Modelling the Perfect Penalty Kick

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This paper aims to identify a region where a player may direct the ball for a guaranteed goal, duing penalty shoot-outs. We also determine the initial velocity and spin that must be imparted to the ball for scoring the said goal.

Abstract

FIFA World Cup is a very popular event in which many countries play against each other. No fans want the matches to be a tie. The penalty kick is a key factor which is often used to determine the outcome of such matchs, and when used in its ‘ shoot-out’ format, resolves tied games during the knock-out stages. The historical significance of penalty kicks (and penalty shootouts) is unmatched. Since the introduction of penalty kick shootouts to the World Cup in 1982 (before that, games were replayed to decide a winner), about 20% of knockout stage matches went to penalties. There are basically three broad techniques to score through a penalty kick. Your first strategy could be to go with brute force and just try to force the ball in directly with a powerful kick. If you're a little experienced, you could use some technique to curl the ball around any obstacles using the inside or outside of your foot. And if you really have some skills, you can spin that ball and direct it to a spot where even the world's tallest goalkeeper can not stop it. This study involved a complete theoretical analysis of the third strategy, by analysing the variables involved in a penalty kick. We attempt to devise the best method to kick a penalty to ensure a very high success rate. This was done by treating the soccer ball as a sphere moving in a viscous fluid and then applying fluid dynamics to derive the relationships between the forward movement of the ball, the Magnus effect and rotational velocity, drag, linear velocity and gravity. Upon establishing the relationships, we created a simulation and ran it to observe the initial velocities and spins which should be imparted to the soccer ball to land in the unreachable area to make it a perfect penalty shot that the goalkeeper can not physically save. This paper provides a useful method in predicting the trajectory of the soccer ball with various parameters to give the conditions for the most perfect shot.

Contents

Nomenclature

K_d	Coefficient of Air resistance	$\frac{Kg}{m}$
C_D	Drag Coefficient	
C_M	Magnus Coefficient	
C_L	Lift Coefficient	
I	Moment of Inertia of the Football	Kgm^2
F_T	Total Force on the moving ball	N
F_W	Weight of the ball	N
F_D	Drag Force on the moving ball	N
F_M	Magnus Force on the moving ball	N
F_G	Force due to gravity on the moving ball	N
F_S	Sideways component of magnus force	N
F_L	Lifting component of magnus force	N
A	Area of cross section of the Football	m^2
$\vec{S}$	Constant vector in case of magnus force	
m	Mass of the Football	Kg
r	Radius of the Football	m
v	Magnitude of net velocity of the moving football	$\frac{m}{s}$
a	Acceleration of the moving football	$\frac{m}{s^2}$
s	Displacement of the moving football	m
g	Acceleration due to gravity	$\frac{m}{s^2}$
ρ	Density of air	$\frac{Kg}{m^3}$
ω	Angular velocity of the moving ball	radian per second
α	Polar angle	
ϕ	Azimuthal Angle	

1 Introduction

Soccer is a worldwide sport that attracts millions of fans. Fans like to see the goal. However, the score of a game is usually low. The 0-0 game is frequently seen, indicating it is not easy to shoot a ball into a goal. This makes fans disappointed. Outstanding soccer players can shoot much more balls on target. Some of them are goals. Therefore, shooting a ball into a goal requires very high skills. [4]

1.1 Penalty Kicks and Why do they matter?

One of the most dramatic and often decisive instances of association football is the penalty kick. It is considered a golden opportunity for the kicker to register a goal. The kicker is virtually unchallenged by any opposing player except the goalkeeper who stands on the goal-line 11 meters away. Therefore, the kicker has an overwhelming advantage. Maximising on this advantage is of paramount importance since penalties in many instances, determine the outcome of games.

Penalty shootouts were only introduced to the World Cup in 1978 and 26 had taken place before the 2018 tournament started. The likelihood of any penalty being scored is around 70 per cent, a stat perfectly reflected by the World Cup penalty shootout success rate, which counts 170 successes from 240 penalties taken from shootouts before the tournament in Russia. But why twelve yards? Simple: that's what the FA decided in 1891. And it's likely never been changed because scoring seven out of every ten penalties gives a good mix of risk, reward and drama. [6].

Scoring a penalty shot in match play more than doubled a team's chance of winning, while missing the same shot more than tripled a team's chance of losing, further highlighting the importance of the penalty shot in post 1997 football. The prevalence of the penalty shot is very high; they are awarded in 25% of tournament matches, and one of the two teams in the final of either the World Cup or the European Championships is likely to have been involved in a penalty shootout en-route to the final.

1.2 FIFA Guidelines

FIFA 2022 has the following guidelines which we have incorporated in our model. [7]

1.2.1 Law 2 : The Ball

All balls must be:

- spherical
- made of suitable material
- of a circumference of between 68 cm (27 ins) and 70 cm (28 ins)
- between 410 g (14 oz) and 450 g (16 oz) in weight at the start of the match
- of a pressure equal to 0.6 – 1.1 atmosphere (600 – 1,100g/cm2) at sea level (8.5 lbs/sq in – 15.6 lbs/sq in)

1.2.2 Law 14 : The Penalty Kick

- The ball must be stationary on the penalty mark and the goalposts, crossbar and goal net must not be moving.
- The defending goalkeeper must remain on the goal line, facing the kicker, between the goalposts, without touching the goalposts, crossbar or goal net, until the ball has been kicked.
- of a circumference of between 68 cm (27 ins) and 70 cm (28 inThe player taking the penalty kick must kick the ball forward; backheeling is permitted provided the ball moves forward.
- When the ball is kicked, the defending goalkeeper must have at least part of one foot touching, in line with, or behind, the goal line.

2 Assumptions

The factors affecting the trajectory of the ball are many including but not limited to the weight of the ball, drag force on the ball, the force due to Magnus effect on the ball and velocity. Considering all these factors would overly complicate the differential equations governing the trajectory of the football. To make the mathematics cleaner and physics easy to interpret we have made the following reasonable assumptions.

- The ball is assumed to be perfect sphere.
- The shape of the ball does not deform in any way and remains a perfect sphere during the whole flight.
- All the factors : m, r, A, K_d and the pressure remain constant during the whole flight time.
- As per the FIFA World Cup 2022 guidelines [7] the defending goalkeeper must remain on the center of the goal line, facing the kicker, between the goalposts, without touching the goalposts, crossbar or goal net, until the ball has been kicked.
- The Wind velocity is assumed to be zero for mathematical simplicity.
- The drag coefficient is taken to be constant throughout the flight of the ball.(John Eric Goff Gold Medal Physics: The Science of Sports [8] [9] [10]
- The coefficient of the Magnus force was also assumed to be constant and equal to one [8].
- Ball conditions such as wetness or how well it is pumped should also affect the results, however, we ignore these microscopic effects.

2.1 The Ball

As specified, this situation is arising in any of the FIFA matches. Thus, we have mapped all the principle properties of our ball based on the rules and norms regarding the ball used in a FIFA match. The parameters of ball that were taken into consideration after satisfying all the assumptions mentioned above were :

$$Mass\ of\ the\ Ball\ (m) = 0.43\ Kg$$

$$Radius\ of\ the\ Ball\ (r) = 0.1146\ m$$

$$Area\ of\ cross\ section\ (A) = 0.0390\ m^2$$

$$Air\ Resistance\ coefficient\ (K_d) = 00.0062\ \frac{Kg}{m}$$

(The calculations for this is shown in the later section.) The pressure of the ball was not taken into consideration anywhere owing to the assumption that the ball does not deform and the pressure inside remains constant.

It has been assumed that the ball acts like a perfect sphere, making the moment of inertia (I):

$$I = \frac{2}{5}mr^2$$

2.2 Field Measurements

We have fixed various parameters on the field in which the penalty shot is being played. Here are the specifications of the goalpost:

1. Height of the goalpost = 2.44 m
2. Length of the goalpost = 7.32 m
3. The Ball is initially at the origin and is 11 m away from the centre of the goalpost.

