Gmail Anirudh Nagulapalli <munna.anirudh@gmail.com>

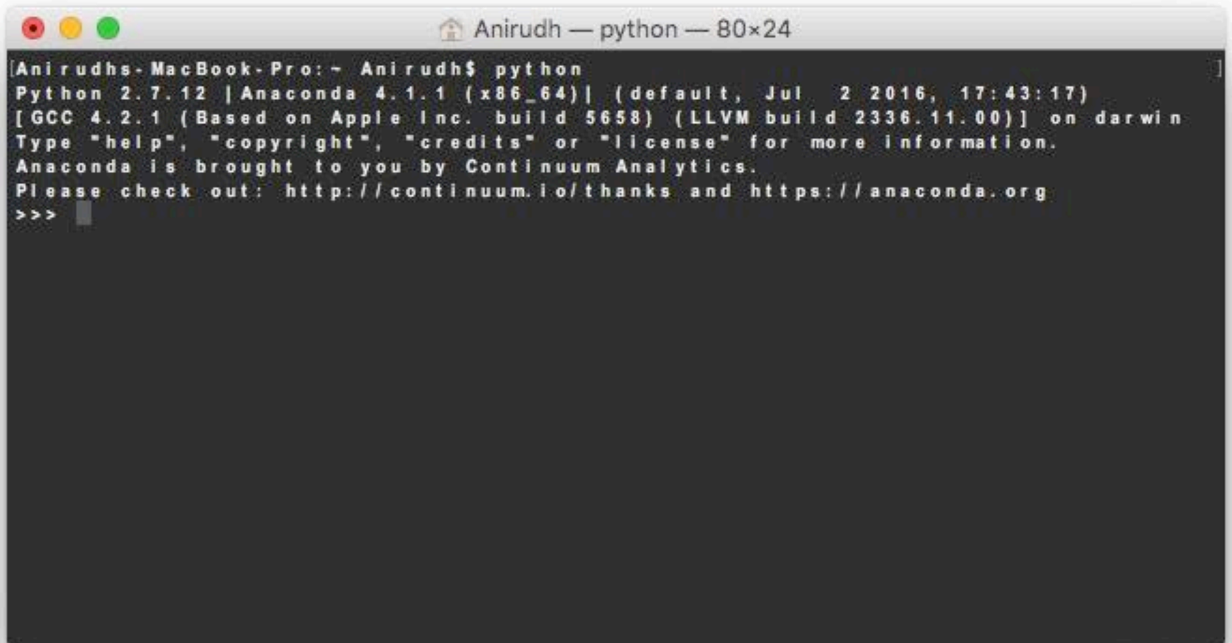
HOME WORK 00

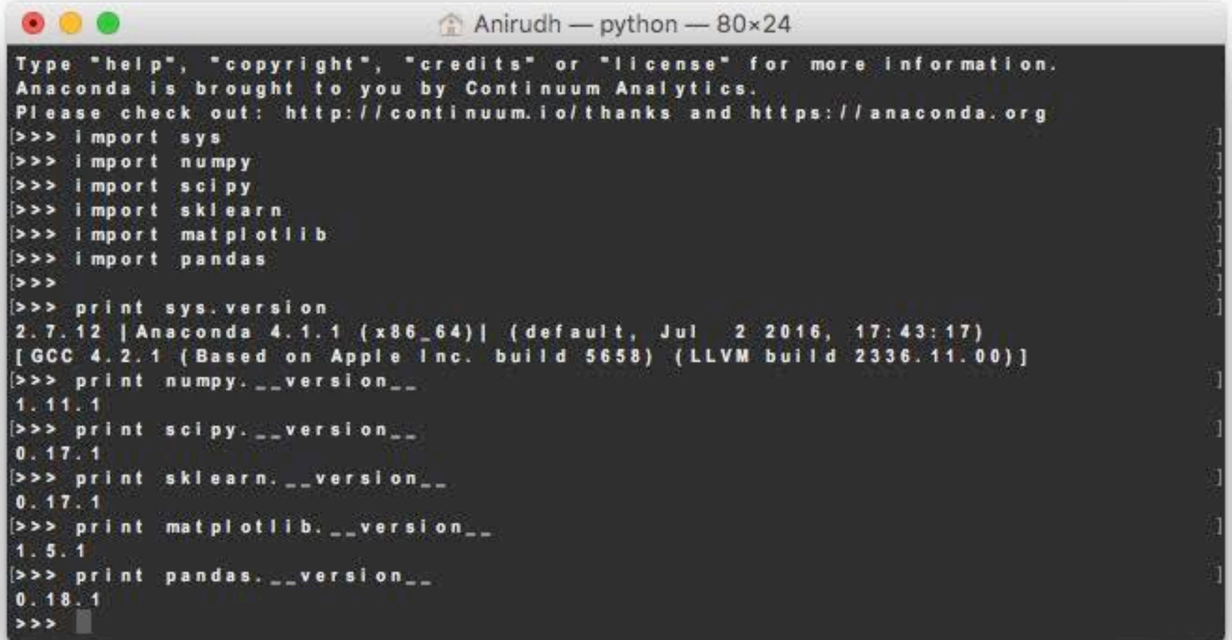
1 message

munna.anirudh@gmail.com <munna.anirudh@gmail.com> Tue, Sep 6, 2016 at 10:26 AM
To: anirudh.nagulapalli1@marist.edu

