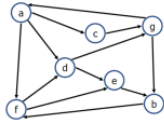


- A digraph G is strongly connected, if for any two vertices u and v of G :
u reaches v and v reaches u.
- Each vertex can reach all other vertices:



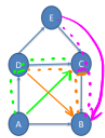
Transitive closure

- The transitive closure provides reachability information about a digraph.
- Given a digraph G, the transitive closure of G is the digraph G* such that
 - ✓ G* has the same vertices as G
 - ✓ If G has a directed path from u to v (u ≠ v), G* has a directed edge from u to v.

Reachability

- A fundamental issue with directed graphs is the notion of reachability
- Reachability, which deals with determining where we can get to in a directed graph.
- Given vertices u and v of a digraph G, we say that u reaches v (and v is reachable from u) if G has a directed path from u to v.
- A vertex v reaches an edge (w,z) if v reaches the origin vertex w of the edge.

Transitive Closure & Reachability Matrix



Adjacency Matrix A:

	A	B	C	D	E
A	0	1	0	1	0
B	0	0	0	0	0
C	0	1	0	0	0
D	0	0	1	0	1
E	0	0	1	0	0

Reachability Matrix/
Transitive Closure T:

	A	B	C	D	E
A	1	1	1	1	1
B	0	1	0	0	0
C	0	1	1	0	0
D	0	1	1	1	1
E	0	1	1	0	1

Warshall's Algorithm

Algorithm: warshall (A[1..n,1..n])
 //implements warshall's algorithm for computing the transitive closure.
 //input: the adjacency matrix A of a digraph with n-vertices
 //output: the transitive closure of the digraph.

```

R(0) ← A
for K ← 1 to n do
  for i ← 1 to n do
    for j ← 1 to n do
      R(k)[i,j] ← R(k-1)[i,j] or (R(k-1)[i,k] and R(k-1)[k,j])
return R(n)

```

$$R^k[i,j] = R^{(k-1)}[i,j] \text{ or } (R^{(k-1)}[i,k] \text{ and } R^{(k-1)}[k,j])$$

So,
 k=0, Adjacency Matrix

In [3]:

```

try:import networkx as nx
except:
    !pip install networkx
    import networkx as nx

try:import matplotlib.pyplot as plt
except:
    !pip install matplotlib
    import matplotlib.pyplot as plt

try:from prettytable import PrettyTable
except:
    !pip install prettytable
    from prettytable import PrettyTable

adm = [[0,1,0,1,0],
        [0,0,0,0,0],
        [0,1,0,0,0],
        [0,0,1,0,1],
        [0,0,1,0,0]]

x = PrettyTable()
x.field_names = [''] + [chr(65+i) for i in range(len(adm))]
i=0
for r in adm:
    x.add_row([chr(65+i)]+r)
    i+=1
print(x)

G = nx.DiGraph()
for i in range(len(adm)):
    for j in range(len(adm)):
        if adm[i][j] == 1: G.add_edge(i,j)

nx.draw(G,with_labels=True)
plt.show()

```

```

+---+---+---+---+---+
|   | A | B | C | D | E |
+---+---+---+---+---+
| A | 0 | 1 | 0 | 1 | 0 |
| B | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 1 | 0 | 0 | 0 |
| D | 0 | 0 | 1 | 0 | 1 |

```

```

graph LR
    1((1)) --> 0((0))
    1((1)) --> 2((2))
    0((0)) --> 3((3))
    2((2)) --> 3((3))
    2((2)) --> 4((4))
    3((3)) --> 4((4))