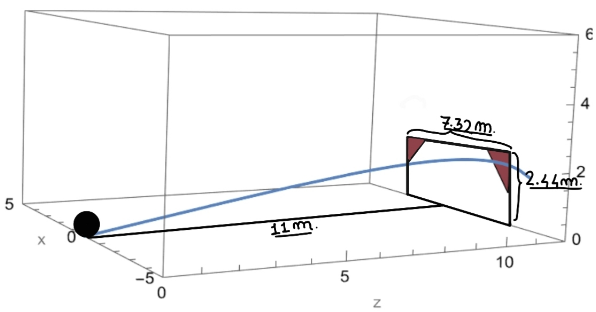


Figure 2.1: Dimensions of the field and a possible trajectory a ball might take

3 Establishing our model

3.1 Constants and Variables

The drag coefficient has been assumed to be constant throughout the journey of the ball. Pragmatically speaking the drag coefficient would vary over the course of the flight, experimentally the coefficient has been determined to fall between 0.23 and 0.29. [3] We have assumed that the drag coefficient would have a constant value of 0.27 over the entire journey.

$$C_D = 0.27$$

The air resistance coefficient K_d is given by the formula:

$$K_d = 0.5C_D(\rho)A$$

Assuming standard air density, ρ=1.2754 Kg/m^3 and cross sectional area A=0.0390m^2 we get the air resistance coefficient as:

$$K_d = 0.0062Kg/m$$

The Magnus coefficient C_M also remains constant throughout the journey of the flight and has been set to a constant value of 1. [8]

$$C_M = 1$$

3.2 Coordinate system

Before hitting the ball, it is at the origin of our assumed coordinate system. The goalkeeper as per FIFA guidelines is at point (0,0,11) and remains at that position till the player kicks the ball. Our final aim as the striker is to direct the ball towards the triangles enclosed by the lines :

$$x = 3.66\ ;\ y = 2.44\ ;\ and\ y = -1.093x + 5.2304$$

$$or$$

$$x = -3.66 \; ; \; y = 2.44 \; ; \; and \; y = 1.093x + 5.2304$$

at z=11. These triangles drawn on the plane z=11 are significant in the sense that they specify a perfect zone, directing the ball towards these zones would result in a guaranteed goal. We elaborate on these perfect zones in the following section.

3.3 The Perfect Zone

We first define a diving envelope- a region where the goalkeeper has a chance to stop the ball.Our shots must be outside the diving envelope — which could not be defended by a goalkeeper when facing a penalty kick. The aim of the striker is to score a goal. We propose a region where the striker is guaranteed to score a goal, referred to as the perfect zone throughout the paper. Without loss of generality we take the height of the goalkeeper to be 2.05m equivalent to the tallest goalkeeper in FIFA 2022. The inaccessible region we define would therefore work for every goalkeeper, provided they are not more than 2.05 m in height - which is the case.

3.3.1 The diving envelope

The tallest goalkeeper in the FIFA 2022 world cup has a height of 2.05m, along with his arm span the goalkeeper can effectively save any goals in a 2.84m radius around his position. The goalkeeper can also move around to save the ball, however traversing this distance would result in elapsed time which would restrict the arm span. The goalkeeper can take a stride of 1.1m either to the left or to the right, to be on the safe side and to save ourselves from superhuman goalkeepers we ignore the time elapsed to traverse the 1.1m distance and still assume an arm span of 2.84m. In the real world the average time taken by the ball to reach the goalpost is 0.642 ± 0.014 seconds. If the goal keeper were to move from his spot it would elapse 66% of the time taken by the ball to reach the goal post. We define the diving envelope and the goal post at z=11m as follows :

$$x = -3.66m$$

$$x = 3.66m$$

The above two equations represent the side-poles of the goalpost and encase a length of 7.32m between them.

$$y = 2.44m$$

this equation would represent the crossbar making the height of the goalpost 2.44 m in accordance with FIFA guidelines. Region spanned by the goalkeeper while at the center spot is

$$x^2 + y^2 = (2.84)^2 \tag{3.1}$$

Region spanned by the goalkeeper after traversing 1.1 m either to the right or to the left is given by the equations:

$$(x - 1.1)^2 + y^2 = (2.84)^2$$

$$(x + 1.1)^2 + y^2 = (2.84)^2 \tag{3.2}$$

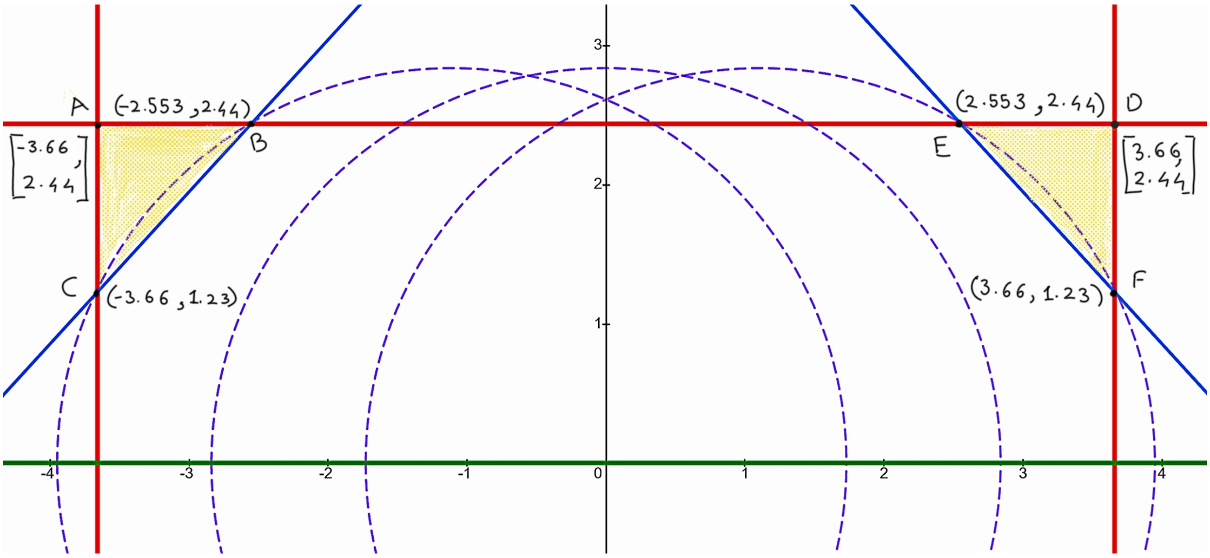


Figure 3.1: Graphical representation of the diving envelope

The two right angle triangles ABC and DEF make the inaccessible region while the rest constitute the diving envelope.

3.3.2 The inaccessible region

The region enclosed by the lines :

$$x = 3.66 \; ; \; y = 2.44 \; ; \; and \; y = -1.093x + 5.2304$$

$$and$$

$$x = -3.66 \; ; \; y = 2.44 \; ; \; and \; y = 1.093x + 5.2304$$

is the inaccessible region. It is constituted by the points :
A:

$$(-3.66, 2.44, 11)$$

B:

$$(-2.553, 2.44, 11)$$

C:

$$(-3.66, 1.23, 11)$$

D:

$$(3.66, 2.44, 11)$$

E:

$$(2.553, 2.44, 11)$$

F:

$$(3.66, 1.23, 11)$$

If the player/ striker could direct the ball to this particular region it will definetely result in a goal.

The region has been approximated to a right angle triangle, this however **does not** compromise our inaccessible region as we had not considered the effects of the elapsed time while evaluating the diving zone.

All of this defines the top corner of a goalpost.

3.4 The unsafe zone

According to FIFA guidelines the goalkeeper can not move from his position i.e. (0,0,11) until the player strikes the ball. However, once the ball is hit he or she may move to protect the goalpost. In the process they might jump or span sideways this was considered in the above section while we defined the top corner. It would be unwise for the goalkeeper to go back into the goalpost because once the ball crosses the plane z=11 it is considered a goal according to FIFA guidelines [7]. The goalkeeper may move forward in an attempt to prevent the ball from entering the plane z=11, this comes at the cost of elapsed time. Considering the same calculations as above we assume that a stride/ jump of 1.1 m would elapse 66% of the time it takes for the ball to reach to the post. Moreover the arm-span of the goalkeeper would provide him further reach whence he traverses 1.1 m in any direction from the point (0,0,11) . The arm span is assumed to be half the height of the keeper. in our case where we consider the tallest keeper whoose height is 2.05 m the arm span is 1.025 m, meaning the keeper can effectively protect in a cylinder whose base is centered at (0,0,11) and has a radius of 2.125 m.

