Session 3 - Problem Statement

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1 Objectives

The objectives of this session are:

1. Complete the code and search for session 2 based on the solutions provided.
2. Find Moment of Inertia (MOI) values for satellites of 1U to 16U.
3. Find typical values of mass for satellites of 1U to 16U.
4. Find coatings used for radiation protection in space based on session 2 solution.
5. Find suppliers for coating materials and their contact information.
6. Run the code from session 2 for each size from 1U to 16U for cells that are smaller than the smallest dimension.
7. Get lenopen, widopen, and mtot values for all to calculate MOI based on given equation.
8. Plot MOI vs size in U for each size for the different configurations.
9. Plot ptot/vtot and ptot/mtot vs size in U for different configurations.

2 Search pointers

2.1 Satellite MOIs

Search in search engines and in AI models using the terms "Typical Moment of Inertia of CubeSats". Try different combinations with 1U, 3U, 16U CubeSats. This paper [1] describes the process for the 1U satellite however, the paper is too complex. The only relevant value for us is the given MOI ($J = (0.002, 0.002, 0.002)kgm^2$) for the 1U craft mentioned on page 6. This paper [2] gives the values on page 5 in table 1 as Ixx = 0.0067 and $Iyy = Izz = 0.0333kgm^2$.

Here is a testing apparatus built by NTU to get MOI. Unfortunately, it does not measure an actual satellite.

If there isn't enough information available to compare to, we have to rely more heavily on our own calculations and values.

2.2 Mass

The Wikipedia page on CubeSats states that the mass per unit (per U) of CubeSats is generally 2kg. So 1U will be 2kg, 16U will be 32kg, and the mass will be in between. But do make sure to search through multiple sources of mass estimates to get the average mass of a CubeSat of a given size.

2.3 Coatings

Refer to the material shared in the solution for session 2. If you narrow it down to a few materials that have been used in space for radiation protection, then we can look for specific suppliers for those materials.

3 Fundamentals of the code

The code for session 2 was the most intense code we will have. The coming codes are more complex for me for deriving the equations but the implementation should be simpler. Use the code from session 2 and run it for different sizes. Pick a cell size that gives at least one cell such that widopen and lframe are both < 100mm. If this does not exist, then only do this for 6U and above or wherever is it possible to fit the panel frames on the walls.

3.1 Inputs to the code from text files

The outputs from the previous session are the inputs for this code. Save the outputs with different names in a new folder so you can read them as needed. For instance, save the output files from session 2 as "$1U_folded_input$" and then read all the files with different output files in one loop into an array.

The new key inputs from a new file are:

1. Body dimensions L, W, H
2. U: The size of the craft in U to keep track
3. mass: total mass of the CubeSat of given size

Get these from a properties file or a separate file of reference material that is known and not calculated.

The key inputs from session 2 outputs are:

1. ptot
2. mtot
3. vtot
4. lenopen
5. widopen

Lenopen and Widopen are in a different format for folded vs outer cover configuration. Thus, we can easily separate the input file names and make different read functions for the 2 different configurations. However, the final outputs need to be compared in a graph, so must be stored in the same file.

It may be ideal to make object arrays that store all the variables for one craft in one object and run them in a loop to read input values and write output values.

3.2 Intermediate values to be calculated and stored

The intermediate values that come up in the process of calculating the MOI are:

1. mbody: remaining mass of the body
2. dcom: distance of the center of mass from the base
3. moipanels: MOI for the panels in their plane
4. moibody: MOI for the body assuming mass is concentrated on the opposite face
5. mpa: mass per unit area for the panels for outer cover configuration

3.3 Outputs to be written in text files

This time, it is not just output values in text files, we also have graphs (plots) made using the plotly library in Python. Those are more important.

1. MOI = Moment of Inertia for the 3 axes (Ixx, Iyy, Izz)
2. powmass = Mass density of power generated (ptot/mtot)
3. powvol = Volume density of power generated (ptot/vtot)

We plot all 3 of these variables against the craft size designation (U) and see if the lines for the folded and outer cover mechanisms ever intersect. Intersections show that after that point, one configuration may become more effective/ efficient than the other. Depending on the priority of the satellite mission, we can use this graph to select the one with the lowest MOI or highest Mass density of power or volume density of power. Depending on the mission objectives, different values may be more important and the graph allows us to compare.

