

AI1103 - Assignment 5

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Download latex-tikz codes from

<https://github.com/Anirudh-Srinivasan-CS20/AI1103/blob/main/Assignment-5/Assignment-5.tex>

QUESTION

For random variables X and Y , show that:

$$\text{Var}[Y] = E[\text{Var}(Y|X)] + \text{Var}[E(Y|X)]$$

SOLUTION

Let the abbreviations LE and LIE denote linearity of expectations and law of iterated expectations respectively.

$$\text{Var}[Y] = E[Y^2] - [E(Y)]^2 \text{ (definition)} \quad (0.0.1)$$

$$= E[E(Y^2|X)] - (E[E(Y|X)])^2 \text{ (LIE)} \quad (0.0.2)$$

$$= E[E(Y^2|X)] - (E[E(Y|X)])^2 - E([E(Y|X)]^2) + E([E(Y|X)]^2) \quad (0.0.3)$$

$$= E[E(Y^2|X)] - E([E(Y|X)]^2) + E([E(Y|X)]^2) - (E[E(Y|X)])^2 \text{ (LE \& LIE)} \quad (0.0.4)$$

$$= \text{Var}[E(Y|X)] + E[\text{Var}(Y|X)] \text{ (definition)} \quad (0.0.5)$$

Hence, proved.