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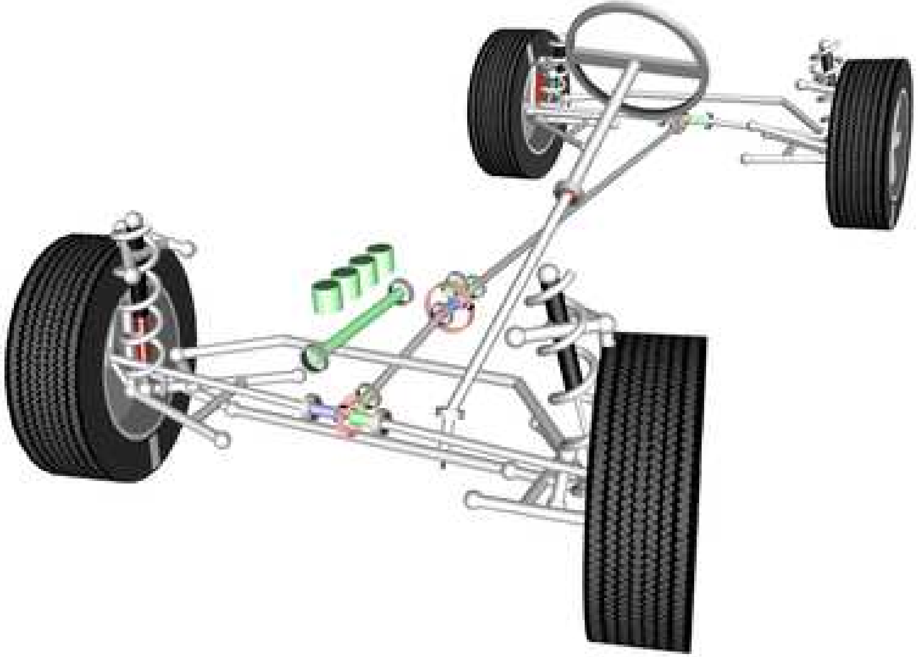


Dynamique du Véhicule

P. Brejaud

Summary

- **Chapter 1 : Definitions , background and basics**

1 – The official SAE Vehicle Reference Frame

2 –Definitions

Roll

Pitch

Bounce

Yaw

Attitude Angle

Track

Wheelbase

Angles and geometry of a drive train : Caster, King pin camber, Inclusive Angle, toe in, toe out , ground offset, thrust axis, set-back angle etc.

- **Chapter 2 : The tire**

1- Constitution and generalities

2 –Simplistic Physical Model : pressure inside contact Area, aquaplaning , adhesion, effects of the inflation pressure and vertical load on the tire

3 –Tire slip versus friction.

4 –Slip Angle

5 –Camber thrust

6 -Introduction to design plan of an Axle : Camber change In case of pure Roll

7- PACJEKA model

- **Chapter 3 :** Axles

1 – Basics and Background

2 –Theory of mechanism applied to an Axle : kinematic scheme and degrees of freedom

3 - Exercises : Front Axle, Rear Axle …

4 – Roll Center of an Axle

- **Chapter 4 :** Suspension and Vertical Behavior

1 Simple Oscillator

Resonant Frequency

Critical damping

Forced oscillations

2 Shock AbsorberTechnology

Oil Shock Absorber

Gas Shock Absorber

Dissymmetry compression / detente.

3 Comfort curves

4 Half Vehicle modelling

Bounce center/ Bounce frequency

Pitch Center/ Pitch frequency

- ## Chapter5 : Transversal Behavior & Cornering

  1 - Ackermann geometry And “Jeantaud” design
  2 - Cornering of a bicycle model.
  Under Steer Coefficient
  Critical and Characteristic Speed
  Lateral acceleration Gain
  influents parameters on Understeer and Oversteer
  3 Four wheels vehicle modeling
  Load effect and vertical load distribution
  Lateral Load Transfer Distribution (LLTD)
  Anti Roll Bar design

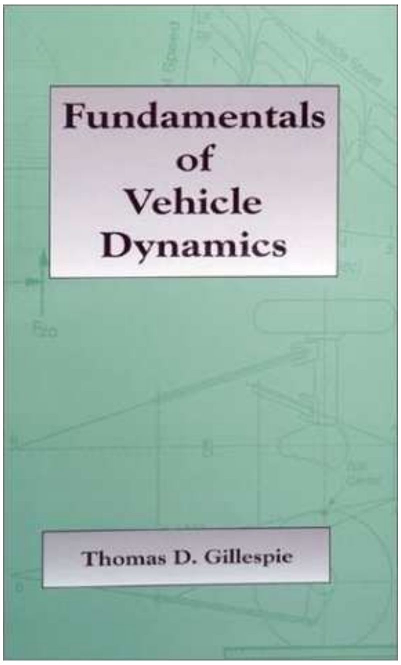
- **Synthesis Exercise :** Numerical Design of a single seater racing car.

- **Numerical Simulations :** 4hours Practical Work “DYNA4” -> Unsteady behavior of a vehicle.

- **Practical Works :**

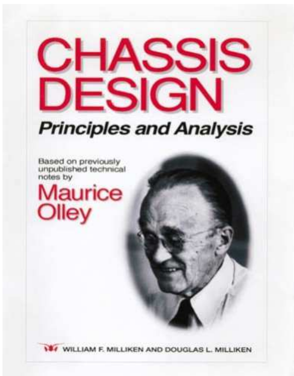
  -Rear Axle : 4 hours – Modeling and experimental measurement .
  -Mac-Pherson Axle : 4 hours

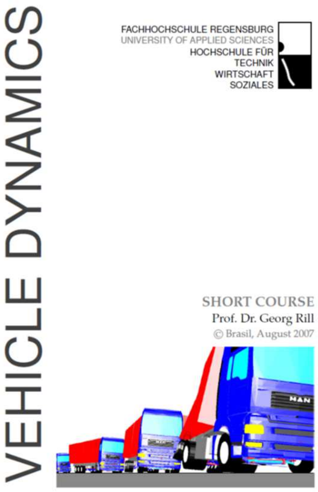
BIBLIOGRAPHY



Fundamentals of Vehicle Dynamics
SAE (Editor)

by Thomas D. Gillespie (Author)
85 USD / Amazon

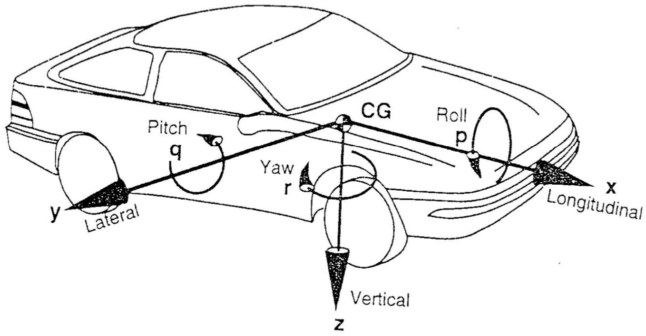




Vehicle Dynamics
by Georg RILL (Author)
Available on Internet

Chapter 1
-
Definitions , background and basics

1 – The S.A.E Vehicle Frame: (International standard)



The origin is the center of gravity (C.G)

X, is the **longitudinal axis**, is horizontal, and is contained in the plane of symmetry of the vehicule (if it exists). It is positive towards the front of the vehicle.

Z, is the **vertical axis** , is as its name suggests , vertical, and is contained (contained in the plane of symmetry of the vehicule (if it exists). . **It is positive towards the ground.**
L'axe

Y is the **transversal axis** , is given so that the triad (X,Y,Z) is direct.

1 – From a practical point of view, the origin of the vehicle reference frame is not necessarily the center of gravity. Indeed, during the design of the vehicle the weight distribution is unknown , thus C.G is undetermined. Car manufacturers choose as the origin the center point of the front axle. (Point O) (Case encountered in 3D CAD file)

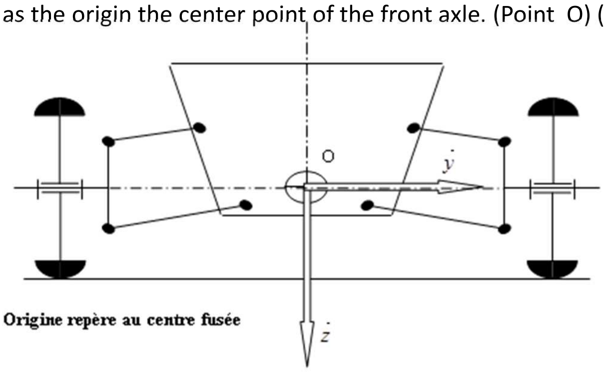
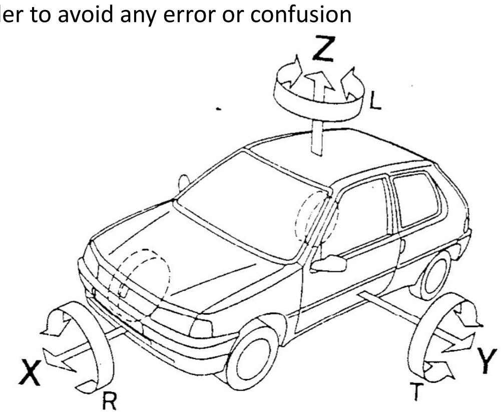


Figure 2 : Origine repère au centre fusée

2 – In Europe , practice for Z axis is to be positive towards the sky, despite the international standard . It is then needed to be cautious in order to avoid any error or confusion



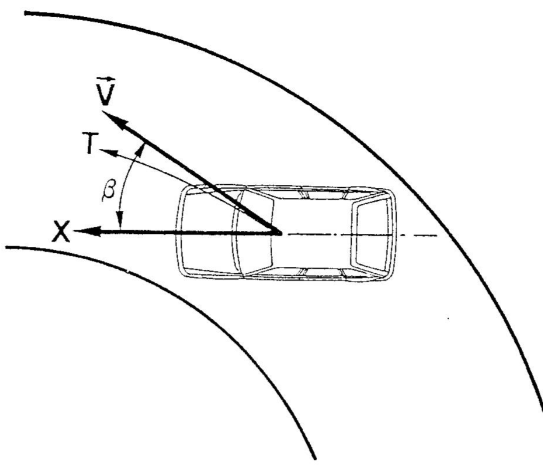
2 – Angles and movements :

The vehicle moving from the ground has degrees of freedom(3 translations Along X,Y,Z axis and 3 rotations Along X,Y,Z axis. Most of these elementary movements get name.

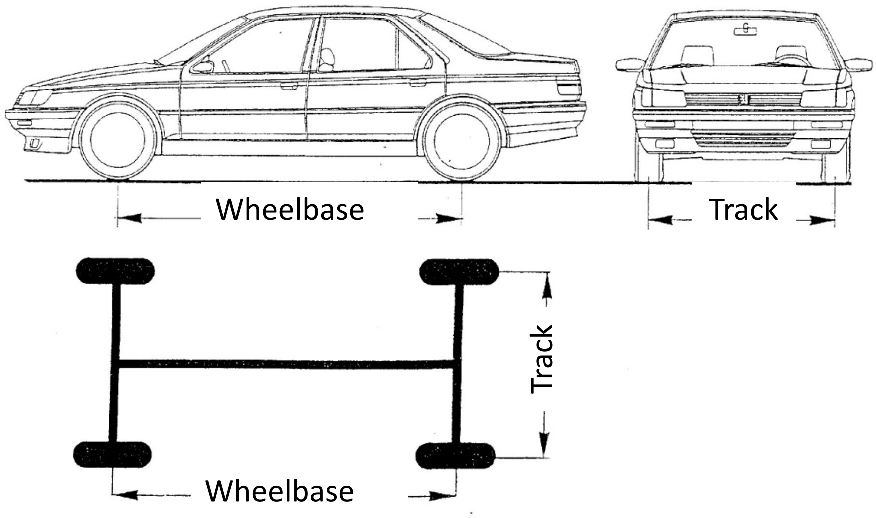
Axis System S.A.E.	Elementary Movement	Name	Observations
Axis X	Rotation	ROLL	Key point Movement in reference to the handling of the vehicle and to the sensations of the driver.
	Translation	Braking Acceleration	
Axis Y	Rotation	PITCH	Nose up while accelerating Diving while braking. Unpleasant but easy movement to control (anti dive bar)
	Translation	----	This movement must not exist.
Ais Z	Rotation	YAW	Evolves only when changing trajectory, like the roll. For the same reason it participated in "handling feeling", but it is not felt by the driver because the range of motion is low.
	Translation	BOUNCE / ELEVATING / HEAVING	Not dangerous, unpleasant and easily controlled by the suspension system. Compression / bounce : The body moves toward the ground. Spring back / relaxing : The body moves the opposite direction.

Total slip angle : β

Angle between the longitudinal axis (X) and the tangent to the trajectory (T), ie the instantaneous velocity of the center of gravity (vector V in the figure).



3 – Track and Wheelbase :



Remarks :

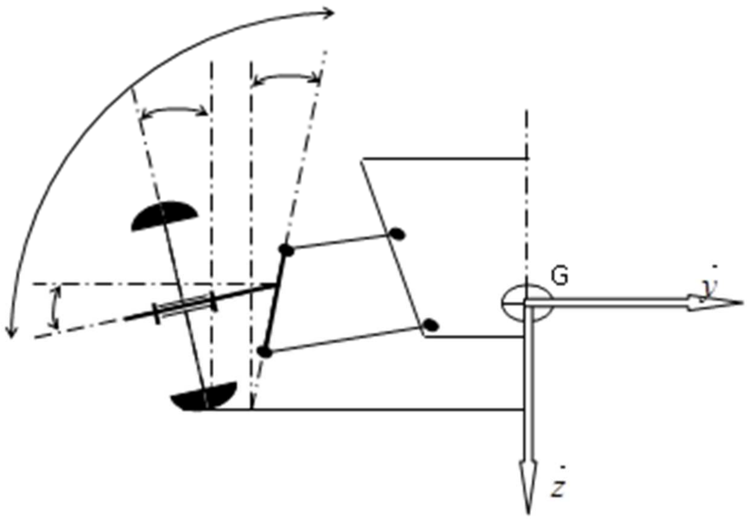
1 – The Front Track is often different from the Rear Track

2 – Vehicle should be symetrical, thus the left wheelbase should be equal to the right wheelbase.
(Famous Counter Example: Renault Super 5).

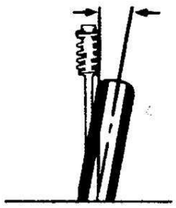
3 – The longer the wheelbase, the better stability , the lower the pitch.

4 – Axle Geometry :

Let's define the angle according to a double wishbone Front Axle.
The different axles will be more detailled in chapter 3



CAMBER ANGLE



CAMBER

Definition :
Angle between the vertical and the midplane of the wheel projected in the transverse plane (YZ plane)

The angle is positive when the wheel tends to deviate in its highest point in reference to the vehicle.

Functions :

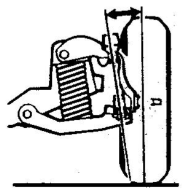
- The positive camber tends to reduce stress on the suspension components and steering by reducing the lever arm.
- The positive camber leads to better absorption of the inequalities of the road.
- The positive camber results in improved the straight line stability.
- **The negative camber to work the function of the tire in good conditions** while cornering(ie schematically the tire is almost vertical to the ground) and **compensate the roll angle** taken by the vehicle..

On modern vehicles the angle is generally quite low (from 0 ° to -2 ° max), it is almost always negative. This is a key point in order that the tire works in good conditions while cornering!

Consequences in case of trouble :

- Irregular Tire Wear
- Deviation of the vehicle on one side

KING PIN ANGLE :



KING PIN

Definition :
Angle formed by the pivot axis of the wheels projected in the transverse plane (YZ plane) with the vertical to the ground.
The angle is positive when the pivot axis is inclined towards the center of the vehicle when climbing along this axis.

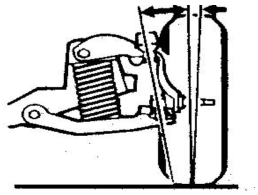
Functions :

- Exactly the same as the camber. Indeed it resumes a negative camber going "seek" the point of contact on the ground.
- more flexible and stable direction during cornering and steering.
- **Prevents SHIMMY** (unstable in a straight line)

Consequences in case of trouble :

- Same as Camber

INCLUSIVE ANGLE :



INCLUSIVE ANGLE

Definition :

algebraic sum of the camber angle and the king pin angle of inclination. (Be carreful at signs!)

Sometimes the practice is to sum camber + king pin + 90 °.

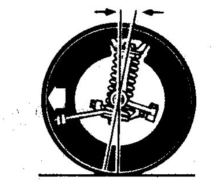
Functions :

- Detecting strain elements of the drive train (hub carrier, suspension, ...) following a shock.

Consequences in case of trouble

- Change pieces !

CASTER ANGLE:



CASTER

Definition :
Angle between the vertical and the pivot axis of the wheel projected in the longitudinal plane (XZ plane)

The angle is positive when the pivot is inclined in the opposite direction of normal travel of the vehicle.

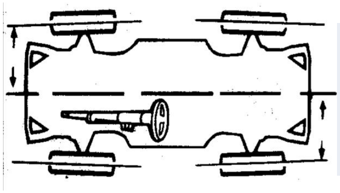
Functions :

- **Steering stability and natural return to the straight direction.**
- smooth steering cornering and steering.
- Influences (with the pivot angle) the camber variations while steering. See Pratical Work on vehicle dynamics.

Consequences in case of trouble :

- Deviation on the side where the angle is lowest.
- Bad return of the steering system in the straight direction.
- Hard steering system…

INDIVIDUAL TOE ANGLE :



TOE

Definition :
Angle between the wheel median plane and the axis of symmetry of the vehicle projected into the horizontal plane (XY plane)
It is called “TOE IN” when the wheels converge toward the front, and “TOE OUT”when it converges towards the rear.

Functions :

- The initial value (measured while vehicle is stopped), ensures a parallel position of the vehicle wheels in motion, due to deformation of the "silent block" under the riding stresses.

Propulsion	Traction
TOE IN	TOE OUT.

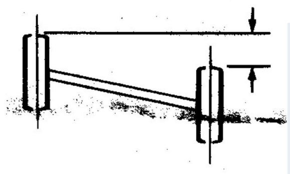
- **Compensate irregular wear of the tire caused by the camber.** An TOE OUT angle compensates a positive camber. A TOE IN compensate a negative camber. This is the reason why toe in is found in a traction vehicle despite the previous rule.

Consequences in case of trouble

- Irregular tire tread.
- Over fuel consumption
- Vehicle does not hold the road correctly !

It is important to note that this is very often the only possible setting angle on a axle.

SET BACK Angle :



DIFFÉRENCE D'AXE
(SET BACK)

Definition :
Angle between the axle and the perpendicular to a wheel axle to this, all projected in the horizontal plane (XY plane)

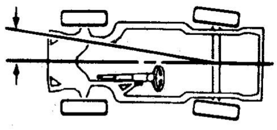
Fonctions :

- Detection an Axle that is in trouble

Consequences in case of trouble :

- Change Axle , or Chassis !