```

```

from pprint import pprint
from IPython.display import display, Math
superscript = str.maketrans("0123456789", "0 1 2 3 4 5 6 7 8 9")
def warshall(adm):
    R = adm
    ll = len(adm)
    for k in range(ll):
        #R.append(adm)
        print('for k = {}'.format(k+1))
        for i in range(ll):
            print('for i = {}'.format(i+1))
            for j in range(ll):
                prevRij = R[i][j]
                R[i][j] = R[i][j] or (R[i][k] and R[k][j])
                display(Math("R^{#{}}@[{},{}] = R^{#{}}@[{},{}] or (R^{#{}}@[{},{}] and R^{#{}}@[{},{}]".format(k+1,i+1,j+1,k,i+1,j+1,k,i+1,k+1,k,k+1,j+1).replace('#','{') + '{{} or ( {} and {} ) = {}'.format(prevRij,R[i][k],R[k][j]) + ' '))
            print('-'*10)
    x = PrettyTable()
    x.field_names = ['R{}'.format(k+1).translate(superscript)] + [chr(65+i) for i in range(ll)]
    i=0
    for r in R:
        x.add_row([chr(65+i)]+r)
        i+=1
    print(x)
    print('-'*20)

```

```

for k = 1
for i = 1

$$R^1[1, 1] = R^0[1, 1] \text{or} (R^0[1, 1] \text{and} R^0[1, 1]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[1, 2] = R^0[1, 2] \text{or} (R^0[1, 1] \text{and} R^0[1, 2]) = 1 \text{or} (0 \text{and} 1) = 1$$


$$R^1[1, 3] = R^0[1, 3] \text{or} (R^0[1, 1] \text{and} R^0[1, 3]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[1, 4] = R^0[1, 4] \text{or} (R^0[1, 1] \text{and} R^0[1, 4]) = 1 \text{or} (0 \text{and} 1) = 1$$


$$R^1[1, 5] = R^0[1, 5] \text{or} (R^0[1, 1] \text{and} R^0[1, 5]) = 0 \text{or} (0 \text{and} 0) = 0$$


```

```

-----
for i = 2

$$R^1[2, 1] = R^0[2, 1] \text{or} (R^0[2, 1] \text{and} R^0[1, 1]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[2, 2] = R^0[2, 2] \text{or} (R^0[2, 1] \text{and} R^0[1, 2]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[2, 3] = R^0[2, 3] \text{or} (R^0[2, 1] \text{and} R^0[1, 3]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[2, 4] = R^0[2, 4] \text{or} (R^0[2, 1] \text{and} R^0[1, 4]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[2, 5] = R^0[2, 5] \text{or} (R^0[2, 1] \text{and} R^0[1, 5]) = 0 \text{or} (0 \text{and} 0) = 0$$


```

```

-----
for i = 3

$$R^1[3, 1] = R^0[3, 1] \text{or} (R^0[3, 1] \text{and} R^0[1, 1]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[3, 2] = R^0[3, 2] \text{or} (R^0[3, 1] \text{and} R^0[1, 2]) = 1 \text{or} (0 \text{and} 1) = 1$$


$$R^1[3, 3] = R^0[3, 3] \text{or} (R^0[3, 1] \text{and} R^0[1, 3]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[3, 4] = R^0[3, 4] \text{or} (R^0[3, 1] \text{and} R^0[1, 4]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[3, 5] = R^0[3, 5] \text{or} (R^0[3, 1] \text{and} R^0[1, 5]) = 0 \text{or} (0 \text{and} 0) = 0$$


```

```

-----
for i = 4

$$R^1[4, 1] = R^0[4, 1] \text{or} (R^0[4, 1] \text{and} R^0[1, 1]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[4, 2] = R^0[4, 2] \text{or} (R^0[4, 1] \text{and} R^0[1, 2]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[4, 3] = R^0[4, 3] \text{or} (R^0[4, 1] \text{and} R^0[1, 3]) = 1 \text{or} (0 \text{and} 0) = 1$$


$$R^1[4, 4] = R^0[4, 4] \text{or} (R^0[4, 1] \text{and} R^0[1, 4]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[4, 5] = R^0[4, 5] \text{or} (R^0[4, 1] \text{and} R^0[1, 5]) = 1 \text{or} (0 \text{and} 0) = 1$$


