2023

Stochastic MPC for Charging Control of Lithium Ion Battery

AuE8930: Robust Predictive Control (Fall 23)

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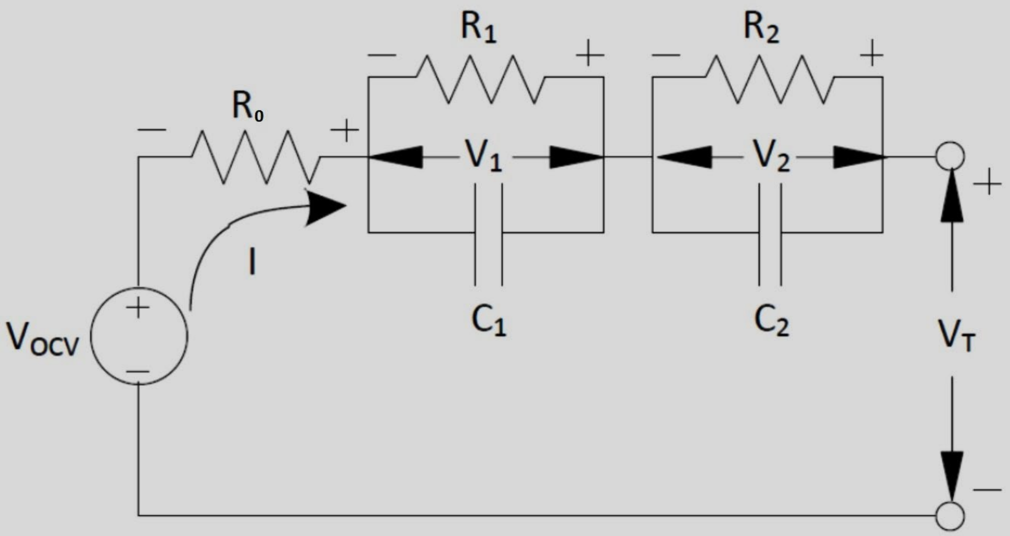
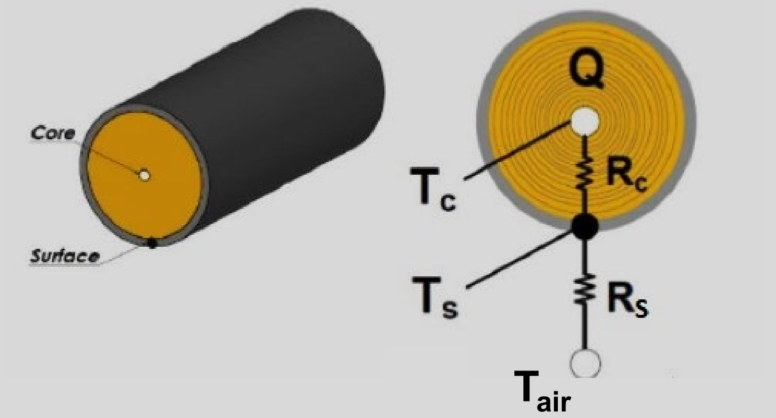
Battery Model Introduction

- An equivalent circuit model was used for electrical dynamics
  - Favoured over an electrochemical model for simplicity and tractability
- #RC refers to the number of parallel resistor and capacitor segments
  - 1RC and 2RC are common
  - 2RC was chosen for greater accuracy and to consider fast and slow dynamics
- Tracks SOC and V_1 and V_2 relating to applied current
- Thermal dynamics were modelled with a cylindrical cell, lumped model
- Determines surface and core temperature conditions
- Relates T_c and T_s states to "layers" of thermal resistance and internal heat generation, Q_{gen}
- Assuming passive cooling with, T_{air}





ECM and thermal model figures adapted from [9]

Coupled Electrical and Thermal Model

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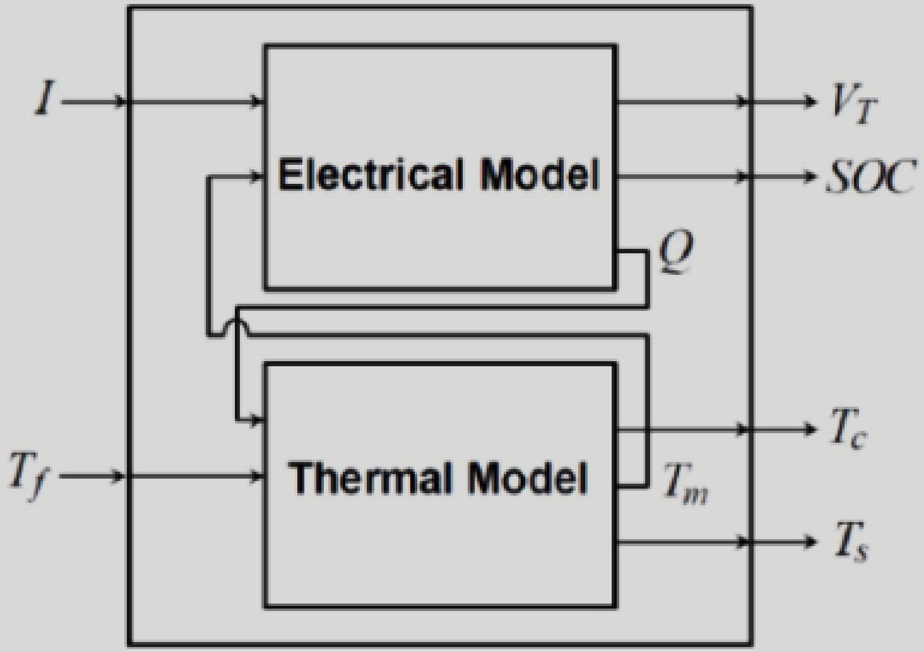
- Batteries exhibit strong coupling between electrical and thermal domains.
- Heat is generated during charging

$$Q_{gen} = I(V_{ocv} - V_t)$$

$$T_m = \frac{T_s + T_c}{2}$$

- Changes in temperature affect the electrical parameters



Coupling model figure adapted from [9]

