AI1103 - Assignment 3

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Download all python codes from

```
https://github.com/Anirudh−Srinivasan−CS20/
    AI1103/tree/main/Assignment−3/Codes
```

and latex-tikz codes from

```
https://github.com/Anirudh−Srinivasan−CS20/
    AI1103/blob/main/Assignment−3/Assignment
    −3.tex
```

Question

Let X and Y be two independent Poisson random variables with parameters 1 and 2 respectively. Then, $\Pr(X = 1|X + Y = 4)$ is

A) 0.426
B) 0.293
C) 0.395
D) 0.512

Solution

Given, $X \sim \mathcal{P}(\lambda)$ and $Y \sim \mathcal{P}(\mu)$. The probability mass functions (PMFs) of random variables X and Y are given by:

$$p_X(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!}, & \text{for } x = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases} \tag{0.0.1}$$

$$p_Y(y) = \begin{cases} \dfrac{e^{-\mu}\mu^y}{y!}, & \text{for } y = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases} \tag{0.0.2}$$

where: the parameters $\lambda = 1$ and $\mu = 2$. As X and Y are independent, we have for $k \geq 0$, the distribution function $p_{X+Y}(k)$ is a convolution of distribution functions $p_X(k)$ and $p_Y(k)$:

$$p_{X+Y}(k) = \Pr(X + Y = k) = \Pr(Y = k - X) \tag{0.0.3}$$

$$= \sum_i \Pr(Y = k - i|X = i) \times p_X(i) \tag{0.0.4}$$

After unconditioning, as X and Y are independent:

$$\Pr(Y = k - i|X = i) = \Pr(Y = k - i) = p_Y(k - i) \tag{0.0.5}$$

$$p_{X+Y}(k) = p_Y(k) * p_X(k) \tag{0.0.6}$$

$$= \sum_{i=0}^{k} p_Y(k - i) \times p_X(i) \tag{0.0.7}$$

$$= \sum_{i=0}^{k} e^{-\mu}\frac{\mu^{k-i}}{(k - i)!}e^{-\lambda}\frac{\lambda^i}{i!} \tag{0.0.8}$$

$$= e^{-(\mu+\lambda)}\frac{1}{k!}\sum_{i=0}^{k}\frac{k!}{i!(k - i)!}\mu^{k-i}\lambda^i \tag{0.0.9}$$

$$= e^{-(\mu+\lambda)}\frac{1}{k!}\sum_{i=0}^{k} {}^kC_i\mu^{k-i}\lambda^i \tag{0.0.10}$$

$$= \frac{(\mu + \lambda)^k}{k!} \times e^{-(\mu+\lambda)} \tag{0.0.11}$$

Hence, $X + Y \sim \mathcal{P}(\mu + \lambda)$.

$$\Pr(X = 1|X + Y = 4) = \frac{\Pr(X = 1, Y = 3)}{\Pr(X + Y = 4)} \tag{0.0.12}$$

$$= \frac{\Pr(X = 1) \times \Pr(Y = 3)}{\Pr(X + Y = 4)} \tag{0.0.13}$$

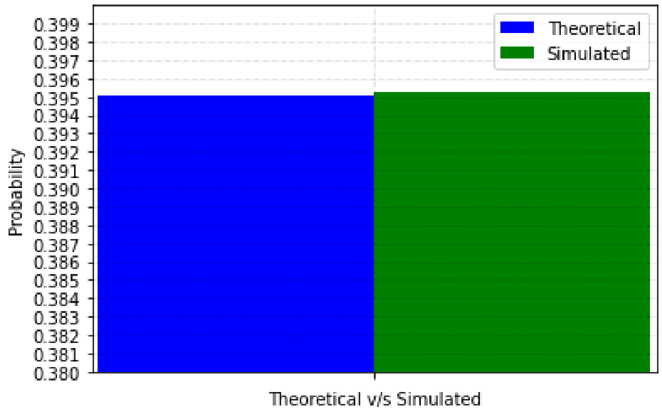
$$= \frac{\dfrac{e^{-1} \times 1^1}{1!} \times \dfrac{e^{-2} \times 2^3}{3!}}{\dfrac{e^{-3} \times 3^4}{4!}} \tag{0.0.14}$$

$$= 4 \times \frac{(1)(2)^3}{(3)^4} \tag{0.0.15}$$

$$= \frac{32}{81} \tag{0.0.16}$$

$$= 0.39506172839 \tag{0.0.17}$$

Answer: Option (C)


Theoretical
Simulated
Probability
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0.398
0.397
0.396
0.395
0.394
0.393
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0.390
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0.388
0.387
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0.383
0.382
0.381
0.380
Theoretical v/s Simulated