```

```

-----
for i = 5

$$R^1[5, 1] = R^0[5, 1] \text{or} (R^0[5, 1] \text{and} R^0[1, 1]) = 0 \text{or} (0 \text{and} 0) = 0$$


$$R^1[5, 2] = R^0[5, 2] \text{or} (R^0[5, 1] \text{and} R^0[1, 2]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[5, 3] = R^0[5, 3] \text{or} (R^0[5, 1] \text{and} R^0[1, 3]) = 1 \text{or} (0 \text{and} 0) = 1$$


$$R^1[5, 4] = R^0[5, 4] \text{or} (R^0[5, 1] \text{and} R^0[1, 4]) = 0 \text{or} (0 \text{and} 1) = 0$$


$$R^1[5, 5] = R^0[5, 5] \text{or} (R^0[5, 1] \text{and} R^0[1, 5]) = 0 \text{or} (0 \text{and} 0) = 0$$


```

```

+-----+-----+-----+-----+-----+-----+
| R1 | A | B | C | D | E |
+-----+-----+-----+-----+-----+-----+
| A   | 0 | 1 | 0 | 1 | 0 |
| B   | 0 | 0 | 0 | 0 | 0 |
| C   | 0 | 1 | 0 | 0 | 0 |
| D   | 0 | 0 | 1 | 0 | 1 |
| E   | 0 | 0 | 1 | 0 | 0 |
+-----+-----+-----+-----+-----+

```

```

-----
for k = 2
for i = 1

$$R^2[1, 1] = R^1[1, 1] \text{or} (R^1[1, 2] \text{and} R^1[2, 1]) = 0 \text{or} (1 \text{and} 0) = 0$$


$$R^2[1, 2] = R^1[1, 2] \text{or} (R^1[1, 2] \text{and} R^1[2, 2]) = 1 \text{or} (1 \text{and} 0) = 1$$


```

```

R2[1,3] = R1[1,3]or(R1[1,2]andR1[2,3]) = 0or(1and0) = 0
R2[1,4] = R1[1,4]or(R1[1,2]andR1[2,4]) = 1or(1and0) = 1
R2[1,5] = R1[1,5]or(R1[1,2]andR1[2,5]) = 0or(1and0) = 0
-----
for i = 2
R2[2,1] = R1[2,1]or(R1[2,2]andR1[2,1]) = 0or(0and0) = 0
R2[2,2] = R1[2,2]or(R1[2,2]andR1[2,2]) = 0or(0and0) = 0
R2[2,3] = R1[2,3]or(R1[2,2]andR1[2,3]) = 0or(0and0) = 0
R2[2,4] = R1[2,4]or(R1[2,2]andR1[2,4]) = 0or(0and0) = 0
R2[2,5] = R1[2,5]or(R1[2,2]andR1[2,5]) = 0or(0and0) = 0
-----
for i = 3
R2[3,1] = R1[3,1]or(R1[3,2]andR1[2,1]) = 0or(1and0) = 0
R2[3,2] = R1[3,2]or(R1[3,2]andR1[2,2]) = 1or(1and0) = 1
R2[3,3] = R1[3,3]or(R1[3,2]andR1[2,3]) = 0or(1and0) = 0
R2[3,4] = R1[3,4]or(R1[3,2]andR1[2,4]) = 0or(1and0) = 0
R2[3,5] = R1[3,5]or(R1[3,2]andR1[2,5]) = 0or(1and0) = 0
-----
for i = 4
R2[4,1] = R1[4,1]or(R1[4,2]andR1[2,1]) = 0or(0and0) = 0
R2[4,2] = R1[4,2]or(R1[4,2]andR1[2,2]) = 0or(0and0) = 0
R2[4,3] = R1[4,3]or(R1[4,2]andR1[2,3]) = 1or(0and0) = 1
R2[4,4] = R1[4,4]or(R1[4,2]andR1[2,4]) = 0or(0and0) = 0
R2[4,5] = R1[4,5]or(R1[4,2]andR1[2,5]) = 1or(0and0) = 1
-----
for i = 5
R2[5,1] = R1[5,1]or(R1[5,2]andR1[2,1]) = 0or(0and0) = 0
R2[5,2] = R1[5,2]or(R1[5,2]andR1[2,2]) = 0or(0and0) = 0
R2[5,3] = R1[5,3]or(R1[5,2]andR1[2,3]) = 1or(0and0) = 1
R2[5,4] = R1[5,4]or(R1[5,2]andR1[2,4]) = 0or(0and0) = 0
R2[5,5] = R1[5,5]or(R1[5,2]andR1[2,5]) = 0or(0and0) = 0
-----
+---+---+---+---+---+---+
| R2 | A | B | C | D | E |
+---+---+---+---+---+---+
| A | 0 | 1 | 0 | 1 | 0 |
| B | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 1 | 0 | 0 | 0 |
| D | 0 | 0 | 1 | 0 | 1 |
| E | 0 | 0 | 1 | 0 | 0 |
+---+---+---+---+---+---+
-----

