INDIAN INSTITUTE OF TECHNOLOGY, PALAKKAD

ID 3801 : Open Ended Lab Project

"*Design and modeling of lake/river surface cleaning boat*"

End Semester Report

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Date:

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Conceptual design

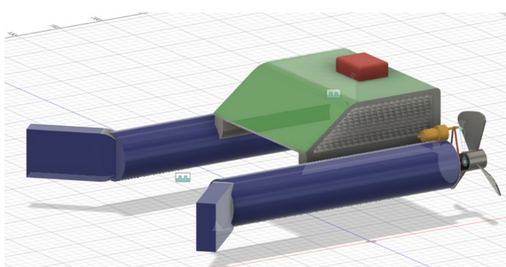
Body of boat is made using pontoons which are of PVC material. There will be rectangular frame on top of pontoons. On top center of that frame there will be small casing in which all electric components, and power supply can be adjusted. Casing will be having small holes for wires to go to propellers, sensors. There will be two propellers at back which will providing required thrust. In space between the pontoons there will be waste trap made by fish nets.

Detailed design

V shaped extended arms are used in front of boat to bring maximum waste in line with waste trap. Pontoons are made of PVC pipes. Frame of boat is also made of PVC. Frame is placed on top of pipes. A casing is placed on top of frame which will contain all electric components and power supply as well. Wires are drawn from this casing to motors. Motors are kept on top of pipes so that they stay above water surface. Propellers are under water surface so for power transmission from motor above water level to propellers under water we use belt drives. So using belt drives power transmission can take place from motor to propellers.

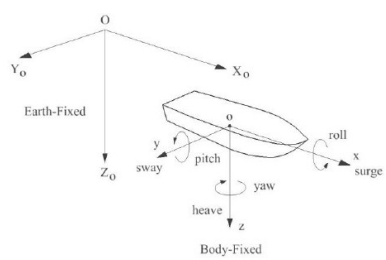
Below given is the link to animated video of my CAD design, made on Fusion360.

https://drive.google.com/file/d/14wS7dfw525o-tp03Bwa-CoNuva8MxI8c/view?usp=sharing



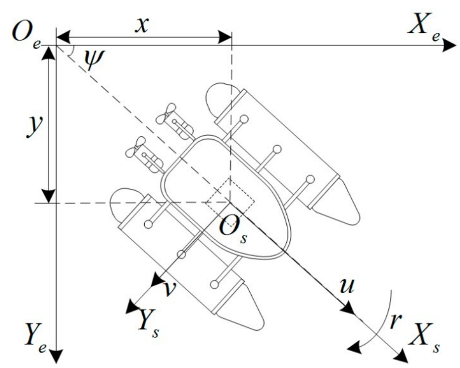
Mathematical Model

Within field of kinematics, a ship moves in 6 degrees of freedom which are defined by: surge, sway, heave, roll, pitch, and yaw.



However 6 degree of freedom system is complicated. In USV (unmanned surface vehicle) we care only about 3 degrees of freedom which are surge, sway and yaw because it is a surface vehicle it cannot have heave motion unlike submarines and we assume boat to be stable in water so we neglect pitch and roll also. Therefore the six degree of freedom model can be simplified to three degree of freedom model to describe planar motions of USV in surge, sway and yaw.

To determine the equations of motion, two reference coordinate systems are considered: the inertial or fixed to earth frame $O_eX_eY_e$ that may be taken to coincide with the USV fixed coordinates in some initial condition and the body-fixed frame $O_bX_bY_b$. Refer to figure below.



Since the motion of the earth hardly affects the USV (different from air vehicles), the earth-fixed frame $O_eX_eY_e$ can be considered inertial. O_sX_s is longitudinal axis, O_sY_s is transverse axis.

The typical USV kinematic model in planar motion without presence of disturbances can be expressed as:

$$\boxed{\dot{\eta} = J(\eta)\zeta}$$

where $\eta = [x\ y\ z]^T$

$\dot{\eta} = [\dot{x}\ \dot{y}\ \dot{z}]^T$

$\zeta = [u\ v\ r]$,

u : velocity in x(surge)

v : velocity in y(sway),

r : rate of turn(yaw)

$$J(\eta) = [\cos(\Psi)\ \ -\sin(\Psi)\ \ 0;$$
$$\sin(\Psi)\ \ \cos(\Psi)\ \ 0;$$
$$0\ \ \ 0\ \ \ 1]$$

Ψ = yaw angle.

Dynamic modeling

Dynamics is study of system that undergo changes of state as time evolves. In mechanical systems, change of states involves motion.

In inverse dynamics for given trajectory (η, η_1, η_2) we are trying to find required input vector τ.

Note : η_1, η_2 are first and second order derivatives of η with respect to time.

Lagrange Euler Method : To derive equations of motion we use Lagrange Euler method.