Thrust Angle :



Définition :
Angle between by the thrust axis of the rear axle and the axis of symmetry of the vehicle, all projected in the horizontal plane (XY plane)

Fonctions :

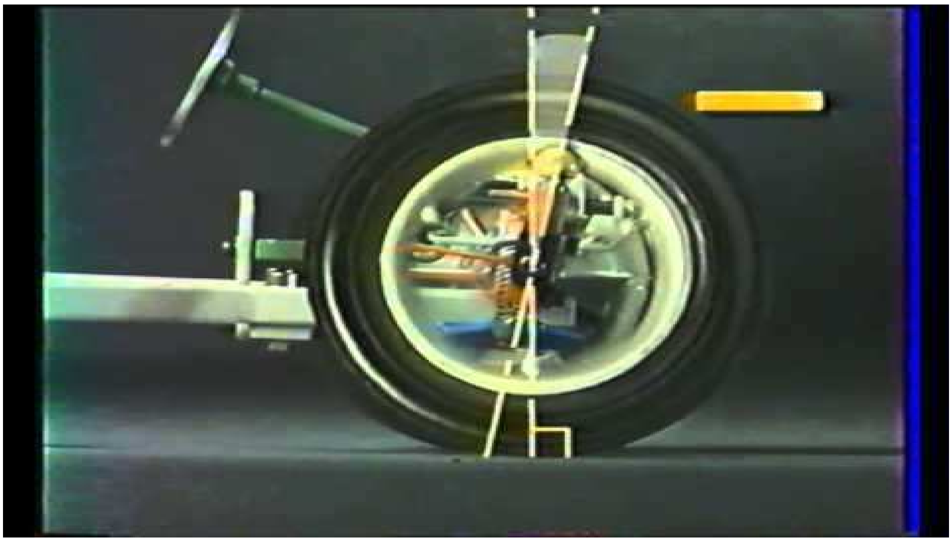
- Defines the trajectoire of the rear axle
- Used as a reference for straight line direction of the front axle

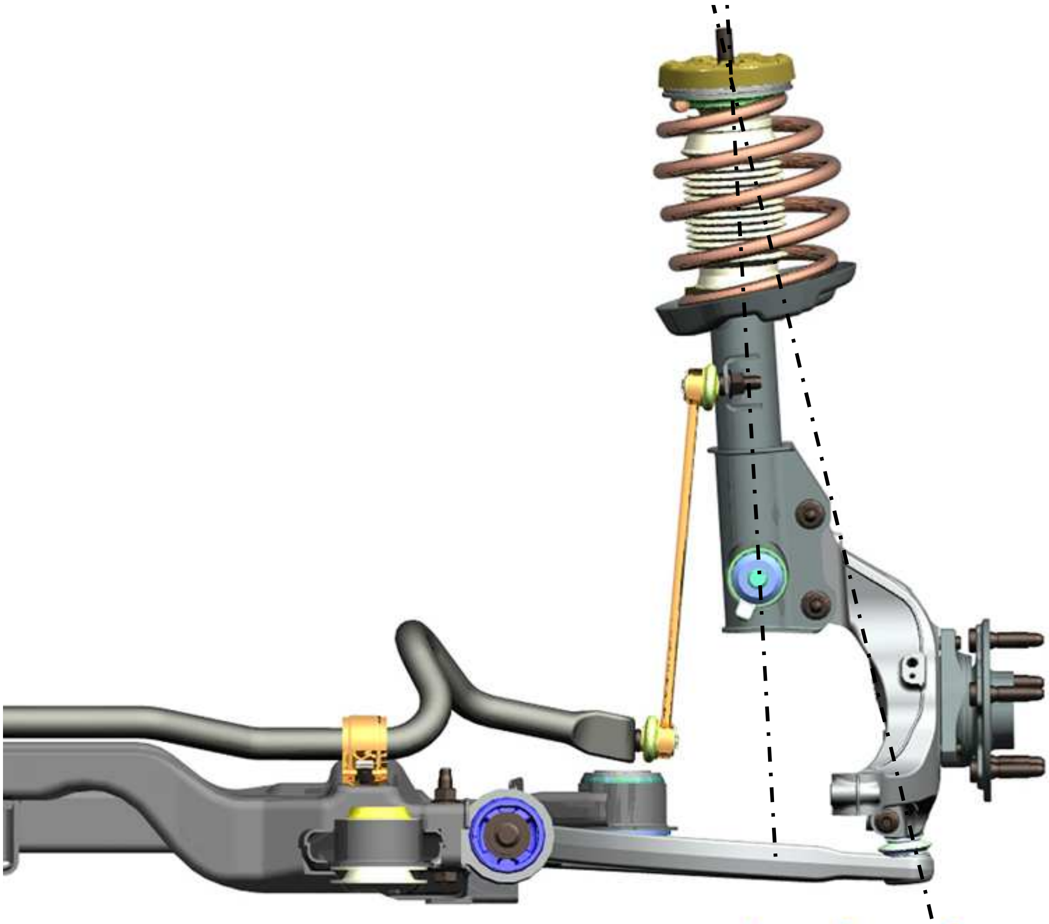
Consequences in case of trouble

- Tire wear.
- Misalignment of steering system
- vehicle slip in the opposite direction of the thrust axis.

Front Axle Video

https://www.youtube.com/watch?v=SMAYOH5P7qk

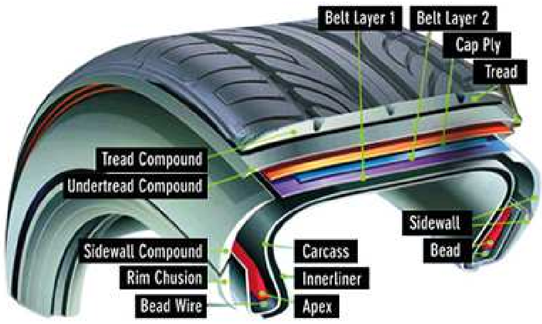


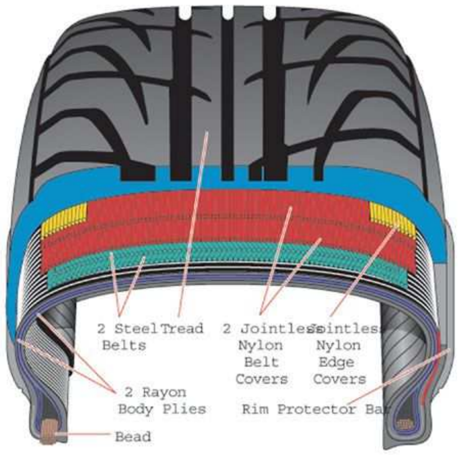


Insignia

Chapter 2
-
The Tire

1 – Tire anatomy and basics :

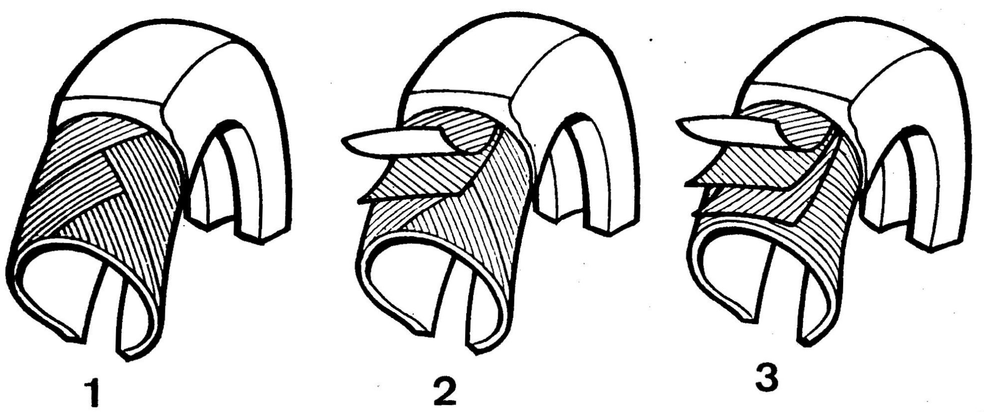




The **bead seat** (or bead) is operable to transmit by adhesion the engine or braking torque to the rim.

The **carcass ply** made of steel or textile gives the "resistance" to the tire and enable it to resume the important efforts which subjected the tire (vertical load, slip thrust etc.)

The tread and its sculptures ensures the tire traction / road, drain the water, ensure grip on snow, and resist wear ... Each context has its own sculpture .



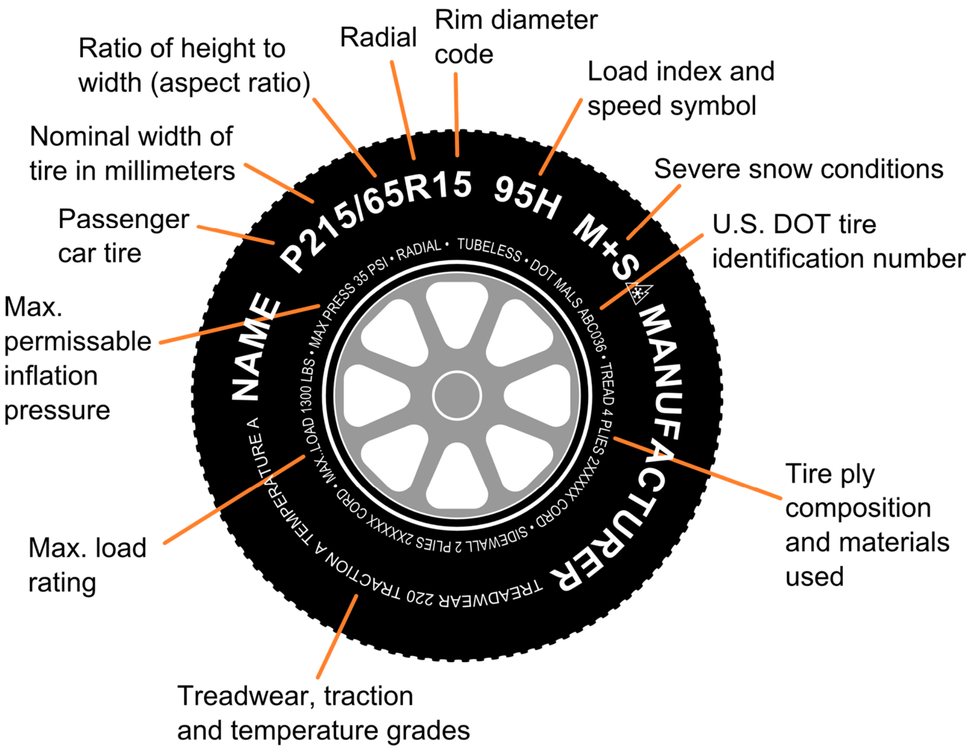
1 :Diagonal structure

2 : Bias belt structure

3 : Radial structure

Tire code :

12/10/2016 Tire_code_-_en.svg



Tire code :

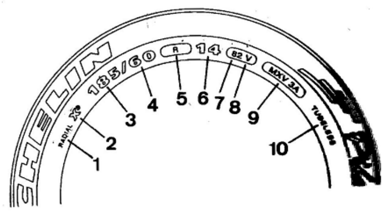


FIG. 2.27 Marquage technique d'un pneumatique.

1. type de structure 2. marque déposée 3. section du pneu 4. série du pneu 5. structure du pneu radial 6. diamètre intérieur du pneu (à comparer à celui de la jante) 7.indice de charge 8. code de vitesse 9. désignation de la confection 10. pneu sans chambre.

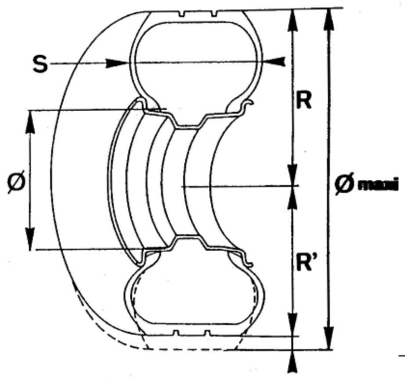


FIG. 2.28 Cotes géométriques d'un pneumatique.

ø : diamètre du pneu ; R : rayon libre ; R' : rayon sous charge ; S : section du pneu

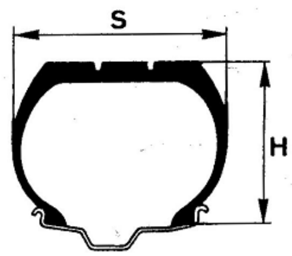


FIG. 2.29 Série d'un pneumatique.

C'est le rapport (H/S) x 100

Exemple : **175/70 R13 S**

-Nominal Width : S = 175 mm

-Ratio Height/widht :70 thus H/S = 0.70

-Tire structure (D,B or R) : R = radial

-Rim diameter : 13 inches

(1 inch = 25.4 mm)

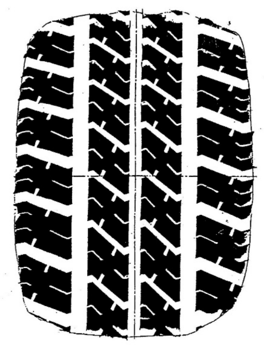
-Max Speed index : S= 180 km/h

Max Speed code	Max speed
Q	160 Km/h
R	170 Km/h
S	180 Km/h
T	190 Km/h
U	200 Km/h
H	210 Km/h
V	240 Km/h
VR	>210 Km/h
ZR	>240Km/h

Ratio	H/S
70	0.7
60	0.6
40	0.4

2 – Behavior and simplified model of a tire :

2.1 : Contact Area :



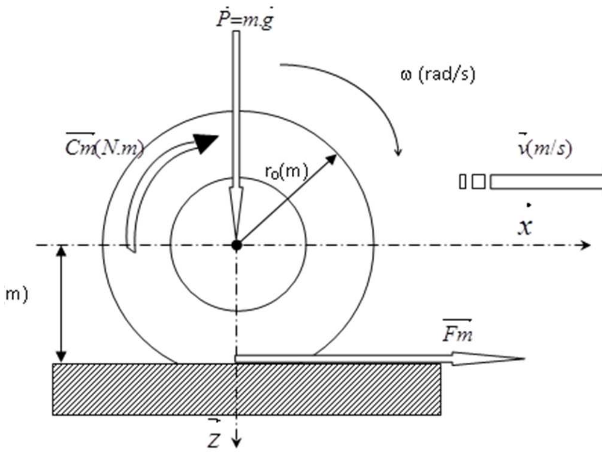
Static footprint of a tire

Dynamic footprint of motorcycle tire on impregnated water with fluoriscine.

It is in the contact area, an area the size of a postcard, that occur **adhesion phenomena**, **evacuating the water** and **tire wear**. Vehicle dynamics thus originates in the contact area because it is the only link between the vehicle and the ground.

In order that a tire works in good conditions (adhesion, guiding and wear) it is needed that the pressure distribution within the contact area is as uniform as possible, both in straight line and cornering situations

2.2 Longitudinal Slip Phenomenon :



Notations	
r_0	Free radius of the tire(m)
r	Effective radius under charge (m)
Ld	Developed length of the tire (m/tour)
Cm	Torque applied to the wheel (N.m)
Fm	Longitudinal Force (tractive or resistant) (N)
P	Vertical Load (N)
ω	Angular speed (rad/s)
v	Vehicle forward velocity (m/s)

Usual mechanical laws :

$$v\left(\frac{m}{s}\right)=w\left(\frac{rad}{s}\right)r(m)$$

$$Ld\left(\frac{m}{tr}\right)=2.\pi.r(m)$$

$$N\left(\frac{tr}{\min}\right)=\frac{30.\omega\left(\frac{rad}{s}\right)}{\pi}$$

Stationnary / Steady state Conditions

$$Cm(N.m)=Fm(N).r(m)$$

One notes :

ω	REAL angular speed of the wheel (rad/s)
ω_0	THEORICAL or IDEAL angular speed of the speed without any slip . (rad/s)

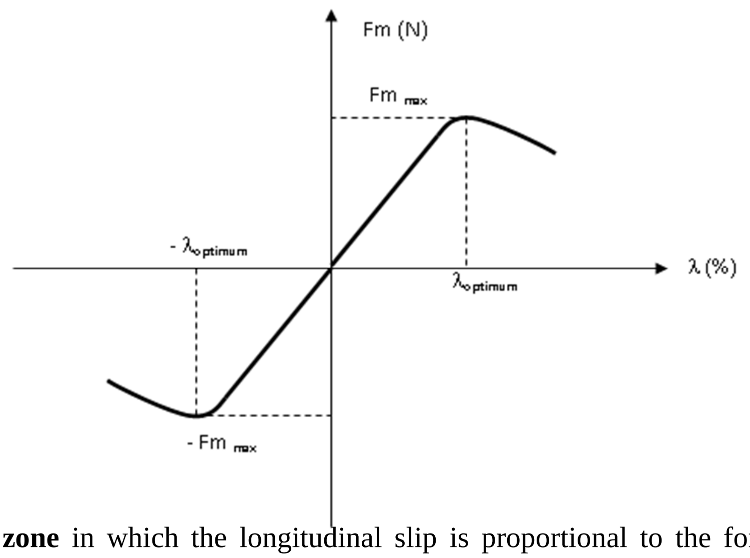
Experience shows that as soon as a torque C is applied to the wheel :
#-1 the tire will develop , as a reaction, a tractive force Fm on ground.
#-2 a longitudinal slip of the tre is observed, meaning that : $\omega \neq \omega_0$

The longitudinal slip is then defined as :

$$\boxed{\lambda(\%) = 100.\frac{\omega - \omega_0}{\omega_0}}$$

Free wheel	$\omega=\omega_0$	$\lambda = 0\ \%$
Blocked wheel at braking	$\omega=0$ mais $\omega_0 \neq 0$	$\lambda = -100\ \%$
Skating wheel at start (burn)	$\omega \neq 0$ mais $\omega_0 = 0$	$\lambda = +\infty$

Imagine that Fm force is varying ground (By varying the torque applied to the wheel), and measure the slip of the wheel λ, a typical "S- shapped" curve in then obtained as follows:

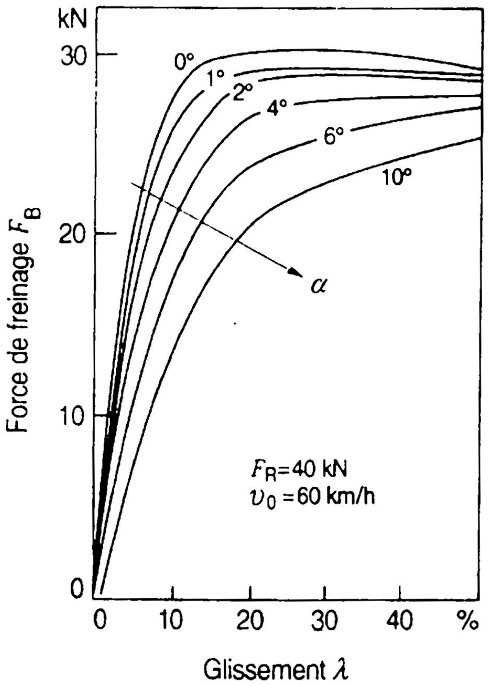


One notices a **linear zone** in which the longitudinal slip is proportional to the force applied to the ground.

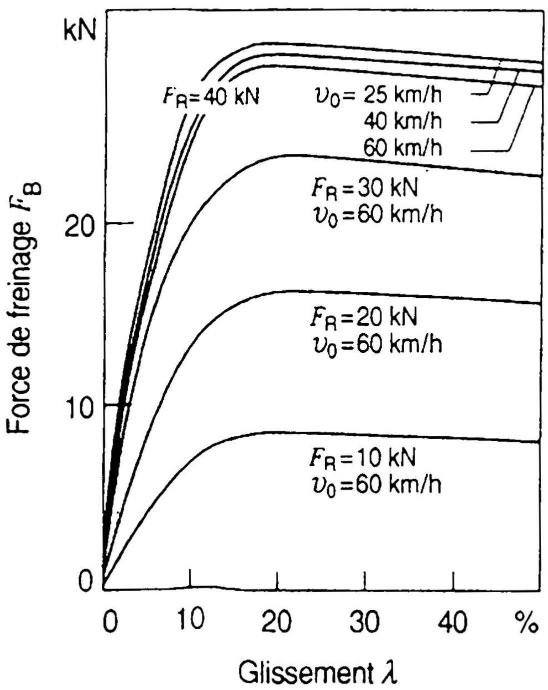
The maximum thrust (thus braking force or maximum acceleration force) is not obtained at blocking or skating conditions . It is therefore necessary to control slip for maximum performances. It is the function and purpose of the traction control and ABS.

In practice this curve is very strongly influenced by the vertical load P applied to the wheel, and moderately affected by the lateral slip angle and the forward speed of the wheel:

Force de freinage en fonction du glissement (figure I).
Paramètre : angle de dérive.



Force de freinage en fonction du glissement (figure E).
Paramètres : charge verticale de la roue et vitesse.



The higher the load applied to the wheel increases (P or Fr) , the higher the force that the tire can develop the ground (drag Fm) is important. It is experimentally observed that a quasi-linearity exists between the MAXIMUM thrust that can develops a given tire and the vertical load P that is applied to it:

$$Fm_{max} \text{ (N)} = \mu . P \text{ (N) or } \mu = \frac{Fm}{P}$$

- μ , is called the **coefficient of adhesion** of the tire. Caution: do not confuse the coefficient of adhesion with a coefficient of friction. The phenomena involved are different .