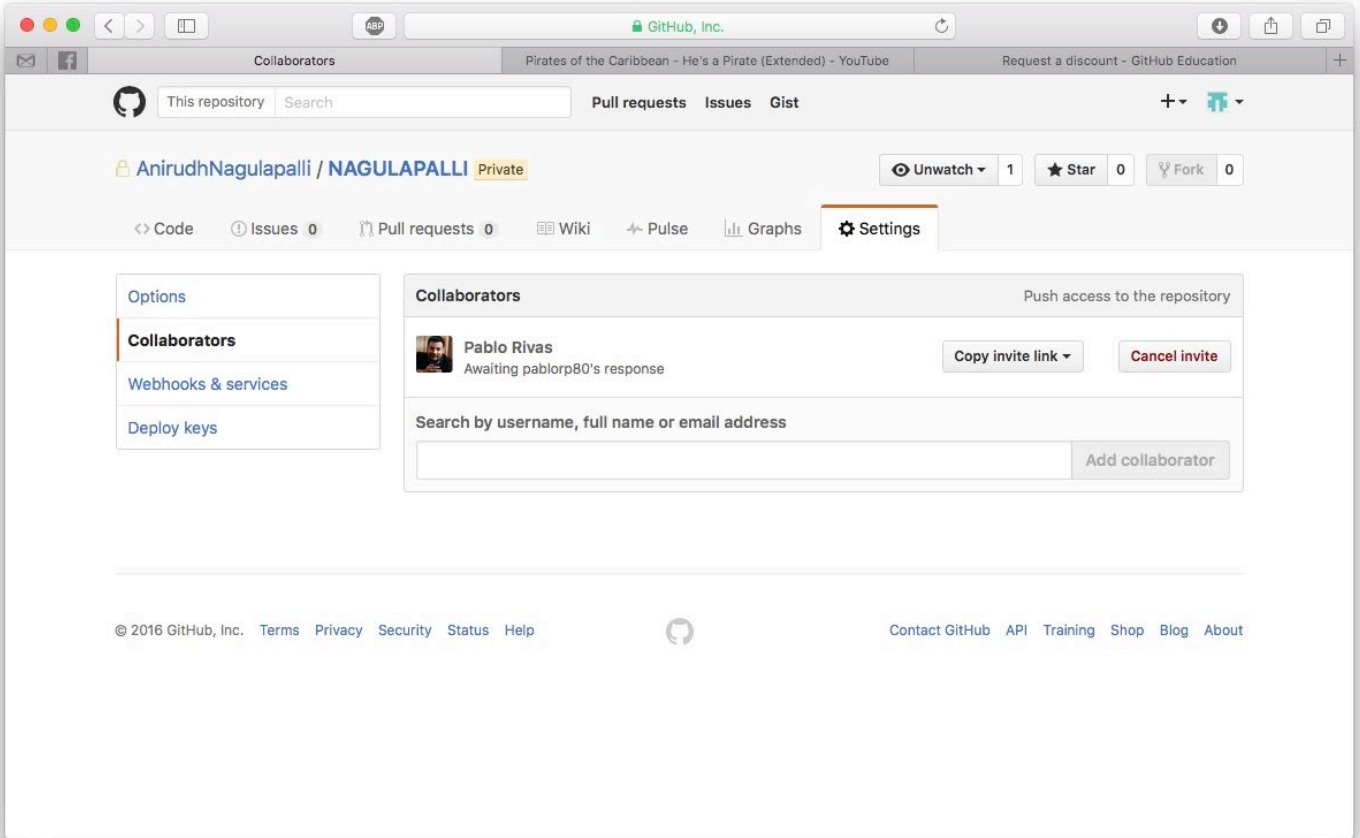
HOME WORK 00

Deliverable 1



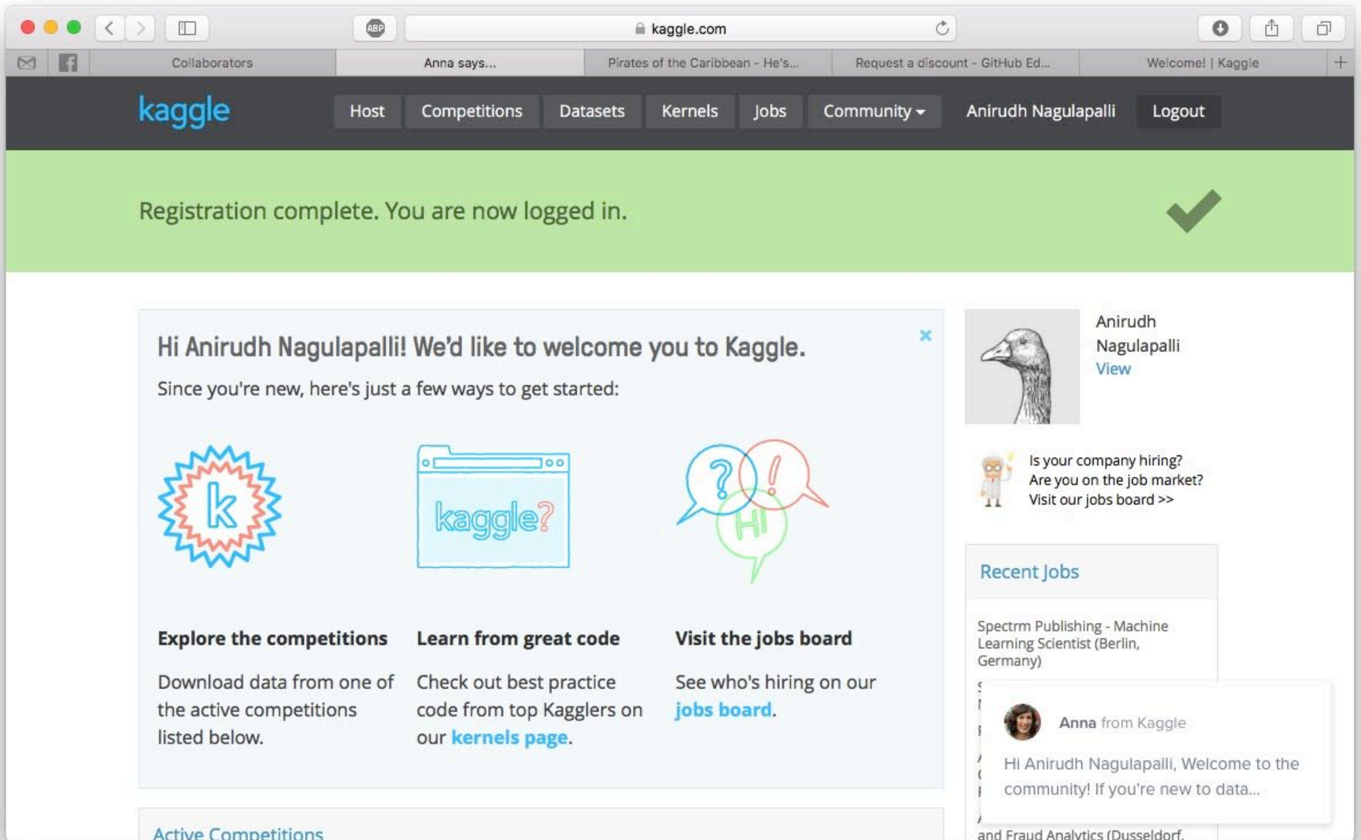


Deliverable 2
Github Username: Anirudh Nagulapalli
Link to repository: https://github.com/AnirudhNagulapalli/NAGULAPALLI.git