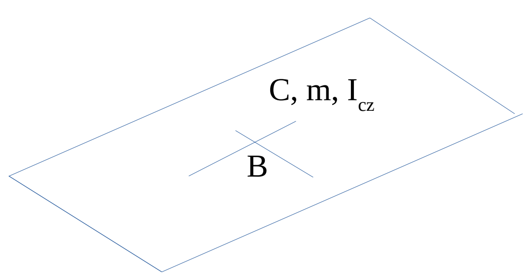
$$\frac{d}{dt}*\frac{dL}{dv}-\frac{dL}{dx}=F \quad \text{(equation1)}$$

where L = K.E. - P.E. , difference of kinetic and potential energy of system.

Assuming my boat to be as rectangular box. B represents body frame which is at center of geometrical shape. I have taken general case where body center does not coincide with center of gravity.

X_{bc} and y_{bc} are the x and y distances of center of gravity from body frame center B respectively.

X direction is along transverse axis, Y direction is along longitudinal axis.



As said in discussing kinematics u,v,r are velocities with respect to body frame.

K.E. = $\frac{1}{2}*m*(\dot{x}^2+\dot{y}^2)$ (equation2)

where x and y are velocities with respect to ground frame.

And where $\dot{x}$ = u − y_{bc}*r

and $\dot{y}$ = v + x_{bc}*r .

r is the rate of turn about z axis (perpendicular to plane axis) (equation3)

Therefore , **K.E. =** $\boxed{\frac{1}{2}*m*(\dot{x}^2+\dot{y}^2)}$ + $\boxed{\frac{1}{2}*Iz*r^2}$ and P.E. = 0 . (equation4)

L = K.E. - P.E. (equation5)

Combining equation 3 and 4 , and put them into equation 1.

After solving we get ,

$$\frac{dL}{du} = m*u - m*r*y_{bc} ;$$

$$\frac{dL}{dv} = m*v + m*r*x_{bc} ;$$

$$\frac{dL}{dr} = mu*y_{bc} + m*v*x_{bc} + m*r*[\ x_{bc}^2+y_{bc}^2\ ] + I_z*r \ ;$$

$$\frac{d}{dt}\frac{dL}{du}=m*\dot{u}-m*\dot{r}*y_{bc}-m*r*\dot{y}_{bc} \quad ;$$

$$\frac{d}{dt}\frac{dL}{dv}=m*\dot{v}+m*\dot{r}*x_{bc}+m*r*\dot{x}_{bc} \quad ;$$

$$\frac{d}{dt}\frac{dL}{dr}=-m*\dot{u}*y_{bc}-m*u*\dot{y}_{bc}+m*\dot{v}*x_{bc}+m*v*\dot{x}_{bc}+m*\dot{r}*[x_{bc}^2+y_{bc}^2]+2*m*r*[2*x_{bc}*\dot{x}_{bc}+2*y_{bc}*\dot{y}_{bc}]+I_z*\dot{r}$$

Now using Lagrange Euler equation we get F_x , F_y and M_z , which implies :

$$\boxed{F_x=m*(\dot{u}-v*r-x_{bc}*r^2-y_{bc}*\dot{r})}$$

$$\boxed{F_y=m*(\dot{v}+u*r-y_{bc}*r^2+x_{bc}*\dot{r})}$$

$$\boxed{M_z=I_{cz}*\dot{r}+m*(x_{bc}*[\dot{v}+u*r]-y_{bc}*[\dot{u}-v*r])+m*\dot{r}*(x_{bc}^2+y_{bc}^2)}$$

however in our case , in marine boats there will be two additional components in F_x which are drag force and skin friction.

$Drag\,force=\frac{1}{2}*\rho*C_d*A*\dot{x}^2$ and $Friction=\beta*\dot{x}$ which will give F_x .

So,

$$\boxed{F_x=m*(\dot{u}-v*r-x_{bc}*r^2-y_{bc}*\dot{r})+\frac{1}{2}*\rho*C_d*A*u^2+\beta*u}$$

where ρ = 1000 kg/m^3 for water, β = 0.5 and C_d = (0.3-0.5). F_y and M_z does not change.

These equations can be re-written in matrix form as follows :

$$\boxed{\overbrace{\begin{matrix} m & 0 & -m*y_{bc} \\ 0 & m & m*x_{bc} \\ -m*y_{bc} & m*x_{bc} & I_z \end{matrix}}^{InertiaMatrix}} * \boxed{\begin{matrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{matrix}} + \boxed{\overbrace{\begin{matrix} -m*r*(v+x_{bc}*r)+\frac{1}{2}*\rho*C_d*A*u^2+\beta*u \\ m*r*(u-y_{bc}*r) \\ m*r*(x_{bc}*u+y_{bc}*v) \end{matrix}}^{n\ vector}} = \boxed{\begin{matrix} F_x \\ F_y \\ M_z \end{matrix}}$$

let D be inertia matrix ,

$$\ddot{\eta} = \boxed{\begin{matrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{matrix}} ,$$

n vector represents other effects ,

$$\boldsymbol{\tau} = \boxed{\begin{matrix} F_x \\ F_y \\ M_z \end{matrix}}$$

so finally equation becomes,

$$\boxed{D_\eta*\ddot{\eta}+n_\eta=\tau_\eta}$$

Now since, $\dot{\eta}=J(\eta)\zeta$ we use principal of conservation of power.