$$\underbrace{\dot{x} = f(x,u) = \begin{bmatrix} S\dot{O}C \\ \dot{V}_1 \\ \dot{V}_2 \\ \dot{T}_c \\ \dot{T}_s \end{bmatrix} = \begin{bmatrix} -\frac{1}{Q_{\text{batt}}} I \\ -\frac{1}{R_1C_1}V_1 + \frac{1}{C_1}I \\ -\frac{1}{R_2C_2}V_2 + \frac{1}{C_2}I \\ \frac{V_1I + V_2I + R_0.I^2}{C_c} + (T_s - T_c)\left(\frac{1}{R_cC_c}\right) \\ \frac{T_{\text{air}} - T_s}{C_sR_s} - \frac{T_s - T_c}{C_sR_c} \end{bmatrix}}_{\text{System dynamics}}$$

$$\underbrace{y = h(x,u) = \begin{bmatrix} V_t \\ T_s \end{bmatrix} = \begin{bmatrix} V_{ocv} - V_1 - V_2 - IR_0 \\ T_s \end{bmatrix}}_{\text{Measurement model}}$$

Estimation and Disturbance model

- Noise
  - Process noise, w_t and Measurement noise, v_t
  - Assumed additive
  - $w_t \sim \mathcal{N}(0, Q_w)$ and $v_t \sim \mathcal{N}(0, R_v)$
- Disturbance
  - Disturbance in ambient temperature

$$T_{amb} = T_{amb,ref} + A_d \sin(\omega t)$$

- SOC, T_c (V_{OCV}) are not measurable directly
  - Extended Kalman Filter estimator is chosen due to system dynamics nonlinearity
  - Linearize the system around the current state using F_k and H_k matrices

Extended Kalman Filter

System:

$$\dot{x}(t) = f(x,u) + w_t$$
$$y(t) = h(x,u) + v_t$$

Prediction:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1|k-1})$$
$$\hat{P}_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_{k-1}$$

Kalman Gain:

$$K_k = P_{k-1|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R_k\right)^{-1}$$

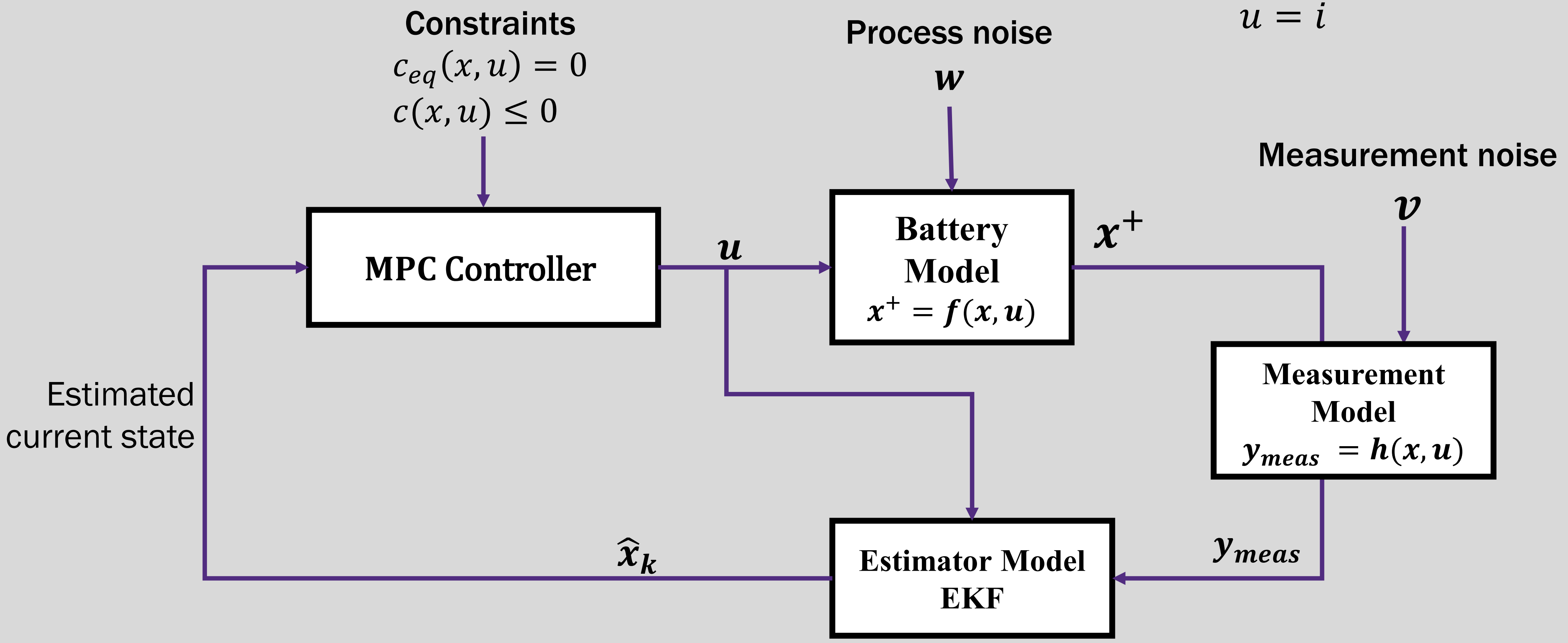
Correction:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(y_k - h\left(\hat{x}_{k|k-1}, u_{k-1}\right)\right)$$
$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}, u_{k-1}} \qquad H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k|k-1}}$$

Flow Chart for formulation of MPC

$x = [SOC, V1, V2, T_c, T_s]'$
$u = i$



Nominal MPC Formulation

- Optimization posed as a *pseudo-min-time* problem
- Aims to move the SOC to match the target as fast as possible
- Constraints based on empirical data and common practice
- Temperature will avoid regions of large degradation and dangerous thermal runaway
  - Ageing accelerates moderately until ~65°C ambient
  - Also accelerates below 25°C ambient
  - Handoff between ageing mechanisms
- Applied current is constrained to match the charging mode
  - 1C charges in 1 hour, 2C charges in 0.5 hour
  - 10Ah battery → 10A (1C), 20A (2C), 50A (5C)
- SOC avoids regions of pronounced degradation
  - Increased ageing with large depths of discharge and maximum charges

$$x = [SOC, V1, V2, T_c, T_s]'$$
$$u = i$$

$$\min_{x_k, u_k} \sum_{k=0}^{N-1} ||SOC_{target} - SOC(k)||_Q$$

Subject to,

$$x^+ = f(x, u)$$
$$20°\text{C} \le T_c \le 65°\text{C}$$
$$0°\text{C} \le T_s \le 45°\text{C}$$
$$-10A \le u_{1C} \le 10A$$
$$15\% \le SOC \le 90\%$$