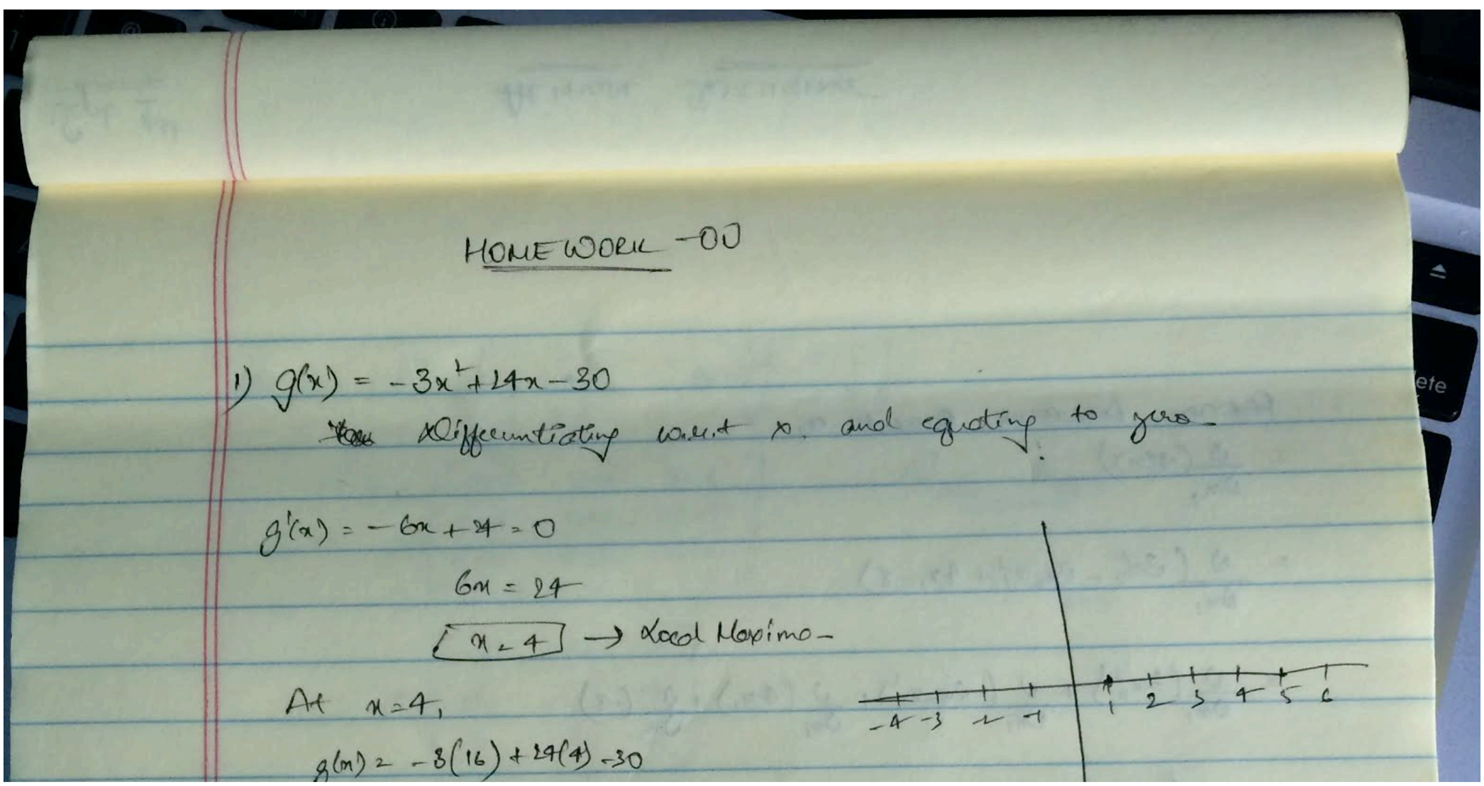


Deliverable 3

Kaggle username: AnirudhNagulapalli



Deliverable 4



HOME WORK - 00

1) $g(x) = -3x^2 + 24x - 30$

Differentiating w.r.t x and equating to zero.