```

```

for k = 3
for i = 1
R3[1,1] = R2[1,1]or(R2[1,3]andR2[3,1]) = 0or(0and0) = 0
R3[1,2] = R2[1,2]or(R2[1,3]andR2[3,2]) = 1or(0and1) = 1
R3[1,3] = R2[1,3]or(R2[1,3]andR2[3,3]) = 0or(0and0) = 0
R3[1,4] = R2[1,4]or(R2[1,3]andR2[3,4]) = 1or(0and0) = 1
R3[1,5] = R2[1,5]or(R2[1,3]andR2[3,5]) = 0or(0and0) = 0
-----
for i = 2
R3[2,1] = R2[2,1]or(R2[2,3]andR2[3,1]) = 0or(0and0) = 0
R3[2,2] = R2[2,2]or(R2[2,3]andR2[3,2]) = 0or(0and1) = 0
R3[2,3] = R2[2,3]or(R2[2,3]andR2[3,3]) = 0or(0and0) = 0
R3[2,4] = R2[2,4]or(R2[2,3]andR2[3,4]) = 0or(0and0) = 0
R3[2,5] = R2[2,5]or(R2[2,3]andR2[3,5]) = 0or(0and0) = 0
-----
for i = 3
R3[3,1] = R2[3,1]or(R2[3,3]andR2[3,1]) = 0or(0and0) = 0
R3[3,2] = R2[3,2]or(R2[3,3]andR2[3,2]) = 1or(0and1) = 1
R3[3,3] = R2[3,3]or(R2[3,3]andR2[3,3]) = 0or(0and0) = 0
R3[3,4] = R2[3,4]or(R2[3,3]andR2[3,4]) = 0or(0and0) = 0
R3[3,5] = R2[3,5]or(R2[3,3]andR2[3,5]) = 0or(0and0) = 0
-----
for i = 4
R3[4,1] = R2[4,1]or(R2[4,3]andR2[3,1]) = 0or(1and0) = 0
R3[4,2] = R2[4,2]or(R2[4,3]andR2[3,2]) = 0or(1and1) = 1
R3[4,3] = R2[4,3]or(R2[4,3]andR2[3,3]) = 1or(1and0) = 1
R3[4,4] = R2[4,4]or(R2[4,3]andR2[3,4]) = 0or(1and0) = 0
R3[4,5] = R2[4,5]or(R2[4,3]andR2[3,5]) = 1or(1and0) = 1
-----
for i = 5
R3[5,1] = R2[5,1]or(R2[5,3]andR2[3,1]) = 0or(1and0) = 0
R3[5,2] = R2[5,2]or(R2[5,3]andR2[3,2]) = 0or(1and1) = 1
R3[5,3] = R2[5,3]or(R2[5,3]andR2[3,3]) = 1or(1and0) = 1
R3[5,4] = R2[5,4]or(R2[5,3]andR2[3,4]) = 0or(1and0) = 0
R3[5,5] = R2[5,5]or(R2[5,3]andR2[3,5]) = 0or(1and0) = 0
-----
+---+---+---+---+---+---+
| R3 | A | B | C | D | E |
+---+---+---+---+---+---+
| A | 0 | 1 | 0 | 1 | 0 |
| B | 0 | 0 | 0 | 0 | 0 |

