Gravitation Wave Identification

Using Metropolis Hastings Algorithm

Group 1

Anirudh Bhat

Samyak Rai

Shanmukh Machiraju

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Problem Statement

Problem Statement

Given a time series strain data with added noise with the structure of a gravitational wave as given below

$$h(t) = \alpha e^t[1 - \tanh\{2(t - \beta)\}]\sin(\gamma t)$$

α, β, γ are parameters that signify the physical properties of the given wave. There value ranges are

$$0 < \alpha < 2$$
$$1 < \beta < 10$$
$$1 < \gamma < 20$$

We need to determine the parameter values using a Metropolis Hastings Random walk algorithm in the 3 dimensional space.

Understanding Wave Parameters

Let us visualize how the parameters α, β, and γ influence the waveform

1. α controls the amplitude of the signal
2. β shifts the signal in time
3. γ controls the oscillation frequency

Animation Paramater effects

Methodology

Random Walks

1. **Initialization:** We start with initial parameter values at the midpoints of the given ranges so

$$\alpha = 1, \beta = 5, \gamma = 10$$

2. **Random Walk:**

For each iteration we propose a new set of parameters using

$$\theta_{\text{new}} = \text{normal}\left(\theta_{\text{initial}}, \sigma^2\right) \quad \text{where} \;\; \sigma = [0.01, 0.07, 0.07]$$

The new value is discarded or chosen based on an Acceptance Probability defined as

$$A(\theta_{\text{new}}, \theta_{\text{initial}}) = \min\left(1, \frac{\text{Posterior}(\theta_{\text{new}})}{\text{Posterior}(\theta_{\text{initial}})}\right)$$

The Posterior function is defined as the following

```
def likelihood_reduced(y_data: np.ndarray, y_prior: np.ndarray):
y_err = 0.1 * np.std(y_data)
Y = np.mean((y_data - y_prior) ** 2) / y_err**2
return -0.5 * Y
```

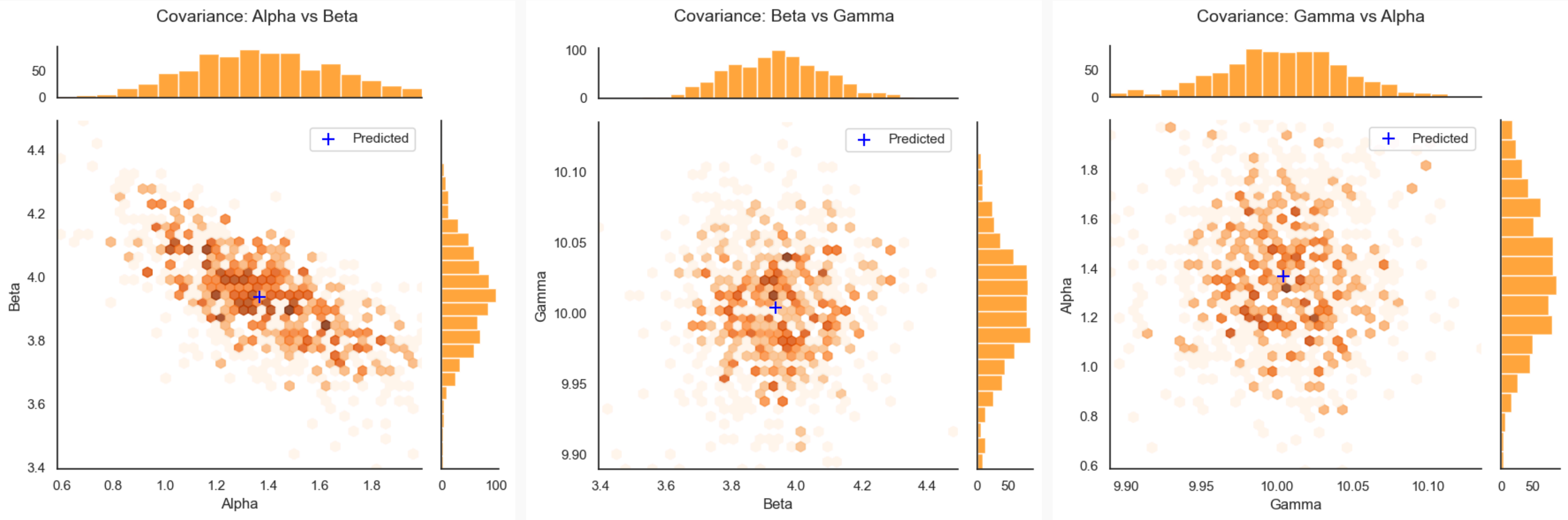
This is different from the function provided in the problem statement, we will explain why this is better in Section 4

Results

Numerical Analysis

Parameter	α (alpha)	β (beta)	γ (gamma)
Median Value	1.36	3.94	10.00
95% CI	0.86 - 1.91	N/A	N/A

Covariance Scatter Plots



Optimization

Scale Selection

Explain step size selection with variance plot

Likelihood Function Selection

Explain why our likelihood function is better

Conclusion