Assignment 2

1. A particle is moving on a hilly landscape where the height at (x, y) is given by $h(x, y) = xy^2 + x^3y$. Find a direction $\boldsymbol{v}$, at the point $(1, 2)$ such that the height of the particle does not change for a small movement along $\boldsymbol{v}$? Can you find another direction $\boldsymbol{w}$ that is orthogonal to $\boldsymbol{v}$ such that the height of the particle does not change for a small movement along $\boldsymbol{w}$?

2. Find the plane tangent to $xyz = \alpha$ at the point (β, β, β).

3. Let $z(x, y) = \frac{f(x-y)}{y}$ where f is differentiable and $y \neq 0$. Calculate $z + y\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$, and find the simplest expression for it.

4. Consider the quadratic function $g(x, y) = x^2 + 3y^2$. Manually calculate the iterates x_k of the gradient descent algorithm to find the minimum of this function for $1 <= k <= 6$. Assume a starting point of $[3 \;\; 8]$. You may assume a step size of $\alpha = 0.1$. Also find the result by assuming a different starting point given by $[1 \;\; 0]$ for the same number of iterations using $\alpha = 0.1$ as step size.

5. A professor informed a student that the function $y^4 - 62y^2 + \alpha y + 9$ attains its maximum value at $y = 1$ on the interval $[0 \; 2]$. Derive the value of α.

6. Compute the derivative $\frac{df}{dx}$ of the following function

$$f(x) = \sin(\sqrt{x} + \ln(x)) + \cos(\sqrt{x} + \ln(x)) + (\sqrt{x} + \ln(x))$$

illustrating back propagation method.

7. Black and Black Co. has two units that produce bricks. Unit I can produce 3000 gray bricks, 2000 red bricks and 300 glazed bricks daily. Unit II can produce 2000 gray bricks, 5000 red bricks and 1500 glazed bricks daily.The daily operating cost of Unit I is 400 dollars and Unit II is 600 dollars. Formulate the LPP to find the number of days of operation of Unit I and Unit II such that the operating cost in filling the order 18000 gray, 34000 red and 9000 glazed bricks is minimized. Derive the dual linear program using the Lagrange's duality.

8. Let$f : \mathbb{R}^2 \to \mathbb{R}$ be given as $f(x, y) = x^3y^2 + x + y^3$. What is the quadratic approximation at $(x_0, y_0) = (1, 1)$?

9. Evaluate without technology the cube root of 1002 using quadratic approximation.Especially look how close you are to real value.

10. Given $g(x, y) = (6y^2 - 5)^2(x^2 + y^2 - 1)^2$, define the Surface S by $f(x, y, z) = g(x, y) + g(y, z) + g(z, x) = 3$. The following equation could be derived with the chain rule. You can take this for granted:

$$\nabla \mathbf{f}(\mathbf{1}, -\mathbf{1}, \mathbf{1}) = \begin{bmatrix} g_x(1, -1) + g_y(1, 1) \\ g_x(-1, 1) + g_y(1, -1) \\ g_x(1, 1) + g_y(-1, 1) \end{bmatrix}$$

using this, find the tangent plane S at $(1, -1, 1)$.