```

C	0	1	0	0	0	
D	0	1	1	0	1	
E	0	1	1	0	0	
+-----+-----+-----+-----+-----+-----+						

```
for k = 4
for i = 1
```

$$R^4[1,1] = R^3[1,1]or(R^3[1,4]andR^3[4,1]) = 0or(1and0) = 0$$

$$R^4[1,2] = R^3[1,2]or(R^3[1,4]andR^3[4,2]) = 1or(1and1) = 1$$

$$R^4[1,3] = R^3[1,3]or(R^3[1,4]andR^3[4,3]) = 0or(1and1) = 1$$

$$R^4[1,4] = R^3[1,4]or(R^3[1,4]andR^3[4,4]) = 1or(1and0) = 1$$

$$R^4[1,5] = R^3[1,5]or(R^3[1,4]andR^3[4,5]) = 0or(1and1) = 1$$

```
for i = 2
```

$$R^4[2,1] = R^3[2,1]or(R^3[2,4]andR^3[4,1]) = 0or(0and0) = 0$$

$$R^4[2,2] = R^3[2,2]or(R^3[2,4]andR^3[4,2]) = 0or(0and1) = 0$$

$$R^4[2,3] = R^3[2,3]or(R^3[2,4]andR^3[4,3]) = 0or(0and1) = 0$$

$$R^4[2,4] = R^3[2,4]or(R^3[2,4]andR^3[4,4]) = 0or(0and0) = 0$$

$$R^4[2,5] = R^3[2,5]or(R^3[2,4]andR^3[4,5]) = 0or(0and1) = 0$$

```
for i = 3
```

$$R^4[3,1] = R^3[3,1]or(R^3[3,4]andR^3[4,1]) = 0or(0and0) = 0$$

$$R^4[3,2] = R^3[3,2]or(R^3[3,4]andR^3[4,2]) = 1or(0and1) = 1$$

$$R^4[3,3] = R^3[3,3]or(R^3[3,4]andR^3[4,3]) = 0or(0and1) = 0$$

$$R^4[3,4] = R^3[3,4]or(R^3[3,4]andR^3[4,4]) = 0or(0and0) = 0$$

$$R^4[3,5] = R^3[3,5]or(R^3[3,4]andR^3[4,5]) = 0or(0and1) = 0$$

```
for i = 4
```

$$R^4[4,1] = R^3[4,1]or(R^3[4,4]andR^3[4,1]) = 0or(0and0) = 0$$

$$R^4[4,2] = R^3[4,2]or(R^3[4,4]andR^3[4,2]) = 1or(0and1) = 1$$

$$R^4[4,3] = R^3[4,3]or(R^3[4,4]andR^3[4,3]) = 1or(0and1) = 1$$

$$R^4[4,4] = R^3[4,4]or(R^3[4,4]andR^3[4,4]) = 0or(0and0) = 0$$

$$R^4[4,5] = R^3[4,5]or(R^3[4,4]andR^3[4,5]) = 1or(0and1) = 1$$

```
for i = 5
```

$$R^4[5,1] = R^3[5,1]or(R^3[5,4]andR^3[4,1]) = 0or(0and0) = 0$$

$$R^4[5,2] = R^3[5,2]or(R^3[5,4]andR^3[4,2]) = 1or(0and1) = 1$$

$$R^4[5,3] = R^3[5,3]or(R^3[5,4]andR^3[4,3]) = 1or(0and1) = 1$$

$$R^4[5,4] = R^3[5,4]or(R^3[5,4]andR^3[4,4]) = 0or(0and0) = 0$$

$$R^4[5,5] = R^3[5,5]or(R^3[5,4]andR^3[4,5]) = 0or(0and1) = 0$$

	R ⁴		A		B		C		D		E	
+	-----	+	-----	+	-----	+	-----	+	-----	+	-----	+
	A		0		1		1		1		1	
	B		0		0		0		0		0	
	C		0		1		0		0		0	
	D		0		1		1		0		1	
	E		0		1		1		0		0	
+	-----	+	-----	+	-----	+	-----	+	-----	+	-----	+

```

-----
for k = 5
for i = 1