μ depends on tire (geometry , anatomy etc.) AND on the road where it evoluates :

Typical values for μ	
Dry Road	**0.8**
Wet Road	0.5 à 0.6
Worn and wet asphalt	0.4 à 0.5
Worn and wet asphalt with a thin layer of mud	0.2 à 0.3
Compacted snow	0.1 à 0.3
Ice	0.07 à 0.1

The fact that the tire slips relatively to the ground necessarily mean that the tire wears and that a power consumption exist. The energy consumed by the tire slip phenomenon is a loss. Thus the tire efficiency can be defined :

$$\boxed{\eta_{contact}(\%) = \frac{Pground(W)}{Pwheel(W)} = \frac{\omega_0}{\omega} \approx 1-\lambda}$$

With :

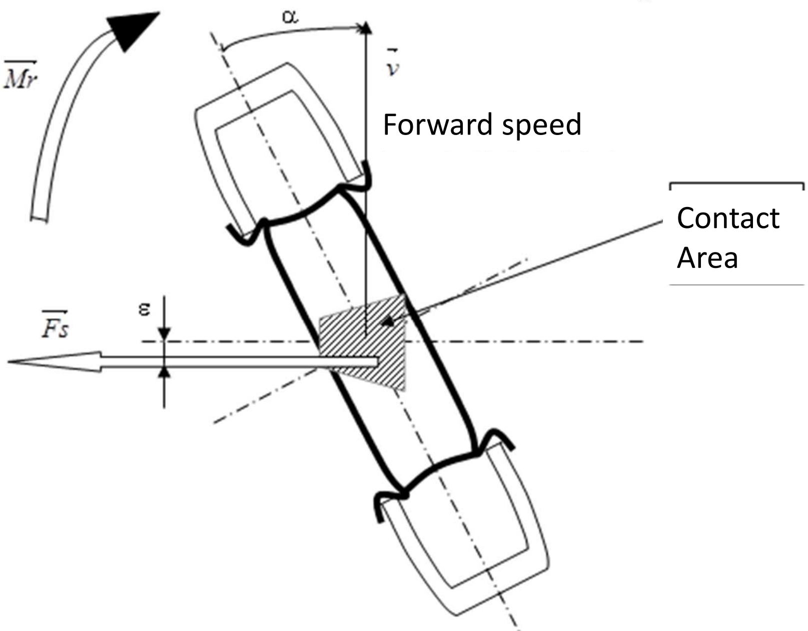
$$v\left(\frac{m}{s}\right) = \frac{Ld\left(\frac{m}{tr}\right)\omega_0\left(\frac{rad}{s}\right)}{2.\pi\left(\frac{rad}{tr}\right)}$$

$$Pground(W) = Fm(N).v\left(\frac{m}{s}\right)$$

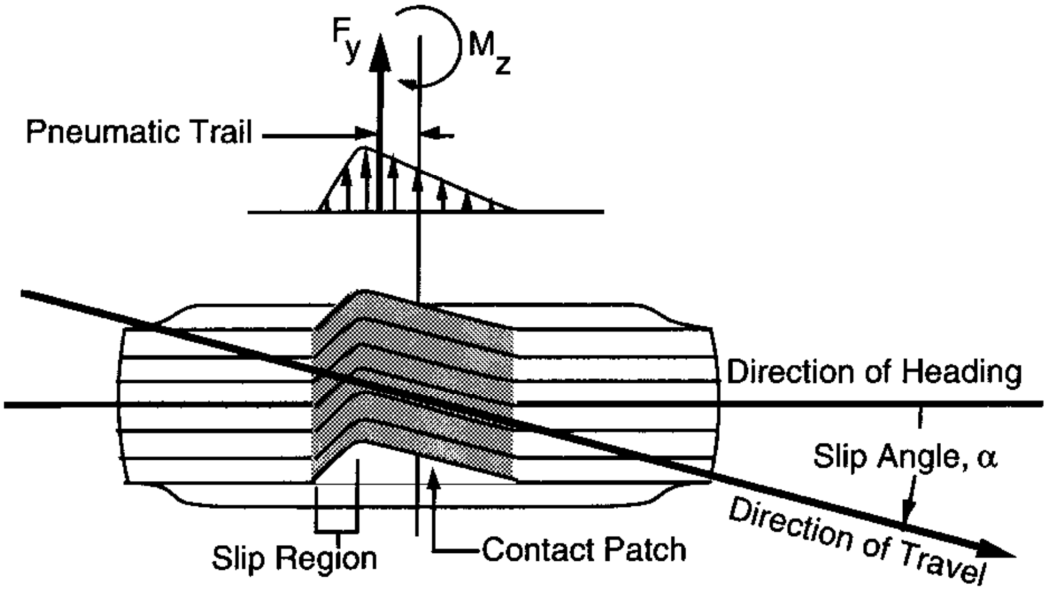
$$Pwheel(W) = Cm(N.m).w\left(\frac{rad}{s}\right)$$

2.3 Lateral Slip Phenomenon :

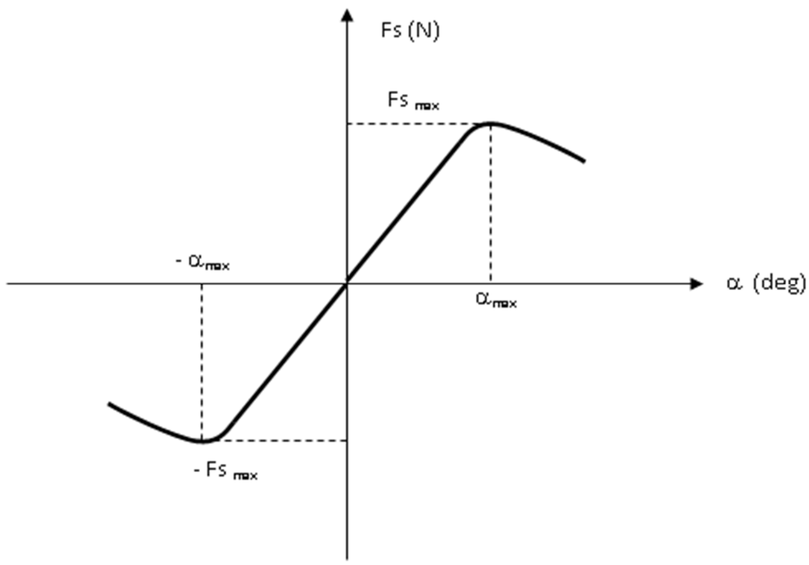
Imagine a riding tire, seen from above (XY plane) . Its direction of heading is inclined with an α angle from its path or direction or travel (vector velocity v). The tire will react to this "unnatural" situation by generating a lateral thrust on the vehicle so that it evolves in the right direction.



v (m/s)	Forward speed in the center of the contact area.
α (rad)	Slip Angle between the direction of heading and the direction of travel.
F_S (N)	Lateral force or slip force
ε (m)	Offset betwen the thrust center and the contact area center.
M_R (N.m)	Auto Aligment wheel Torque , with trivial relationship $\left\Vert\overrightarrow{M_R}\right\Vert = \left\Vert\overrightarrow{F_s}\right\Vert.\varepsilon$



Imagine that the slip angle is varying and that the lateral force is measured. Once again, a typical S shapped courves is obtained :

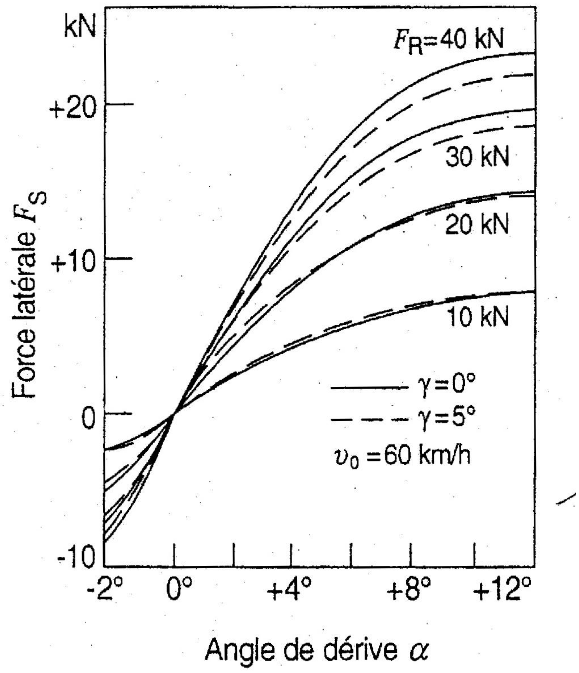


Alinear zone is observed during which the slip thrust is proportional to the slip angle. then passes through a maximum before getting a drop of slip thrust... It clearly appears that the normal range of use fir the tire tire is in the linear region, and that approaching the maximum can be rather dangerous ...
Inside the linear zone, it can be written :

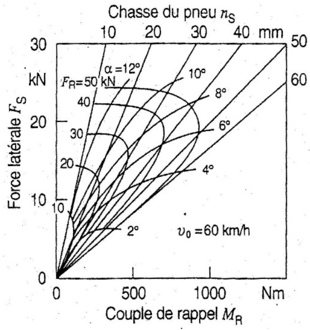
$$\boxed{Fs(N)=K\left(\frac{N}{\deg}\right).\alpha(\deg)}$$

K (deg/N) is defined as the cornering stiffness of the tire
(For a typical tire K evoluates between 1000 and 2500 N/deg.)

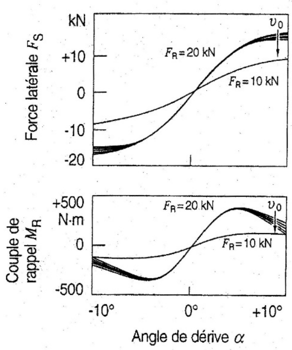
Force latérale en fonction de l'angle de dérive (figure C).
Paramètres : charge verticale de la roue et angle de carrossage.



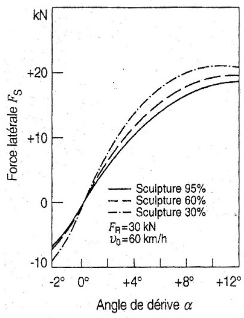
Cartographie de pneumatique selon Gough
(figure B).



***Force latérale et moment d'auto-alignement en fonction de l'angle de dérive** (figure A).*
Paramètre : vitesse
20 km/h ≤ v_0 ≤ 100 km/h.



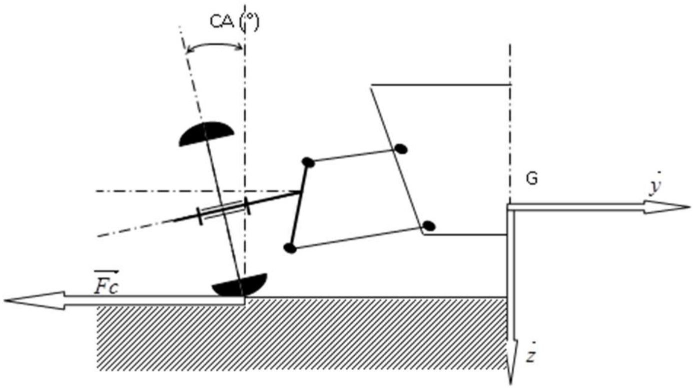
Force latérale en fonction de l'angle de dérive
(figure D).
Paramètre : état des sculptures.



The slip phenomenon tire is the one that allows the mass that constitutes a vehicle to cornering, by counteracting the centrifugal force. We will see that the cornering stiffness is a key point factor for the vehicle behavior during cornering.

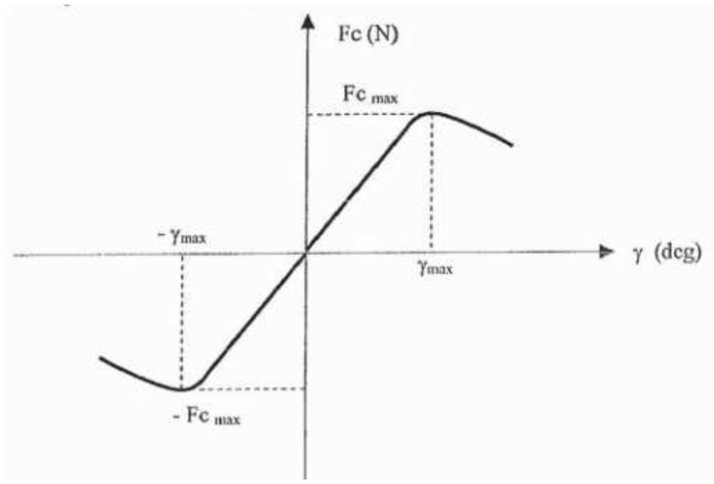
Variation of cornering stiffness K as function as main parameters .	
Vertical Load increases (Fr ou P) ⇒	K increases!
Inflate pressure increases ⇒	K increases!
Tire height decreases ⇒	K slightly increases
Tire width increases ⇒	K does not significantly increase
Wear increases⇒	K slightly increases

2.4 Camber Thrust :



Imagine a riding tire, in front view, therefore, in the YZ plane. The median plan of this tire is inclined with a ϒ from vertical . The tire will respond to this situation by creating a lateral force , called Camber thrust.

Imagine that the camber angle is varying while the lateral force is measured. Once again, a typical S shapped curve is obtained :

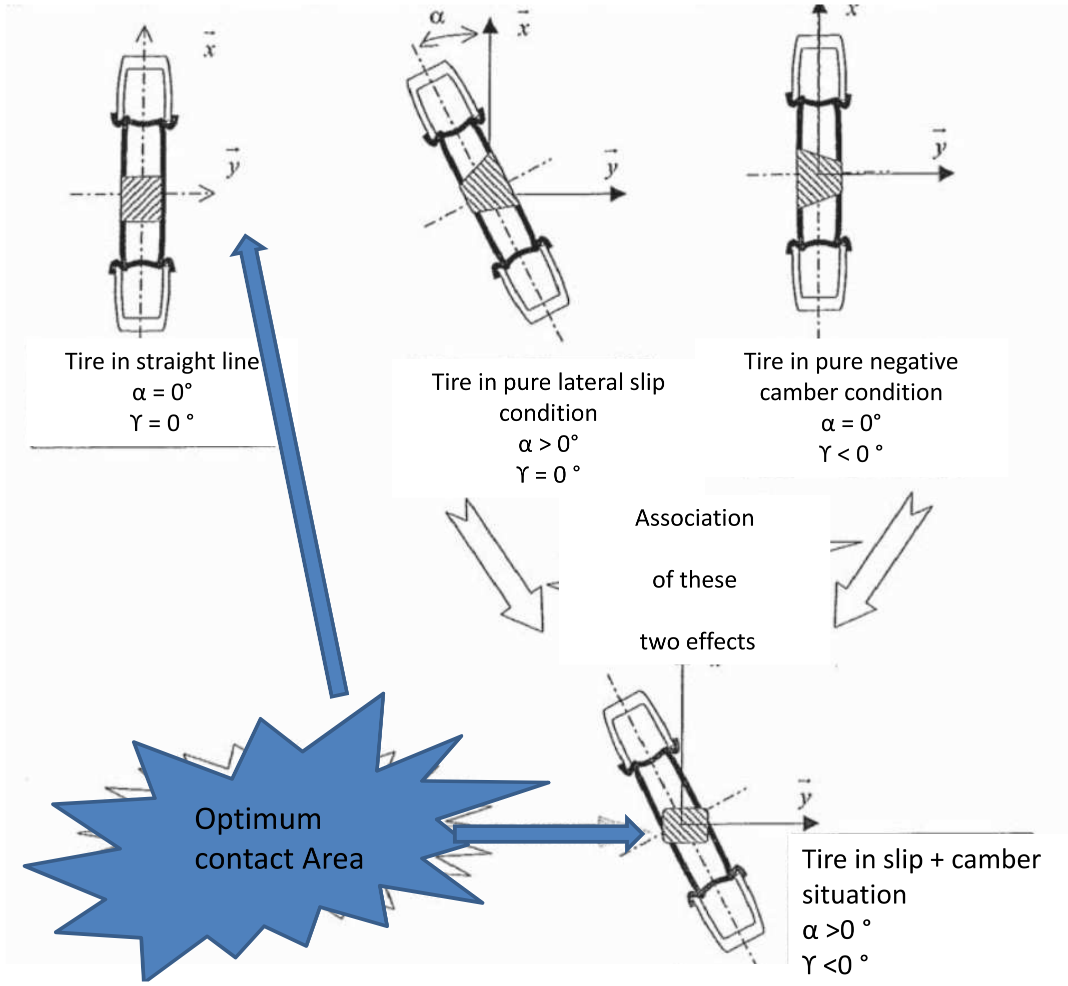


First, a linear zone can be observed. Inside this zone, the camber thrust is proportionnal to the camber angle. Next , a maximum is obtained before observing a decrease of the thrust. It clearly appears that the normal range of use fir the tire tire is in the linear region, and that approaching the maximum can be rather dangerous ...
Inside the linear zone, it can be written :

$$\boxed{Fc(N) = H\left(\frac{N}{\deg}\right).\gamma(\deg)}$$

H (deg/n) is called the camber stiffness.

On a car, the camber thrust contribution to the cornering is low , due to a low value of the camber angle. Nevertheless, controlling the camber is a key point in order to control the contact pressure between gum and road.


α
$\vec{x}$
$\vec{y}$
Tire in straight line
α = 0°
ϒ = 0 °
Tire in pure lateral slip condition
α > 0°
ϒ = 0 °
Tire in pure negative camber condition
α = 0°
ϒ < 0 °
Association
of these
two effects
Optimum contact Area
Tire in slip + camber situation
α >0 °
ϒ <0 °

The PACEJKA "Magic" Formula

P.BREJAUD- University of Orleans

10 octobre 2018

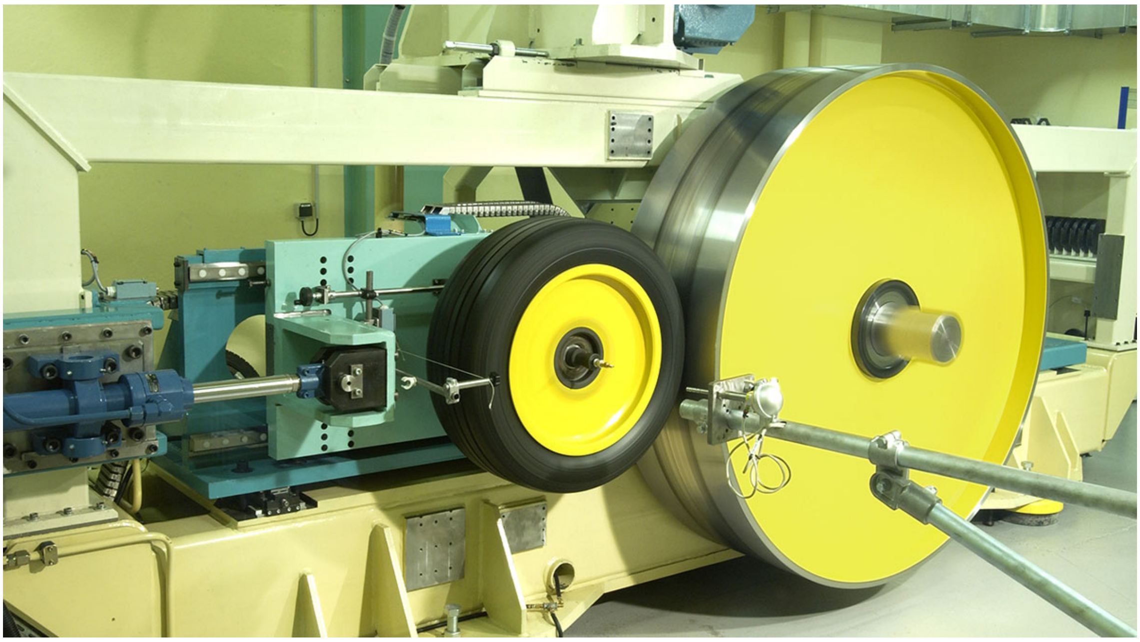


FIGURE 1 – Tire test bench

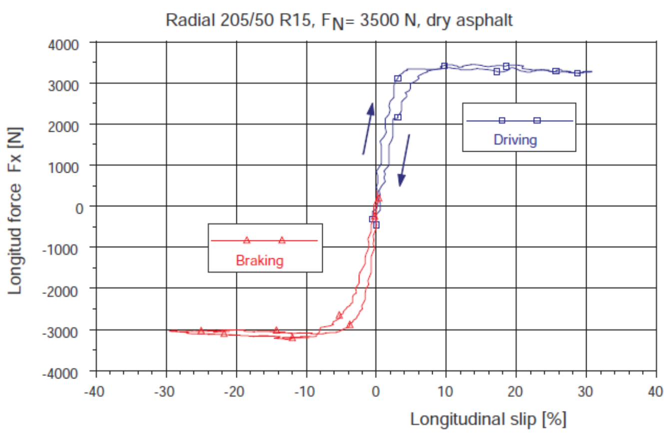


FIGURE 2 – Typical experimental result of a tire test

1 Introduction

The Pacejka model, sometimes called "magic formula" is an empirical model for describing the behavior of a tyre output variable (denoted Y) tire according to an input variable (denoted X). It is used by manufacturers to characterize a tire experimentally.

