FnW Project
Group 10

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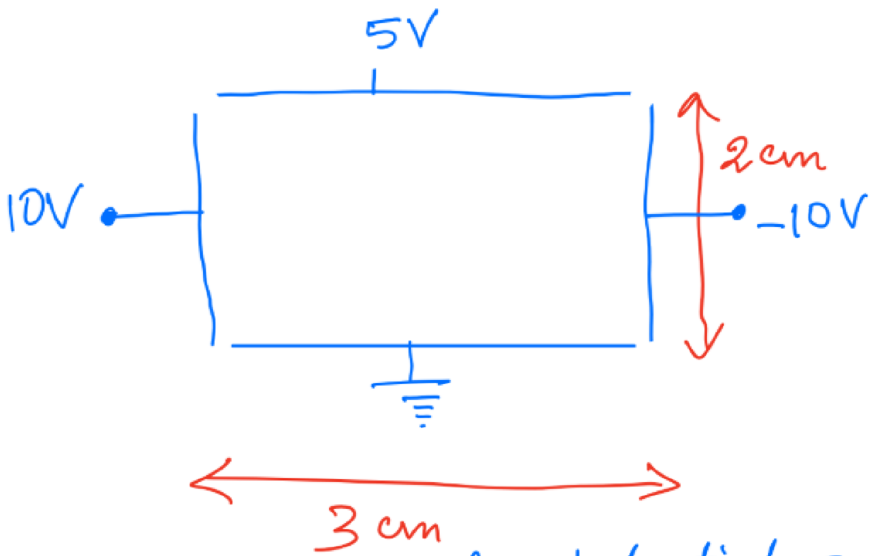
Question



Use FDM to solve for potential and generate a 2D plot.

Finite Difference Method (FDM)

Consider a two-dimensional rectangular region. Given the values of potential at all points on the four boundaries, we want to find how the potential function varies inside the rectangular region.

Solution for Potential Function

We divide the grid into several points on the grid and find a solution for each of them.

Forward difference:

$$\left.\frac{\partial \phi}{\partial x}\right|_{\mathrm{FD}} = \frac{\phi(x+\Delta x) - \phi(x)}{\Delta x}$$

In the grid this can be written as:

$$\left.\frac{\partial \phi}{\partial x}\right|_{\mathrm{FD}} = \frac{\phi_{i+1,j} - \phi_{i,j}}{h}$$

Backward difference:

$$\left.\frac{\partial \phi}{\partial x}\right|_{\mathrm{BD}} = \frac{\phi(x) - \phi(x-\Delta x)}{\Delta x}$$

In the grid:

$$\left.\frac{\partial \phi}{\partial x}\right|_{\mathrm{BD}} = \frac{\phi_{i,j} - \phi_{i-1,j}}{h}$$

Second derivative in x:

$$\begin{aligned}\frac{\partial^2 \phi}{\partial x^2} &= \frac{\left.\frac{\partial \phi}{\partial x}\right|_{\mathrm{FD}} - \left.\frac{\partial \phi}{\partial x}\right|_{\mathrm{BD}}}{h} = \frac{\frac{\phi_{i+1,j}-\phi_{i,j}}{h} - \frac{\phi_{i,j}-\phi_{i-1,j}}{h}}{h} \\ &= \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{h^2}\end{aligned}$$

Second derivative in y:

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{h^2}$$

Substituting into Laplace's equation $\nabla^2 \phi = 0$:

$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{h^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{h^2} = 0$$

$$\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1} - 4\phi_{i,j} = 0$$

$$\phi_{i,j} = \frac{1}{4}\left(\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}\right)$$

The value of ϕ at any point in the interior grid is just the average of the potentials at the four points in the surrounding grid.

Gauss-Seidel Method

The Gauss-Seidel method is an iterative method for solving linear equation systems. We use this to find potentials at the grid points.
We start the iteration process by guessing the value of ϕ at each point on the grid. In each subsequent iteration, we use the updated values from the previous step. We iterate until the maximum relative error becomes less than a small value (epsilon). For simplicity, we assume satisfactory convergence after 1000 iterations.
With each iteration, the maximum relative error reduces and our approximation improves. Thus, the Gauss-Seidel algorithm helps solve the equations derived using the FDM.

Python Code

Below is the Python implementation. We are given a rectangular region 3cm $\times$ 2cm with the following boundary conditions:

- Left boundary: 10V
- Right boundary: -10V
- Top boundary: 5V
- Bottom boundary: 0V

The region is divided into a grid 31×21. The Gauss-Seidel algorithm is then applied for 1000 iterations.

```python
import numpy as np
import matplotlib.pyplot as plt

MAX_ITERATIONS = 1000

width = 3
height = 2
numOfPointsX = 31
numOfPointsY = 21

potential = np.zeros((numOfPointsY , numOfPointsX))

for i in range(numOfPointsX):
    potential[0 , i] = 0
    potential[-1 , i] = 5

for i in range(numOfPointsY):
    potential[i , 0] = 10
    potential[i , numOfPointsX - 1] = -10

for i in range(MAX_ITERATIONS):
    for j in range(1 , numOfPointsY - 1):
        for k in range(1 , numOfPointsX -  1):
```

```
                potential[j , k] = 0.25 * (potential[j + 1 , k] +
                    potential[j - 1 , k] + potential[j , k + 1] +
                    potential[j , k - 1])

x = np.linspace(0, width , numOfPointsX)
y = np.linspace(0 , height , numOfPointsY)
X, Y = np.meshgrid(x , y)

plt.figure()
plt.contourf(X , Y , potential , 100 , cmap = 'plasma')
plt.colorbar()
plt.title("2D Potential Plot Using FDM - Group 10")
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```

2D Plot of Potential