AI1103 - Assignment 5

Anirudh Srinivasan
CS20BTECH11059

Download latex-tikz codes from

https://github.com/Anirudh−Srinivasan−CS20/AI1103/blob/main/Assignment−5/Assignment−5.tex

Question

For random variables X and Y, show that:

$$Var[Y] = E[Var(Y|X)] + Var[E(Y|X)]$$

Solution

Let the abbreviations LE and LIE denote linearity of expectations and law of iterated expectations respectively.

$$Var[Y] = E[Y^2] - [E(Y)]^2 \text{ (definition)} \tag{0.0.1}$$

$$= E[E(Y^2|X)] - (E[E(Y|X)])^2 \text{ (LIE)} \tag{0.0.2}$$

$$= E[E(Y^2|X)] - (E[E(Y|X)])^2 - E([E(Y|X)]^2) + E([E(Y|X)]^2) \tag{0.0.3}$$

$$= E[E(Y^2|X)] - E([E(Y|X)]^2) + E([E(Y|X)]^2) - (E[E(Y|X)])^2 \text{ (LE \& LIE)} \tag{0.0.4}$$

$$= Var[E(Y|X)] + E[Var(Y|X)] \text{ (definition)} \tag{0.0.5}$$

Hence, proved.