The aim of the striker is to direct the ball towards the top corners shaded in red, however while doing so the trajectory of the ball must not pass through the translucent cylinder. If the ball were to pass through it, it would have a chance of being intercepted by the goalkeeper thereby losing the opportunity of a penalty kick.

In the upcoming sections we find values of velocity and spin such that the ball reaches the top corner, we also briefly discuss on the solutions such that the ball reaches the top corner and the trajectory does not cross the cylinder.

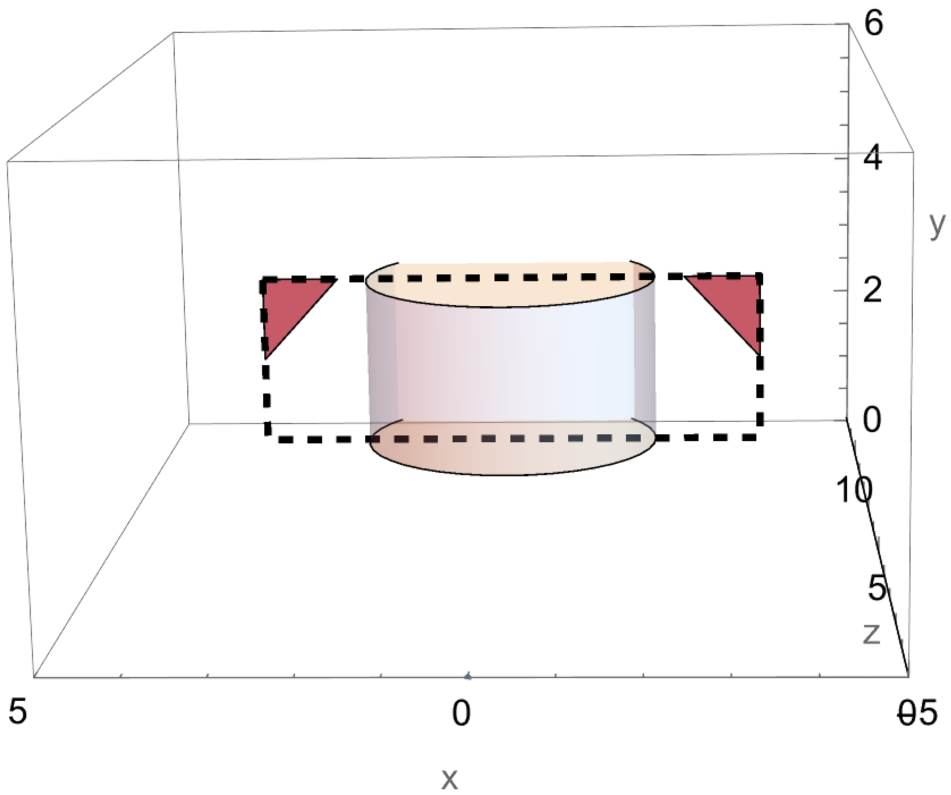


Figure 3.2: Front view of the field : The dark red triangles at the plane z=11 indicate the top corners ; the dark dashed line represents the entire goal post and the translucent cylinder restricts trajectory

4 Aerodynamics

The flight trajectories of sports balls largely depend on the aerodynamic characteristics of the balls. Depending on aerodynamic behavior, the ball can be deviated significantly from the anticipated flight path resulting in a curved flight trajectory [11].Therefore, we consider the aerodynamics of football, specifically, the interaction between a ball in flight and the ambient air. Doing so allows us to account for the characteristic range and trajectories of balls in flight, as well as their anomalous deflections as may be induced by striking the ball with spin.

4.1 The Spin

The spinning of the ball has a stabilizing effect on the flow around it and thereby on its trajectory. [12]

The spin on a football can be used in a penalty kick to deceive the goalkeeper, that the ball is directed away from the goalpost when in fact it will curve in flight and finish up on target. The spin is quantised by the angular velocities along the x, y and z directions namely ω_x, ω_y and ω_z respectively.

This collectively constitutes the angular velocity vector $\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$ A spinning soccer ball moving through the air is subject to different forces that determine the ball's trajectory. Lift force creates a lift on the object due to pressure difference (this is why planes fly). Then there's the side force – this is where Magnus effect comes in.

4.2 The Magnus Effect

"Magnus Effect" is the curving of a spinning spherical object to one side because of the uneven air flow generated around the object from its spin. It explains how a player is able to bend a ball into the goal post during a free kick. When a soccer ball moves through the air, it's the air that begins to take charge and moves — or in this case, curves — the ball to its own liking.

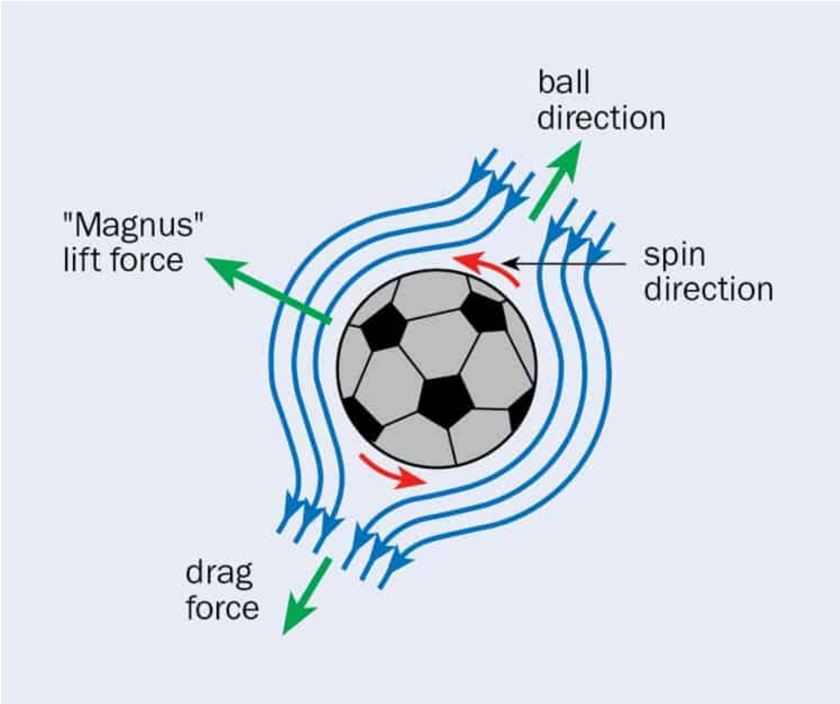


Figure 4.1: A bird's-eye view of a football spinning about an axis perpendicular to the flow of air across it. [14]

To start, the kicker needs to put a spin on the ball as he or she strikes it. Basically, the top-spin of the ball causes air around the top to move less than the air around the bottom half. The tangential velocity of the ball in the top half is opposite the airflow, and the tangential velocity of the bottom half is in the same direction of the airflow.

This imbalance means the pressure on the ball is greater at the top. The Magnus effect becomes more prominent and noticeable as the ball loses velocity — because at some point, the airflow that is moving against the spin is halted. The pressure imbalance is heightened and creates a force on the ball that acts both downward and against the side of the ball that used to create an opposing airflow. When a soccer player's kick creates a counter-clockwise spin, the Magnus force acts left on the ball, causing the ball to curve left — and vice-versa. [13]

Because the Magnus effect increases as the velocity decreases, you notice the curve in the ball happening much more rapidly as the ball begins to make its way back down towards the ground. The curve on the ball gets harder and harder during flight, fooling the goalkeeper from correctly tracking it as it moves down. If the striker imparts just the right amount of initial velocity and spin, it will ensures that the ball will make its way into the unreachable zone. The Magnus force is caused by the pressure differences around an object due to its rotation, and is the main reason here that makes the golf ball to go around the tree and, hopefully, hit the circular green. Theoretically, it is in the form of

$$F_M = \vec{S} \cdot (\vec{\omega} x \vec{v}) \tag{4.1}$$

where $\vec{S}$ is a constant and its magnitude can be expressed as $\frac{\pi^2}{2}\rho C_M r^3$. Here C_M represents a constant determined by the features of the golf ball's surface and the internal friction and viscosity of the air (usually satisfies $C_M \leq 1$).

