AI1103 - Assignment 3

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Download all python codes from

https://github.com/Anirudh-Srinivasan-CS20/AI1103/tree/main/Assignment-3/Codes

and latex-tikz codes from

https://github.com/Anirudh-Srinivasan-CS20/AI1103/blob/main/Assignment-3/Assignment-3.tex

Question

Let X and Y be two independent Poisson random variables with parameters 1 and 2 respectively. Then, $\Pr(X = 1|X + Y = 4)$ is

A) 0.426
B) 0.293
C) 0.395
D) 0.512

Solution

The probability mass functions (PMFs) of random variables X and Y are given by:

$$f_X(x) = \begin{cases} \frac{e^{-\lambda_1}\lambda_1^x}{x!}, & \text{for } x = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{e^{-\lambda_2}\lambda_2^y}{y!}, & \text{for } y = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

where: the parameters $\lambda_1 = 1$ and $\lambda_2 = 2$

$$\Pr(X = 1|X + Y = 4) = \frac{\Pr(X = 1, Y = 3)}{\Pr(X + Y = 4)} \quad (0.0.1)$$

$$= \frac{\Pr(X = 1) \times \Pr(Y = 3)}{\Pr(X + Y = 4)} \quad (0.0.2)$$

$$= \frac{\frac{e^{-1} \times 1^1}{1!} \times \frac{e^{-2} \times 2^3}{3!}}{\frac{e^{-3} \times 3^4}{4!}} \quad (0.0.3)$$

$$= 4 \times \frac{(1)(2)^3}{(3)^4} \quad (0.0.4)$$

$$= \frac{32}{81} \quad (0.0.5)$$

$$= 0.39506172839 \quad (0.0.6)$$

Answer: Option (C)