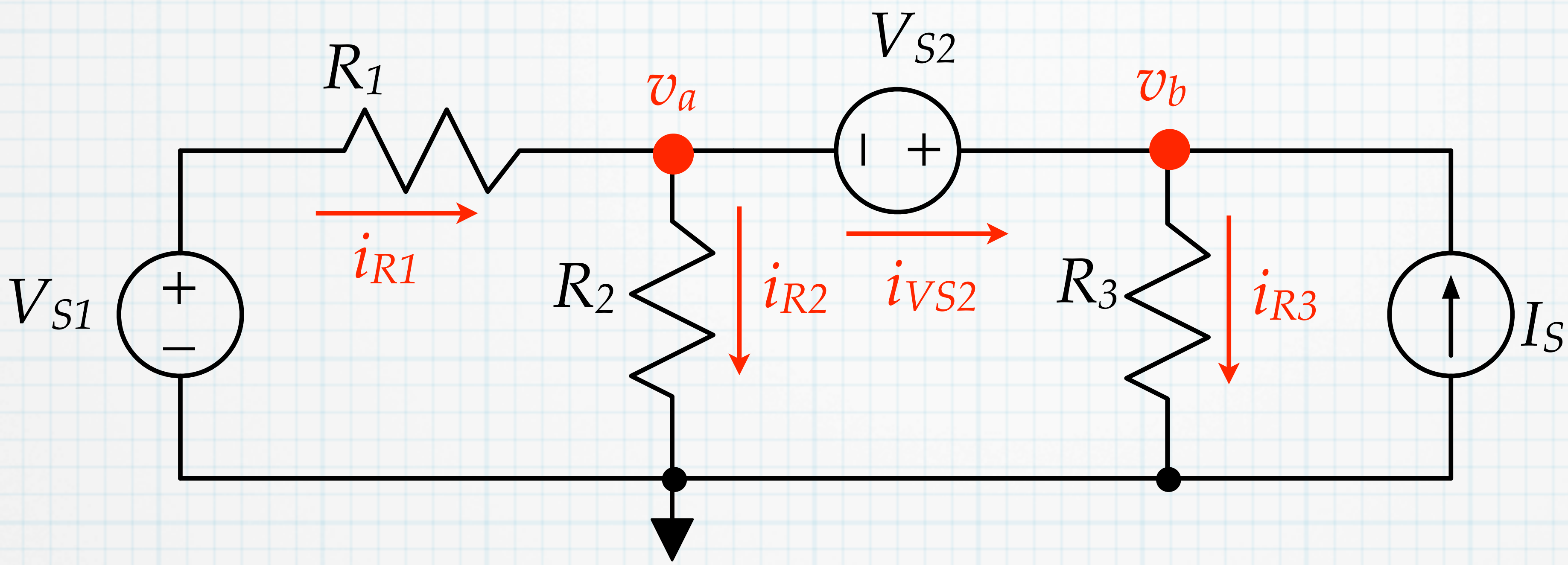


We see that there are four nodes. The problem comes in picking one to be ground. Either 2 or 4 would be a reasonable candidate. We'll flip a coin and pick 4 as ground. Then node 1 is clearly at V_{S1} = 15 V.

Label the two nodes and the currents.



Again, we see the same sort of difficulty. The current through the second source is not known, and we cannot use Ohm's law to relate it to the node voltages on either side. So, in addition to the two unknown node voltages, we have a third unknown, i_{VS2}.



Use KCL to balance currents

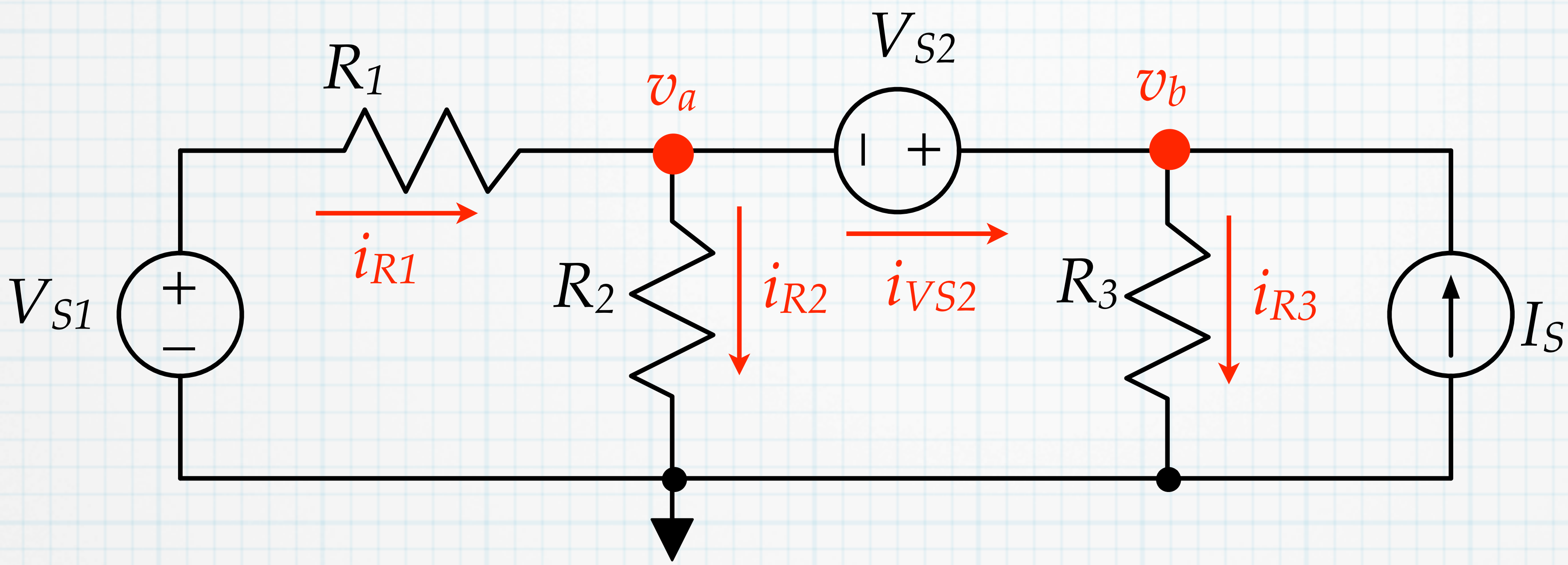
$$i_{R1} = i_{R2} + i_{VS2} \qquad \frac{V_{S1} - v_a}{R_1} = \frac{v_a}{R_2} + i_{VS2}$$