```

$$R^5[1,1] = R^4[1,1]or(R^4[1,5]andR^4[5,1]) = 0or(1and0) = 0$$

$$R^5[1,2] = R^4[1,2]or(R^4[1,5]andR^4[5,2]) = 1or(1and1) = 1$$

$$R^5[1,3] = R^4[1,3]or(R^4[1,5]andR^4[5,3]) = 1or(1and1) = 1$$

$$R^5[1,4] = R^4[1,4]or(R^4[1,5]andR^4[5,4]) = 1or(1and0) = 1$$

$$R^5[1,5] = R^4[1,5]or(R^4[1,5]andR^4[5,5]) = 1or(1and0) = 1$$

```

-----
for i = 2

```

$$R^5[2,1] = R^4[2,1]or(R^4[2,5]andR^4[5,1]) = 0or(0and0) = 0$$

$$R^5[2,2] = R^4[2,2]or(R^4[2,5]andR^4[5,2]) = 0or(0and1) = 0$$

$$R^5[2,3] = R^4[2,3]or(R^4[2,5]andR^4[5,3]) = 0or(0and1) = 0$$

$$R^5[2,4] = R^4[2,4]or(R^4[2,5]andR^4[5,4]) = 0or(0and0) = 0$$

$$R^5[2,5] = R^4[2,5]or(R^4[2,5]andR^4[5,5]) = 0or(0and0) = 0$$

```

-----
for i = 3

```

$$R^5[3,1] = R^4[3,1]or(R^4[3,5]andR^4[5,1]) = 0or(0and0) = 0$$

$$R^5[3,2] = R^4[3,2]or(R^4[3,5]andR^4[5,2]) = 1or(0and1) = 1$$

$$R^5[3,3] = R^4[3,3]or(R^4[3,5]andR^4[5,3]) = 0or(0and1) = 0$$

$$R^5[3,4] = R^4[3,4]or(R^4[3,5]andR^4[5,4]) = 0or(0and0) = 0$$

$$R^5[3,5] = R^4[3,5]or(R^4[3,5]andR^4[5,5]) = 0or(0and0) = 0$$

```

-----
for i = 4

```

$$R^5[4,1] = R^4[4,1]or(R^4[4,5]andR^4[5,1]) = 0or(1and0) = 0$$

$$R^5[4,2] = R^4[4,2]or(R^4[4,5]andR^4[5,2]) = 1or(1and1) = 1$$

$$R^5[4,3] = R^4[4,3]or(R^4[4,5]andR^4[5,3]) = 1or(1and1) = 1$$

$$R^5[4,4] = R^4[4,4]or(R^4[4,5]andR^4[5,4]) = 0or(1and0) = 0$$

$$R^5[4,5] = R^4[4,5]or(R^4[4,5]andR^4[5,5]) = 1or(1and0) = 1$$

```

-----
for i = 5

```

$$R^5[5,1] = R^4[5,1]or(R^4[5,5]andR^4[5,1]) = 0or(0and0) = 0$$

$$R^5[5,2] = R^4[5,2]or(R^4[5,5]andR^4[5,2]) = 1or(0and1) = 1$$

$$R^5[5,3] = R^4[5,3]or(R^4[5,5]andR^4[5,3]) = 1or(0and1) = 1$$

$R^5[5,4] = R^4[5,4]or(R^4[5,5]andR^4[5,4]) = 0or(0and0) = 0$

$R^5[5,5] = R^4[5,5]or(R^4[5,5]andR^4[5,5]) = 0or(0and0) = 0$

+-----+-----+-----+-----+-----+-----+
R ⁵ A B C D E
+-----+-----+-----+-----+-----+-----+
A 0 1 1 1 1
B 0 0 0 0 0
C 0 1 0 0 0
D 0 1 1 0 1
E 0 1 1 0 0
+-----+-----+-----+-----+-----+-----+

we can mark cells with $i == j$ as 1 i.e diagonal