Qualitatively, the rotations of football can be subdivided into backspin (spin along the y-axis) and sidespin (spin along the z-axis), in which Magnus Effect shows different effects. Backspin placed on the ball will allow it to gain lift force, thus it will be able to have a much longer flight. Meanwhile, sidespin placed on the ball will allow it to gain side force, which will pull the ball back towards the goal post when it flies out.

F_M consists of two components namely F_s and F_l. F_s is the sideways component of the Magnus force and F_l is the lifting component of the Magnus force. To further explain F_s and F_l consider a ball that is rotating strictly with topspin or backspin. The ball will have no sideways rotation and hence $F_s = 0$. Likewise, consider a ball that has strictly sideways spin. The Magnus force now has no component in the z direction and thus $F_l = 0$. However when a ball is rotating in more than one axis, F_s and F_l must be considered.

We consider the general case that the spin on the ball may be along the x, y and z axes respectively. Evaluating 4.1 we get the following equations decomposed along the x,y and z coordinates :

$$\begin{aligned}
F_{Mx} &= 0.5\rho C_M Ar(\omega_y(t)z'(t) - \omega_z(t)y'(t)) \\
F_{My} &= 0.5\rho C_M Ar(\omega_z(t)x'(t) - \omega_x(t)z'(t)) \\
F_{Mz} &= 0.5\rho C_M Ar(\omega_x(t)y'(t) - \omega_y(t)x'(t))
\end{aligned} \tag{4.2}$$

4.3 The Drag Force

A ball traveling through the air experiences a backwards force due to the fact that the air pressure on the front of the ball is larger than the force at the back of the ball. This force is called the drag force. The drag force results from the friction of air molecules on the surface (friction drag) and the pressure imbalance due to the airflow around the ball (pressure drag). Drag slows the ball down – a universal aerodynamic phenomenon, be it airplanes, cars or sports balls. The drag force is given by [3]:

$$\vec{F}_D = -0.5 C_D \rho A v^2 \hat{v} \tag{4.3}$$

The drag force can be decomposed into its constituent forces along the x,y and z axes as follows :

$$F_{Dx} = -0.5 C_D \rho A v^2 \hat{v}_x$$
$$F_{Dy} = -0.5 C_D \rho A v^2 \hat{v}_y$$
$$F_{Dz} = -0.5 C_D \rho A v^2 \hat{v}_z \tag{4.4}$$

The unit vector $\hat{v}$ can be written as :

$$\hat{v} = \frac{\vec{v}}{v}$$

We can therefore rewrite these equations as:

$$F_{Dx} = -0.5 C_D \rho A \sqrt{v_x^2 + v_y^2 + v_z^2}\mathbf{v}_x$$
$$F_{Dy} = -0.5 C_D \rho A \sqrt{v_x^2 + v_y^2 + v_z^2}\mathbf{v}_y$$
$$F_{Dz} = -0.5 C_D \rho A \sqrt{v_x^2 + v_y^2 + v_z^2}\mathbf{v}_z \tag{4.5}$$

4.4 Torque due to drag force

If we were to kick a ball it would not perpetually rotate, in real life the drag force applies a torque on the ball in opposite direction thereby reducing its angular velocity. The angular velocity ω therefore varies as a function of time. To derive this function, the forces acting on a rotating ball were considered. A rotating sphere would experience two extremities with respect to the drag force. By the definition of the Magnus effect it can be taken that the side of the ball where the tangential velocity v_t was in the same direction as the drag, the ball's velocity relative to the air would be minimal. On the side where the tangential velocity was opposing the drag, the ball's velocity relative to the air would be maximum. Thus the tangential velocity could be added to and subtracted from the linear velocity to represent the the maximum and minimum net velocity of the ball on either side and hence the maximum and minimum drag forces experienced on either side of the ball. The maximum drag force is:

$$F_{Dmax} = 0.5 C_D \rho A (v + v_t)^2 \tag{4.6}$$

and the minimum drag force is given by

$$F_{Dmin} = 0.5 C_D \rho A (v - v_t)^2 \tag{4.7}$$

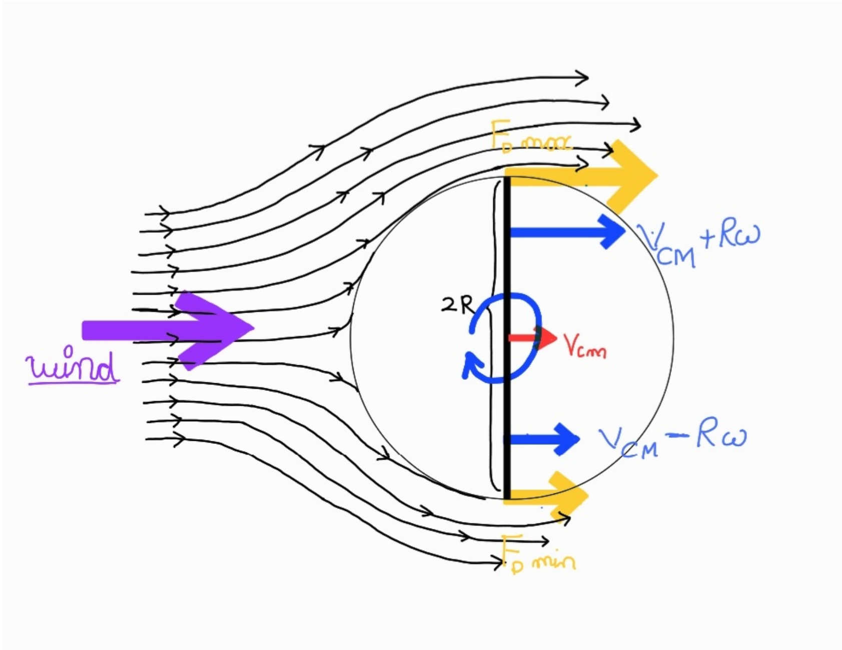


Figure 4.2: Representation of Torque

This couple provides a torque equivalent to :

$$\tau = \Delta F(2r)$$

$$\Delta F = 2\rho A C_D v v_t \tag{4.8}$$

the tangential velocity is equal to $v_t = r\omega$. So the torque on the ball is given by

$$\tau = (2\rho A C_D v \omega r^2) \tag{4.9}$$

in the direction opposite to the angular velocity of the ball

$$\tau = I\alpha$$

Where I is the moment of inertia of the ball and α is the angular deceleration

$$I = \frac{2}{5}mr^2$$

$$\omega'(t) = \frac{\tau}{I}$$

On simplifying this expression we get

$$\omega'(t) = -\frac{5\rho A C_D v \omega}{m} \tag{4.10}$$

Decomposing ω along x,y and z axes we get

$$\omega_x'(t) = -\frac{5\rho A C_D \sqrt{v_x^2 + v_y^2 + v_z^2}\omega_x[t]}{m}$$

$$\omega_y'(t) = -\frac{5\rho A C_D \sqrt{v_x^2 + v_y^2 + v_z^2}\omega_y[t]}{m}$$

$$\omega_z'(t) = -\frac{5\rho A C_D \sqrt{v_x^2 + v_y^2 + v_z^2}\omega_z[t]}{m} \tag{4.11}$$

4.5 Equations of Motion

The total force on the moving football is :

$$F_{net} = F_G + F_D + F_S + F_L$$

where F_T is the total force on the moving football , F_W is the weight of the ball , F_D is the drag force on the ball given by equations 4.5 and F_M is the force due to Magnus effect on the ball given by the equations 4.2.