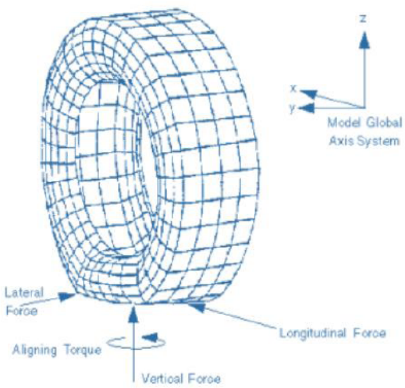


FIGURE 3 – Axis and Forces on a tire

This model can be applied to :

— Lateral slip force (F_y) as a function of the lateral slip angle α.
— Longitudinal force (F_x) as a function of the longitudinal slip λ.
— Aligning torque (M_z) as a function of the lateral slip angle α.

In its basic and original form, the model only depends on 4 parameters : B,C,D,E.

$$Y = D \sin\left[C \arctan(B\varphi)\right] \tag{1}$$

with,

$$\varphi = (1-E)x + \frac{E}{B}\arctan\left[Bx\right] \tag{2}$$

WARNING : In the original Pacejka form , the variable x must be expressed in **radian**.

2 Properties of the PACJEKA "magic" formula

The function derivative $Y = f(x)$ is given :

$$Y' = \frac{dY}{dx} = BCD\frac{(1-E)x + \frac{E}{1+B^2x^2}}{1 + B^2\left[(1-E)x + \frac{E}{B}\arctan(Bx))\right]^2}\cos\left[C\arctan(B\varphi)\right] \tag{3}$$

The derivative at the origin point is expressed as :

$$Y'(0) = \frac{dY}{dx}_{x=0} = BCD; \tag{4}$$

The function goes through its maximum when its derivative is zero $Y'(0) = 0$, what is verified when :

$$C\arctan(B\varphi) = \frac{\pi}{2}; \tag{5}$$

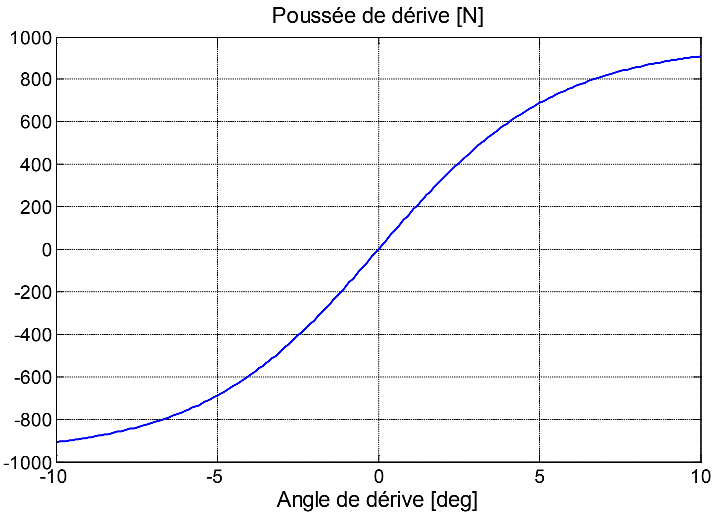


FIGURE 4 – Example of a PACEJKA Curve for B=5,C=2,D=1000,E=1.5

3 Influence of the parameters A,B,C,D

In this section, the influence of different parameters B, C, D and E on the behavior of the function is depicted . The reader of this document is highly invited to set the Pacejka function in a spreadsheet and conduct his own parametric study.

3.1 Influence of the D parameter

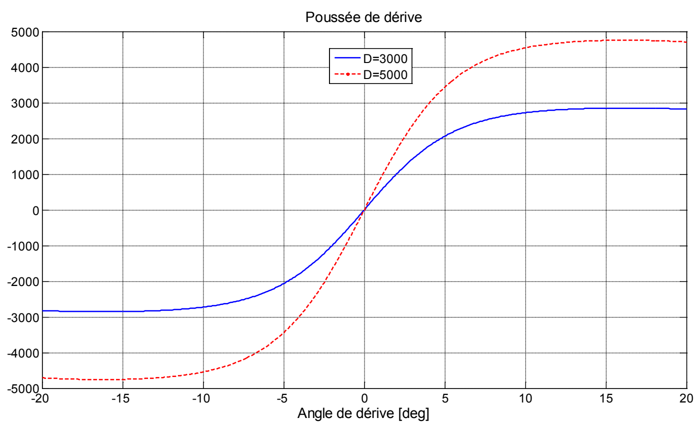


FIGURE 5 – Influence of the D parameter, with B=5,C=2,E=1.5

Equation 1 shows that the parameter D defines directly the maximum amplitude of the curve. Moreover Equation 4 shows that the parameter influences the slope of the curve. Figure 5 depicts these remarks.

3.2 Influence of the C parameter

According to Equation 4,the C parameter directly influences the slope at the origin . Moreover, Equation 5 shows that C changes the value of X that brings the Y function at its maximum. These remarks are illustrated on figure 6.

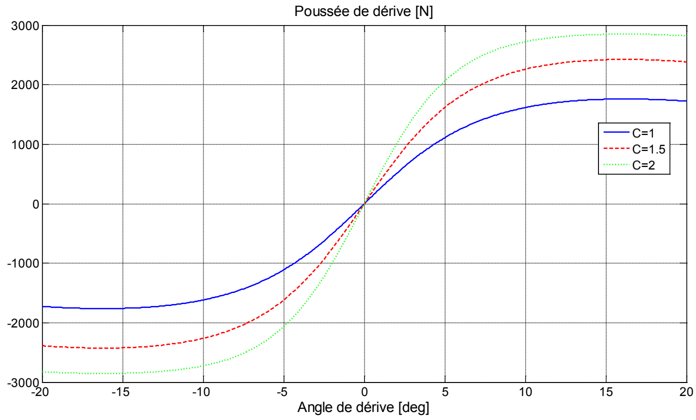


FIGURE 6 – Influence of the C parameter, with B=5,D=3000,E=1.5

3.3 Influence of the B parameter

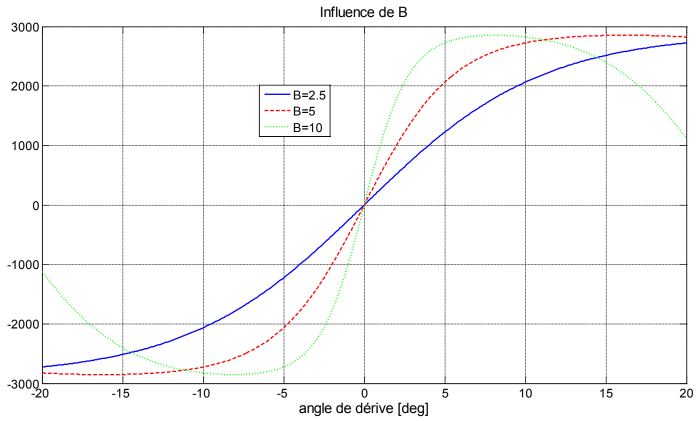


FIGURE 7 – Influence of the B parameter, with C=2,D=3000,E=1.5

See Figure 7. According to equation 4, the B parameter influences the slope at the origin.

3.4 Influence of the E parameter

See figure 8.The E parameter is called the shape coefficient at the top of the curve. It strongly influences the shape of the curve AFTER its maximum. However this area is outside of the range of use . From a practical point of view, it is convenient to simplify and approximate E=0. The model is then greatly simplified and becomes :

$$\varphi = x \tag{6}$$

donc,

$$Y = D \sin\left[C \arctan(Bx)\right] \tag{7}$$

4 First exercise

It is proposed to determine the value of the coefficient B, C, D, and E for the experimental lateral force versus slip angle curve, given in Figure 9 .

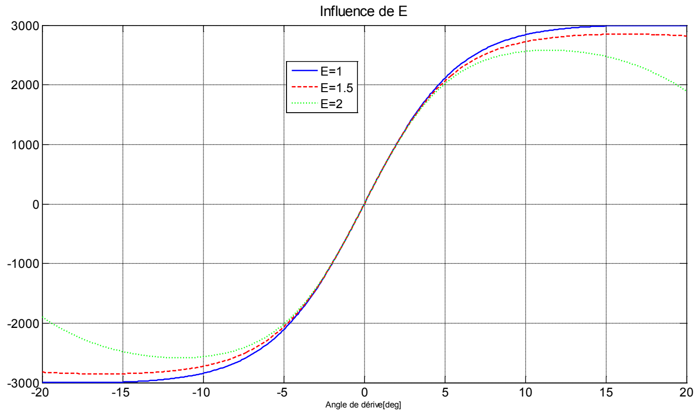


FIGURE 8 – Influence of the E parameter, with B=5,C=2,D=3000

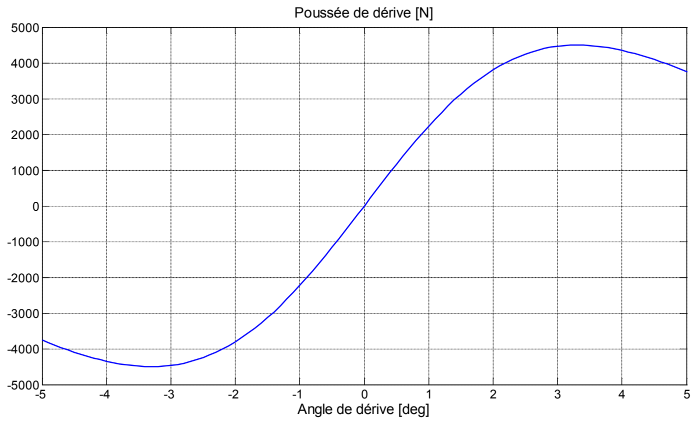


FIGURE 9 – Exercise

5 Second exercise

It is given below the experimental data of a tire***205/55/R16***. The datas are the Lateral Slip Force (Fy) as function of the lateral slip angle (α)in **degrees**, for 5 different vertical loads (Fz).

Import these values into excel, then using the solver tool, it is asked to find, the values of the coefficients B, C, D and E, for each load Fz, which minimize the sum of the squared deviations between the theoretical curve and the experimental data. Then trace the evolutions of the coefficients B, C, D and E according to the vertical load Fz. Conclusion ?

6 Manufacturers modified formula

Over time, the manufacturers have enriched the basic formula from Pacejka, to take into account more parameters, for instance the vertical load Fz and the camber angle γ. It is developed here, the most common formula, called "PACEJKA 96".

6.1 Lateral Force

It is presented here the model giving the Lateral force Fy in kN as a function of lateral slip angle α expressed in degrees. This formulation includes the influence of the vertical load carried by the tire, denoted

Fz[N]	1725	3500	6100	6950	9005
Alpha[deg]	Fy[N]	Fy[N]	Fy[N]	Fy[N]	Fy[N]
0	0	0	0	0	0
0.5	444.357	894.186	1293.54	1601.93	1784.39
1	843.546	1694.5	2452.99	3047.78	3417.9
1.5	1162.09	2327.41	3373.13	4214.48	4780.29
2	1403.45	2800.6	4064.18	5111.02	5873.77
2.5	1580.72	3142.1	4565.24	5779.43	6731.16
3	1707.08	3379.93	4915.41	6261.69	7385.32
3.5	1795.66	3542.12	5153.78	6599.82	7869.06
4	1859.6	3656.67	5319.43	6835.85	8215.23
4.5	1893.83	3713.24	5402.64	6968.31	8439.3
5	1904.16	3724.46	5422.23	7019.99	8564.79
5.5	1896.19	3701.92	5395.45	7011.91	8613.61
6	1875.03	3656.45	5337.38	6962.7	8605.54
6.5	1845.3	3597	5261.2	6887.9	8557.9
7	1810.95	3531.15	5177.14	6800.2	8485.17
7.5	1775.12	3464.91	5092.89	6709.15	8398.94
8	1740.18	3402.12	5013.56	6621.16	8308.19
8.5	1707.8	3345.23	4942.1	6540.06	8219.02
9	1678.67	3295.32	4879.5	6467.91	8135.36
9.5	1653.27	3252.61	4825.96	6405.21	8059.17
10	1631.62	3216.48	4780.78	6351.47	7991.56
12	1574.53	3123.37	4662.96	6207.67	7798.93
14	1546.58	3078	4604.6	6133.95	7693.72
16	1531.91	3054.18	4573.3	6093.89	7634.76
18	1523.64	3040.4	4554.99	6070.17	7599.62
20	1523.64	3040.4	4554.99	6070.17	7599.62

TABLE 1 – Experimental data

Z in kN, the camber angle γ in degree, and of course the lateral slip angle α in degree.
Please note that the model takes into account the hysteresis phenomenon observed experimentally through two 'offset offsets' SV and SH.

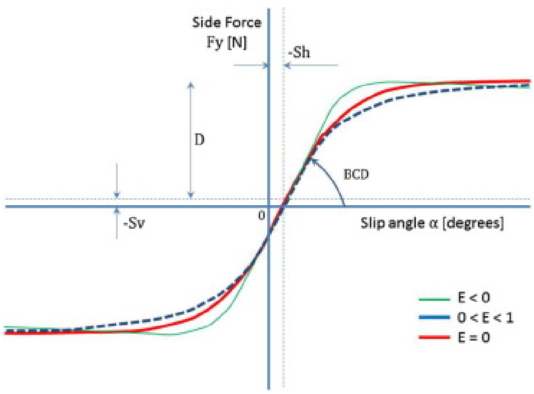


FIGURE 10 – SH and SV parameters

The following equations are used to compute the tire response :

Shape factor	(no unit)	a0
Load influence on lat. friction coeff (*1000)	(1/kN)	a1
Lateral friction coefficient at load = 0 (*1000)	no unit)	a2
Maximum stiffness	(N/deg)	a3
Load at maximum stiffness	(kN)	a4
Camber influence on stiffness	(% /deg/100)	a5
Curvature change with load	(no unit)	a6
Curvature at load = 0	(no unit)	a7
Horizontal shift because of camber	(deg/deg)	a8
Load influence on horizontal shift	(deg/kN)	a9
Horizontal shift at load = 0	(deg)	a10
Camber influence on vertical shift	(N/deg/kN)	a111
Camber influence on vertical shift	(N/deg/kN**2)	a112
Load influence on vertical shift	(N/kN)	a12
Vertical shift at load = 0	(N)	a13

TABLE 2 – Lateral Force

a0	1.3
a1	-49
a2	1216
a3	1632
a4	11
a5	0.006
a6	-0.04
a7	-0.4
a8	0.003
a9	-0.002
a10	0.16
a111	-11
a112	0.045
a12	0.17
a13	-23.5

TABLE 3 – 195/60 HR 15 on 6"1/2 rim at 2.1 bar

$$a_0 = C \tag{8}$$
$$SV_0 = a_{12}Z + a_{13} \tag{9}$$
$$SV = SV_0 + (a_{112}Z^2 + a_{111}Z)\gamma \tag{10}$$
$$SH_0 = a_9 Z + a_{10} \tag{11}$$
$$SH = SH_0 + a_8\gamma \tag{12}$$
$$E = a_6 Z + a_7 \tag{13}$$
$$BCD_0 = a_3(2\arctan\left[\frac{Z}{a_4}\right]) \tag{14}$$
$$BCD = BCD_0(1 - a_5|\gamma|) \tag{15}$$
$$D = a_1 Z^2 + a_2 Z \tag{16}$$
$$\varphi = (1-E)(\alpha + SH) + \frac{E}{B}\arctan\left[B(\alpha + SH)\right] \tag{17}$$
$$Fy = SV + D\sin\left[C\arctan(B\varphi)\right] \tag{18}$$

6.2 Longitudinal force

It is presented here the model giving the longitudinal force Fx in kN as a function of the longitudinal slip λ expressed in %. This formulation includes the influence of the load carried by the tire, denoted Z in kN. One more time, the model takes into account hysteresis through two 'offset offsets' of the curve noted SV and SH.

The following equations are used to compute the tire response :

Shape factor	(no unit)	b0
Load infl. on long. friction coeff (*1000)	(1/kN)	b1
Longitudinal friction coefficient at load = 0 (*1000)	(no unit)	b2
Curvature factor of stiffness	(N/%/kN**2)	b3
Change of stiffness with load at load = 0	(N/%/kN)	b4
Change of progressivity of stiffness/load	(1/kN)	b5
Curvature change with load	(no unit)	b6
Curvature change with load	(no unit)	b7
Curvature at load = 0	(no unit)	b8
Load influence on horizontal shift	(%/kN)	b9
Horizontal shift at load = 0	(%)	b10
Load influence on vertical shift	(N/kN)	b11
Vertical shift at load = 0	(N)	b12

TABLE 4 – Longitudinal Force

b0	1.57
b1	-48
b2	1338
b3	6.8
b4	444
b5	0
b6	0.0034
b7	-0.008
b8	0.66
b9	0
b10	0
b11	0
b12	0

TABLE 5 – 195/60 HR 15 on 6"1/2 rim at 2.1 bar

$$D = b1.Z^2 + b2.Z \tag{19}$$
$$BCD = (b3.Z^2 + b4.Z)/\exp(b5.Z) \tag{20}$$
$$b_0 = C \tag{21}$$
$$B = BCD/(CD) \tag{22}$$
$$E = b_6Z^2 + b_7Z + b_8 \tag{23}$$
$$SH = b_9Z + b_{10} \tag{24}$$
$$SV = b_{11}Z + b_{12} \tag{25}$$
$$\varphi = (1-E)(\lambda + SH) + \frac{E}{B}\arctan\left[B(\lambda + SH)\right] \tag{26}$$
$$Fx = SV + D\sin\left[C\arctan(B\varphi)\right] \tag{27}$$

6.3 Aligning moment

It is presented here the model giving the aligning moment Mz in Nm as a function of laterak slip angle α expressed in degrees. This formulation includes the influence of the vertical load carried by the tire, denoted Z in kN and camber angle γ in degree. One more time, the model takes into account hysteresis through two 'offset offsets' of the curve noted SV and SH.