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Stochastic Chance Constraints

Original Stochastic Problem

$$\min_{x_k,u_k} \sum_{k=0}^{N-1} \|SOC_{target} - SOC(k)\|_Q$$

Subject to:

$$x^+ = f(x,u)$$

$$\Pr\left(G(x,u) \leq g\right) \geq 1 - \varepsilon$$

Constraints tightening

$$[G]_j x_k \leq [g]_j - \gamma_j$$

$$\gamma_j = \sqrt{[G]_j \Sigma_k [G]_j'}\, F^{-1}(\varepsilon),$$

$$j = 1, 2 \ldots J$$

Deterministic Problem

$$\min_{x_k,u_k} \sum_{k=0}^{N-1} \|SOC_{target} - SOC(k)\|_Q$$

Subject to:

$$x^+ = f(x,u)$$

$$G(x,u) \leq g - \gamma$$

$$x = [SOC, V1, V2, T_c, T_s]'$$

$$u = i$$

- Probabilistic constraints violation $\varepsilon = 0.05$
- Assume Gaussian process noise and measurement noise
- $F^{-1}(\varepsilon)$ is a quantile function
- Process Noise Covariance

$$\sigma_w^2 = [10^{-4}, 10^{-2}, 10^{-2}, 1, 1]$$

- Measurement Noise Covariance

$$\sigma_v^2 = [4 \times 10^{-2}, 1]$$

$$\min_{x_k,u_k} \sum_{k=0}^{N-1} \|SOC_{target} - SOC(k)\|_Q$$

Subject to:

$$x^+ = f(x,u,w)$$

$$21.2℃ \leq T_c \leq 63.8℃$$

$$1.2℃ \leq T_s \leq 43.9℃$$

$$-9.6A \leq u_{1C} \leq 9.6A$$

$$16.2\% \leq SOC \leq 88.8\%$$

State of Health Formulation

- SoH is an important parameter but cannot be measured directly
    - Slow dynamics (Calendar ageing and Cyclic ageing)
    - Normally requires hundreds of cycles to analyse

- Focus on quantifying a single charging cycle impact

- A semi-empirical method based on Arrhenius equation is adopted
    - Relates capacity-fade as a function of ampere-hour throughout the lifetime
    - S_{loss} is accumulated capacity loss

Arrhenius Equation: $k = A\, e^{\frac{E_a}{RT}}$

$$f_c(SOC, T_m) = A_c(SOC, T_m) \cdot \exp\left(-\frac{E_{ac}}{R_g \cdot T_m}\right)$$

$$S_{loss}(Ah) = f_c(SOC, T_m) \cdot Ah^z$$

$$S(Ah) = S_{in}\left(1 - \frac{S_{loss}(Ah)}{100}\right)$$

Simulation Parameters

- Battery Capacity : 10Ah
- $SOC_{initial} = 0.25, \quad SOC_{target} = 1 \ (pseudo\ min\ time)$
- Ambient temperature, $T_{amb} = 25 + 5\sin(10^3 \pi t) \quad °C$
- Sampling time : 1 second
- MPC horizon
  - 15 time steps for single charge cycle
  - 4 time steps for degradation (to reduce computation demand)
- Probabilistic violation, $\varepsilon = 0.05$
- Charging rates: $1C\ (10A), 2C\ (20A)$ and $5C\ (50A)$
- Process Noise Covariance

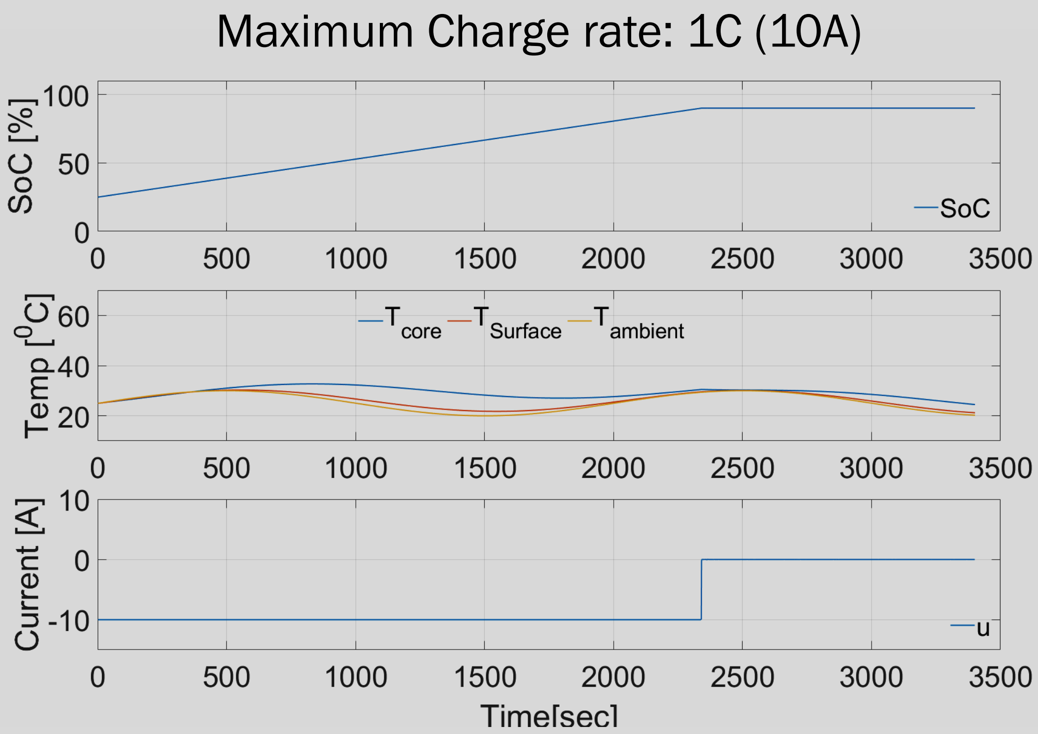
$$\sigma_w^2 = [10^{-4}, 10^{-2}, 10^{-2}, 1, 1]$$