$$F_{net} = ma$$

$$F = m\frac{dv}{dt} = m\frac{d^2s}{dt^2} = ms''(t)$$

$$F_{mx} = mx''(t)\ (lateral\ direction)$$

$$F_{my} = my''(t)\ (vertical\ direction)$$

$$F_{mz} = mz''(t)\ (forward\ direction)$$

We get the following equations of motion in the x, y and z directions

$$mx''(t) = -0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}x'(t) - 0.5\rho C_M Ar(\omega_y(t)z'(t) - \omega_z(t)y'(t))$$

$$my''(t) = -0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}y'(t) - 0.5\rho C_M Ar(\omega_z(t)x'(t) - \omega_x(t)z'(t))$$

$$mz''(t) = -0.5C_D\rho A\sqrt{v_x^2 + v_y^2 + v_z^2}z'(t) - 0.5\rho C_M Ar(\omega_x(t)y'(t) - \omega_y(t)x'(t)) - mg \tag{4.12}$$

5 Simulations and Results

5.1 Solution of differential equations

After analysing the problem and taking into account all the possible forces we get six coupled differential equations namely equations 4.11 and equations 4.12. The solutions to these give us x,y,z coordinates of the ball after we give it certain velocity and spin in x,y,z directions. We make use of the NDSolve and Manipulate modules in Mathematica to obtain our desired results. The results were plotted in a 3 Dimensional coordinate system and it was verified visually, whether the ball went to our 'Perfect Zone' or not. 6 parameters, namely the velocities and spins in the x,y,z directions were varied manually using Mathamatica's Manipulate function and the data of the successful trajectories (trajectories that go to our 'Perfect Zone') was noted and tabulated.

V_x	V_y	V_z	W_x	W_y	W_z	V	ϕ	α
5.4	3.35	22.35	2.6	-7.7	-7.2	23.23585591	8.28943462	13.58296353
4.9	3.8	23.35	12.1	0.75	-9.2	24.15931497	9.04958818	11.85155338
5.2	2.85	20.2	13.15	2.5	-12.85	21.05237516	7.78040022	14.43597498
5.1	1.6	19.95	14	-0.05	-9.6	20.65363164	4.44305345	14.3399285
4.8	2.25	20.55	16.6	1.4	-8.6	21.22274723	6.08583947	13.14724238
4	2.95	18.8	0.5	-2	0.7	19.44588645	8.72563286	12.01147839
4.3	2.7	18.8	-0.25	-1.5	0.8	19.47357183	7.96970321	12.88327651
3.9	2.65	18.8	1.45	-1.3	0.8	19.38227283	7.85825703	11.71959909
7.5	4.05	27.1	4.95	-5.2	-3.1	28.40884545	8.19608091	15.46957477
7.7	3.9	28.1	2.3	-1.9	-3.1	29.39574799	7.62403879	15.32410817

Figure 5.1: Values for the Top Left corner of the goalpost

V_x	V_y	V_z	W_x	W_y	W_z	V	ϕ	α
-4.6	1.85	18.5	14.95	-12.05	4.95	19.15287185	5.542913469	-13.9633481
-4.6	1.75	18.3	1.85	-9.6	-5.75	18.95026385	5.29864269	-14.1098884
-6.8	3.6	25.95	8.4	-3.15	2.5	27.06663075	7.643280419	-14.6837722
-4.9	3.62	22.3	1.85	8.3	2.5	23.11718841	9.00922211	-12.3927135
-5.8	2.7	22.3	12.85	1.6	6.3	23.19956896	6.683312686	-14.5790294
-5.8	3.25	22.3	12.5	-3	0.5	23.26999141	8.028453382	-14.5790294
-3.4	3	17.95	-10.3	9.2	-16.6	18.51384617	9.325376323	-10.7256157
-6.6	2.75	23.85	13.8	7.9	21.9	24.89869474	6.341115887	-15.4683397
-5.6	4.45	24.35	-16.2	26.2	-3.7	25.37882976	10.09861886	-12.9516471
-6.6	3.45	24.35	20.5	-17.2	13.6	25.46340511	7.786871678	-15.1654852

Figure 5.2: Values for the Top Right corner of the goalpost

Below are some examples of the trajectory a ball follows when subject to varying initial conditions of V_x, V_y , V_z and ω_x , ω_y and ω_z.