$g'(x) = -6x + 24 = 0$

$6x = 24$

$\boxed{x = 4} \rightarrow$ Local Maxima

At $x = 4$,

$g(x) = -3(16) + 24(4) - 30$

$= -48 + 96 - 30$

$= \underline{18}$

Second Derivative $\rightarrow g''(x) = \underline{-3} < 0$

So, Local Maxima = Global Maxima

$x = 0 \rightarrow g(x) = -30$

$x = -1 \rightarrow g(x) = -57$

$x = -2 \rightarrow g(x) = -90$

$x = 1 \rightarrow g(x) = -9$

$x = 2 \rightarrow g(x) = 6$

$x = 3 \rightarrow g(x) = 15$

$x = 4 \rightarrow g(x) = 18$

$x = 5 \rightarrow g(x) = 15$

2) $f(x) = 3x_0^3 - 2x_0 x_1^2 + 4x_1 - 8$

PARTIAL DERIVATIVE w.r.t x_0,

$$\frac{\partial}{\partial x_0} f(x)$$

$$= \frac{\partial}{\partial x_0}\left(3x_0^3 - 2x_0 x_1^2 + 4x_1 - 8\right)$$

$$= \frac{\partial}{\partial x_0}(3x_0^3) + \frac{\partial}{\partial x_0}(2x_0 x_1^2) + \frac{\partial}{\partial x_0}(4x_1) + \frac{\partial}{\partial x_0}(-8)$$

$$= 3\frac{\partial}{\partial x_0}(x_0^3) + -2x_1^2\frac{\partial}{\partial x_0}(x_0) + 4x_1\frac{\partial}{\partial x_0}\left(\frac{1}{x_0}\right) - 8\frac{\partial}{\partial x_0}(1)$$

$$= 3(3x_0^2) - 2x_1^2(1) + 0 + 0 \Rightarrow \underline{9x_0^2 - 2x_1^2}$$

PARTIAL DERIVATIVE w.r.t x_1

$$\frac{\partial}{\partial x_1}(f(x))$$

$$= \frac{\partial}{\partial x_1}\left(3x_0^3 - 2x_0 x_1^2 + 4x_1 - 8\right)$$

$$= \frac{\partial}{\partial x_1}(3x_0^2) + \frac{\partial}{\partial x_1}(-2x_0x_1^2) + \frac{\partial}{\partial x_1}(4x_1) + \frac{\partial}{\partial x}(-8)$$

$$= 3x_0^2 \frac{\partial}{\partial x_1}(1) + -2x_0 \frac{\partial}{\partial x_1}(x_1^2) + 4\frac{\partial}{\partial x_1}(x_1) - 8\frac{\partial}{\partial x}(1)$$

$$= 0 - 2x_0(2x_1) + 4(1) - 0 \Rightarrow \underline{\underline{-4x_0x_1 + 4}}$$

3) $A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix}$

(a) We Cannot multiply.

For two matrices A, B to be multiplicable,

If $A_{p\times q}$ and $B_{r\times s}$

$\Rightarrow$ Number of ~~rows~~ Columns in A should be equal to number of rows in B.

$$A_{2\times 3} \quad B_{2\times 3}$$

Not Matching

(b) $A^T = \begin{bmatrix} 1 & 2 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix}$

$A^T \cdot B = \begin{bmatrix} -2+0 & 0-2 & 5+8 \\ -8+0 & 0+1 & 20-4 \\ 6+0 & 0-3 & -15+12 \end{bmatrix}$ $A^T_{3\times \boxed{2}} \cdot B_{\boxed{2}\times 3}$

$= A^T \cdot B_{3\times 3}$

$\begin{bmatrix} -2 & -2 & 13 \end{bmatrix}$

-12
48
36

$$= \begin{bmatrix} -8 & 1 & 16 \\ 6 & -3 & -3 \end{bmatrix}$$

RANK $\longrightarrow$ $A^T \cdot B = \begin{bmatrix} -2 & -2 & 13 \\ -8 & 1 & 16 \\ 6 & -3 & -3 \end{bmatrix}$ $R_2 \leftarrow 4R_1 - R_2$, $R_3 \leftarrow 3R_1 - R_3$

$$= \begin{bmatrix} -2 & -2 & 13 \\ 0 & -9 & \cancel{68}\,36 \\ 0 & -3 & 42 \end{bmatrix} R_3 \to 3R_3 - R_2$$

