Model Order Reduction for $\sum\triangle$ converter

Dual Degree Project 2012 - 13

Bhargava CV
EE08B037

Under the guidance of Prof. Vinita Vasudevan

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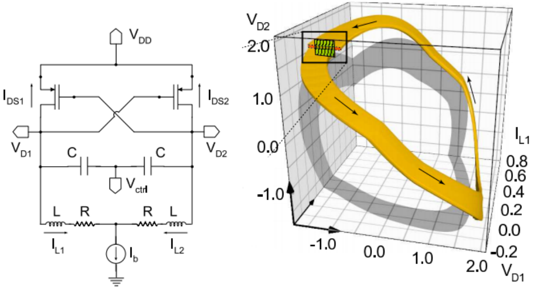
Objective

- Have a robust framework for MOR of non-linear analog circuits
- Achieve this by working on MOR for $\sum\triangle$ converter

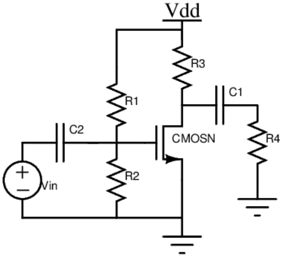
- Some circuits can be approximated to be working in a lower dimensional space within their phase space



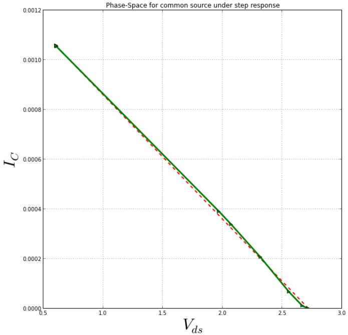
(a) states: V_{d1}, V_{d2}, I_{L1} (b) approx 2D annular soln

Figure: Oscillator with 3 D phase space but approx. 2 D solution

- Some circuits operate in that manner under special conditions
  - Common source amplifier with step input



(a) states: V_{d1}, I_{C2}



(b) approx 1-D solution

Figure: Common source amplifier 2D phase space and approx 1D soln

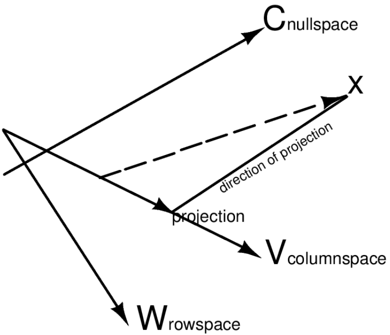
$$x_{projection} = Vx_{coeff}$$
$$W^T\left(x - Vx_{coeff}\right) = 0$$
$$W^T x = W^T V x_{coeff}$$
$$\left(W^T V\right)^{-1} W^T x = x_{coeff}$$

For orthogonal projection $W = V$

if V_{basis} are orthonormal $V^T V = I$

Projection matrix reduces to

$$x_{coeff} = V^T x$$



Actual k dimensional state space equations

$$\dot{x} = (A_{k\times k})x + (B_{k\times p})u$$
$$y = (C_{o\times k})x$$

$q \lneqq k$ dimensional state space equations $x_{k\times 1} \approx V_{k\times q}(x_{coeff})_{q\times 1}$

$$(V\dot{x_{coeff}}) = A(Vx_{coeff}) + Bu$$
$$y = C(Vx_{coeff})$$

Multiplying with V^T,we get

$$\dot{x}_{coeff} = \left(V^T AV\right)x_{coeff} + \left(V^T B\right)u$$
$$y = (CV)x_{coeff}$$

$$\dot{x}_{coeff} = A_{reduced}x_{coeff} + B_{reduced}u$$
$$y = C_{reduced}x_{coeff}$$

$$A_{reduced} = \left(V^T AV\right)_{q\times q}$$
$$B_{reduced} = \left(V^T B\right)_{q\times p}$$
$$C_{reduced} = (CV)_{o\times q}$$

The system has now reduced to a $q - dimensional$ linear system. For linear systems reduction in model order $\implies$ reduction in computational complexity

Non linear state space equations

$$\dot{x} = f(x) + B(x)u$$
$$y = Cx$$

After projection and order reduction

$$\dot{x}_{coeff} = V^T f(Vx_{coeff}) + V^T B(Vx_{coeff})u$$
$$y = CVx_{coeff}$$

Because of non-linear integration, in non linear systems
Order reduction to $q \nRightarrow$ reduction in computational complexity