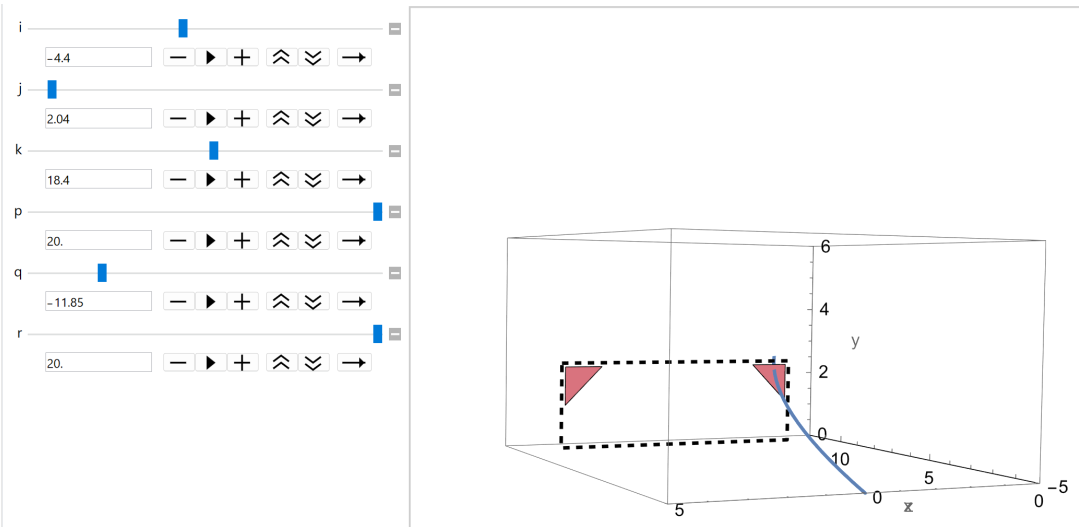


Figure 5.3: Test Case 1: Initial Velocity=19.14$\frac{m}{s}$

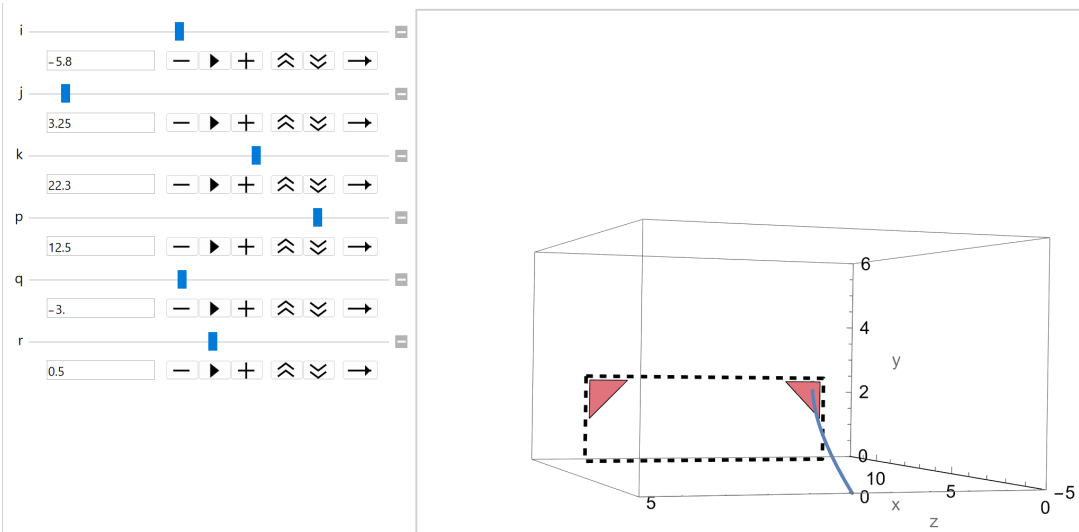


Figure 5.4: Test Case 2: Initial Velocity=23.269$\frac{m}{s}$

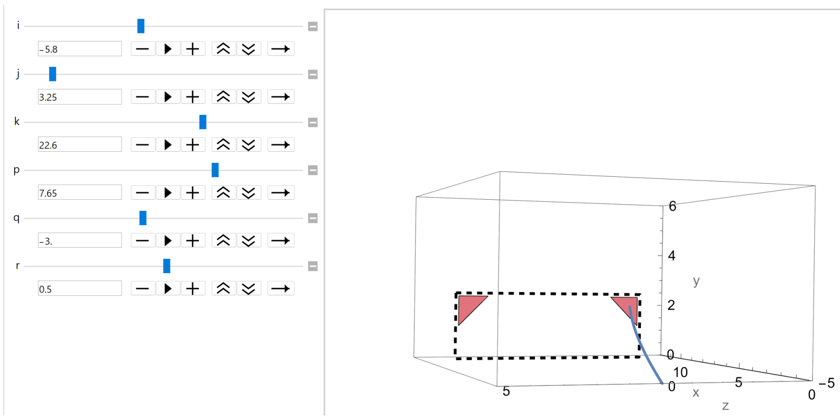


Figure 5.5: Test Case 3: Initial Velocity=24.169$\frac{m}{s}$

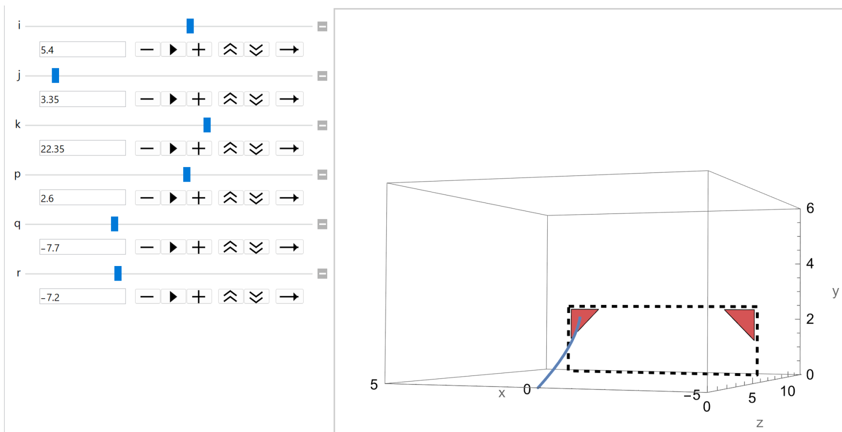


Figure 5.6: Test Case 4: Initial Velocity=23.235$\frac{m}{s}$

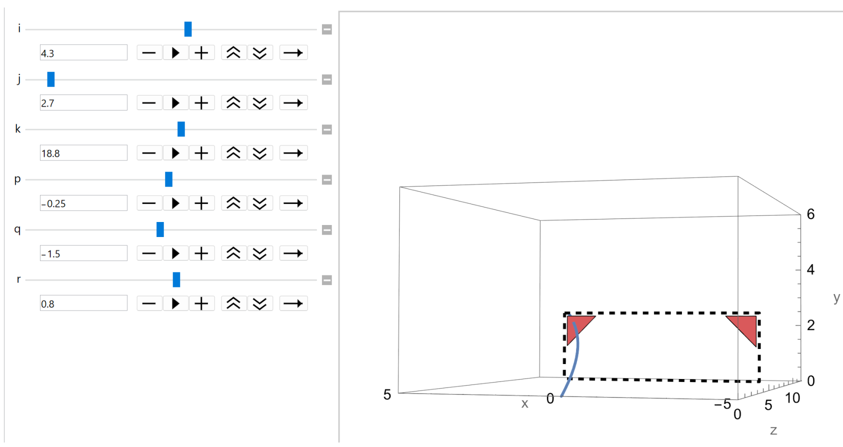


Figure 5.7: Test Case 5: Initial Velocity=19.47$\frac{m}{s}$

6 Restricting trajectories

We have calculated the velocities, azimuthal angle and polar angle for directing the ball to the top corner. However for the ball to reach the goal post without being obstructed by the goalkeeper the trajectory must not pass through the cylinderical zone where the goalkeeper might block it. We have generated a list of some values of velocities and spin for which the ball can reach the top corner without crossing the the cylinderical zone. This list is in no way exhaustive since there will be many more values of V, ϕ and α for which this would hold.

V_x	V_y	V_z	W_x	W_y	W_z	V	ϕ	α
-3.4	3	17.95	-10.3	9.2	-16.6	18.51385	9.325376	-10.7256
-6.6	2.75	23.85	13.8	7.9	21.9	24.89869	6.341116	-15.4683
-5.6	4.45	24.35	-16.2	26.2	-3.7	25.37883	10.09862	-12.9516
-6.6	3.45	24.35	20.5	-17.2	13.6	25.46341	7.786872	-15.1655