4 Diagrams and Equations

The main thing to calculate is the MOI only. We use the equation for the MOI of a rectangle to get the MOI of the spread-out panels. In this, we take advantage of the fact that the mass distribution is assumed to be in a plane. So we can simply use the perpendicular axis theorem to get the MOI along the axis perpendicular to the plane. Then we use the Parallel axis theorem to get the MOI of the craft about the actual center of mass of the craft. Here are the references:

- MOI for planar sheet about 3 axes as given by testbook.
- MOI of simple shapes from the Wikipedia page on MOI
- MOI about a distant point given by the Wikipedia page on the parallel axis theorem
- MOI for a planar object normal to the plane is given by Wikipedia page on the perpendicular axis theorem

The following figure shows the equations and some derivations.

To compute the moment of inertia of a rectangular plate, we use the following formulas;

For the x-axis;

$I_x = (1/12)\, mb^2$

For the y-axis;

$I_y = (1/12)\, ma^2$

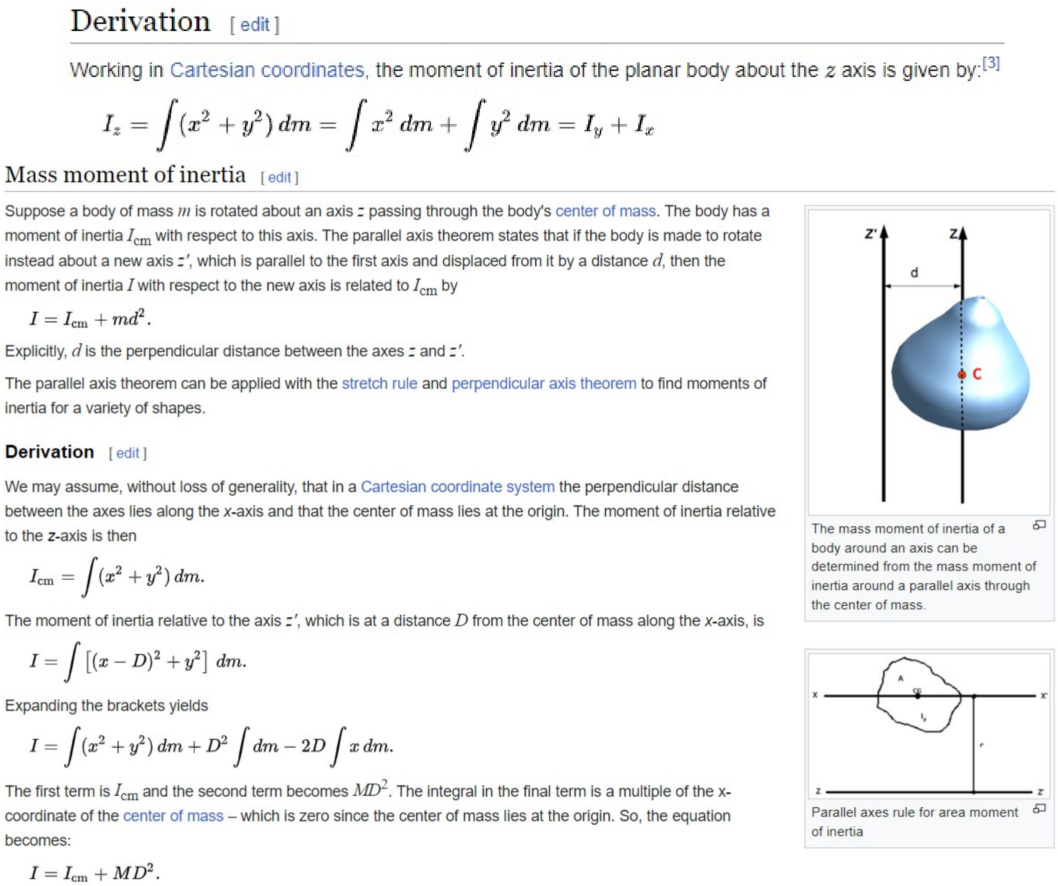
And for the z-axis;

$I_z = (1/12)\, m (a^2 + b^2)$

Sectional areas moment calculated thus[21] [edit]

1. Square: $I_{xx} = I_{yy} = \frac{b^4}{12}$
2. Rectangular: $I_{xx} = \frac{bh^3}{12}$ and; $I_{yy} = \frac{hb^3}{12}$
3. Triangular: $I_{xx} = \frac{bh^3}{36}$
4. Circular: $I_{xx} = I_{yy} = \frac{1}{4}\pi r^4 = \frac{1}{64}\pi d^4$

Figure 1: (a) MOI for 3 axes for a thin sheet (planar object) (b) MOI equations for area moment of inertia from Wikipedia