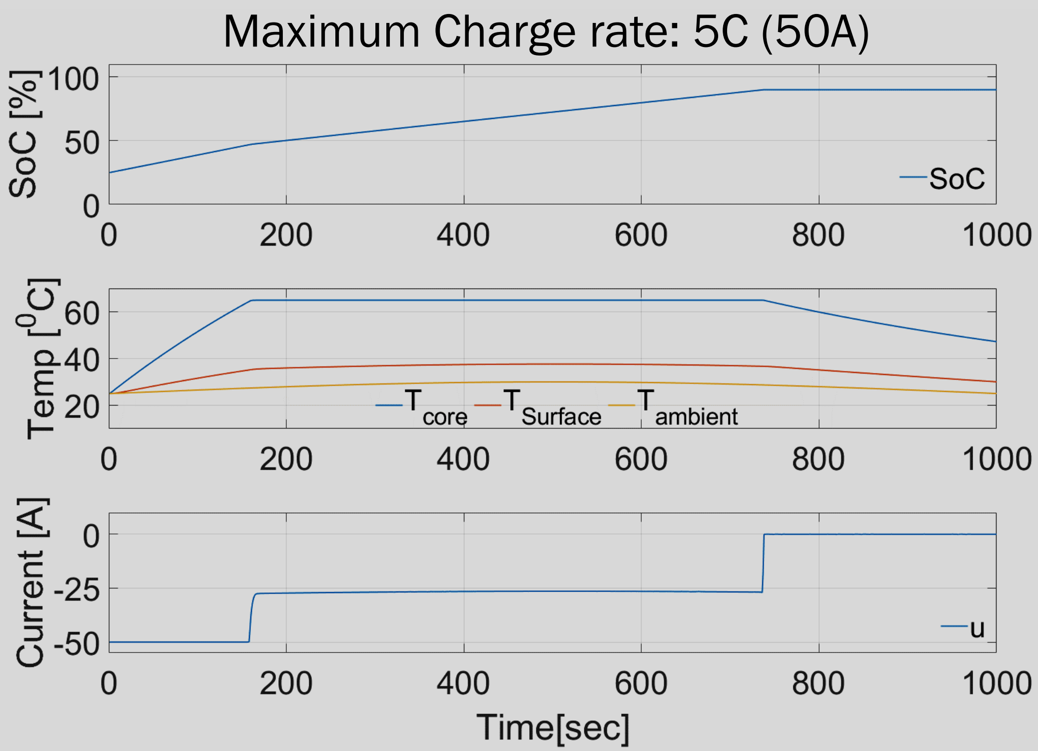
- Measurement Noise Covariance

$$\sigma_v^2 = [4 \times 10^{-2}, 1]$$

Parameters	Value	Units
$R_{0,\text{nom}}$	0.0055	Ohms
R_1	0.0016	Ohms
R_2	0.0113	Ohms
C_1	523.215	Farad
C_2	6.2449×10^4	Farad
Q_{batt}	10	Ah
R_c	7.4013	K/W
C_c	44.07	J/K
R_s	2.0751	K/W
C_s	4.5	J/K
$T_{\text{amb,ref}}$	298	Kelvin
A_d	5	Kelvin
ω	0.0031	rad/sec
$A_c(\cdot)$	557	-
E_{ac}	22406	-
R_g	8.314	J/mol K
z	0.48	-
S_{in}	1	-
$u_{\min}$	10, 20, 50	Ampere
$u_{\max}$	10, 20, 50	Ampere
$\text{SOC}_{\min}$	0.15	-
$\text{SOC}_{\max}$	0.9	-
$T_{c,\min}$	293 (20°C)	Kelvin
$T_{c,\max}$	338 (65°C)	Kelvin
$T_{s,\min}$	0	Kelvin
$T_{s,\max}$	318 (45°C)	Kelvin
ε	5%	-