39 $\quad 13 \times 4 = 52,\ 52 - 16 = \cancel{68}\ 36$

$$= \begin{bmatrix} -2 & -2 & 13 \\ 0 & -9 & 36 \\ 0 & 0 & 90 \end{bmatrix}$$

$42 \times 3 = 126,\ 126 - 36 = 90$

RANK $\to$ 3

$96 - 24 = 72$

$18 \times 13 = 54 + 180 = 234$

Determinant $\longrightarrow -2(-3+48) + 2(24-96) + 13(24-6)$

$\Rightarrow -90 - 144 + 234$

$= 0$

So, RANK = 3

$-y = u$

$dy = -du$

(c) $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$AB^T + C^{-1} = ?$

$$AB^T = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & -1 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0-15 & 0-4-12 \\ -4+0+15 & 0+1+12 \end{bmatrix} = \begin{bmatrix} -17 & -16 \\ 11 & 13 \end{bmatrix}$$

$$C^{-1} = \frac{1}{|C|} \times [\operatorname{adj} C]$$

$$= \frac{1}{(2-0)} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$AB^T + C^{-1} = \begin{bmatrix} -17 & -16 \\ 11 & 13 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} -16 & -16 \\ 11 & 13.5 \end{bmatrix}$$

8) $P(Y=y) = \begin{cases} e^{-y} & ; y \geq 0 \\ 0 & ; y < 0 \end{cases}$

(a) $\int_{-\infty}^{\infty} P(Y=y) \cdot dy = 1$

$\Rightarrow \int_{-\infty}^{0} P(y)\,dy + \int_{0}^{\infty} P(y)\,dy$

$\Rightarrow 0 + \int_{0}^{\infty} e^{-y}\,dy \quad u + [e^{-u}]$

if y

ther

$-y = u$

$dy = -du.$

$\Rightarrow -\int_0^{\infty} e^u . du$

$\Rightarrow -[e^u]_0^{-\infty}$

$\Rightarrow -[e^{-\infty} - e^0] \Rightarrow 0-(-1) = 1$

$\because e^{\infty} \to \infty$ and $e^{-\infty} \to 0.$

(b) $\mu_Y = E[Y] = \int_{y=-\infty}^{\infty} P(Y=y).y.dy$

$\Rightarrow \int_{y=-\infty}^{0} 0.y.dy + \int_0^{\infty} e^{-y}.y.dy$

$\Rightarrow 0 + \int_0^{\infty} e^{-y}.y.dy$

~~Integrating by parts,~~

$\Rightarrow \cancel{y.\int_0^{\infty} e^{-y} - \int \frac{d(y)}{dy} . \int_0^{\infty} e^{-y}dy}.$

$\Rightarrow \cancel{y.(1) - \int 1.1.dy}$

$\cancel{e^{-y} \text{ let } e^{-y} = u}$

$\Rightarrow \cancel{e^{-y}.(-y).dy = du.}$

$\Rightarrow \cancel{y.e^{-y}dy = -du.}$

$\Rightarrow \cancel{\int^0 -du}$

$\Rightarrow \cancel{(1-0) = 1}$

~~Integrating by parts.~~

$\Rightarrow \cancel{y.\int_0^{\infty} e^{-y}dy - \int_0^{\infty}\left(\frac{d(y)}{dy} . \int_0^{\infty} e^{-y}.dy\right)dy}$

$\Rightarrow \cancel{y.(1) - \int_0^{\infty}(1.1)dy.}$

$\Rightarrow y - [y]_0^{\infty}$

INTEGRATING BY PARTS

$\Rightarrow y\int_0^{\infty} e^{-y}dy - \int_0^{\infty}\left(1. \int_0^{\infty} e^{-y}dy\right)dy$

$\Rightarrow y(1)\Big|_0^{\infty} - \int_0^{\infty} 1.dy$

$\Rightarrow [y]_0^{\infty} - [y]_0^{\infty} = 0$

(c) $\sigma^2 = Var[Y] = \int_{y=-\infty}^{\infty} p(Y=y)\,(y-\mu_Y)^2\,dy$

$\Rightarrow \int_{-\infty}^{0} p(Y=y)\cdot(y-\mu_Y)^2 dy + \int_{0}^{\infty} p(Y=y)\cdot(y-\mu_Y)^2 dy$

$\Rightarrow \int_{-\infty}^{0} 0\cdot(y-1)^2 dy + \int_{0}^{\infty} e^{-y}(y-1)^2 dy$

$\Rightarrow 0 + \int_{0}^{\infty} e^{-y}(y^2-2y+1)\,dy$

$\Rightarrow \int_{0}^{\infty} e^{-y}\cdot y^2 dy + 2\int_{0}^{\infty} e^{-y}\cdot y\,dy + \int_{0}^{\infty} e^{-y}\cdot dy$

We know, $\int_{0}^{\infty} e^{-y}\cdot y\cdot dy = 0$; $\int_{0}^{\infty} e^{-y} dy = 1$

