AI1103 - Challenging Problem 19

Anirudh Srinivasan
CS20BTECH11059

Download latex-tikz codes from

https://github.com/Anirudh−Srinivasan−CS20/AI1103/blob/main/Challenging−Problem−19//Challenging−Problem−19.tex

Question

Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E\left(X_1 + X_2 + X_3 + X_4\right)^4$ equals

A) 40
B) 76
C) 16
D) 12

Solution

Since, X_1, X_2, X_3, X_4 are i.i.d random variables

$$\Pr\left(X_i = 1\right) = \frac{1}{2} \text{ for i = 1,2,3,4} \tag{0.0.1}$$

$$\Pr\left(X_i = -1\right) = \frac{1}{2} \text{ for i = 1,2,3,4} \tag{0.0.2}$$

Consider the random variable $Y = |X_1 + X_2 + X_3 + X_4|$
As X_i can take only the values 1 or -1, we have:

n(1)	n(-1)	Y	$\Pr(Y)$
0	4	4	${}^4C_0 \times \frac{1}{16} = \frac{1}{16}$
1	3	2	${}^4C_1 \times \frac{1}{16} = \frac{4}{16}$
2	2	0	${}^4C_2 \times \frac{1}{16} = \frac{6}{16}$
3	1	2	${}^4C_3 \times \frac{1}{16} = \frac{4}{16}$
4	0	4	${}^4C_4 \times \frac{1}{16} = \frac{1}{16}$

Table 4: This table shows probability associated with each value that the random variable Y take where n(1) represents the number of X_i which takes the value 1 and n(-1) represents the number of X_i which takes the value -1.

$\implies$ For Y, we have:

$$\Pr\left(Y = 0\right) = \frac{3}{8} \tag{0.0.3}$$

$$\Pr\left(Y = 2\right) = \frac{1}{2} \tag{0.0.4}$$

$$\Pr\left(Y = 4\right) = \frac{1}{8} \tag{0.0.5}$$

$\implies$ For Y^4, we have:

$$\Pr\left(Y = 0\right) = \frac{3}{8} \tag{0.0.6}$$

$$\Pr\left(Y = 16\right) = \frac{1}{2} \tag{0.0.7}$$

$$\Pr\left(Y = 256\right) = \frac{1}{8} \tag{0.0.8}$$

$$E(Y^4) = \sum_{\forall y} \Pr\left(Y = y\right) \times y \tag{0.0.9}$$

$$= \frac{3}{8} \times 0 + \frac{1}{2} \times 16 + \frac{1}{8} \times 256 \tag{0.0.10}$$

$$= 40 \tag{0.0.11}$$

$$\implies \text{E}(X_1 + X_2 + X_3 + X_4)^4 = 40$$

Answer: Option (A)