Shape factor	(no unit)	c0
Load influence of peak value	(Nm/kN**2)	c1
Load influence of peak value	(Nm/kN)	c2
Curvature factor of stiffness	(Nm/deg/kN**2)	c3
Change of stiffness with load at load = 0	(Nm/deg/kN)	c4
Change of progressivity of stiffness/load	(1/kN)	c5
Camber influence on stiffness	(%/deg/100)	c6
Curvature change with load	(no unit)	c7
Curvature change with load	(no unit)	c8
Curvature at load = 0	(no unit)	c9
Camber influence of stiffness	(no unit)	c10
Camber influence on horizontal shift	(deg/deg)	c11
Load influence on horizontal shift	(deg/kN)	c12
Horizontal shift at load = 0	(deg)	c13
Camber influence on vertical shift	(Nm/deg/kN**2)	c14
Camber influence on vertical shift	(Nm/deg/kN)	c15
Load influence on vertical shift	(Nm/kN)	c16
Vertical shift at load = 0	(Nm)	c17

TABLE 6 – Lateral Force

c0	2.46
c1	-2.77
c2	-2.9
c3	-0
c4	-3.6
c5	-0.1
c6	0.0004
c7	0.22
c8	-2.31
c9	3.87
c10	0.0007
c11	-0.05
c12	-0.006
c13	0.33
c14	-0.04
c15	-0.4
c16	0.092
c17	0.0114

TABLE 7 – 195/60 HR 15 on 6"1/2 rim at 2.1 bar

The following equations are used to compute the tire response :

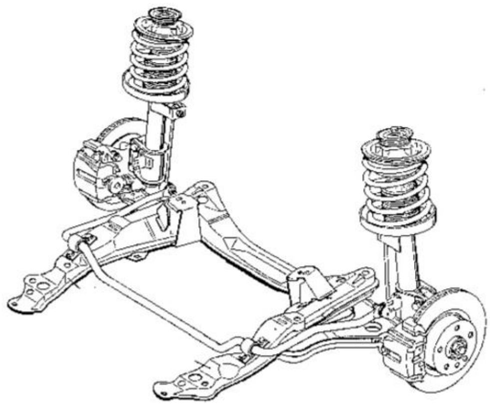
$$
\begin{aligned}
D &= c1.Z^2 + c2.Z & (28)\\
BCD_0 &= (c1.Z^2) + c4.Z/\exp(c5.Z) & (29)\\
BCD &= BCD_0(1 - c_6|\gamma|) & (30)\\
c_0 &= C & (31)\\
B &= BCD/(CD) & (32)\\
E_0 &= c_7Z^2 + c_3Z + c_{13} & (33)\\
E &= E_0(1 - c_{10}|\gamma|) & (34)\\
SH_0 &= c_{10}Z + c_{13} & (35)\\
SH &= SH_0 + c_{11}\gamma & (36)\\
SV_0 &= c_{16}Z + c_{17} & (37)\\
SV &= SV_0 + (c_{14}Z^2 + c_{15}Z)\gamma & (38)\\
\varphi &= (1 - E)(\alpha + SH) + \frac{E}{B}\arctan\left[B(\alpha + SH)\right] & (39)\\
Y &= SV + D\sin\left[C\arctan(B\varphi)\right] & (40)
\end{aligned}
$$

7 Third exercise

On spreadsheet (type Excel) draw the curves of tire response (Lateral Force, Longitudinal force and Aligning Moment), for three different loads of 100, 200 and 300 Kg.

Chapter 3
-
Axles

1 –Basics and Definitions :



An axle is the set of components (**unsprung mass**) that connect the wheel to the chassis (**srung mass**) and allow its guidance. The suspension elements (shock absorber, springs, transmission shafts etc.) do not form the axles except in special cases (ex MacPherson), but are in practice inseparable.

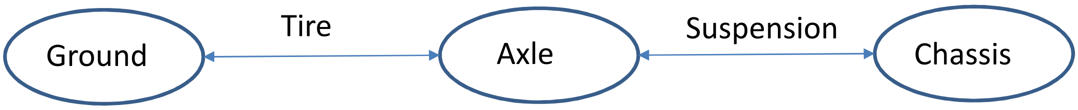
The technical performance of an axle is judged in two areas:

- Filtering
- Guiding

The filtering represents the ability of an axle to transmit a vibratory (or acoustic) signal of amplitude less than that generated at the tire / ground interface to the chassis. This function is provided by three elements:

- The tire
- The suspension elements
- The elastic filter elements of the axle, called "SILENT-BLOC“

Guidance is the ability of the axle to provide the vehicle a STABLE , SAFE and EFFICIENT behavior. The axle achieves the tire / chassis interface, ie its main function is to correctly place the tire despite the inevitable movements of the body due to the filtering and roll effect,



/!\ Tire and Suspension are IDEALLY not part of the Axle /!\

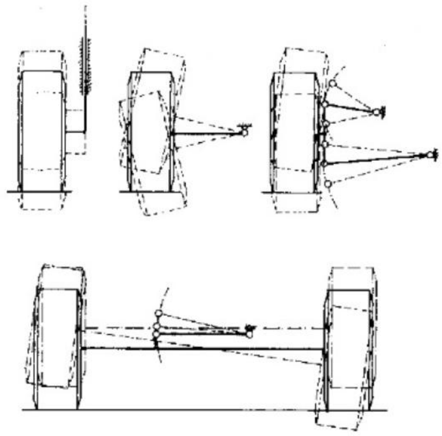
The guidance performance is judged according to two main criteria:

In a **straight line**, the trajectory of the vehicle must not be disturbed by the profile of the road and lateral winds. Tires should wear as little as possible.

When **cornering**, the vehicle must be positioned as quickly as possible, stabilize and exit as quickly as possible, with no tail heads (regardless of speed, vehicle load), without significant tire wear, Roll and respond to the abrupt movements of a miserable driver.

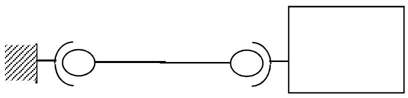
2 –Kinematic Analysis of an Axle :

To ensure the filtering function, each wheel must have at least 1 degree of freedom (ddl) enabling the wheel to pump. If the axle is rigid two degrees of freedom are required (roll + pump) so that each wheel has its own degree od freedom

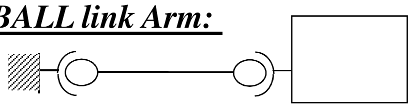


Obviously, for the steering wheels, each wheel must have two degrees of freedom so that the steering movement exists . **When the analysis of an axle is made, it is considered that the steering wheel is blocked so the degree of freedom of steering is lost.**

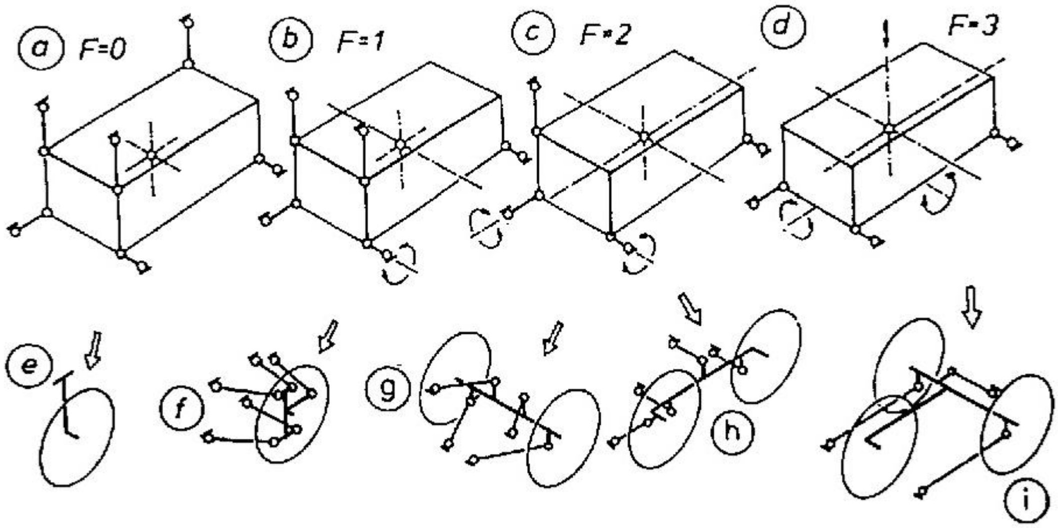
The hub carrier, as any solid in space, has 6 dofs (3 translations + 3 rotations). To eliminate ONE dof, basicaly a simple connecting rod link can be used.



Connecting Rods or Pivot BALL link Arm:

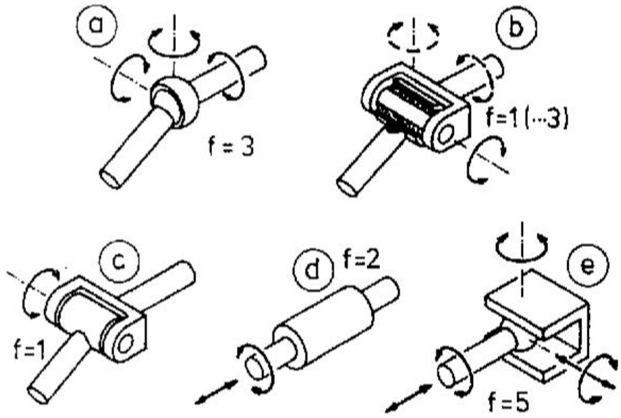


These are bars with ball joints at their ends. Each link removes 1 degree of freedom, precisely the translation along their longitudinal axis. With six links, it is therefore possible to completely immobilize a solid provided that none of the axes of the connecting rods intersect each other, the system then becoming unstable.



Classical Kinematic Joints

(a) BALL JOINT 3dof, (c) turning joint 1 dof, (d) turning-and-sliding joint 2 dof , (e) Ball and Socket joint 5 dof and (b) RUBBER JOINT (silent bloc)(b) 1 dof + 2 dof « elastic » low amplitude

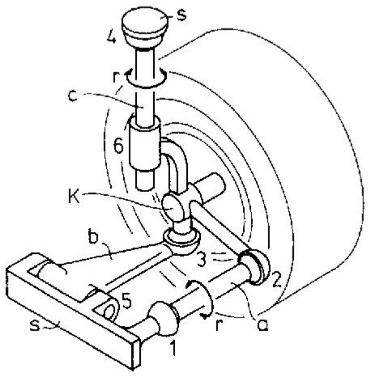


By associating these joints with connecting rods , it will be possible to obtain an almost infinite number of axles. In the general case, the theory of mechanisms teaches us that the degree of freedom of an axle will be given by the following relation

$$F = 6.(k+l-g) - r + \sum_{1}^{g} f_i$$

k : number of independant hub carrier that constituting the axle
l : Number of solid excepted Hub Carrier and Chassis
g : number of kinematic joint
r : Number of individual rod rotation without effects on the hub carrier
f_i : number of dof for the kinematic joint i

<u>Example 1 :</u> Mac Pherson strut



k=1 , one hub carrier noted K

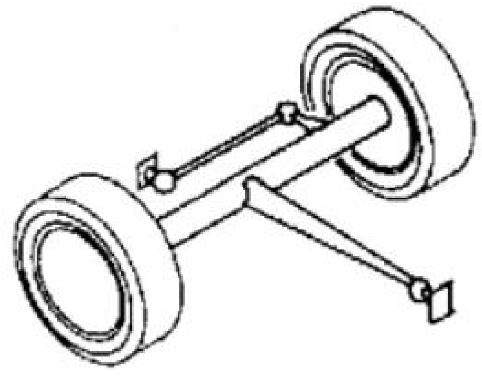
1 connecting rod (a)	2 ball joints at ends(1) and (2) ; f=3
1 triangle (b)	1 turning joint (5) f=1
	1 ball joint (3) f=3
1 shock absorber (c)	1 ball joint (4) f=3
	1 turning-and-sliding joint(6) f=2
Total : **l = 3 solids**	Total : **g=6 joints**

r=2 because both connecting rods (a) and tube of the shock absorber (c) can turn along their own axis without any change

Finally

F = 6*(1+3-6)- 2 +3 +3 +1 +3 +3 +2 = -12 – 2 + 15 = 1 (pomping / bouncing)

<u>Exemple 2 :</u> Rigid Axle with a « panhard » bar .



k= 1 Because the axle is rigid, the two wheels rotate around the axle directly.

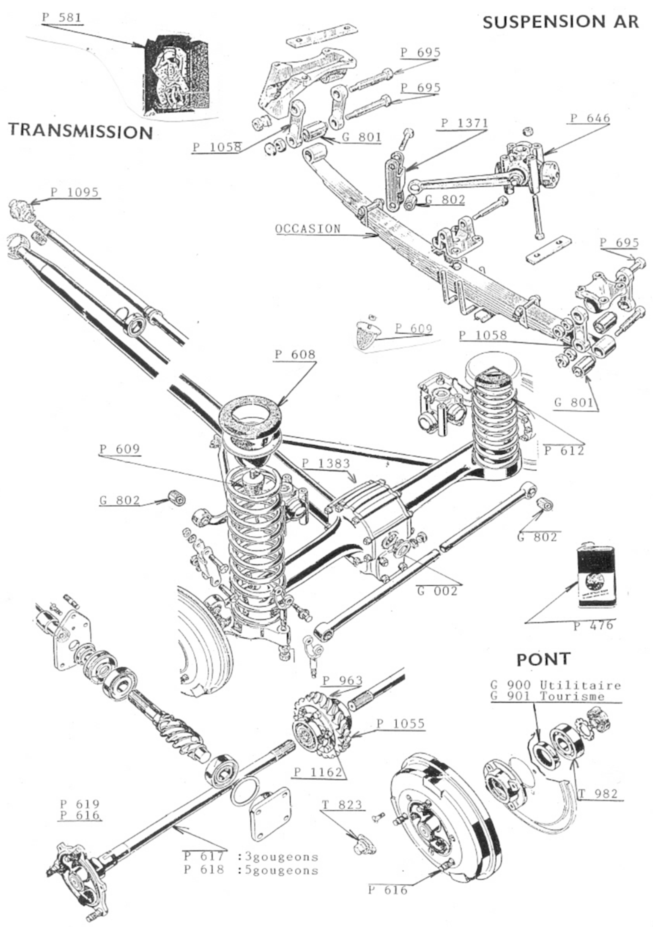
l =1 , one solid : the « panhard » bar

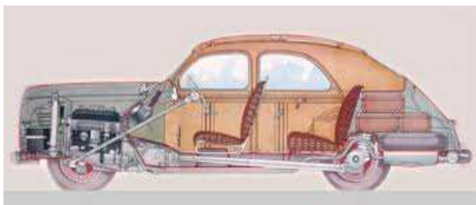
g=3 three ball joints (2 at ends of the panhard bar and one at the end of the thrust tube.

r=1 , rotation along the « panhard » bar axis, without any effect on the system

Thus :

F= 6*(1+1-3)-1+3+3+3 = 2 , Which is normal because there are two wheels! (Pumping + rolling)

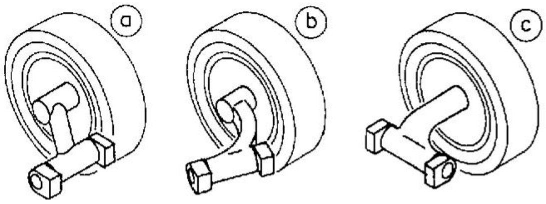




3 –Basic types of wheel suspensions :

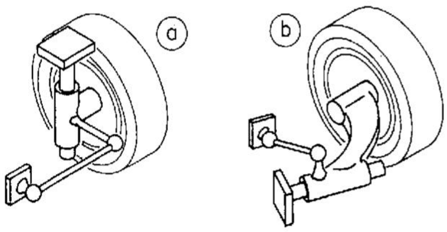
Now that we have figured out how to analyze an axle, let's investigate the main families of existing solutions to date. Obviously the list is not exhaustive because the variants are almost infinite.

a) *Suspension based on a turning joint :*

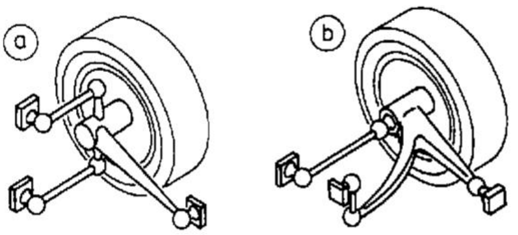


Remark: it is in this family that we will classify the rear trains with pulled arms or H frame swing axle , cases (a) and (b)

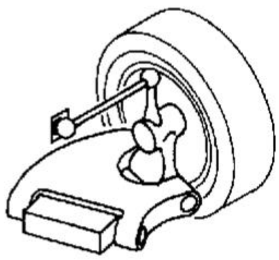
b) *Suspension based on a turning-and-sliding joint joint :*