Results for Nominal MPC

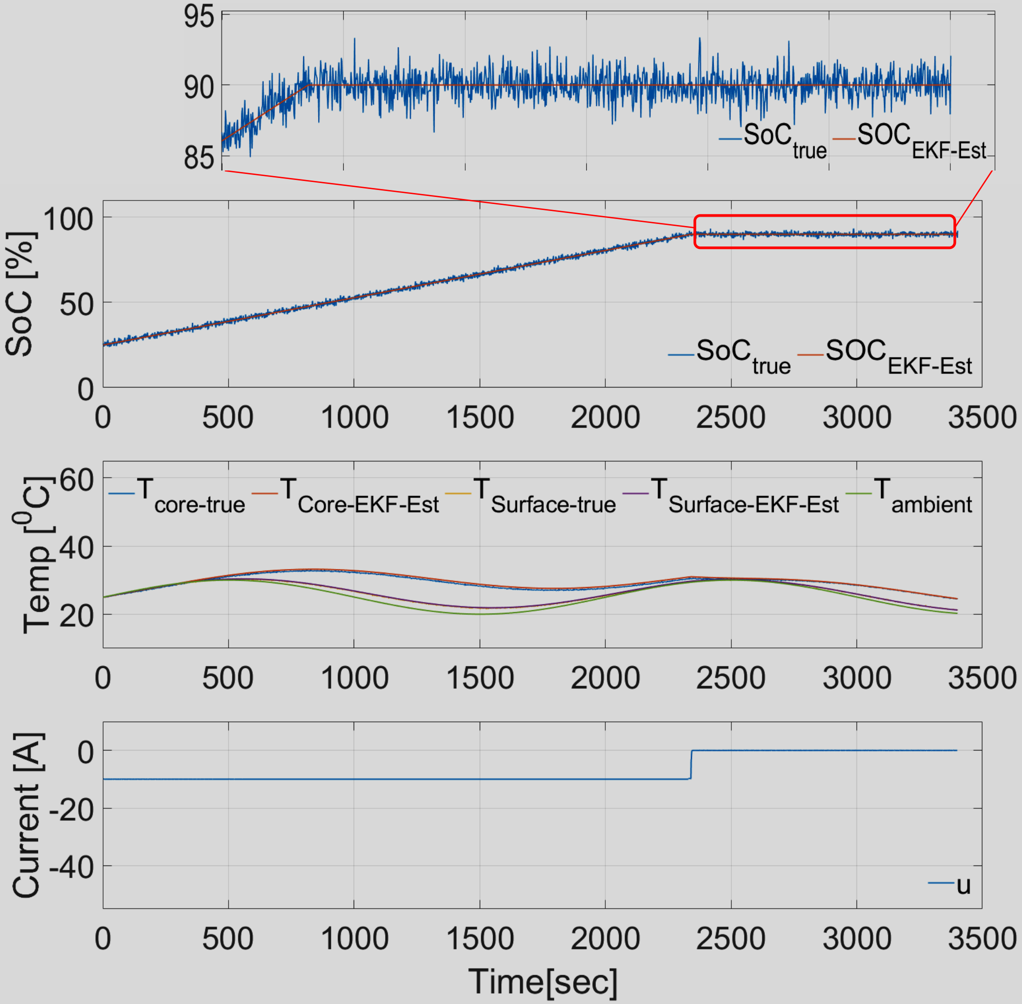


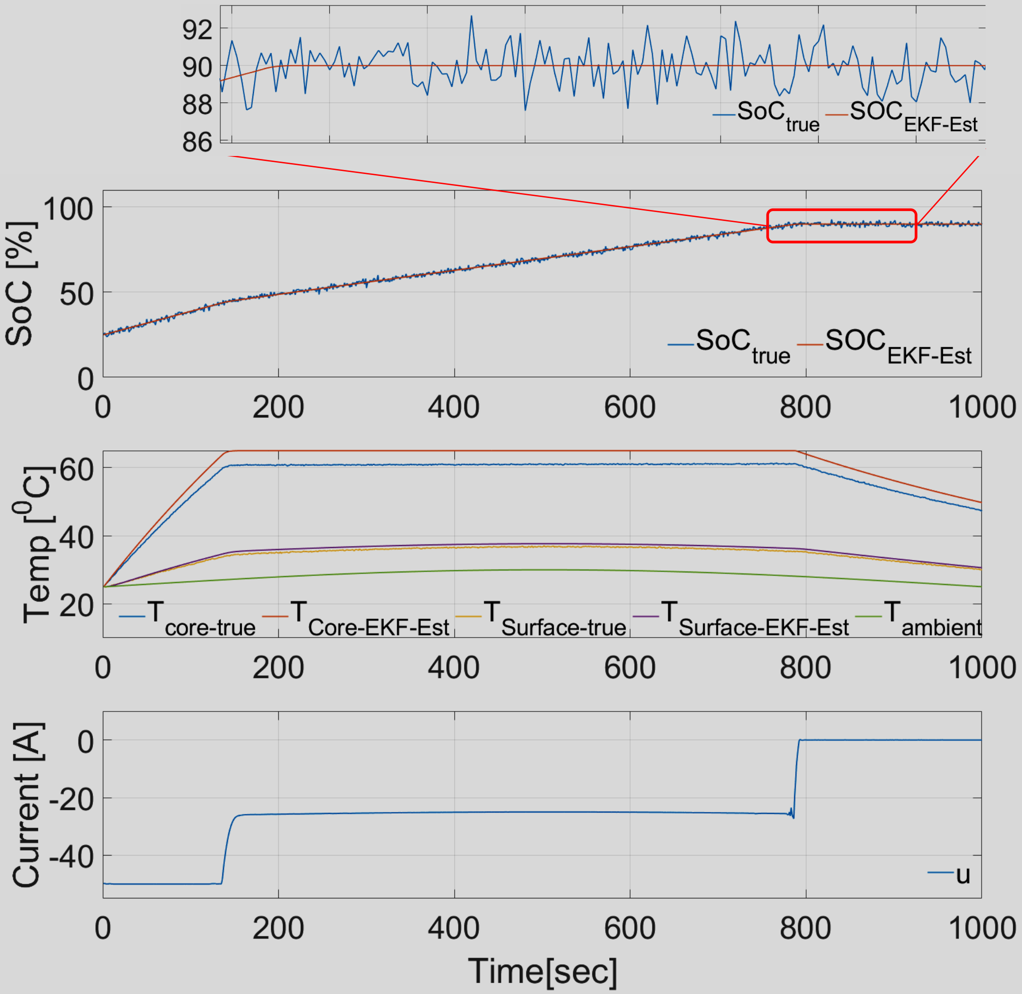


- Slow-charging (1C) always
- Only SOC constraint active

- Fast-charging (5C) initially
- Temperature constraints active
- Core temp > Surface temp as expected (heat gen.)
- Shorter charging time than 1C (but not 1/5th)

