AI1103 - Assignment 2

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Download all python codes from

```
https://github.com/Anirudh-Srinivasan-CS20/
    AI1103/tree/main/Assignment-2/Codes
```

and latex-tikz codes from

```
https://github.com/Anirudh-Srinivasan-CS20/
    AI1103/blob/main/Assignment-2/Assignment
    -2.tex
```

Question

The lifetime of a component of a certain type is a random variable whose probability density function is exponentially distributed with parameter 2. For a randomly picked component of this type, what is the probability that its lifetime exceeds the expected lifetime (rounded to 2 decimal places) ?

Solution

Given, The lifetime of a component $X \sim \exp(2)$. The probability density function (PDF) of random variable X is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } 0 < x < \infty \\ 0, & \text{otherwise} \end{cases} \tag{0.0.1}$$

where: the parameter $\lambda = 2$. Since, the PDF of the random variable is exponentially distributed, Expected Lifetime $= E(X) = \frac{1}{\lambda} = \frac{1}{2} = 0.5$

$$\Pr(X > E(X)) = \int_{\frac{1}{\lambda}}^{\infty} \lambda e^{-\lambda x}\, dx \tag{0.0.2}$$

$$= -e^{-\lambda x}\Big|_{\frac{1}{\lambda}}^{\infty} \tag{0.0.3}$$

$$= \lim_{x\to\infty}(-e^{-\lambda x}) - (-e^{-\lambda \times \frac{1}{\lambda}}) \tag{0.0.4}$$

$$= \frac{1}{e} \tag{0.0.5}$$

$$= 0.36787944117 \tag{0.0.6}$$

The probability that the lifetime of an exponentially distributed random variable exceeds the expected lifetime $= \frac{1}{e}$, irrespective of the parameter λ. Therefore, for $\lambda = 2$,

$$\Pr(X > 0.5) = 0.36787944117 \approx 0.37 \tag{0.0.7}$$

Answer: 0.37