Figure 6.1: Values for the Top Left corner of the goalpost

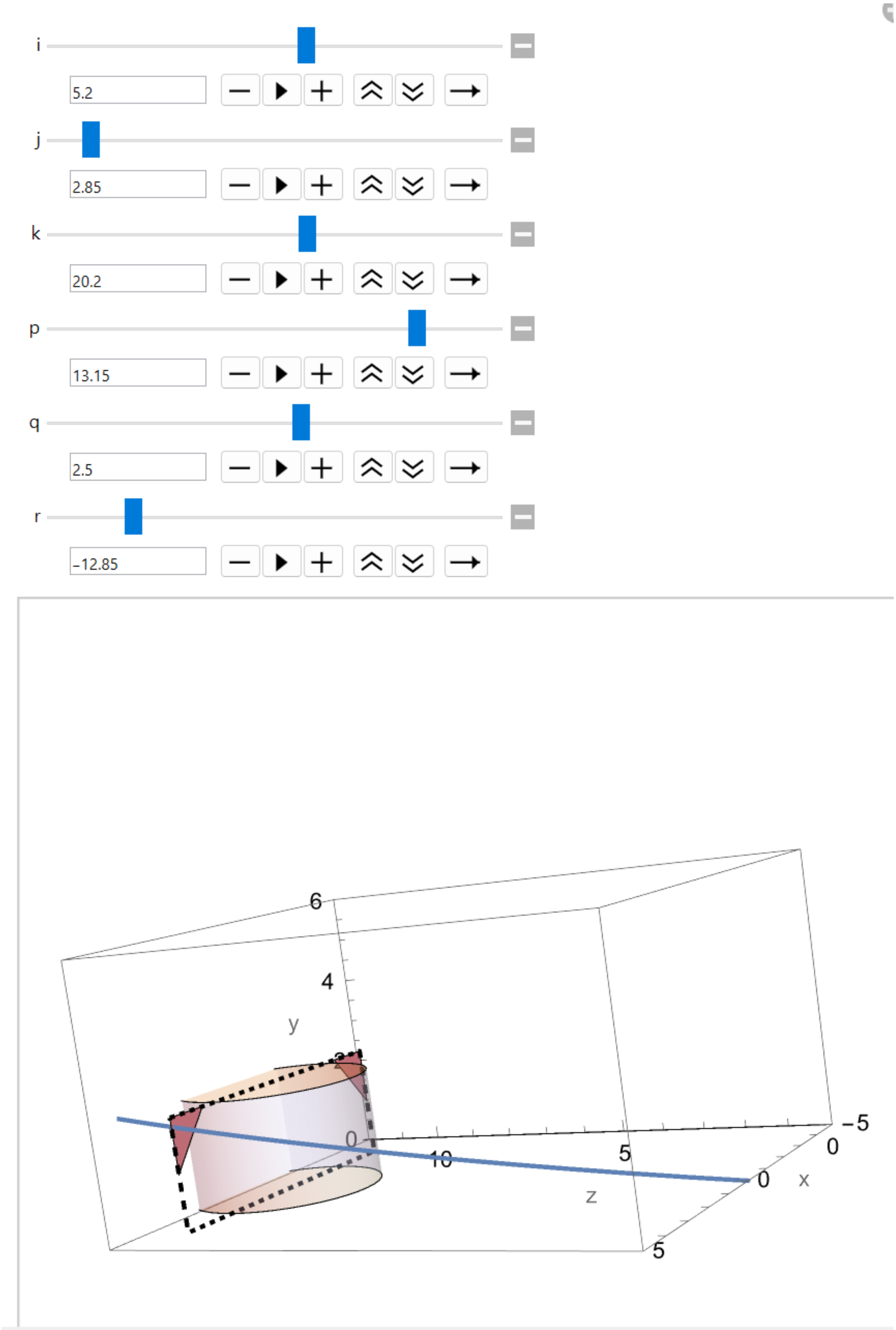


Figure 6.2: Velocity = 21.05 m/s ; ϕ=7.78deg; α=14.43deg

i

-6.6

j

2.75

k

23.85

p

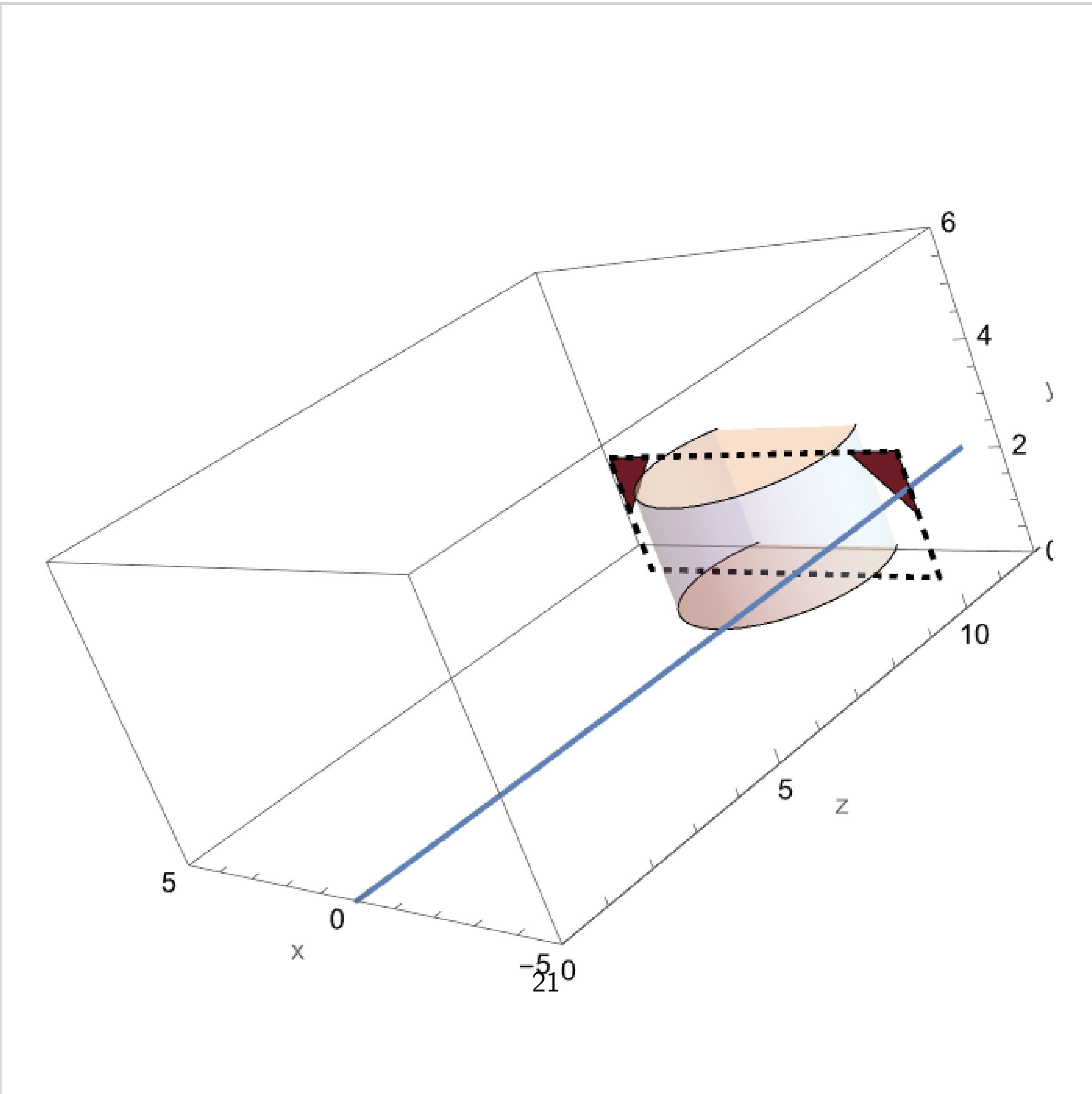
13.8

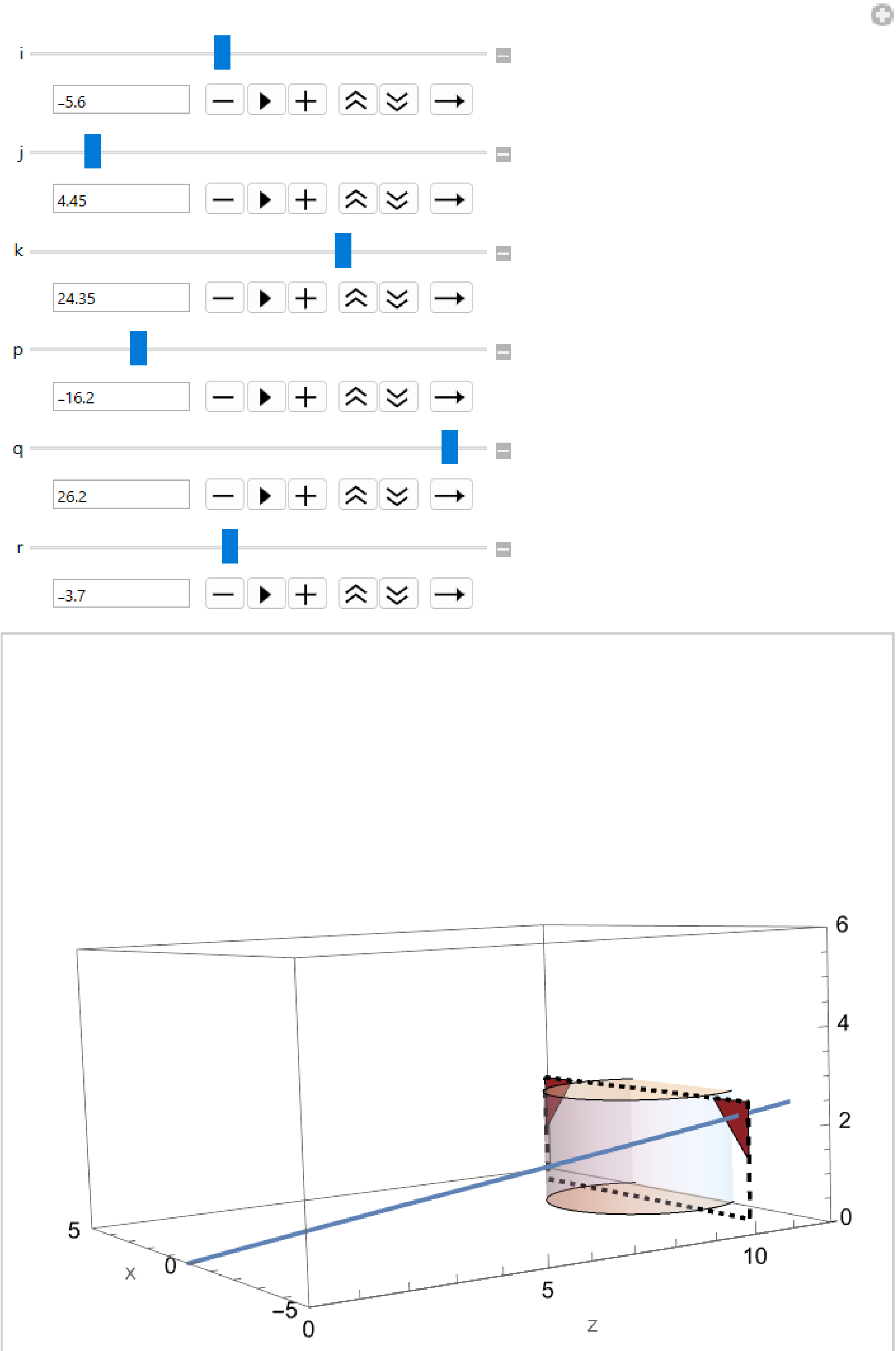
q

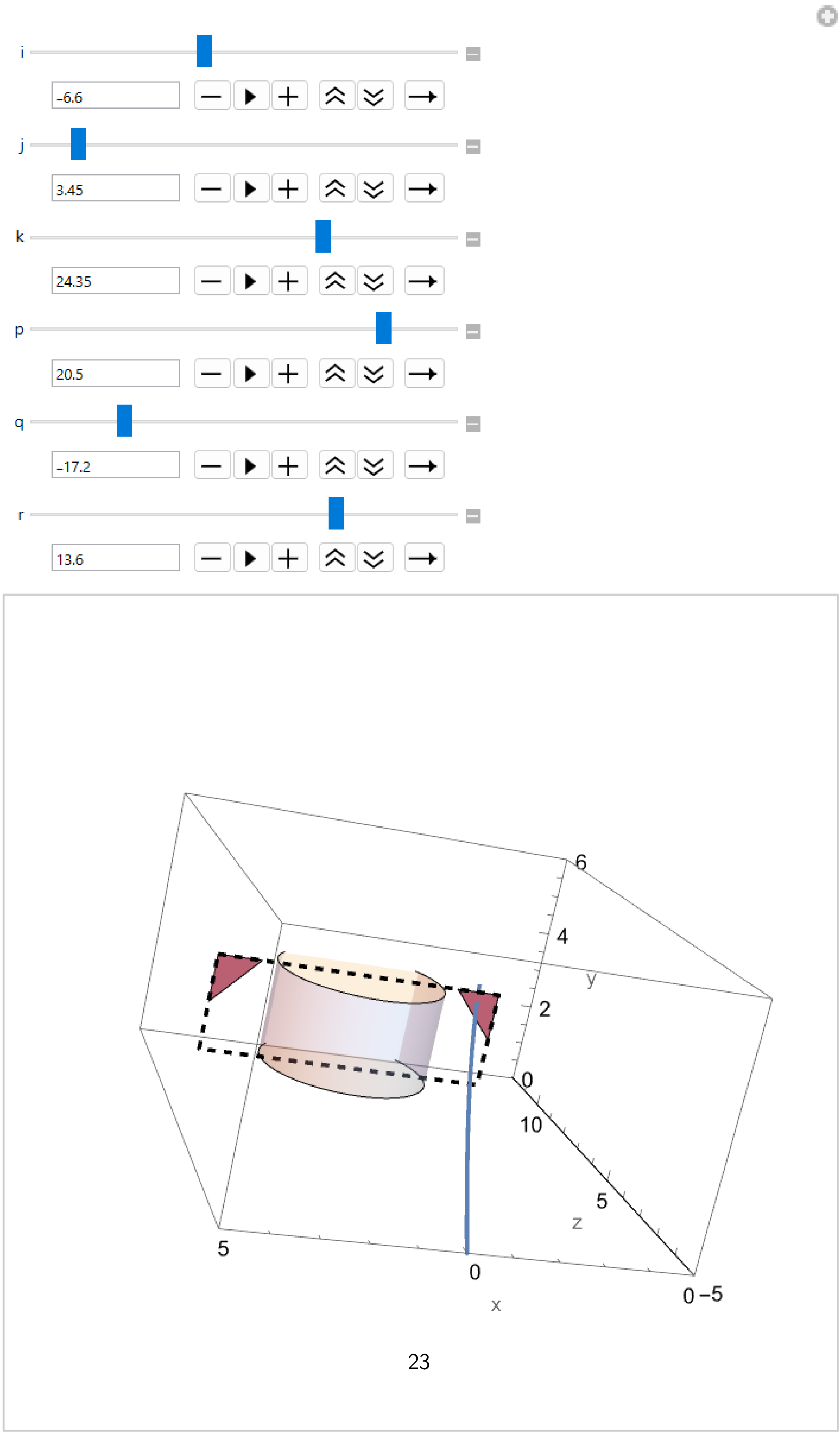
7.9

r

21.9




i
–5.6
j
4.45
k
24.35
p
–16.2
q
26.2
r
–3.7
6
4
2
0
5
x
0
–5
0
5
10
z


i
-6.6
j
3.45
k
24.35
p
20.5
q
-17.2
r
13.6
6
4
y
2
0
10
5
z
5
0
x
0 −5
23

7 Conclusion

We have produced a table of values of valid initial velocities and spin for the player to direct the ball to the top corner of the goal post. Both the questions in Problem B has been solved by proving a thorough theoretical analysis of the situation. The theoretical analysis is also backed-up by running simulations on Mathematica.

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A1 Appendix

A1.1 Mathematica code for solving coupled differential equations 4.11 and 4.12

```
post = Line[{{3.66, 2.44, 11}, {-3.66, 2.44, 11}, {-3.66, 0, 11}, {3.66, 0, 11}, {3.66, 2.44, 11}}];
Safe1 = Polygon[{{-2.553, 2.44, 11}, {-3.66, 2.44, 11}, {-3.66, 1.23, 11}}];
Safe2 = Polygon[{{2.553, 2.44, 11}, {3.66, 2.44, 11}, {3.66, 1.23, 11}}];