Results for Nominal MPC with disturbance

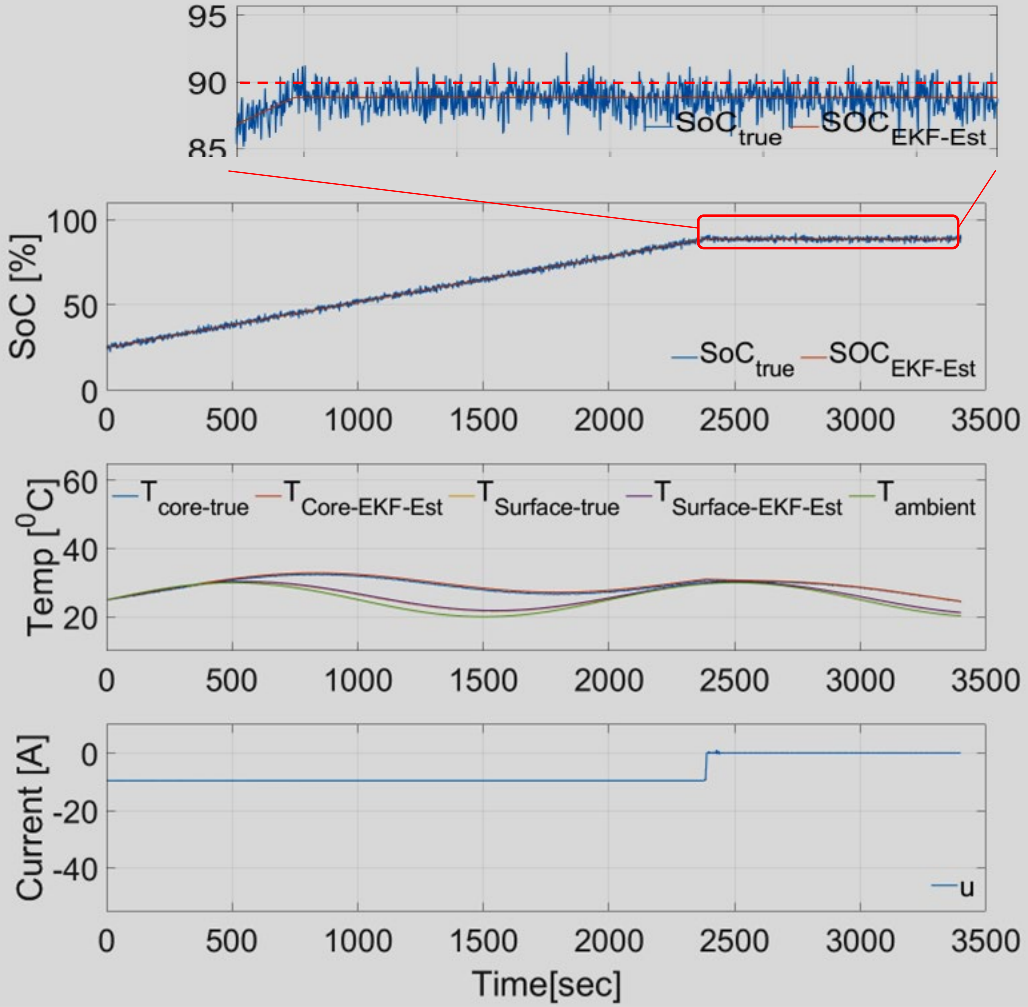




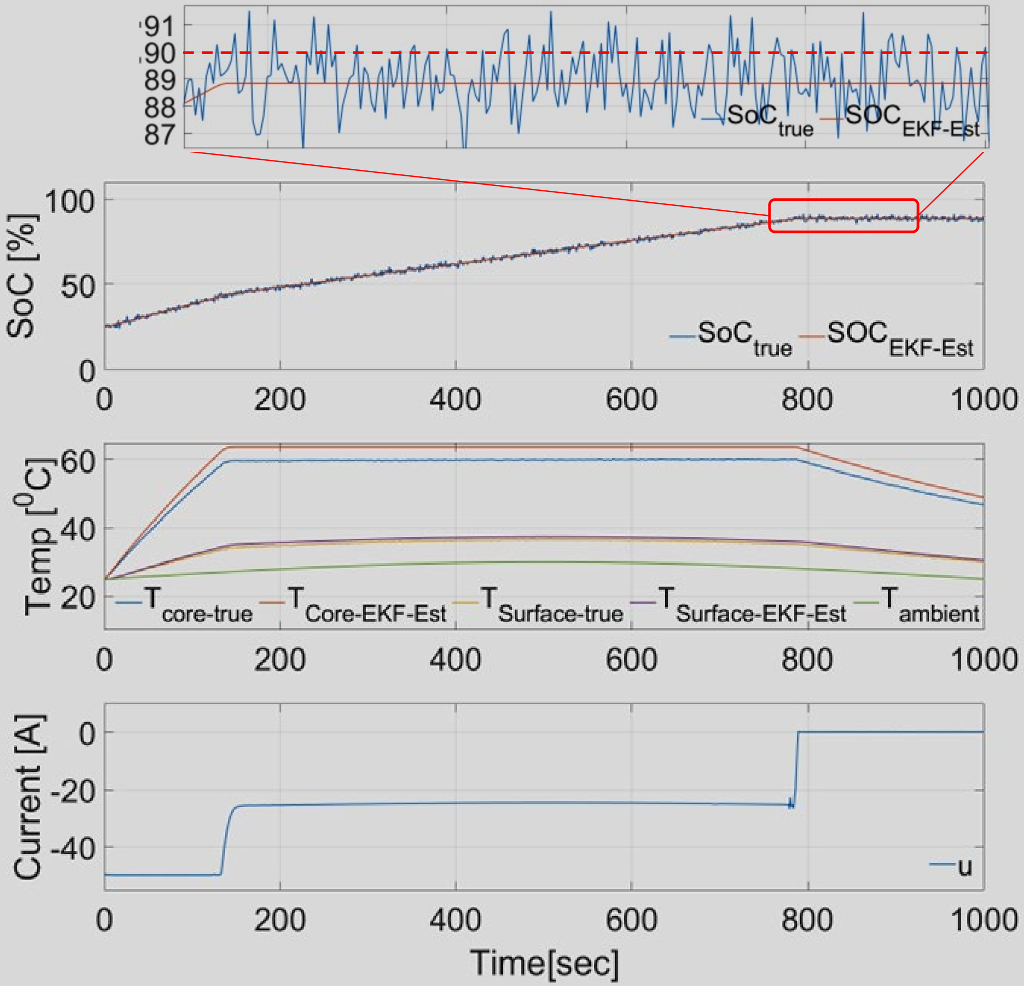
- Temperatures are exacerbated by disturbances
- Only the SOC constraints active
- Nominal may be sufficient for these applications

- EKF slightly overestimates temperature throughout
- Est. T_c constraints are active earlier than SOC constraints
- True T_c constraint is not violated due to overestimation

Results for Stochastic MPC



- Slow-charging constraints inactive
- True SOC_{max} is violated less as compared to the nominal case



- EKF slightly overestimates temperature throughout
- T_c constraints act the same as with nominal
- True SOC_{max} is violated less than nominal