- The bottle-neck is non-linear integration. Involves the following at every point
  - $\tilde{x} = Vz \rightarrow \mathcal{O}(Nq)$
  - calculation of $f(x) \rightarrow \mathcal{O}(N^{\alpha})$
  - calculation of $V^T\tilde{x} \rightarrow \mathcal{O}(Nq)$
  - Jacobian evaluation $V^T J_f(z) V \rightarrow \mathcal{O}(Nq)$
  - Backward Euler integration takes p iterations to converge
  - Assuming we need k time steps to perform the whole operation
- Cost of solving reduced order system $\rightarrow \mathcal{O}\left(pk\left(Nq + N^{\alpha} + q^3\right)\right)$
- Solving reduced order system is dependant on N

Avoiding non-linear integration $\Longrightarrow$ Linearize at chosen points

- Polynomial expansion at some state x_0

$$f(x) = f(x_0) + A_0(x - x_0) + \frac{1}{2}W_0(x - x_0) \circledast (x - x_0) + \ldots\ldots$$

Where, A_0 is the Jacobian and B_0 is the hessian at x_0

- This linear approximation reduces the state equations to

$$\frac{d}{dt}(Vz) = V^T f(x_0) + W^T A_0 V(x - x_0) + V^T B(x_0) u$$

- Solving and storing $A_0 \rightarrow \mathscr{O}(Nq^2)$
- The non-linear integration will not have implicit equations to solve. (during backward euler)

Moment of a transfer function

$$\eta_m(s_0) = (-1)^m \frac{d^m}{ds^m} H(s) \,|_{s=s_0}$$

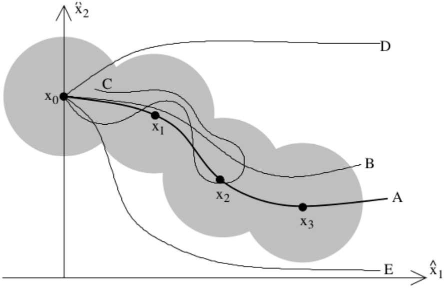
Moments from Taylor series expansion

$$\begin{aligned} H(s) &= H(s_0) + \left(\frac{d}{ds}H(s)\right)_{s=s_0} \frac{(s-s_0)}{1!} + .. + \left(\frac{d^m}{ds^m}H(s)\right)_{s=s_0} \frac{(s-s_0)^m}{m!} + . \\ &= \eta_0(s_0) - \eta_1(s_0)\frac{(s-s_0)}{1!} + .. + (-1)^m \eta_m(s_0)\frac{(s-s_0)^m}{m!} + .. \end{aligned}$$

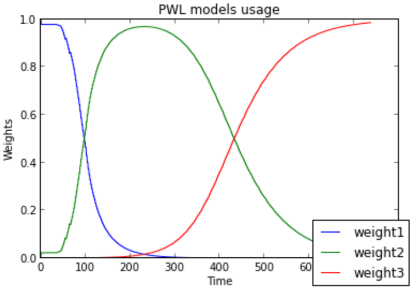
Without proof

$$span(V) = span\left\{B, AB, ...A^{k-1}B\right\}$$

- Choosing points along the trajectory in phase space
- Integration is done using these linear models



(a) Choice of Points



(b) Weighing functions

Figure: Trajectory Based PWL models

Theory

- TPWL model + Order reduced equation

$$\frac{d}{dt}\dot{x} = \sum_{i=0}^{s-1} w_i(x)\left(f(x_i) + (A_i)_r (x - x_i) + (B_i)_r u\right)$$

Progress

- Non-linear Transmission line model $\rightarrow$TPWL + MOR
- Common source Amplifier $\rightarrow$TPWL
- Differential Amplifier $\rightarrow$Implementing TPWL