unsafe0 = Cylinder[{{0, 0, 11}, {0, 2.44, 11}}, 2.125];
Manipulate[
 soln = NDSolve[{a'[t] == (-(0.1492) ((x'[t]^2) + (y'[t])^2 + (z'[t])^2)^0.5) a[t],
    b'[t] == (-(0.1492) ((x'[t]^2) + (y'[t])^2 + (z'[t])^2)^0.5) b[t], c'[t] == (-(0.1492) ((x'[t]^2) + (y'[t])^2 + (z'[t])^2)^0.5) c[t],
    x''[t] == (-0.01492 ((x'[t]^2) + (y'[t])^2 + (z'[t])^2)^0.5) x'[t] - (0.0062 (b[t] × z'[t] - c[t] × y'[t])),
    y''[t] == (-0.01492 ((x'[t]^2) + (y'[t])^2 + (z'[t])^2)^0.5) y'[t] - (0.0062 (c[t] × x'[t] - a[t] × z'[t])),
    z''[t] == (-0.01492 ((x'[t]^2) + (y'[t])^2 + (z'[t])^2)^0.5) z'[t] - (0.0062 (a[t] × y'[t] - b[t] × x'[t])) - 9.81, x[0] == 0,
    y[0] == 0, z[0] == 0, x'[0] == i, y'[0] == j, z'[0] == k, a[0] == p, b[0] == q, c[0] == r}, {x, y, z}, {t, 0, 3}];
 Show[ParametricPlot3D[{x[t], y[t], z[t]} /. soln, {t, 0, 3}, AxesLabel → {"x", "y", "z"}, PlotRange → {{-5, 5}, {0, 6}, {0, 12}}],
  Graphics3D[Line[{Scaled[{0, 0, 0}], Scaled[{0, 0, 11}]}], Axes → True], Graphics3D[{Black, Thick, Dashed, post}],
  Graphics3D[{Pink, Safe1}], Graphics3D[{Pink, Safe2}], Graphics3D[{Opacity[0.3], unsafe0}]], {i, -35, 35}, {j, 0, 35},
 {k, 0, 35}, {p, -30, 30}, {q, -30, 30}, {r, -30, 30}]
```

Figure A1.1: Mathematica Code