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Parameter Evaluation:-

Q1. $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}}$

$x_1, x_2, x_3 ---- x_n \Rightarrow$ sample of size n

$L(x_1, x_2, x_3 ---- x_n)$

$= f(x_1).f(x_2) ---- f(x_n)$

$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-u)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-u)^2}{2\sigma^2}}\right) \cdot ----$

taking ln on both sides.

$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-u)^2}{2\sigma^2}\right)$

$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{\cancel{2}(x_i-u)}{2\sigma^2}\right) = 0$

$\Rightarrow \sum_{i=1}^{n} (x_i - u) = 0$

$n\bar{x} - n\mu = 0$

$\bar{x} = \mu \qquad \Rightarrow \theta_1 = \bar{x}$ Ans

$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i-u)^2}{2\sigma^2} = 0$

$= -n + \sum_{i=1}^{n} -\frac{(x_i-u)^2}{\sigma^2} = 0$

$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i - \mu)^2\right) \Rightarrow \theta_2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i - u)^2\right)$ Ans

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Q2. $^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x}$

$$L = \prod_{i=1}^{n} {^nC_{x_i}}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

taking log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log(^nC_{x_i}) + \log \theta^{x_i} + \log(1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^{n} \left(\log(^nC_{x_i}) \right) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n - x_i)$$

$$\frac{d \log L}{d\theta} = 0$$

$$\Rightarrow \frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\frac{1}{\cancel{1}\,\theta\cancel{(\theta-1)}} \Sigma x_i = \frac{n^2}{\cancel{1-\theta}}$$

$$\theta = \frac{\Sigma x_i}{n^2}$$

Ans