$$i_{VS2} + I_S = i_{R3} \qquad i_{VS2} + I_S = \frac{v_b}{R_3}$$

Three unknowns.

Can eliminate i_{VS2}:

$$\frac{V_{S1} - v_a}{R_1} = \frac{v_a}{R_2} + \frac{v_b}{R_3} - I_S$$



Use information from the second source: $v_b - v_a = V_{S2}$

Substitute into the other equation:

$$\frac{V_{S1} - v_a}{R_1} = \frac{v_a}{R_2} + \frac{v_a + V_{S2}}{R_3} - I_S$$

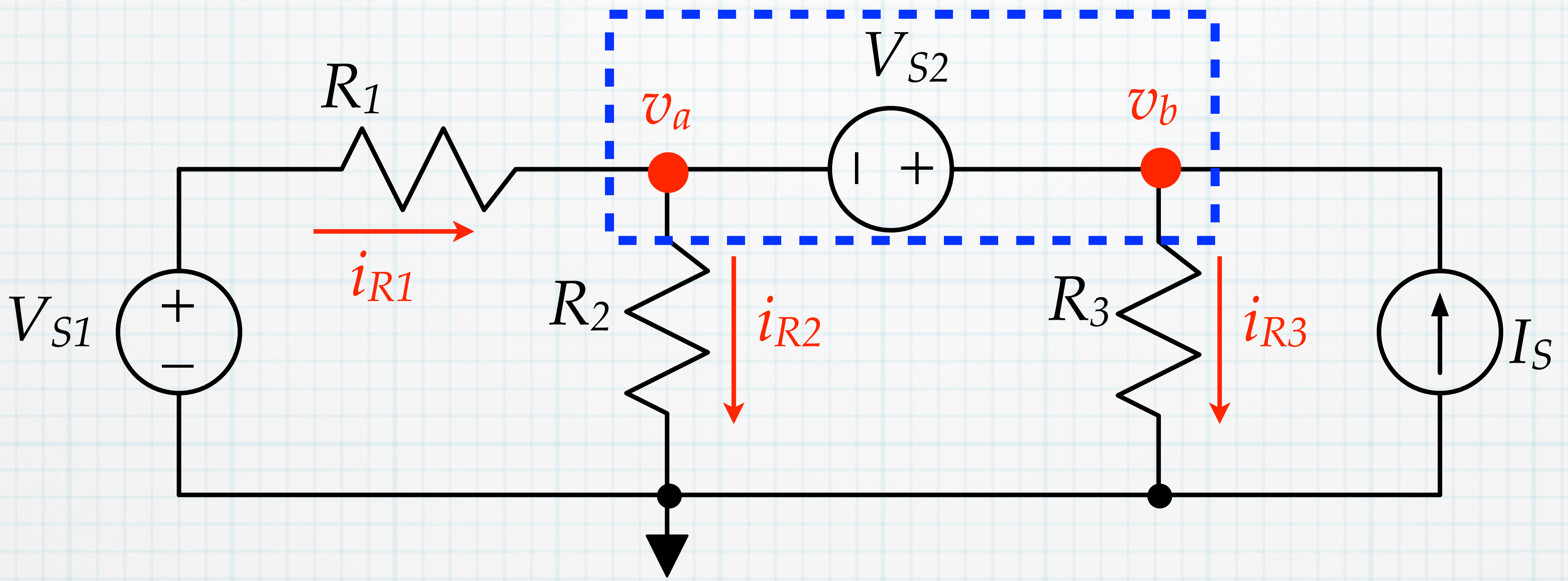
Solve for v_a:

$$v_a \left[1 + \frac{R_3}{R_1} + \frac{R_2}{R_2} \right] = \frac{R_3}{R_1} V_{S1} - V_{S2} + R_3 I_S$$

$v_a = 8.36\text{ V}$

$v_b = 13.36\text{ V}$

An alternative approach that gets to the same point in one less step is the *super node*. Enclose the other voltage source, along with the two nodes connected in an imaginary container – call it a super node. KCL applies to the super node – what goes in must come out. Note the super node is not all at one voltage.



Write a KCL equation equation for the currents crossing the boundary of the super node:

$$i_{R1} + I_S = i_{R2} + i_{R3}$$

Relate resistor currents to the voltages:

$$\frac{V_{S1} - v_a}{R_1} + I_S = \frac{v_a}{R_2} + \frac{v_a + V_{S2}}{R_3}$$

Relate v_a and v_b using V_{S2}:

$$v_b - v_a = V_{S2}$$

Right where we were before without having to mess with i_{VS2}.