B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2024.

First Semester

LINEAR ALGEBRA AND CALCULUS

(Common to I Semester AI, CE, CSE, ECE and ME)

(RU 23 Regulations)

Time : 3 Hours *Max. Marks : 70*

SECTION — A

(Compulsory Question) **(10 × 2 = 20 Marks)**

Answer the following.

1. (a) Find the value of k for which the system of equations $(3k-8)x+3y+3z=0$, $3x+(3k-8)y+3z=0$, $3x+3y+(3k-8)z=0$ has a non - trivial solution.

(b) Find the Rank of $\begin{bmatrix} 2 & 0 & -1 \\ 4 & 1 & 2 \\ 1 & 5 & -4 \end{bmatrix}$.

(c) Find the Eigen values and Eigen vectors of the Matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

(d) Find the nature of the quadratic form $x^3+5y^3+z^2+2xy+2yz+6zx$.

(e) Verify Rolle's theorem for the function $y=x^2+1$, $a=-1$ and $b=1$.

(f) Find 'c' of Cauchy's Mean Value Theorem for the functions $f(x)=2.\ln x$ and $g(x)=x^2-1$ in the interval [2, 3].

(g) If $z=e^{ax+by}f(ax-by)$ prove that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz$.

(h) Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u\log u$ where $u=e^{x^2}+e^{y^2}$.

(i) Evaluate $\int_0^5\int_0^{x^2} x(x^2+y^2)dxdy$.

(j) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\ dxdydz$.

Turn Over

SECTION — B

Answer ONE full question from each unit; All questions carry equal marks.

(5 × 10 = 50 Marks)

UNIT I

2. For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ find non- singular matrices P and Q such that PAQ is in the normal form and hence find Rank of A. (10)

Or

3. Solve the system of equations $x_1 - x_2 + x_3 + x_4 = 2$, $x_1 + x_2 - x_3 + x_4 = -4$, $x_1 + x_2 - x_3 - x_4 = 4$, $x_1 + x_2 + x_3 + x_4 = 0$ by finding the inverse by elementary row operations. (10)

UNIT II

4. Using Caylay-Hamilton theorem find A^{-1} for the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ hence find A^4. (10)

Or

5. Reduce the quadratic form $2 + 2yz + 2zx$ to canonical form by orthogonal transformation and hence find rank, index, signature and the nature of the quadratic form. (10)

UNIT III

6. Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1$. (10)

Or

7. Evaluate $\iint_R (x + y)^2 \, dxdy$ where R is the Parallelogram in the XY - plane with vertices (1, 0), (3, 1), (2, 2), (0, 1) using the transformation $u = x + y, v = x - 2y$. (10)

UNIT IV

8. If $x + y = 2e^{\theta} \cos \varphi$ and $x - y = 2ie^{\theta} \sin \varphi$ show that $\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \varphi^2} = 4xy \frac{\partial^2 u}{\partial x \partial y}$. (10)

Or

9. Expand $f(x, y) = \tan^{-1}(y/x)$ in powers of $x - 1$ and $y - 1$ upto third degree terms Hence compute $f(1, 1, 0.9)$ approximately. (10)

UNIT V

10. Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1$. (10)

Or

11. Evaluate $\iint_R (x + y)^2 \, dxdy$ where R is the Parallelogram in the XY – plane with vertices (1, 0), (3, 1), (2, 2), (0, 1) using the transformation $u = x + y, v = x - 2y$. (10)