Derivation [edit]

Working in Cartesian coordinates, the moment of inertia of the planar body about the z axis is given by:[3]

$$I_z = \int (x^2 + y^2)\, dm = \int x^2\, dm + \int y^2\, dm = I_y + I_x$$

Mass moment of inertia [edit]

Suppose a body of mass m is rotated about an axis z passing through the body's center of mass. The body has a moment of inertia I_{cm} with respect to this axis. The parallel axis theorem states that if the body is made to rotate instead about a new axis z', which is parallel to the first axis and displaced from it by a distance d, then the moment of inertia I with respect to the new axis is related to I_{cm} by

$$I = I_{\mathrm{cm}} + md^2.$$

Explicitly, d is the perpendicular distance between the axes z and z'.

The parallel axis theorem can be applied with the stretch rule and perpendicular axis theorem to find moments of inertia for a variety of shapes.

The mass moment of inertia of a body around an axis can be determined from the mass moment of inertia around a parallel axis through the center of mass.

Parallel axes rule for area moment of inertia

Derivation [edit]

We may assume, without loss of generality, that in a Cartesian coordinate system the perpendicular distance between the axes lies along the *x*-axis and that the center of mass lies at the origin. The moment of inertia relative to the *z*-axis is then

$$I_{\mathrm{cm}} = \int (x^2 + y^2)\, dm.$$

The moment of inertia relative to the axis z', which is at a distance D from the center of mass along the *x*-axis, is

$$I = \int \left[(x - D)^2 + y^2\right]\, dm.$$

Expanding the brackets yields

$$I = \int (x^2 + y^2)\, dm + D^2 \int dm - 2D \int x\, dm.$$

The first term is I_{cm} and the second term becomes MD^2. The integral in the final term is a multiple of the x-coordinate of the center of mass – which is zero since the center of mass lies at the origin. So, the equation becomes:

$$I = I_{\mathrm{cm}} + MD^2.$$

Figure 2: (a) Perpendicular axis theorem that states Izz = Ixx + Iyy (b) Parallel axis theorem derivation from Wikipedia

4.1 Figures and Variables Labelled

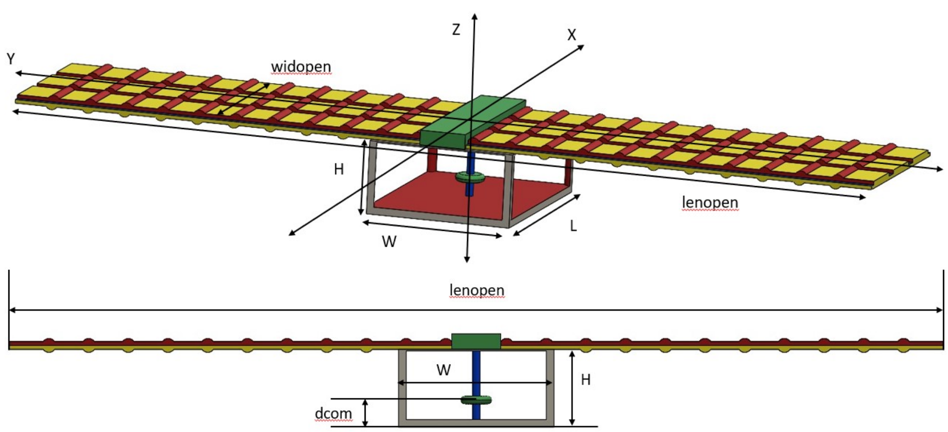


Figure 3: (a) 3D representation of COM position (b) Labelled dimensions for COM

These are the reference figures we use for applying the MOI formulas and the parallel and perpendicular axes theorems to get the MOI about the actual COM that is dcom away from the face opposite to the solar panels.