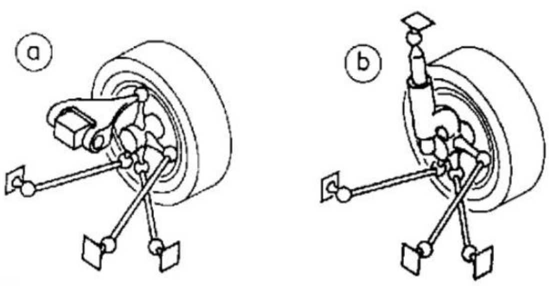
c) *'Spherical ' suspension mechanism*



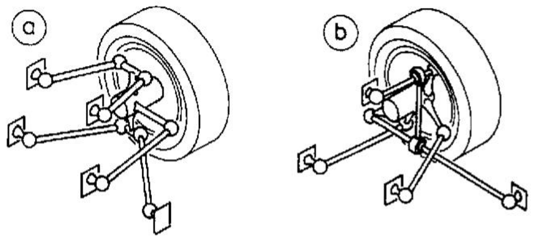
d) *Trapézoïdal Link:*



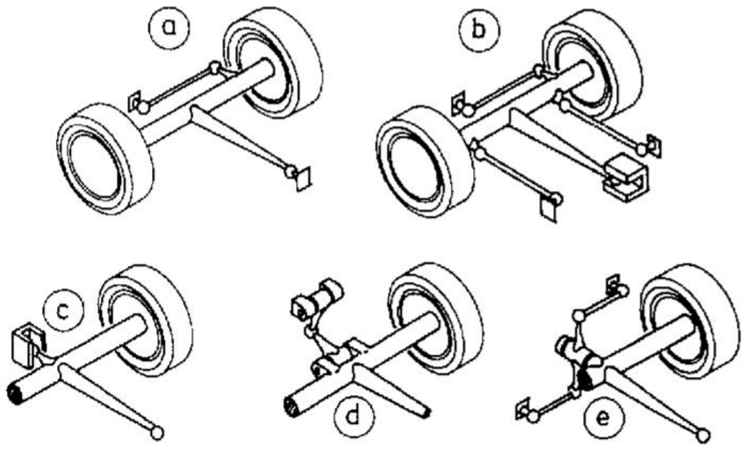
e) *Four Link Suspension*



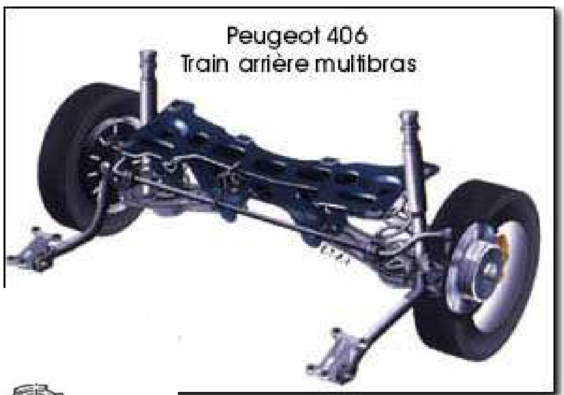
g) <u>Five Links :</u>

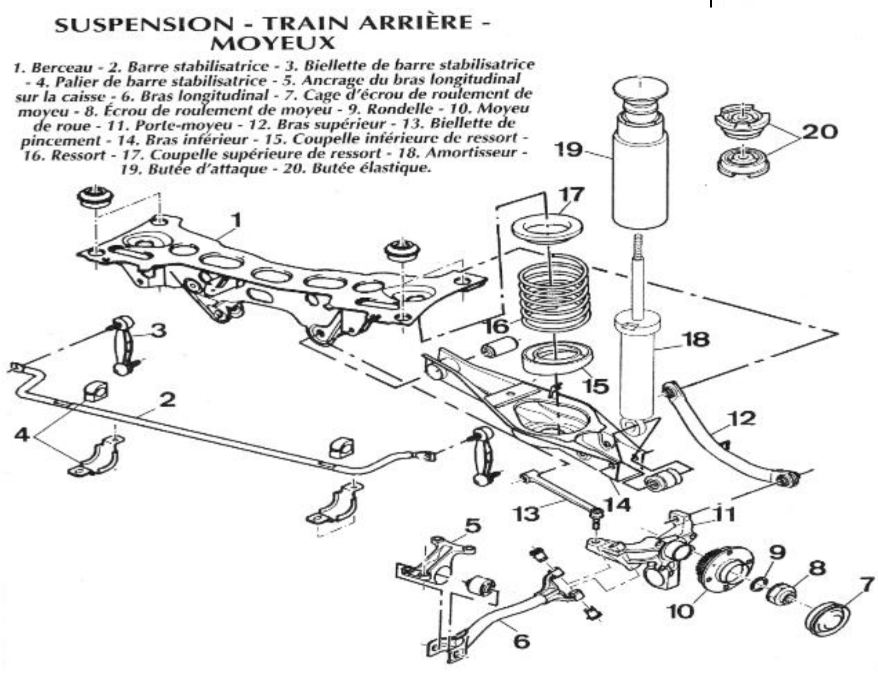


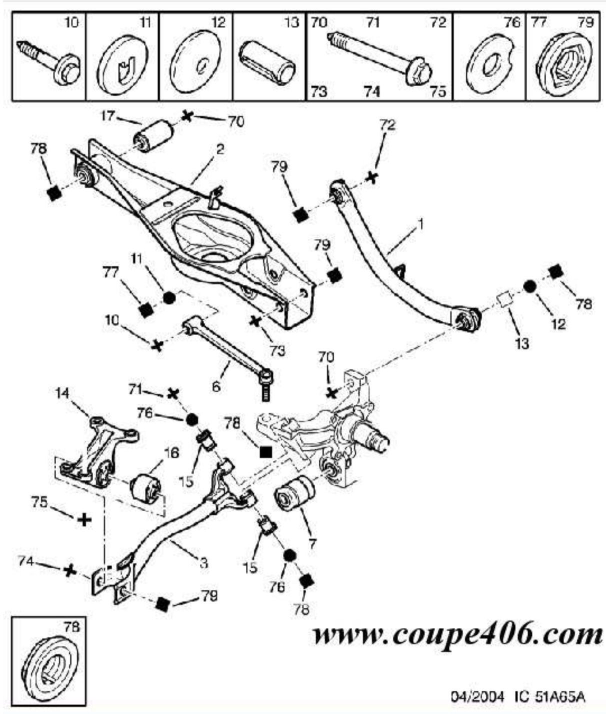
h) <u>Rigid Axle:</u>



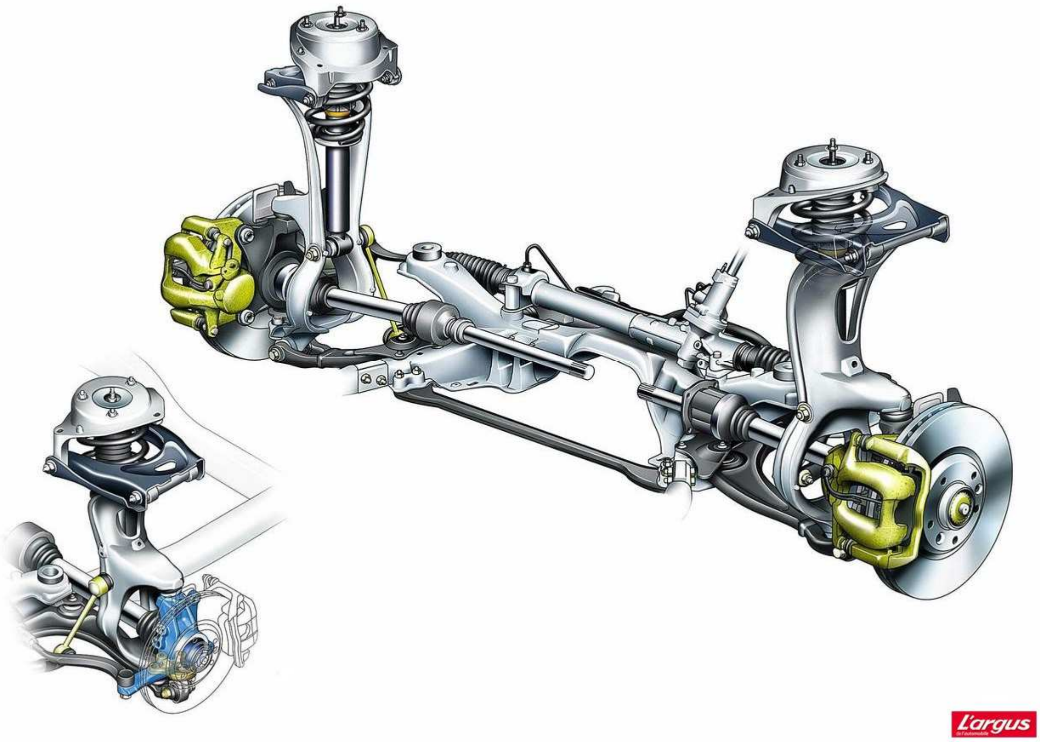
Exercice 1 : Rear Axle Peugeot 406

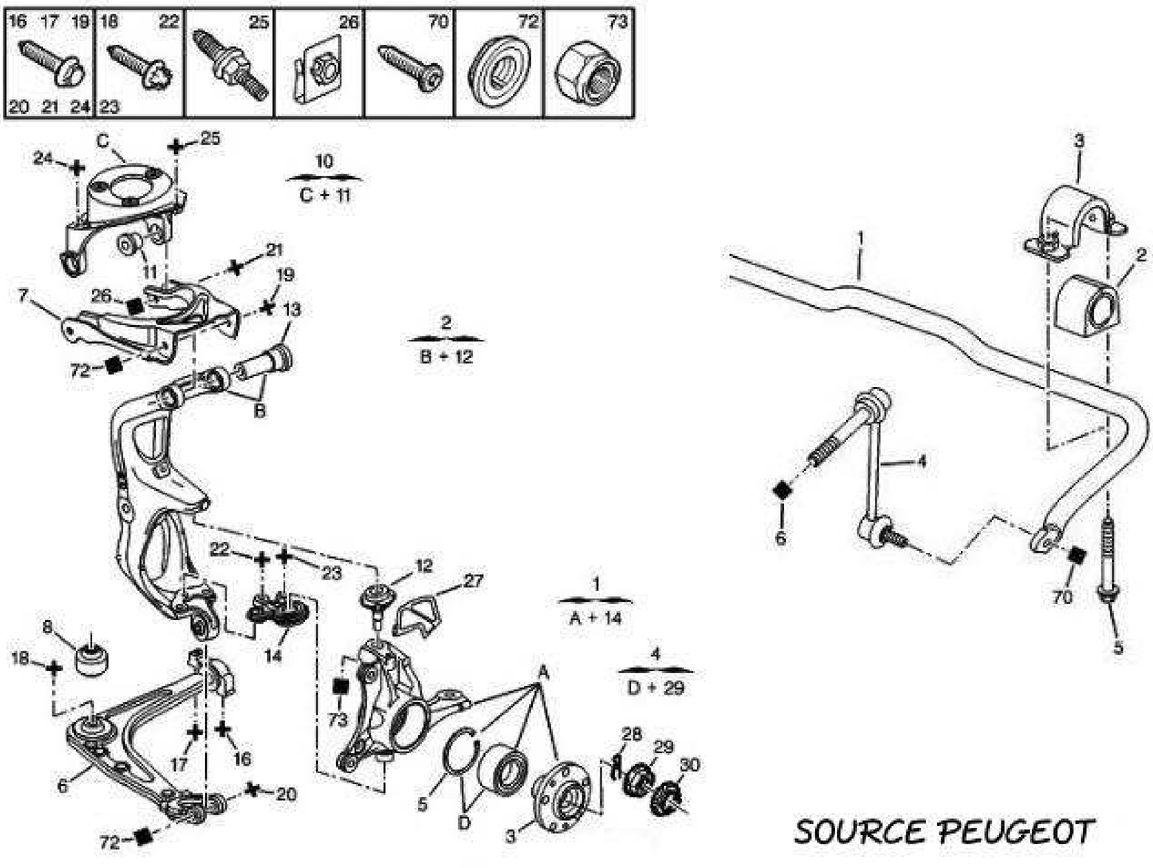






Exercice 2 : Front axle Peugeot 407

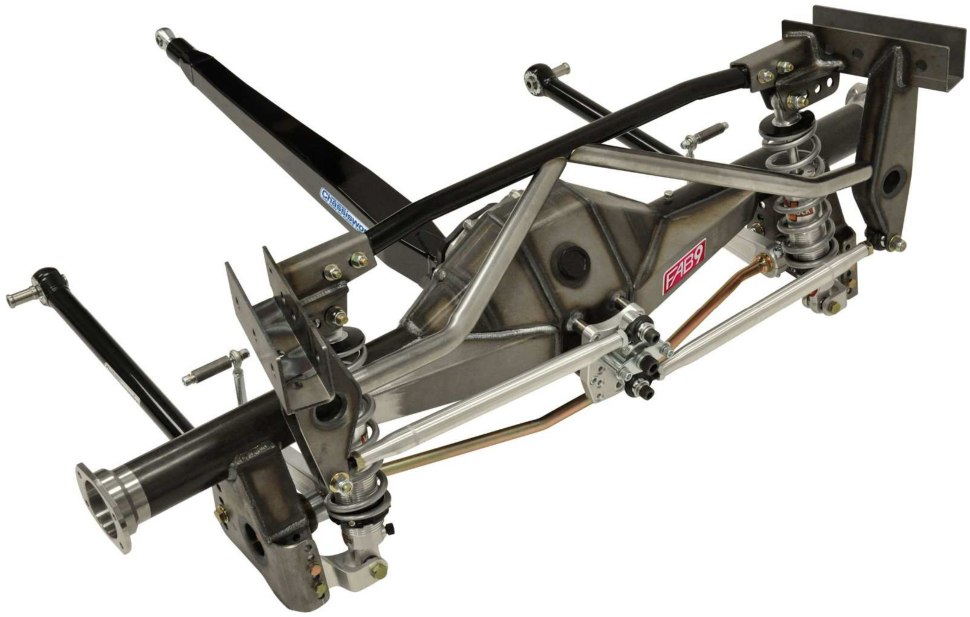




SOURCE PEUGEOT

Exercice 3 : Rear axle based on a Watt Bar (USA)

http://www.pro-touring.com/threads/116066-New-Chassisworks-Torque-Arm-Rear-Suspension



Chapter 3 bis
-
ROLL CENTER

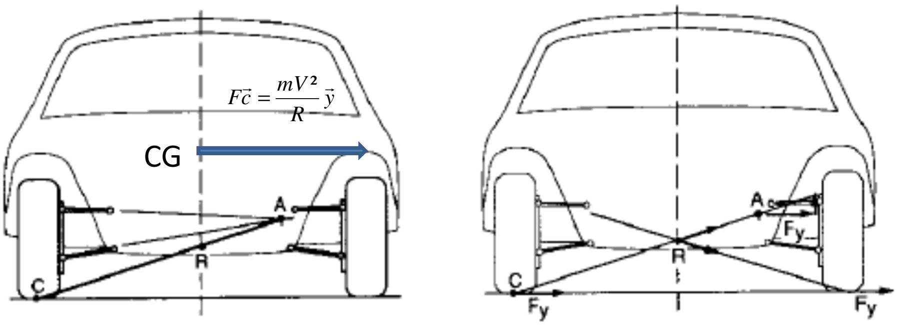


Fig. 7.19 Roll center analysis of an independent suspension.

From an ideal point of view, it is needed that distance between the Center of gravity (CG) and the Roll Center (R) to be as short as possible so that the torque created by the Centrifugal force is minimum and finally the Roll Angle is minimum too.

Moreover, it is needed in order to minimize the lateral load transfer from the inside wheel to the outside wheel that the distance between CG and the ground is minimum.

Thus finally, an IDEALIZED CAR supposes that CG and R are both located at ground level.

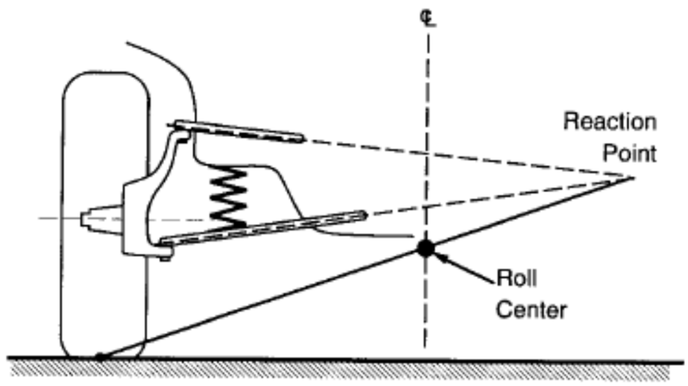


Fig. 7.20 Positive swing arm independent suspension.

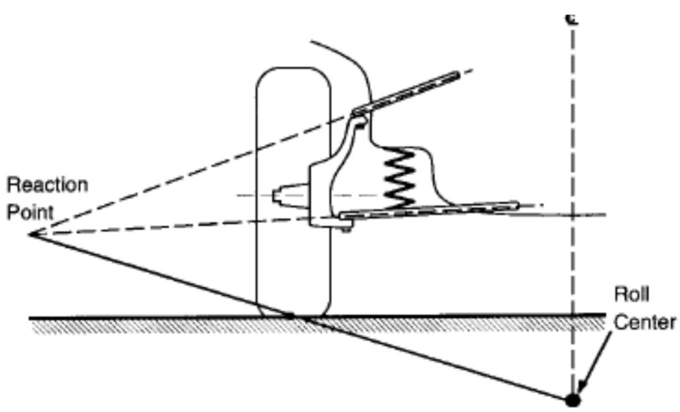


Fig. 7.21 Negative swing arm independent suspension.

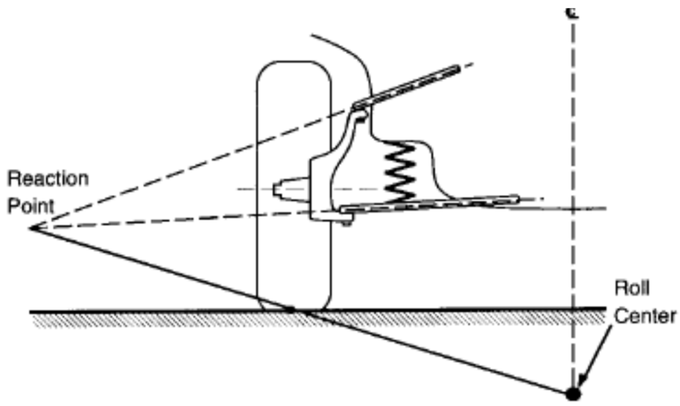


Fig. 7.21 Negative swing arm independent suspension.

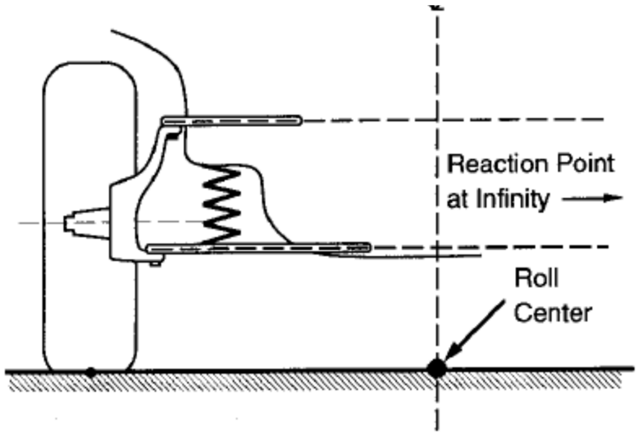


Fig. 7.22 Parallel horizontal link independent suspension.

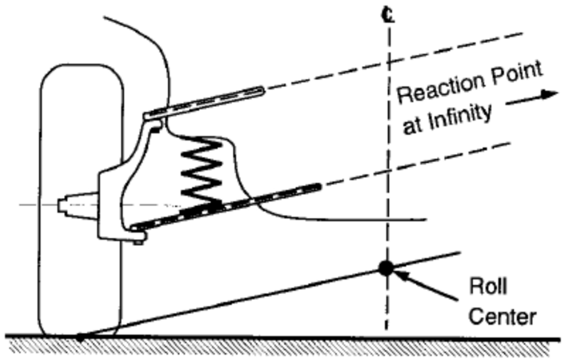


Fig. 7.23 Inclined parallel link independent suspension.

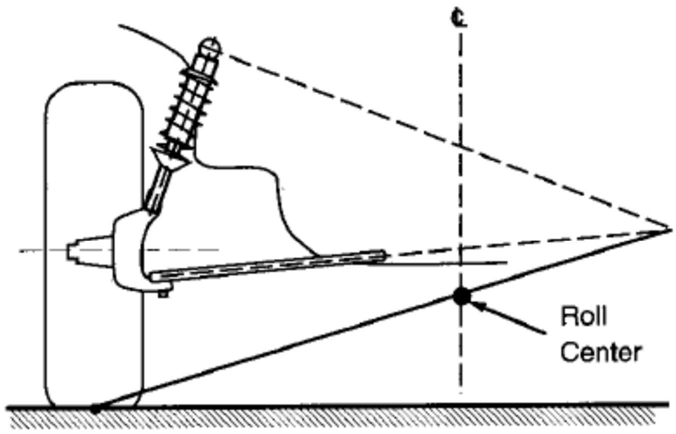


Fig. 7.24 MacPherson strut independent suspension.

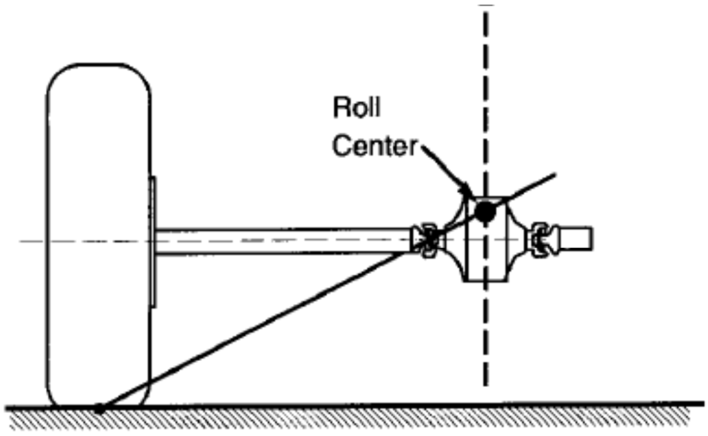


Fig. 7.25 Swing axle independent suspension.