According to **power conservation,**

$$\boxed{\begin{matrix} \tau_\eta.\dot{\eta}=\tau.\zeta \\ \tau_\eta^T*\dot{\eta}=\tau^T*\zeta \\ \tau_\eta^T*J(\eta)*\zeta=\tau^T*\zeta \\ \tau_\eta^T*J(\eta)=\tau^T \\ \tau=J^T(\eta)*\tau_\eta \\ so\ \tau_\eta=\tau \end{matrix}}$$

so **finally** these two equations we need to solve

$$\boxed{\begin{matrix} D_\eta*\dot{\zeta}+n(\zeta)=\tau \\ \dot{\eta}=J(\eta)*\zeta \end{matrix}}$$

taking inverse gives,

$$\boxed{\dot{\zeta} = D^{-1}(\tau - n(\zeta))}$$

So we converted our equations from body frame to inertial frame of reference. However we will solve in body frame itself in Matlab as our forces will be with respect to body.

SOLUTION TO EQUATION OF MOTION

To find solution we need to use numerical integration technique. We will use Euler integration method which is also called as polygonal integration method because it approximates solution of differential equation with a series of connected lines (polygon).

According to Euler integration method ,

$$\boxed{x_{i+1} = x_0 + \delta t * \dot{x}_i + \frac{1}{2} * \delta t^2 * \dot{x}_i}$$

this technique we will use in our Matlab code.

FINDING SOLUTION AND SIMULATION USING MATLAB

```
%% Dynamic simulation of marine robot
clear all;
close all;
clc;
%% Simulation parameters
dt = 0.1;             % step size
ts = 10 ;             % simulation time
t = 0:dt:ts ;         % time span
%% Initial Conditions
eta0 = [0;0;0];                 % initial position and orientation of the vehicle
zeta0 = [0;0;0];                % initial vector of input commands.
eta(:,1) = eta0 ;
zeta(:,1) = zeta0 ;
%% Boat parameters
m = 10 ;                              % mass of vehicle is 10 kgms
Iz =  0.1 ;                           % Inertia of vehicle
```

```
xbc=0; ybc=0 ;                    % coordinates of mass center
rho = 1000;                       % density of water
cd = 0.4;                         % coefficient of drag
dia = 0.2;                        % diameter of hull in meter
area = pi*(dia^2)/4 ;             % calculating area of hull
beta= 0.5;                        % skin friction factor
 %% State propagation
 for i=1:length(t)
   u = zeta(1,i); v = zeta(2,i); r = zeta(3,i) ;
   %% Inertia matrix, N vector
   D = [m,0,-m*ybc;
        0,m,m*xbc;
        -m*ybc,m*xbc,Iz+m*(xbc^2+ybc^2)];

 n_v = [-m*r*(v+xbc) + 2*0.5*rho*cd*area*(u^2) + beta*u;
          m*r*(u-ybc*r);
          m*r*(xbc*u-ybc*v)] ;

   %% input vector
   tau(:,i) = [1;0.5;0];
   %% Jacobian matrix
   psi = eta(3,i);
   J_eta = [cos(psi),-sin(psi),0;
            sin(psi),cos(psi), 0;
             0,      0,      1];
  zeta_dot(:,i) = inv(D)*(tau(:,i) - n_v) ;
  zeta(:,i+1) = zeta(:,i) + dt*zeta_dot(:,i);                  % euler method of integration
  eta(:,i+1) = eta(:,i) + dt*( J_eta*(zeta(:,i) + dt*zeta_dot(:,i))) ;   % euler method of integration to find eta
                                                               have to integrate twice so dt^2 term comes.
end

%% Animation

l = 1.2 ;                 % length of boat
w = 1.2 ;                 % width of boat
```

```
bo_co = [-l/2,l/2,l/2,-l/2,-l/2;
        -w/2,-w/2,w/2,w/2,-w/2];                 % boat coordinates, will use these coordinates to make rectangular
                                                  box which will represent our boat in animation.

figure

for i=1:length(t)
   psi = eta(3,i);
   R_psi = [cos(psi),-sin(psi);
          sin(psi),cos(psi);];                     % rotation matrix

   v_pos = R_psi*bo_co ;
   fill(v_pos(1,:) + eta(1,i),v_pos(2,:) + eta(2,i),'g');          % filling rectangular box with green color.
   hold on, grid on;
   axis([-1 3 -1 3]), axis square
   plot(eta(1,1:i),eta(2,1:i),'b-');
   legend('MR','Path'),set(gca,'fontsize',24)
   xlabel('x,[m]');
   ylabel('y,[m]');
   pause(0.1);
   hold off
end
```

RESULTS

- I was able to design a mechanical model of lake/river surface cleaning boat, and did detailed CAD modeling of it on FUSION360.
- I was able to mathematically model the system and derive its equations of motion using Lagrange Euler method.
- I used simple numerical integration technique to solve system of differential equation in Matlab.
- Results obtained from simulation can now be used to compare performances, and choose proper actuator for our boat.

REFERENCES

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