SOH degradation results

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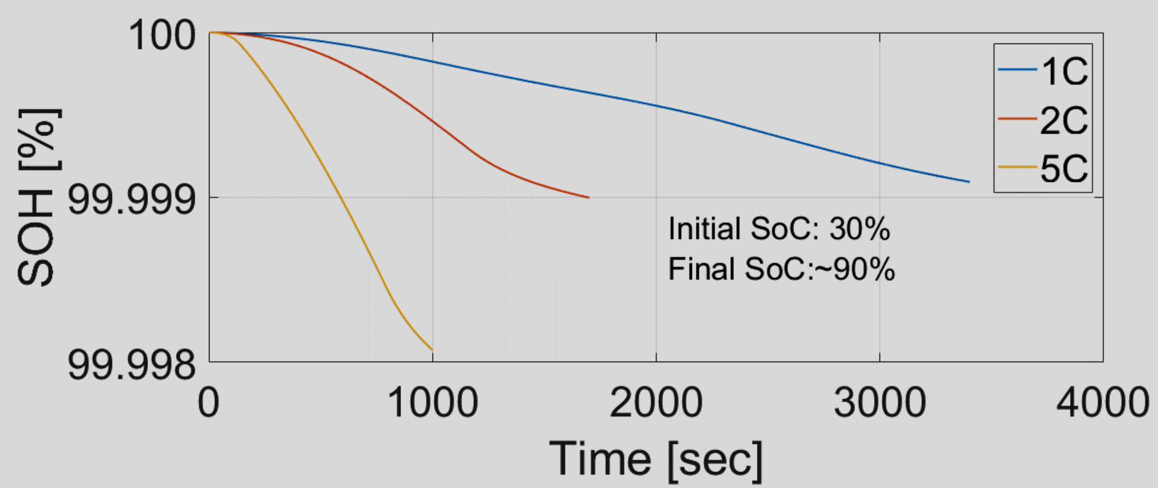
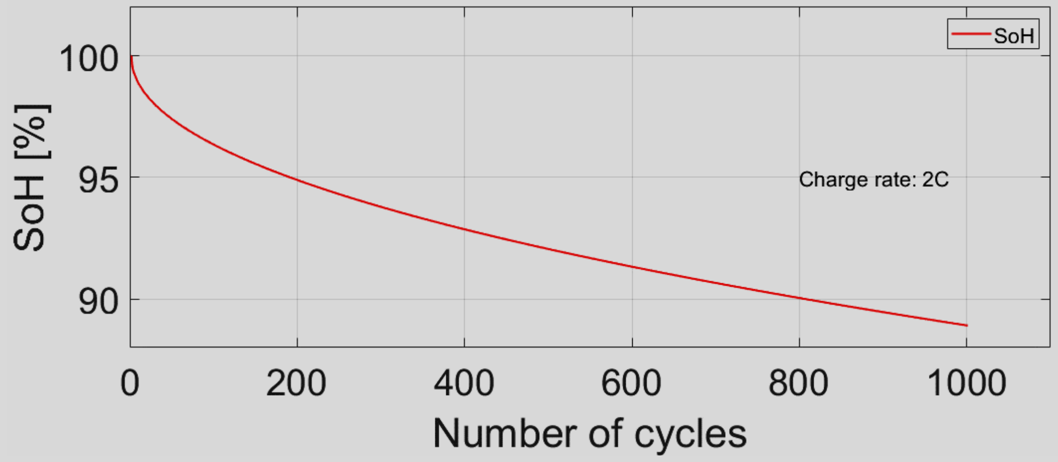
Degradation in one charging cycle



- Degradation rate increases with charging rate
- Similar total degradation between 1C and 2C due to increased cycle time at 1C
- Suggests 2C may be used without incurring additional battery lifetime penalty

Degradation in 1000 charging cycle



- ~12% degradation after 1000 cycles at 2C
- Degradation magnitude aligns with empirical data
- Initial ageing is nonlinear followed by a linear “tapering” of ageing
- Shape agrees with some studies
- 1.7M calculation loops (~18 hrs computation time at Palmetto computing cluster with 40 cpu cores and 125gb memory)

Conclusions

- Stochastic Model Predictive Control enhances lithium-ion battery performance, showcasing adaptability under disturbances.
- Fast charging without active cooling doubles battery degradation, highlighting the trade-off between speed and battery health.
- Slow-charging at 1C prioritizes State of Health (SOH), ensuring consistent performance.
- A 2C charging rate balances speed and longevity, proving to be a more appropriate charging strategy.
- Degradation analysis supports 2C as an better charging rate for a trade-off between time efficiency and battery health.
- The study supports the importance of a balanced charging strategy, considering factors like temperature constraints and degradation risks, to ensure the safe, efficient, and long-term operation of lithium-ion batteries.