Chapitre 4

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Suspension and damping

1- Simple Oscillator :

a) Undamped Spring Mass System :

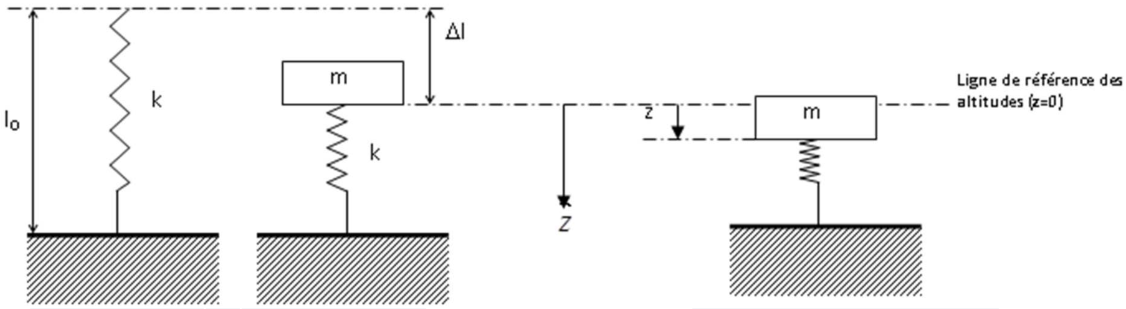


Fig. 1 : Unloaded spring

Fig. 2 : Load spring at rest

Fig. 3 : System in oscillation

Consider a spring k (N/m) with an unloaded length l_0 (Voir **Fig. 1**).
Let it carry a mass m (Kg) purely vertical.
The spring is compressed Δl (m) that is called **static deflection** . (**Voir Fig.2**)

The fundamental law of dynamics gives for steady state :

$$\overrightarrow{Fspring} + m\vec{g} = \vec{0}$$

In projection on the z axis, one obtains: :

$$-k.\Delta l + mg = 0$$

$$\boxed{\Delta l = \frac{m.g}{k}} \quad \text{(Eq 1)}$$

Consider now Fig. 3 which illustrates the same system subjected to free oscillations. This time the mass is no longer at steady state, and the writing of the PFD brings us the following equation:

$$\overrightarrow{Fressort} + m\vec{g} = m.\vec{\gamma} = m.\ddot{z}.\vec{z}$$

In projection on the z axis :

$$-k.(\Delta l + z) + m.g = m.\ddot{z}$$

$$-k.\Delta l - k.z + m.g = m.\ddot{z}$$

If equation 1 is injected in the preceding relation, the differential equation which governs the motion then becomes:

$$\boxed{m.\ddot{z} + k.z = 0} \quad \text{(Eq 2)}$$

The solution of such an equation is a pure sinusoidal function of the form $z(t) = Z.\sin(\omega.t)$ where Z represents the amplitude (m) and ω (rad/s) the pulsation (rad/s).

$$\dot{z}(t) = +Z.\omega.\cos(\omega.t)$$

$$\ddot{z}(t) = -Z.\omega^2.\sin(\omega.t)$$

Equation 2 becomes :

$$-m.Z.\omega^2.\sin(\omega.t)+k.Z.\sin(\omega.t)=0$$

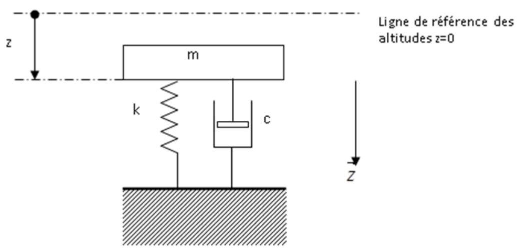
$$-m.Z.\omega^2+k.Z=0$$

Thus $\boxed{\omega=\sqrt{\frac{k}{m}}}$ **et** $\boxed{f=\frac{\omega}{2.\pi}=\frac{1}{2.\pi}\sqrt{\frac{k}{m}}}$

This frequency is called the natural frequency of the system. It can also be calculated as a function of the static deflection:

$$\boxed{\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{k.g}{m.g}}=\sqrt{\frac{g}{\Delta l}}}$$

b) System dampened by a pure viscous damper :



In this case, the mass is subjected to three forces : the gravity, the spring force , and the damper force. The writing of the PFD gives:

$$\overrightarrow{Fd}+\overrightarrow{Fs}+\overrightarrow{m.g}=m.\vec{\gamma}$$

Projection on the z axis: $-k.(\Delta l+z)-c.\dot{z}+m.g=m.\ddot{z}$

$$-k.\Delta l-k.z-c.\dot{z}+m.g=m.\ddot{z}$$

After simplification with equation (1) : $\boxed{m.\ddot{z}+c.\dot{z}+k.z=0}$

If the system were not damped, in other words (c = 0 N.s / m) , the natural natural frequency of the system should be : $\boxed{\omega_n=\sqrt{\frac{k}{m}}}$

The damper will change this pulse to the new value

$$\omega_d=\omega_n.\sqrt{1-\xi^2}$$

With ξ = critical damping ratio = $\frac{c}{\sqrt{4.k.m}}=\frac{c}{c_c}$

Cc=critical damping= $\sqrt{4.k.m}$

However, the shock absorbers used in the automobile are more complex than the simplistic damper shown above. Indeed the damping laws are different between compression and expansion:

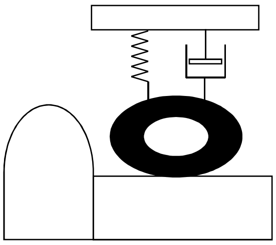


Fig.4 : Compression

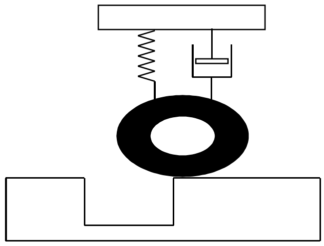


Fig. 5 : Expansion

It is evident that, in compression (FIG. 4), the damper is required to be "flexible" in order to prevent passengers from being seriously shaken. In the case of expansion(FIG. 5), if the shock absorber is too soft, the wheel will strike very hard at the bottom of the hole and will generate a hudge shock. It is for this reason that the automotive dampers are asymmetrical between the expansion and the compression, and it is generally:

$$\boxed{C_{détente} = 3 * C_{compression}}$$

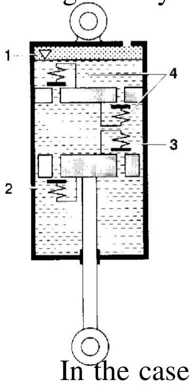
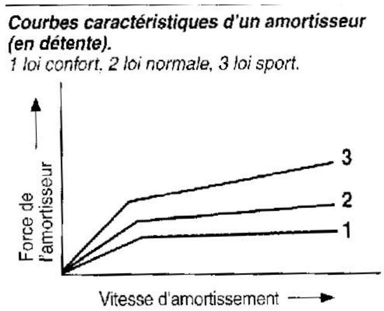
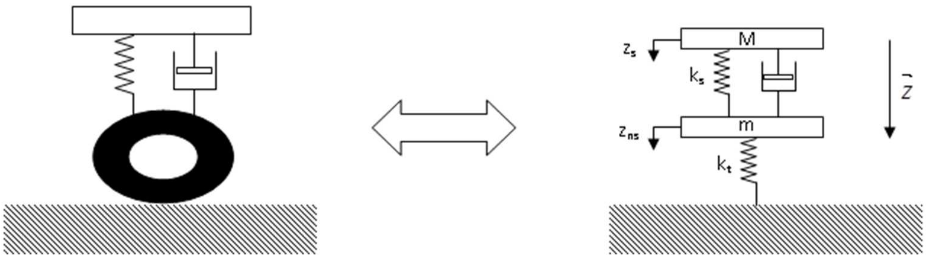


Fig.6: Oil Monotube Damper



In the case of an oil damper, complex laws are obtained by means of different spring-loaded relief valves and different diameter holes in the case of an oil damper.

2 – Modeling of half an Axle :



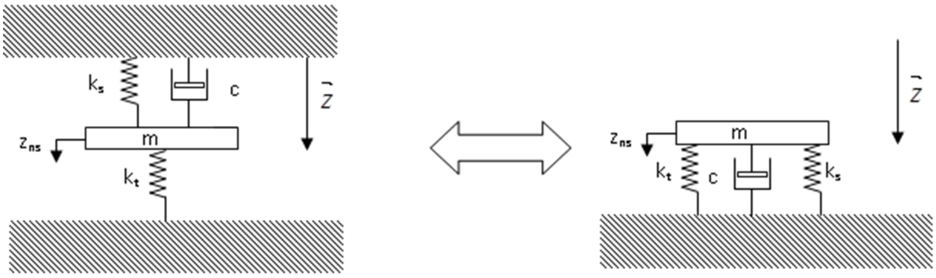
Notations :

M	Srpung Mass(Kg)
m	Unsprung Mass (Kg)
k_t	Vertical stiffness of the tire (N/m)
k_s	Spring stiffness (N/m)
c	Expansion damping constant(N.s/m)
z_s	Altitude of sprung mass(m)
z_{ns}	Altitude of unsprung mass(m)

It is obvious that the system presents two simple oscillators stacked one on the other, so two frequencies of distinct resonances are obtained , one due to the sprung mass and the other to unsprung mass.

a) ***Resonance frequency of unsprung mass:***

To simplify the reasoning, it is considered that the suspended mass is blocked with respect to the ground. The half axle can then be modeled as follows:



Thus a simple oscillator is obtained , with:

Resonance frequency of unsprung mass : $\boxed{\omega_{nns} = \sqrt{\frac{k_t + k_s}{m}}}$

Critical damping : $\boxed{C_{cns} = \sqrt{4.(k_t + k_s).m}}$

Typical values :

With a typical unsprung mass m = 50 Kg, a suspension stiffness ks of 17500 N / m and a tire stiffness kt of 175000 N / m gives a resonant frequency

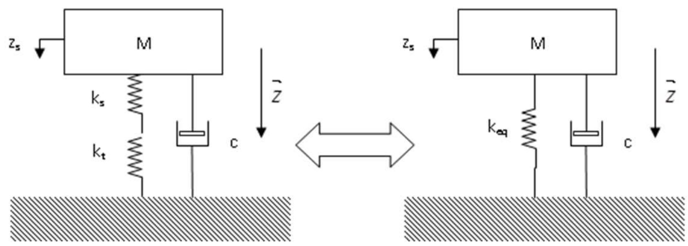
$$\omega_{nns} = \sqrt{\frac{17500 + 175000}{50}} = 62 rad/s$$

$$f_{nns} = \frac{\omega_{ns}}{2\pi} = \frac{62}{2.\pi} \approx 10 Hz$$

If the ripples of the road associated with the speed of the vehicle bring the unsprung mass into resonance this can prove extremely dangerous because the wheel can leave contact with the ground ...

b) ***Resonance frequency of sprung mass:***

In this case, in order to simplify the reasoning, the effect of the unsprung mass is neglected,: it is purely and simply suppressed. Thehalf axle can then be modeled as follows:



The two springs works in series, they can be assimilated to an equivalent spring of stiffness:

$$k_{eq} = \frac{k_s k_t}{k_s + k_t}$$

The simple oscillator laws give :

Resonance frequency of sprung mass: $\boxed{\omega_{ns} = \sqrt{\frac{k_{eq}}{M}} = \sqrt{\frac{k_s k_t}{M.(k_s + k_t)}}}$

Critical damping : $\boxed{C_{cs} = \sqrt{4.k_{eq}.M} = \sqrt{\frac{4.k_s.k_t.M}{(k_s + k_t)}}}$

Typical value for a passenger car :

1 Hz < f_{ns} <1.5 Hz

$2.\pi < \omega_{ns} < 3.\pi$ rad/s

$\frac{g}{9.\pi^2} < \Delta l < \frac{g}{4.\pi^2}$ **0.10 m < Δl < 0.25 m**

Indeed below a frequency of 1 Hz the static deflection will be greater than 0.25 m which is obviously unacceptable when the wheels will be pendent (case of repairs). Above 1.5 Hz the suspension will be judged to be hard and uncomfortable

However for a sports car one will choose 1.5 Hz <fns <3 Hz, sacrificing of course the comfort to the "speed of reaction" of the suspensions.

Fréquence (Hz)	Subjectif	Tolérance
5 à 3	Tightly painful	Quickly intolerable
1.5 à 3	Dry, sporty	At the limit of the tolerable
1.5 à 1	Comfortable , soft	Good tolerance
1 à 0.7	« boat » feeling	Nausea

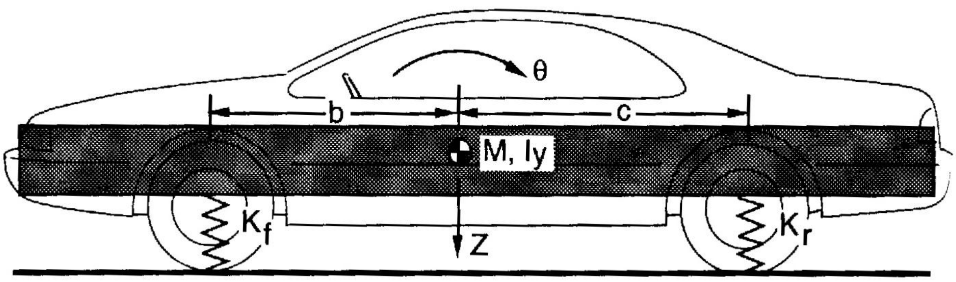
c) Damping Constant Value :

In order to choose a shock absorber, it is necessary to fix the shock and the compression. Of course, a shock absorber can not be optimum for both the suspended mass and the unsprung mass. The choice of a shock absorber is therefore the art of compromise, however one tries to respect the following rules:

$$\boxed{\begin{array}{c} C_{expansion} = 2 \text{ à } 4\ C_{compression} \\ C_{expansion} > 1 \text{ à } 1.5\ C_{cs} \end{array}}$$

$$C_{cs} < C_{expansion} < C_{cns}.$$

3 – Pumping and Pitch frequencies :



For simplicity, the tire and the suspension are considered as a single equivalent stiffness: $k_{eq} = \frac{k_s . k_t}{k_s + k_t}$

The effects of shock absorbers and unsprung mass are neglected.

One poses : $\boxed{\alpha = \frac{Kf + Kr}{M}}$ $\boxed{\beta = \frac{Kr.c - Kf.b}{M}}$ $\boxed{\gamma = \frac{Kf.b^2 + Kr.c^2}{M.k^2}}$

With

Kf	Equivalent Stiffness to the entire FRONT axle (N.m)
Kr	Equivalent Stiffness to the entire REAR axle (N.m)
M	Sprung Mass (Kg)
Iy	Moment of inertia in G about the y-axis(Kg.m²)
k	Giration radius k = $\sqrt{(Iy/M)}$ (m)
b	Distance from front axle to center of gravity G (m)
c	Distance from rear axle to center of gravityG (m)
DI	Dynamic Index = k²/(bc) =Iy/(M.b.c) (no unit)

Writing the theorem of the dynamic resultant and the dynamic moment in G brings the following two equations:

$$\ddot{z}+\alpha.z+\beta.\theta=0 \quad \text{(Equation a)}$$

$$\ddot{\theta}+\beta\frac{z}{k^2}+\gamma.\theta=0 \quad \text{(Equation b)}$$

Without damping, the solutions of the preceding differential equations will be of sinusoidal form, that is to say of the form:

ou

$$\mathbf{z(t) = z.sin(\omega.t)} \Rightarrow \quad \ddot{z}(t)=-z.\omega^2.\sin(\omega.t)$$

$$\mathbf{\theta(t)=\theta.sin(\omega.t)} \Rightarrow \quad \ddot{\theta}(t)=-\theta.\omega^2.\sin(\omega.t)$$

If these solutions are injected into equation (a), this becomes:

$$-z.\omega^2.\sin(\omega.t)+\alpha..z.\sin(\omega.t)+\beta.\theta.\sin(\omega.t)=0$$

thus $(\alpha-\omega^2).z+\beta.\theta=0$

finally $$\boxed{\frac{Z}{\theta}=\frac{-\beta}{\alpha-\omega^2}} \quad \text{(Equation c)}$$

Same approach on equation (b) leads to :

$$-\omega^2.\theta.\sin(\omega.t)+\frac{\beta}{k^2}z.\sin(\omega.t)+\gamma.\theta.\sin(\omega.t)=0$$

$$\theta.(\gamma-\omega^2)+\frac{\beta}{k^2}Z=0$$

$$\boxed{\frac{Z}{\theta}=\frac{-k^2(\gamma-\omega^2)}{\beta}} \quad \text{(Equation d)}$$

We then write the equality between equations c and d, which brings:

$$\frac{-\beta}{\alpha-\omega^2}=\frac{-k^2(\gamma-\omega^2)}{\beta}$$

$$\omega_4-(\alpha+\gamma).\omega^2+\alpha.\gamma-\frac{\beta^2}{k^2}=0$$

This equation is in fact a polynomial of degree two whose two solutions are:

$$(\omega_1)^2=\frac{\alpha+\gamma}{2}+\sqrt{\frac{(\alpha+\gamma)^2}{4}-(\alpha.\gamma-\frac{\beta^2}{k^2})}$$

$$(\omega_2)^2=\frac{\alpha+\gamma}{2}-\sqrt{\frac{(\alpha+\gamma)^2}{4}-(\alpha.\gamma-\frac{\beta^2}{k^2})}$$

After simplification

$$\boxed{\omega_1=\sqrt{\frac{\alpha+\gamma}{2}+\sqrt{\frac{(\alpha-\gamma)^2}{4}+\frac{\beta^2}{k^2}}}} \quad \text{(Equation e)}$$

$$\boxed{\omega_2=\sqrt{\frac{\alpha+\gamma}{2}-\sqrt{\frac{(\alpha-\gamma)^2}{4}+\frac{\beta^2}{k^2}}}} \quad \text{(Equation f)}$$

The values for ω_1 et ω_2 allow to calculate the amplitude ratios : $\frac{Z}{\theta}(\omega_1)$ **et** $\frac{Z}{\theta}(\omega_2)$

Using equation c) ou d),**. The 2 obtained values are opposite sign/**

Case 1 : $\frac{Z}{\theta}$ **Is positive:**

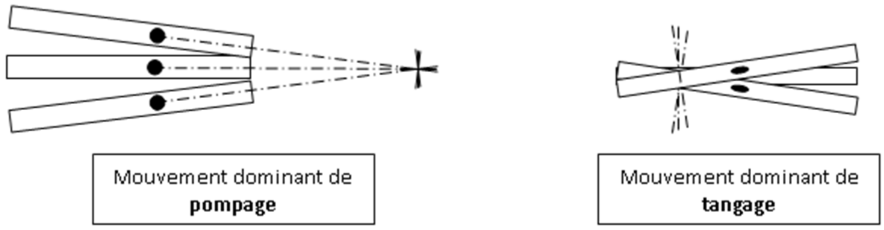
This means that Z and θ are both positive or negative . In this case, the center of the oscillations will be located BEFORE the center of gravity G at a distance x = Z/ Θ

Case 2 : $\frac{Z}{\theta}$ **Is negative**

This means that Z et θ are opposite sign . . In this case, the center of the oscillations will be located BEFORE the center of gravity G at a distance x =Z/Θ

A numerical study shows that one of the two distances will be located beyond the wheelbase of the vehicle and the other is small enough to be included in the wheelbase.

When the center of the oscillation is outside the wheelbase, the predominant movement will be pumping. In the case where the center of the oscillations is located in the wheelbase, the "predominant" movement will be pitching.



The positions of the two centers of oscillation depend on the relative values of the eigen frequencies of the rear and front suspensions which, if one neglects the effect of tires and shock absorbers:

$$f_f = \frac{1}{2.\pi}\sqrt{\frac{K_f}{m_f}}$$

$$f_r = \frac{1}{2.\pi}\sqrt{\frac{K_r}{m_r}}$$

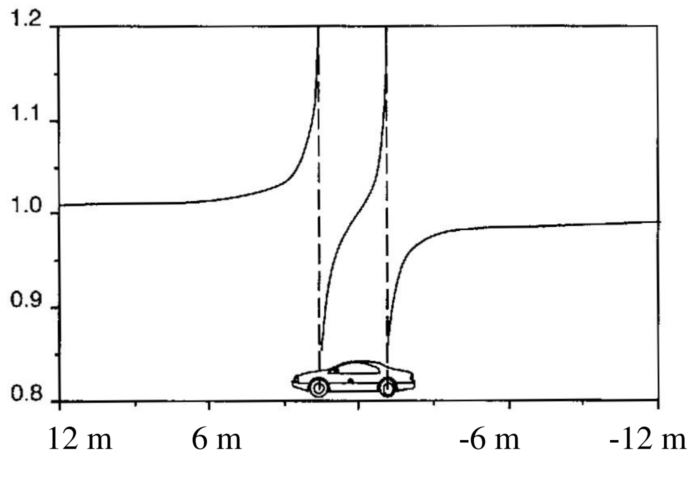
m_f : mass carried by the FRONT axle $\qquad m_f = \frac{M.c}{b+c}$

m_r : mass carried by the REAR axle $\qquad m_r = \frac{M.b}{b+c}$

The following figure shows the position of the oscillation centers as a function of the ratio of the natural frequencies r = ff / fr. For the equal frequencies (r = 1) a center of oscillation is situated on the center of gravity G and the other is rejected at infinity. This case corresponds to a cutting of pumping and pitching which are two movements purely independent..

With a front natural frequency greater than that of the rear axle (r> 1), the two movements are coupled with a pumping center located forward of the front axle and a center of pitch behind the rear axle.

In the opposite case, where the natural frequency of the front axle is smaller than that of the rear axle (r <1), the pumping center is positioned behind the rear axle and the center of pitch will be very close to the axle before. It is this latter configuration (r <1) that was identified by Maurice Olley in the 1930s as the best compromise of suspension.



Moreover, an experimental study with the famous vehicle of variable inertia (k^2-rig) conducted by Maurice OLLEY has stated 4 fundamental criteria, which are:

1) *The stiffness of the front suspension (Kf) must be 30% less than that of the rear axle (Kr).*

2) *The eigen frequencies of pitching and pumping must be close: the pumping frequency must be less than 1.2 times that of pitching. For a higher frequency ratio, "interference strokes" resulting from the superposition of the two movements can occur in a common way.*

3) *Neither of the two frequencies (pumping and pitching) shall be greater than 1.3 Hz, which means that the static deflection of the suspensions must not exceed 150 mm.*

4) *The roll frequency (see Chapter 5) should be approximately equal to that of rolling and pumping*

4 – Application Exercises :

Exercise n°1 :

Equivalent stiffness for FRONT axle	K_f = 23 500 N/m
Equivalent stiffness for REAR axle	K_r= 17 000 N/m
Wheelbase	L = 2.555 m
Carried Mass by one front tire	m_f= 435 Kg
Carried Mass by one rear tire	m_r= 331 Kg
Dynamic Index	DI = 1.1 (no unit)

Question 1: Calculate the position of the pumping and pitch center and the associated natural frequencies for a car with the above characteristics.