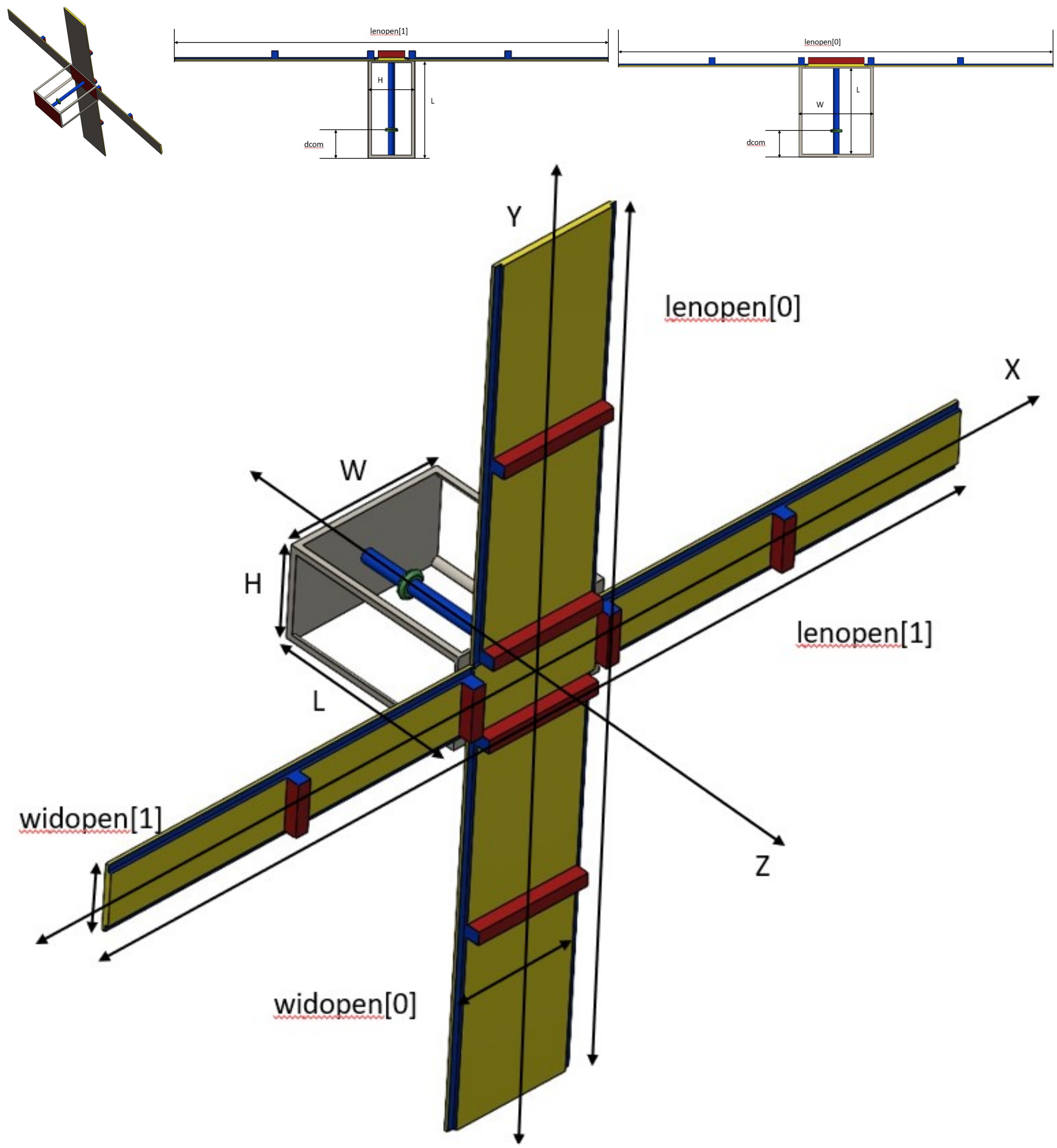


Figure 4: (a) 3D representation of COM position (b) Labelled dimensions for COM

For the folded panels, both the panels and the body are simple rectangles, so the MOI formula from 1 can be used directly for both the panels as well as the body plate.

However, for the outer cover panels, only the body is a simple rectangle. So, we creatively divide the plus shape into simple rectangles centered at the axis in question. For instance, for Ixx, we take the rectangle $lenopen[0] \times widopen[0]$ and 2 other rectangles that are effectively $(lenopen[1] - widopen[0])/2 \times widopen[1]$. On either side of our original rectangle, together, they act as one rectangle about the x-axis that is $(lenopen[1] - widopen[0]) \times widopen[1]$ in size.

The equations are described in the subsequent section.

4.2 Equations for intermediate values

Calculating the common variable, mbody:

$$mbody = mass - mtot \tag{1}$$

Mass of the rest of the body subtracted from the mass of the solar panels. Note that this mass should not turn out to be negative. Check if it is negative, and if so, then this configuration cannot be used for that size.

4.2.1 Folded Configuration

Calculating dcom:

$$dcom = (mtot/mass) \cdot H \tag{2}$$

This is derived from the equation for the center of mass.