$\Rightarrow \int_{0}^{\infty} e^{-y} y^2\cdot dy + 0 + 1$

~~$e^{-y} = u. \Rightarrow \log_e u = -y \Rightarrow y = -\log_e u$~~

~~$\Rightarrow -e^{-y}\cdot dy = du$~~

$\Rightarrow$ Integrating by parts

$\Rightarrow y^2 \int_{0}^{\infty} e^{-y} dy - \int_{0}^{\infty} \left(2y\cdot \int_{0}^{\infty} e^{-y}\cdot dy\right) dy$

$\Rightarrow \left[y^2(1)\right]_0^{\infty} - 2\int_{0}^{\infty} y\cdot(1)\,dy.$

$\Rightarrow y^2 - \cancel{2}\left[\frac{y^2}{\cancel{2}}\right]_0^{\infty} = 0$

$\Rightarrow \sigma^2 = \underline{1}$

(d) $E[Y|Y \geq 10] \Rightarrow \int_{10}^{\infty} e^{-y} \cdot y \cdot dy$

$\Rightarrow \int y \int_{10}^{\infty} e^{-y} dy - \int_{10}^{\infty}\int_{10}^{\infty} e^{-y} dy \cdot dy.$

$\Rightarrow [y(1)]_0^{\infty} - \int_{10}^{\infty} (1)\, dy.$

$\Rightarrow y(1)_{10}^{\infty} - (y)_{10}^{\infty} = \underline{0}.$

4) Normal Distribution (or) Gaussian Distribution

Probability Density =

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu \rightarrow$ Mean

$\sigma \rightarrow$ Standard Deviation

$\sigma^2 \rightarrow$ Variance

Conditions for a Vector to have Multivariate Gaussian Distribution

$\rightarrow$ Every linear combination of its components is normally distributed.

$\rightarrow$

$\rightarrow \varphi_x(u) = \exp\left(iu'\mu - \frac{1}{2}u'\Sigma u\right)$

Bernoulli Distribution

$Pr(x=1) = 1 - Pr(x=0) = 1-q = p$

Probability Mass function $\rightarrow f(k;p) = \begin{cases} p & \text{if } k=1 \\ 1-p & \text{if } k=0 \end{cases}$

$\rightarrow f(k;p) = p^k(1-p)^{1-k}$ for $k \in \{0,1\}$

Binomial Distribution

$f(k;n,p) = Pr(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$

where, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Exponential Distribution :- P.D.F $\rightarrow$ $f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

5) Bernoulli Distribution is a subset of Binomial Distribution.

Bernoulli Random Variable has two outcomes 0 or 1.

So, if Prob. of 0 $\rightarrow$ p

Prob. of 1 $\rightarrow$ (1-p)

$f(k;p) = p^k (1-p)^{1-k} \rightarrow$ P.M.F

On the other hand,

Binomial Distribution takes the sum of independent and identically distributed Bernoullis Random Variable.

So, $f(k;n,p) = \frac{n!}{k!\,(n-k)!} \cdot p^k \cdot (1-p)^{1-k}$

7) We know that,

$$d = \sqrt{(a_{1x}-a_{2x})^2 + (a_{1y}-a_{2y})^2}$$

Given,

$$x^* = \arg_x \min \|x-y\|_2^2 \quad ; \quad x \in Z$$

We know,

L_2 Norm $= \|a_1 - a_2\| = \left((a_1-a_2)^T (a_1-a_2)\right)^{1/2}$

Here, $a_1 = x$; $a_2 = y$

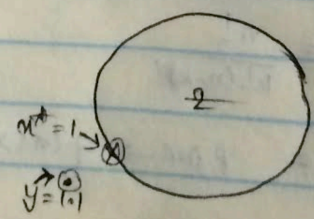
(a) If $y = 1.1$; $Z \in N$; $x^* = ?$

$$\Rightarrow x^* = \arg \min \|x - 1.1\|_2^2$$
$$= \min\left(\sqrt{(x-1.1)}\right)^2 \Rightarrow x - 1.1 = 0$$
$$x = 1.1$$

But, $x \in ...$

as $x \notin Z$. So, $x = \underline{1}$

(b)



6) $X \sim N(2,3)$

Expected Value of X is Zero as there are no natural numbers between 2 and 3.

$X \sim N(2,3) \longrightarrow$ Not Inclusive.