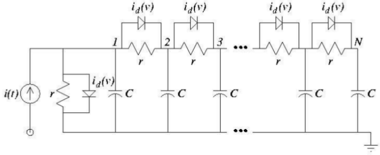


Figure: Non-Linear Transmission Line

$$\dot{v} = \begin{pmatrix} -2 & 1 & 0 & . & . & 0 \\ 1 & -2 & 1 & 0 & . & 0 \\ 0 & 1 & -2 & 1 & 0 & . \\ . & . & . & . & . & . \\ 0 & . & . & 1 & -2 & 1 \\ 0 & . & . & . & 1 & -1 \end{pmatrix} v + \begin{pmatrix} 2 - e^{10v_1} - e^{10(v_1 - v_2)} \\ e^{10(v_1 - v_2)} - e^{10(v_2 - v_3)} \\ . \\ . \\ . \\ e^{10(v_9 - v_{10})} - 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} i$$

$$y = \begin{pmatrix} 0 & 0 & . & . & . & 1 \end{pmatrix} v$$

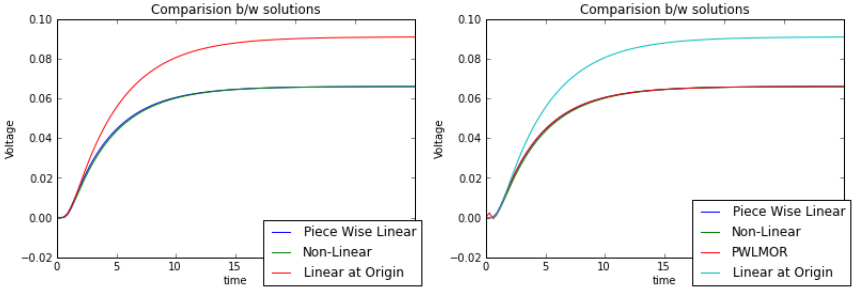


Figure: Results obtained

$$\dot{v} = \left(\left(\frac{V_{dd} - v}{R_3}\right) - I_d\left(M1\right) - I_C\right) * \frac{1}{C_{para}}$$

$$\dot{I_C} = \left(\frac{\left(\left(\frac{V_{dd} - v}{R_3}\right) - I_d\left(M1\right) - I_C\right) * \frac{1}{C_{para}} - I_c}{R_4}\right)$$

$I_d\left(M1\right)$ uses
LEVEL1 current equations.

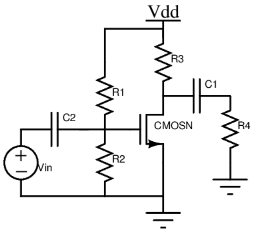
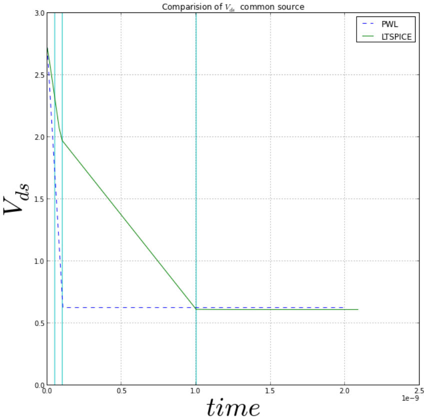


Figure: Common source amplifier



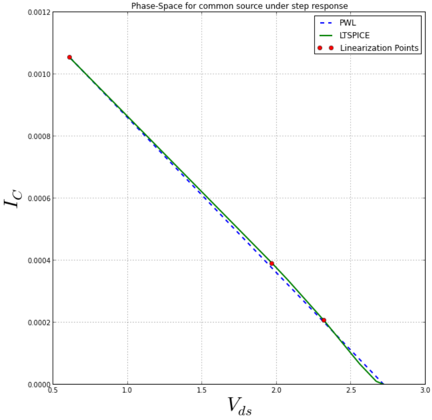


Figure: Results obtained

Looking Ahead

- TPWL + MOR for Differential Amplifier
- Make a robust framework which is easily scalable to any analog circuit
- Theory behind cascaded MOR systems, where systems are built on top of MOR systems
- Use this theory for working on MOR for $\sum\triangle$ converter

References

- Rewienski, Michał Jerzy. *A trajectory piecewise-linear approach to model order reduction of nonlinear dynamical systems*. Diss. Massachusetts Institute of Technology, 2003.
- Frehse, Goran, Bruce H. Krogh, and Rob A. Rutenbar. "*Verifying analog oscillator circuits using forward/backward abstraction refinement*." Proceedings of the conference on Design, automation and test in Europe: Proceedings. European Design and Automation Association, 2006.