Calculating moipanels[0,1,2]:

$$moipanels[0] = (mtot \cdot lenopen^2)/12 \tag{3}$$

This is Ixx for the panels from the equation given in Figure 1 (a).

$$moipanels[1] = (mtot \cdot widopen^2)/12 \tag{4}$$

This is Iyy.

$$moipanels[2] = moipanels[0] + moipanels[1] \tag{5}$$

This is Izz for the panels from the perpendicular axis theorem in Figure 2 (a)

Similarly, calculating moibody[0,1,2]:

$$moibody[0] = (mbody \cdot W^2)/12 \tag{6}$$

This is Ixx for the body.

$$moibody[1] = (mbody \cdot L^2)/12 \tag{7}$$

This is Iyy.

$$moibody[2] = moibody[0] + moibody[1] \tag{8}$$

This is Izz.

4.2.2 Outer Cover Configuration

Calculating dcom:

$$dcom = (mtot/mass) \cdot L \tag{9}$$

Calculating mpa:

$$mpa = \frac{mtot}{lenopen[0] \cdot widopen[0] + lenopen[1] \cdot widopen[1] - widopen[0] \cdot widopen[1]} \tag{10}$$

With mass per unit area, we can use the area MOI equations as shown in Figure 1 (b) by simply multiplying the mass per unit area with the area moment of inertia. You can check that upon multiplication, the equations become the same as Figure 1 (a).

Calculating moipanels[0,1,2]:

$$moipanels[0] = (mpa/12) \times (widopen[0] \cdot lenopen[0]^3 + (lenopen[1] - widopen[0]) \cdot widopen[1]^3) \tag{11}$$

This is Ixx for the panels

$$moipanels[1] = (mpa/12) \times (widopen[1] \cdot lenopen[1]^3 + (lenopen[0] - widopen[1]) \cdot widopen[0]^3) \tag{12}$$

This is Iyy for the panels

$$moipanels[2] = moipanels[0] + moipanels[2] \tag{13}$$

This is Izz for the panels. It is the same equation as the folded panels since the panels are in a single plane in both cases and the MOI about the axis normal to the plane is the sum of the 2 MOIs in the plane.

Calculating moibody[0,1,2]:

$$moibody[0] = (mbody \cdot H^2)/12 \tag{14}$$

This is Ixx for the body.

$$moibody[1] = (mbody \cdot W^2)/12 \tag{15}$$

This is Iyy.

$$moibody[2] = moibody[0] + moibody[1] \tag{16}$$

This is Izz.

4.3 Equations for output values

Only the MOI value equations are needed here. Powmass and Powvol equations are given where they are defined directly.

4.3.1 Folded Panels Configuration

Calculating MOI[0,1,2]:

$$MOI[0] = moipanels[0] + mtot \cdot (H - dcom)^2 + moibody[0] + mbody \cdot dcom^2 \tag{17}$$

This is Ixx for the entire craft found using the parallel axis theorem as shown in Figure 2 (b).

$$MOI[1] = moipanels[1] + mtot \cdot (H - dcom)^2 + moibody[1] + mbody \cdot dcom^2 \tag{18}$$

This is Iyy of the entire craft

$$MOI[2] = moipanels[2] + moibody[2] \tag{19}$$

This is Izz for the entire craft. Note that since the z axis passes through the COM for the panels as well as the body, there is no additional distance to add to get the total MOI using the perpendicular axis theorem.

4.3.2 Outer Cover Configuration

Calculating MOI[0,1,2]:

$$MOI[0] = moipanels[0] + mtot \cdot (L - dcom)^2 + moibody[0] + mbody \cdot dcom^2 \tag{20}$$

This is Ixx for the entire craft

$$MOI[1] = moipanels[1] + mtot \cdot (L - dcom)^2 + moibody[1] + mbody \cdot dcom^2 \tag{21}$$

This is Iyy of the entire craft

$$MOI[2] = moipanels[2] + moibody[2] \tag{22}$$

This is Izz for the entire craft.

5 Tips for making the code

Note that the equations for MOI for both configurations are the same. Try to make a dedicated function that can just be copied over in the two folders.

Make functions for things that are to be done more frequently. Isolate complicated calculations broken down into smaller steps so that it is easier to understand and debug the code.

References

[1] P. M. W. T. . L. R. Li, J. Design of attitude control systems for cubesat-class nanosatellite. *Journal of Control Science and Engineering, 2013. https://doi.org/10.1155/2013/657182*, 2013.

[2] . P. M. A. n. Zeledon, R. A. Attitude dynamics and control of a 3u cubesat with electrolysis propulsion. *American Institute of Aeronautics and Astronautics.*