Days Two and Three: Probability

Task 1

In a single toss of two fair (evenly-weighted) six-sided dice, find the probability that their sum will be at most 9.

Solution

The possible outcomes for this game are the set of numbers from 2 to 12.

There are 6 ways the first dice can land and 6 ways the second dice can land. Since 6 x 6 = 36, there are 36 total ways the dice can land. How many ways have a sum less than or equal to 9?

Instead of looking at the number of ways the sum will be less than 9, we can look at the number of ways the sum will equal more than 9. Then, we can subtract that value from 36 to get our answer.

Outcome	Dice #1	Dice #2	Total
1	6	6	12
2	6	5	11
3	6	4	10
4	5	6	11
5	5	5	10
6	4	6	10

As we can see, there are 6 outcomes that yield a result greater than 9. That means there are 36 - 6 = 30 different ways the sum will be at most 9.

Since we're looking for the probability that the sum will be at most 9, and every outcome is equally likely, the answer to this question is 30/36. This, of course, can be simplified to 5/6.

Final Answer

The probability is 5/6.

Task 2

In a single toss of two fair (evenly-weighted) six-sided dice, find the probability that the values rolled by each die will be different and the two dice have a sum of 6.

Solution

If we don't know how to approach a problem, it's easiest to to just start out by enumerating some of the possibilities and see if a pattern emerges.

Outcome	Dice #1	Dice #2
1	1	5
2	2	4
3	4	2

Outcome	Dice #1	Dice #2
4	5	1

We can actually stop here because we see that since the sum must equal 6, we have actually already listed out all the possibilities. There are only 4 ways the sum of these two dice can equal six. (We must reject the 3 and 3 possibility because they are the same number.) We know from our work in task one that the total number of possible outcomes is 36. Therefore, the answer to this question is $4/36 = 1/9$.

Final Answer

The probability is 1/9.

Task 3

There are 3 urns labeled X, Y, and Z.

Urn X contains 4 red balls and 3 black balls. Urn Y contains 5 red balls and 4 black balls. Urn Z contains 4 red balls and 4 black balls.

One ball is drawn from each of the 3 urns. What is the probability that, of the 3 balls drawn, 2 are red and 1 is black?

Solution

Set up

This may seem tricky at first glance due to the volume of information presented in the question prompt. One place to start is to notice that each draw only has two possible outcomes:

1. We draw a red ball.
2. We draw a black ball.

Since there are only 3 draws, we are left with 6 possible outcomes. (3 x 2 = 6)

Outcome #1	Outcome #2	Outcome #3	Outcome #4	Outcome #5	Outcome #6
Red	Red	Red	Black	Black	Black
Red	Red	Black	Red	Black	Black
Red	Black	Red	Red	Red	Black

We are only interested in the probability of the outcomes where two draws are red, and one draw is black. These are outcomes 2,3, and 4. Let's calculate the likely hood that those will happen. To do this we must first calculate the probability of each individual outcome, and then sum them together.

Probability of Outcome 2

Outcome 2 involves drawing a red ball from Urn X, a red ball from Urn Y, and a black ball from Urn Z.

Since there are 4 red balls and 3 black balls P(Red in X) $= 4/7$. Since there are 5 red balls and 4 black balls P(Red in Y) $= 5/9$. Since there are 4 red balls and 4 black balls P(Black in Z) $= 4/8 = 1/2$.

The probability of Outcome 2 is the product of the probability of getting our desired draw in each Urn. 4/7 x 5/9 x 1/2 = 20/126 = 10/63.

We follow a similar process for the other two outcomes.

Probability of Outcome 3

Outcome 3 involves drawing a red ball from Urn X, a black ball from Urn Y, and a red ball from Urn Z.

Since there are 4 red balls and 3 black balls P(Red in X) = 4/7. Since there are 5 red balls and 4 black balls P(Black in Y) = 4/9. Since there are 4 red balls and 4 black balls P(Red in Z) = 4/8 = 1/2.

Just like when we calculated the P(Outcome 2), the P(Outcome 3) is the product of the probability of getting our desired draw in each Urn.

4/7 x 4/9 x 1/2 = 16/126 = 8/63.

We follow the exact same procedure to calculate P(Outcome 4).

Probability of Outcome 4

Outcome 4 involves drawing a black ball from Urn X, a red ball from Urn Y, and a red ball from Urn Z.

Since there are 4 red balls and 3 black balls P(Black in X) = 3/7. Since there are 5 red balls and 4 black balls P(Red in Y) = 5/9. Since there are 4 red balls and 4 black balls P(Red in Z) = 4/8 = 1/2.

We complete the process of calculating the probability of individual outcomes by calculating the product of the probability of getting our desired draws in each urn.

3/7 x 5/9 x 1/2 = 15/126 = 5/42.

Summing the probability

The last step of the process is to add up the probabilities of each outcome occurring to get the answer to this question: "What is the probability that, of the 3 balls drawn, 2 are red and 1 is black?"

P(Outcome 1) + P(Outcome 2) + P(Outcome 3) = 10/63 + 8/63 + 5/42 = 17/42.

Answer

The probability of drawing 2 red and 1 black is 17/42.

Task 4

Suppose a family has 2 children. Given that at least one of them is a boy, what is the probability that both children are boys?

Solution

For this problem we'll assume gender is binary. This will be a contested assumption, but one that we'll have to make to formulate an answer.

This question is tricky because at first glance the answer seems to be 1/2. We already know one child is a boy, so there is a one hundred percent chance he is a boy. There is a 50% chance the 2nd child is a boy. So the answer might be 100% x 50% or 1/2. But this is the wrong way to look at this conditional probability problem. This is because we don't know which child is a boy.

{Boy, Girl} {Girl, Boy} {Boy, Boy} {Girl, Girl}

Shown above are all the all the possibilities. We are given the information that one of the children is a boy. This means we can eliminate the {Girl, Girl} option. We are left with these three options:

{Boy, Boy} {Boy, Girl} {Girl, Boy}

There are three possible outcomes, and since only one of them has two boy children, the probability is 1/3.

Answer

The Probability of having two boy children knowing that one of them is a boy is 1/3.

Task 5

You draw 2 cards from a standard 52-card deck *without* replacing them. What is the probability that both cards are of the same suit?

Solution

We know there are 52 cards in a deck, 13 cards in each suit, and 4 possible suits. If we draw one card, then it will be one of the 4 possible suits. There will be 51 cards left in the deck, 12 of which are the suit that we have in our hand. The probability of drawing that card is 12/51. Which we can simplify to 4/17 or roughly 23.52%. Pretty good odds actually!

Answer The Probability is 4/17 or approximately 23.52%