Question n°2 : The vertical stiffness of a tire Kt = 100 000 N / m is given. Calculate the stiffness of a suspension spring for the front and rear axles so as to obtain the desired equivalent stiffness.

Question n°3 : Calculate the natural frequency of the unsprung front and rear masses. Considerating a 20 Kg unsprung mass for each front and rear wheel.

Question n°4 : Choose appropriate damping values for rear and front shock absorbers.

Exercise n°2 :

Equivalent stiffness for FRONT axle	K_f = 23 000 N/m
Equivalent stiffness for REAR axle	K_r= 16 300 N/m
Wheelbase	L = 2850 m
Carried Mass by one front tire	m_f= 470 Kg
Carried Mass by one rear tire	m_r= 250 Kg to 450 Kg
Dynamic Index	DI = 1.05 (no unit)

Question 1 : Find the position of the pump and pitch centers as well as the associated natural frequencies by programming an automatic spreadsheet on a spreadsheet:

Question 2 : For what load to the rear axle gets pure pumping and pure pitching?

Transversal behavior of a vehicle in steady state cornering

Pascal Brejaud

IUT Orléans - GMP

Ackermann Steering Geometry

The Ackermann geometry is a kinematic condition so that the 4 wheels of the same vehicle to describe a pure circular motion without any drift or slip of the tires.

Hypothesis

The ideal Ackermann geometry assumes that the vehicle speed is almost zero, so that the centrifugal force $F_c = \frac{mV^2}{R}$ is negligible. Therefore no lateral slip phenomenon of the tire exists and speed of each latter is contained in its median plane.

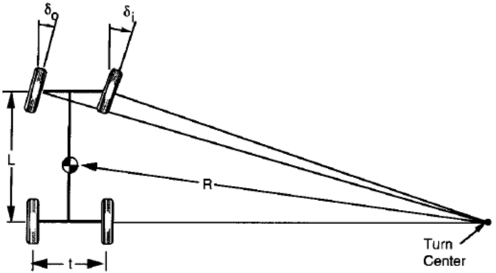


Figure – Ideal Ackermann Steering Geometry

t	Vehicle track[m]
L	Vehicle wheelbase [m]
R	Cornering Radius [m]
δ_o	Outside Wheel Steering Angle [rad]
δ_r	Inside Wheel Steering Angle [rad]

Hypothesis

It is assumed that the effective cornering radius is much higher than the vehicle track. As a consequence the steering angle of the interior and exterior wheel are small.

Figure1 gives :

$$\tan \delta_o \approx \frac{L}{R + \frac{t}{2}} \tag{1}$$

$$\tan \delta_i \approx \frac{L}{R - \frac{t}{2}} \tag{2}$$

Then , expressing the cotangent :

$$cotan(\delta_o) = \frac{1}{\tan \delta_o} \approx \frac{R + \frac{t}{2}}{L} \tag{3}$$

$$cotan(\delta_i) = \frac{1}{\tan \delta_i} \approx \frac{R - \frac{t}{2}}{L} \tag{4}$$

Then finally :

$$cotan(\delta_o) - cotan(\delta_i) = \frac{R + \frac{t}{2}}{L} - \frac{R - \frac{t}{2}}{L} = \frac{t}{L} \qquad (5)$$

Final Ackermann relation :

$$cotan(\delta_o) - cotan(\delta_i) = \frac{t}{L} \qquad (6)$$

Jenteaud's drawing board

Jeantaud is a french engineer that has described a practical solution for a steering mechanism that respects the Ideal Ackermann geometry **for small steering angles.** Jeantaud's design uses a steering bar that rigidly connects steering links. It is this system, not rigorous from the mathematical point of view, but quite satisfactory in practice, from which are derived all the current automobile directions.

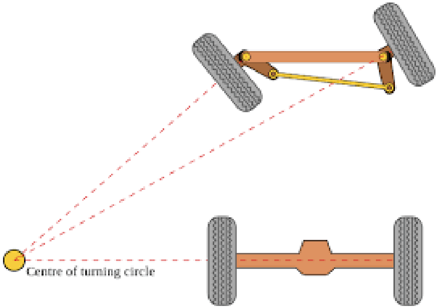


Figure – Jeantaud's bar in action

Jeantaud's condition :

The two lines which pass through the center of the hub carrier pivot and the ball joint at each jeantaud's bar ends must intersect exactly at the middle point of the rear axle

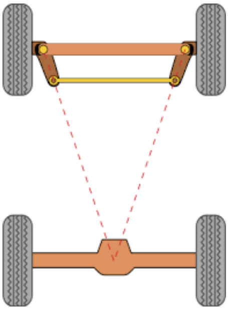


Figure – Jeantaud's design board

Realizations of Steering systems around the Jeantaud's idea

<table>
<tr><th>Historical System</th><th>Modern System</th></tr>
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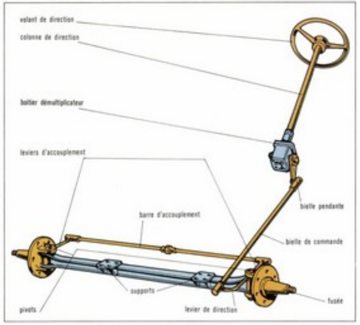

Direction classique.
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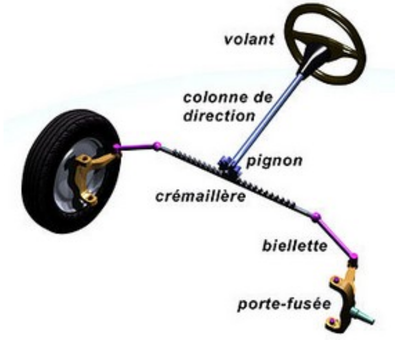

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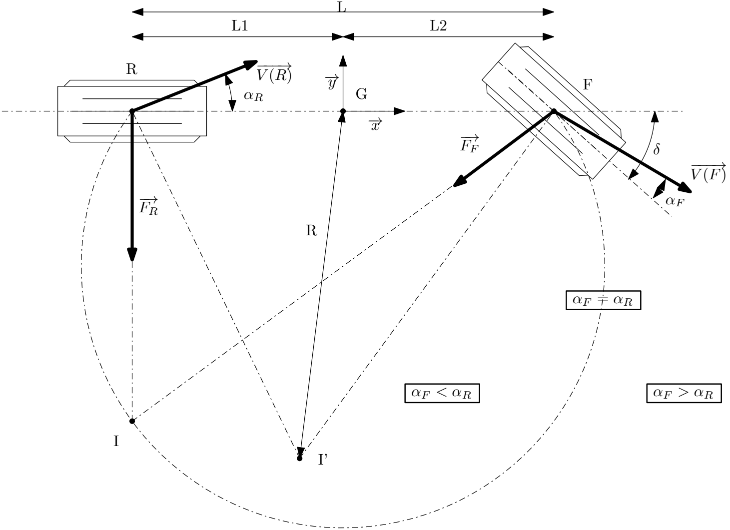
Bicycle Model



Geometric Relations :

$$\widehat{FRI'} = \frac{\pi}{2} - \alpha_R \tag{7}$$

$$\widehat{RI'F} \approx \widehat{RIF} \approx \tan \widehat{RI'F} \approx \frac{L}{R} \tag{8}$$

$$\widehat{RFI'} = \frac{\pi}{2} - (\delta - \alpha_F) = \frac{\pi}{2} - \delta + \alpha_F \tag{9}$$

Since the sum of the angles in a triangle is equal to π radians :

Geometrical relation :

$$\delta = \frac{L}{R} + \alpha_F - \alpha_R \tag{10}$$

The fundamental law of dynamics, applied to the considered bicycle, gives respectively for translation and rotation :

$$\sum \overrightarrow{Force} = m\overrightarrow{\gamma}$$

with :

$$\overrightarrow{\gamma} = m\frac{V^2}{R}\overrightarrow{y}$$

$$\sum \overrightarrow{Moment(G)} = \overrightarrow{0}$$

Thus along the $\overrightarrow{y}$ axis, **considering that the angle are small and expressed in radian**, projection gives :

$$\sum \overrightarrow{Force} \cdot \overrightarrow{y} \approx F_F + F_R = m\frac{V^2}{R} \tag{11}$$

A balance of the moment at G point along the $\overrightarrow{z}$ axis gives :

$$L1 \times F_R - L2 \times F_F = 0 \tag{12}$$

The lateral slip force model of the tire for small slip angles, so near the origin point of the pacjeka model S shaped curve gives :

$$F_R = K_R \times \alpha_R \tag{13}$$

$$F_F = K_F \times \alpha_F \tag{14}$$

The five unknowns values are $\delta, \alpha_F, \alpha_R, F_F, F_R$
Parameters values considered known : $m, L, L1, L2, V, R, K_R, K_F$

Equations set

$$\delta = \frac{L}{R} + \alpha_F - \alpha_R \tag{10}$$

$$F_F + F_R = m\frac{V^2}{R} \tag{11}$$

$$L1 \times F_R - L2 \times F_F = 0 \tag{12}$$

$$F_R = K_R \times \alpha_R \tag{13}$$

$$F_F = K_F \times \alpha_F \tag{14}$$

Resolution of sub-Equations set (11) and (12) directly gives :

$$F_R = \frac{L2}{L} \times \frac{mV^2}{R} \tag{15}$$

$$F_F = \frac{L1}{L} \times \frac{mV^2}{R} \tag{16}$$

Then according to equations(13) and (14)

$$\alpha_R = \frac{F_R}{K_R} = \frac{L2}{L} \times \frac{1}{K_R} \times \frac{mV^2}{R} \tag{17}$$

$$\alpha_F = \frac{F_F}{K_F} = \frac{L1}{L} \times \frac{1}{K_F} \times \frac{mV^2}{R} \tag{18}$$

Resolution (2/3)

Thus, finally :

$$\delta = \frac{L}{R} + \frac{mV^2}{R} \times \frac{1}{L} \times \left[\frac{L1}{K_F} - \frac{L2}{K_R} \right] \tag{19}$$

The Equation 19 can be re arranged in the Maurice OLLEY form :

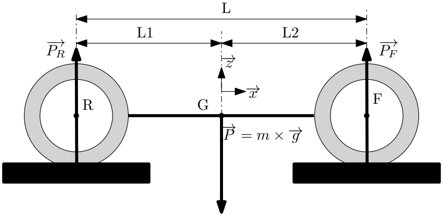


Figure – Side View of the Bicycle Model

Balance of moments for the $\overrightarrow{y}$ axis :

$$\sum \overrightarrow{Moment(G)} \cdot \overrightarrow{y} = 0$$

$$P_R \times L1 - P_F \times L2 = 0$$

$$P_R + P_F = P = m.g$$

Thus two previous equation lead to :

$$P_R = \frac{L2}{L} \times P \tag{20}$$

$$P_F = \frac{L1}{L} \times P \tag{21}$$

Thus a rearrangement of equations (19), (20) and (21) leads to :

Final Equation of bicycle model :

$$\delta = \underbrace{\frac{L}{R}}_{\text{Ackermann Angle}} + \underbrace{\frac{V^2}{R.g}}_{\text{Lateral Acceleration}} \times \underbrace{\left[\frac{P_F}{K_F} - \frac{P_R}{K_R}\right]}_{\text{Understeer Coefficient}} \quad (22)$$

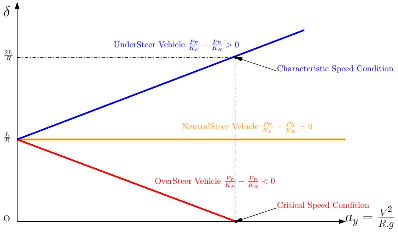


Figure – UnderSteer, Neutral and OverSteer Vehicle

Characteristic Speed for a Understeer Vehicle

$$V_{characteristic} = \sqrt{\frac{L \times g}{\frac{P_F}{K_F} - \frac{P_R}{K_R}}} \tag{23}$$

Critical Speed for a Understeer Vehicle

$$V_{critical} = \sqrt{\frac{-L \times g}{\frac{P_F}{K_F} - \frac{P_R}{K_R}}} \tag{24}$$

Roll effect

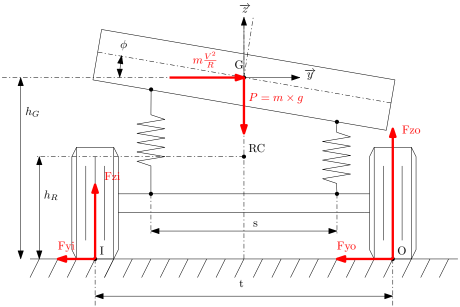


Figure – Axle in Pure ROLL Condition

$$P = F_{zi} + F_{zo} \tag{25}$$

$$m\frac{V^2}{R} = F_{yi} + F_{yo} \tag{26}$$

Roll Stiffness of an Axle

Isolate the sprung mass :

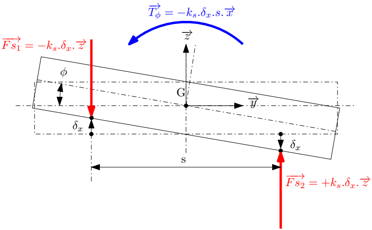


Figure – Sprung Mass in Pure ROLL Condition

$$\tan\phi = \frac{\delta_x}{\frac{s}{2}} = \frac{2\delta_x}{s} \tag{27}$$

The angle ϕ is small and expressed in radian. Thus $\tan\phi \approx \phi$ and Eq.(25) leads to : $\delta_x \approx \frac{s.\phi}{2}$

The torque $\overrightarrow{T_\phi}$ created by the two spring forces $\overrightarrow{F_{S1}}$ and $\overrightarrow{F_{S2}}$ can be expressed as :

$$\overrightarrow{T_\phi} = -\frac{k_s \times s^2}{2} \times \phi \times \overrightarrow{x} \tag{28}$$

The torque magnitude $T_\phi$is proportional to the roll angle ϕ, then it is posed :

Roll Stiffness K_ϕ

$$T_\phi = K_\phi \times \phi \tag{29}$$

with :

$$K_\phi = \frac{k_s \times s^2}{2} \text{ (ROLL STIFFNESS)} \tag{30}$$

Lateral Load Transfer

Isolate the sprung mass :

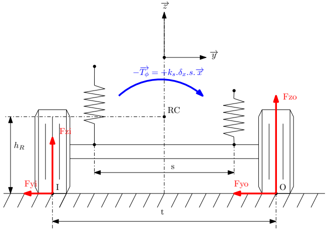


Figure – UnSprung Mass in Pure ROLL Condition

$$\sum \overrightarrow{T} = \overrightarrow{R_c I} \wedge \overrightarrow{F_i} + \overrightarrow{R_c O} \wedge \overrightarrow{F_O} + \overrightarrow{T_\phi} = \overrightarrow{0}$$

$$\begin{vmatrix} 0 \\ -\frac{t}{2} \\ -h_R \end{vmatrix} \wedge \begin{vmatrix} 0 \\ F_{yi} \\ F_{zi} \end{vmatrix} + \begin{vmatrix} 0 \\ +\frac{t}{2} \\ -h_R \end{vmatrix} \wedge \begin{vmatrix} 0 \\ F_{yo} \\ F_{zo} \end{vmatrix} + \begin{vmatrix} T_\phi \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \qquad (31)$$

Along the $\overrightarrow{x}$ axis, previous equation gives :

$$\frac{-t}{2} \times F_{zi} + h_R \times F_{yi} + \frac{t}{2} \times F_{zo} + h_R \times F_{yo} + T\phi = 0 \qquad (32)$$

It is posed :

Lateral Load Transfer

$$\triangle F_z = \frac{F_{zo} - F_{zi}}{2} \qquad (33)$$

Then equations 26,29, 32 and 32 finally lead to :

Lateral Load Transfer

$$\triangle F_z = \frac{1}{t} \times \left[\underbrace{h_R \times \frac{mV^2}{R}}_{\text{Centrifugal Effect}} + \underbrace{K_\phi \times \phi}_{\text{Roll effect}} \right] \tag{34}$$

Roll Axis and Roll Angle

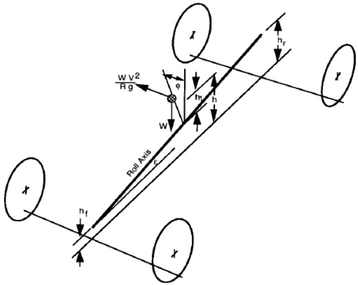


Figure – Roll axis

The total roll torque applied to the sprung mass along the roll axis can be expressed as :

$$T_\phi = \left[m \times g \times h_1 \times \sin\phi + \frac{mV^2}{R} \times h_1 \times \cos\phi\right] \qquad (35)$$

Angle ϕ is small and expressed in radian , thus $\cos\phi \approx 1$ and $\sin\phi \approx \phi$:

$$T_\phi = m \times g \times h_1 \times \left[\underbrace{\phi}_{\text{Roll effect}} + \underbrace{\frac{V^2}{R \times g}}_{\text{Weight effect}}\right] \qquad (36)$$

The total roll torque must be balanced by the roll stiffness of each axle.

$$T_\phi = K_{\phi,f} \times \phi + K_{\phi,r} \times \phi = m \times g \times h_1 \times \left[\phi + \frac{V^2}{R \times g}\right] \qquad (37)$$

Equation 37 leads to :

Roll Angle

$$\underbrace{\phi}_{\text{Roll Angle [rad]}} = \underbrace{\frac{m \times g \times h_1}{K_{\phi,f} + K_{\phi,r} - m \times g \times h_1}}_{F_\phi \text{ : Roll Flexibility [rad/g]}} \times \underbrace{\frac{V^2}{R \times g}}_{a_y \text{ : Lateral acceleration [g]}} \tag{38}$$

Lateral Load Transfer Distribution

According to equation (34) :

$$\triangle F_{zf} = \frac{1}{t} \times \left[h_f \times \frac{P_f \times V^2}{R \times g} + K_{\phi f} \times \phi \right]$$

$$\triangle F_{zf} = \frac{1}{t} \times [h_f \times P_f \times a_y + K_{\phi f} \times F_{\phi} \times a_y]$$

$$\triangle F_{zf} = \frac{a_y}{t} \times [h_f \times P_f + K_{\phi f} \times F_{\phi}] \tag{39}$$

And for the rear axle :

$$\triangle F_{zr} = \frac{a_y}{t} \times [h_r \times P_r + K_{\phi r} \times F_{\phi}] \tag{40}$$

Equations (39) and (40) allow the definition of the LLTD :

Lateral Load Transfer Distribution :

$$LLTD = \frac{\triangle F_{zf}}{\triangle F_{zr}} = \frac{h_f \times P_f + K_{\phi f} \times F_\phi}{h_r \times P_r + K_{\phi r} \times F_\phi} \tag{41}$$

- $LLTD >= 1$: In this case, while cornering, the weight applied to the outside front wheel increases more than the weight applied to the outside rear wheel. Therefore, the under-steer coefficient increases. The behavior of the vehicle remains under-steer.
- $LLTD < 1$: In this case, the weight applied to the outside rear wheel increases more than the weight applied to the outside front wheel. This is a dangerous situation : the under-steer coefficient decreases and can become negative. The vehicle can becomes over-steer.

Exercise 1

A car has a weight of 862 kg on the front axle and 703 kg on the rear axle. The wheelbase is 2.555 m. Is is given the lateral slip stiffness of the tyre as function of the applied weight on the tyre :

Weight [Kg]	Lateral Slip Stiffness $\left(\frac{N}{deg}\right)$
102	298
204	538
306	760
408	1000
510	1143
612	1334

- Ackermann angle for cornering radius of 150 , 60 , 30 and 15 m.
- Under-steer coefficient.
- Critical or characteristic speed.
- Lateral Slip Angle (front and rear) for a cornering radius of

Exercise 2

The front axle of a car is a double wishbone with two springs. The roll stiffness of the front axle is $K_{\phi f} = 170 \frac{N.m}{deg}$. The rear axle is a rigid axle with two springs with a stiffness $K = 20000 \frac{N}{m}$. These springs have a center distance of 1.016 m. The total unsprung mass is 1250 Kg. The distance between the center of gravity and the roll axis is $h_1 = 203$ mm.

- Determine the roll stiffness of the rear axle.
- Determine the roll flexibility.
- Determine the LLTD. Conclusion ?