Gravitation Wave Identification

Using Metropolis Hastings Algorithm

Group 1

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Problem Statement

Problem Statement

Given a time series strain data with added noise with the structure of a gravitational wave as given below

$$h(t) = \alpha e^{t}[1 - \tanh\{2(t - \beta)\}]\sin(\gamma t)$$

α, β, γ are parameters that signify the physical properties of the given wave. There value ranges are

$$0 < \alpha < 2$$
$$1 < \beta < 10$$
$$1 < \gamma < 20$$

We need to determine the parameter values using a Metropolis Hastings Random walk algorithm in the 3 dimensional space.

Understanding Wave Parameters

Let us visualize how the parameters α, β, and γ influence the waveform

1. α controls the amplitude of the signal
2. β shifts the signal in time
3. γ controls the oscillation frequency

Animation Paramater effects

Methodology

Random Walks

1. **Initialization:** We start with initial parameter values at the midpoints of the given ranges so

$$\alpha = 1, \beta = 5, \gamma = 10$$

2. **Random Walk:**

For each iteration we propose a new set of parameters using

$$\theta_{new} = \text{normal}\left(\theta_{initial}, \sigma^2\right) \quad \text{where} \quad \sigma = [0.01, 0.07, 0.07]$$

The new value is discarded or chosen based on an Acceptance Probability defined as

$$A(\theta_{\text{new}}, \theta_{\text{initial}}) = \min\left(1, \frac{\text{Posterior}(\theta_{\text{new}})}{\text{Posterior}(\theta_{\text{initial}})}\right)$$

The Posterior function is defined as the following

```
def likelihood_reduced(y_data: np.ndarray, y_prior: np.ndarray):
y_err = 0.1 * np.std(y_data)
Y = np.mean((y_data - y_prior) ** 2) / y_err**2
return -0.5 * Y
```

This is different from the function provided in the problem statement, we will explain why this is better in Section 4

Stochastic Maximum Likelihood Estimation

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Why not Bayesian Inference ?

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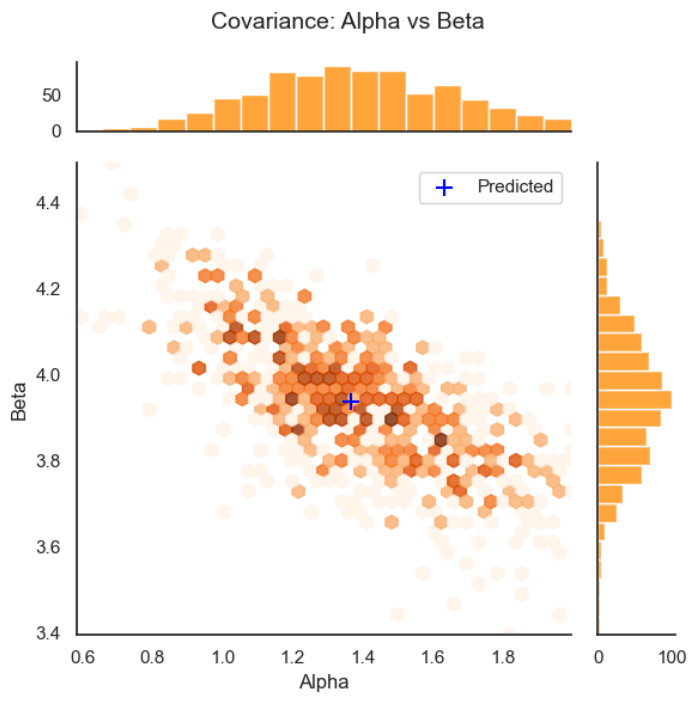
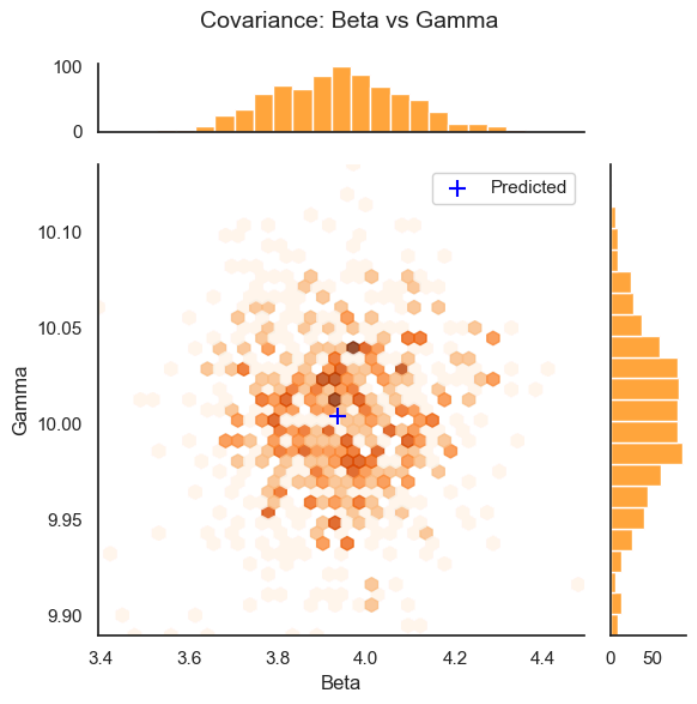
Results

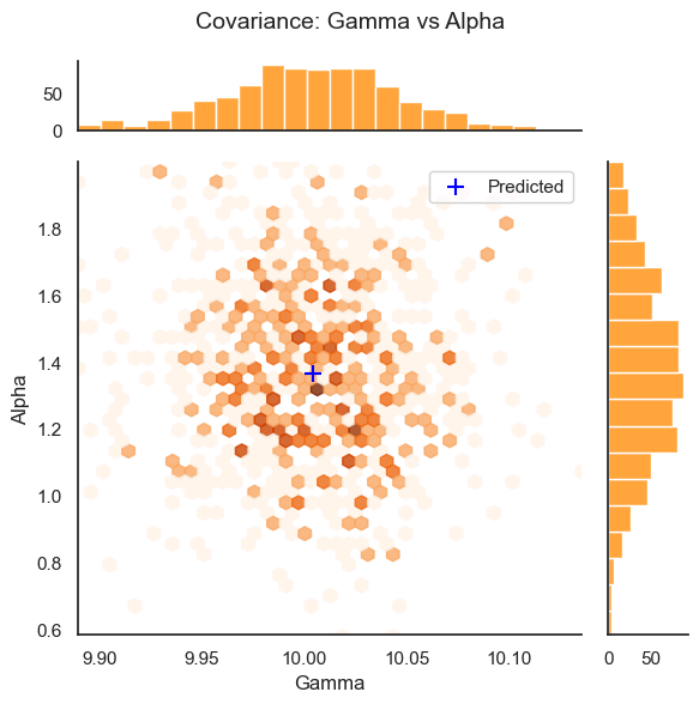
Numerical Analysis

Parameter	α (alpha)	β (beta)	γ (gamma)
Median Value	1.36	3.94	10.00
95% CI	0.86 - 1.91	N/A	N/A

Covariance Scatter Plots







Histograms and Trace Plots

Heloo

Optimization

Scale Selection

Explain step size selection with variance plot

Likelihood Function Selection

Explain why our likelihood function is better

Data Generation for Better Inference

Hello

Conclusion

Thank You

Hello