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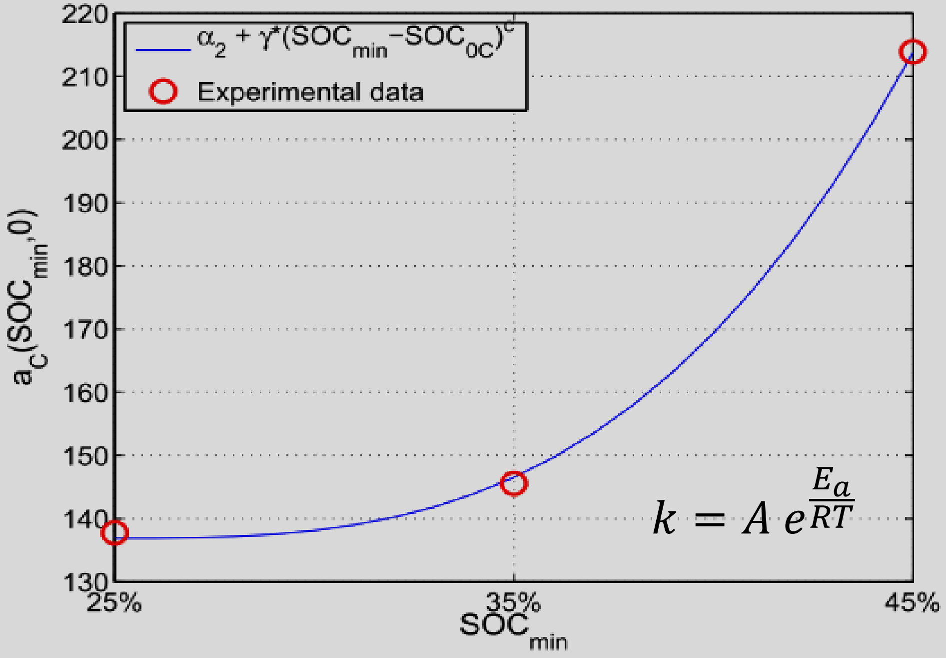
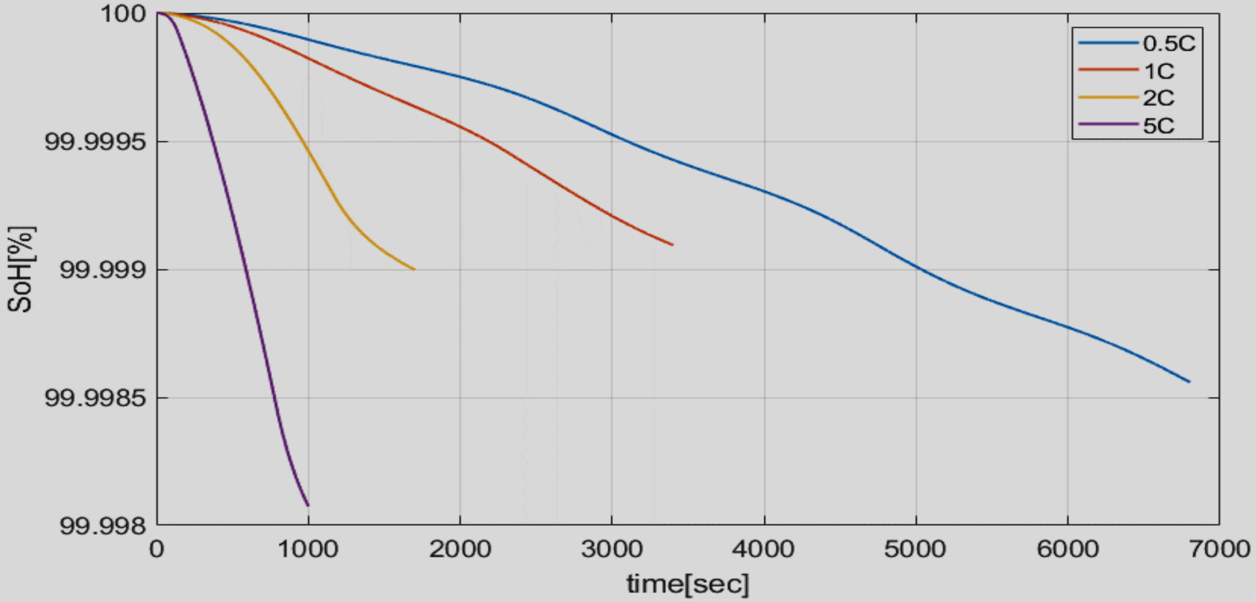
Acknowledgment

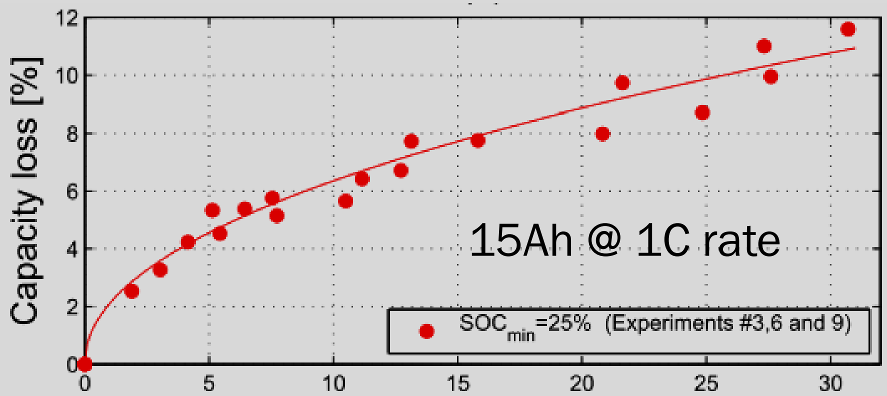
We sincerely appreciate the guidance, support, and encouragement we received from Dr. Beshah Ayalew throughout the AuE8930 Robust Predictive Control course. We also appreciate his exceptional teaching methodologies that have significantly contributed to our understanding of this subject.

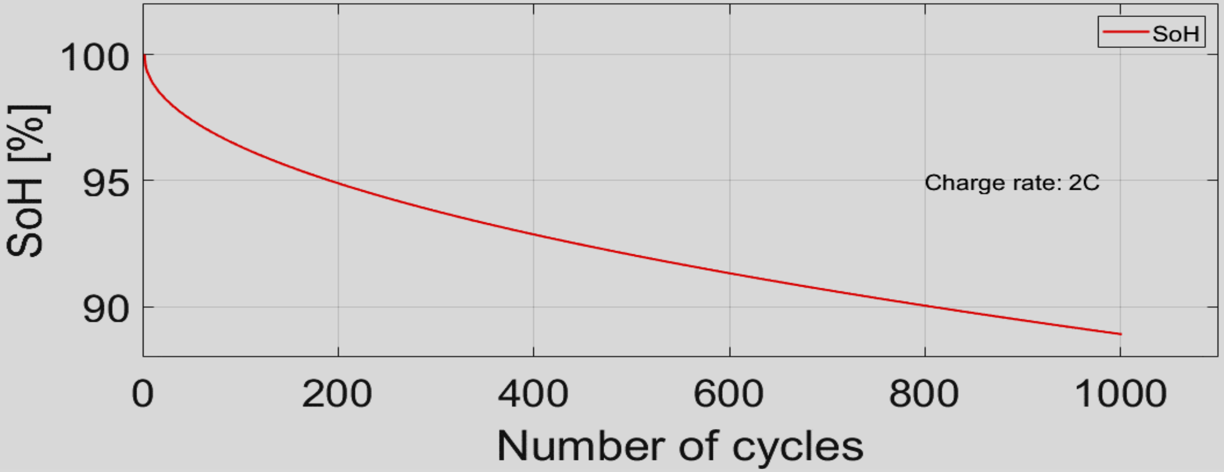
Thank you

Additional slides









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