Wave and Optics

(Module-I)

$\psi(x,t) = A\sin(kx - \omega t + \phi)$ [Calculation]

$\psi(\vec{r},t) = A\sin(k\vec{r} - \omega t + \phi)$ [Notation]

A = amplitude.

k = Wave Propagation vector $= 2\pi/\lambda$

ω = Angular frequency $= 2\pi f$ or $2\pi\nu$

ϕ = Initial phase.

Wave velocity $\leftarrow v = \dfrac{\omega}{k} = \dfrac{2\pi f}{2\pi/\lambda} = f\lambda$

Differential form Of Wave Equation-

$$\psi(x,t) = A\sin(kx - \omega t + \phi) \quad \text{—} (1)$$

$$\frac{\partial\psi}{\partial t}(x,t) = -\omega A\cos(kx - \omega t + \phi)$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2 A\sin(kx - \omega t + \phi)$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi \quad \text{—} (2)$$

$$\frac{\partial\psi}{\partial x} = kA\cos(kx - \omega t + \phi)$$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2 A\sin(kx - \omega t + \phi)$$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi.$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2\psi}{\partial t^2}} \qquad \boxed{\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}}$$

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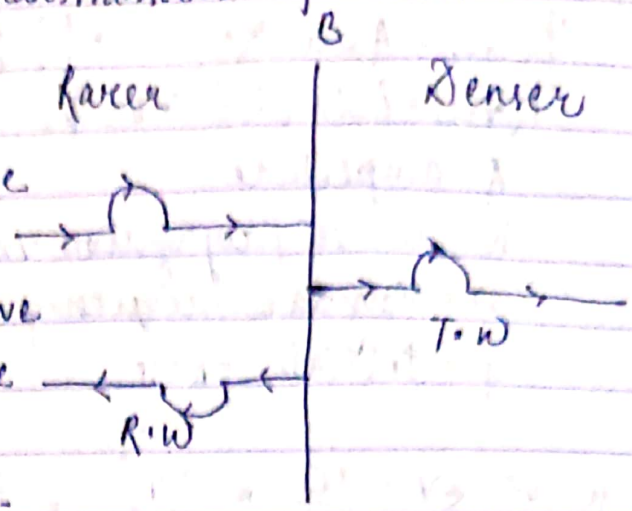
Reflection and Transmission Of Waves-



R·W → Reflected Wave

T·W → Transmitted Wave

I·W → Incident Wave

Rarer to Denser-

	R·W	T·W
(i) Amplitude	Less wrt I·W	"
(ii) Speed	Same wrt I·W.	Less wrt I·W
(iii) Frequency	Same	Same.
(iv) Wave length	Same	less
(v) Phase	Opp.	Same.

Here boundary will serve as obstruction

In case of denser to rarer- boundary will not serve as obstruction.

14/8/19 Superposition of waves-

The principle of superposition states that when 2 or more waves act on a particle of a medium simultaneously. then the resultant displacement of that particle is the algebraic sum of displacemo of the same particle by the individual waves.

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$\psi = \psi_1 + \psi_2$ (if both are in same direction).
$\psi = \psi_1 - \psi_2$ (if both are in opp. direction)

Two types Of Superposition-
(i) Coherent Superposition (Phase diff. is constant).
(ii) In-coherent Superposition (Phase diff. changes frequently and randomly)

Coherent - $I_{res} = A^2 \neq I_1 + I_2$

Incoherent - $I = I_1 + I_2$

H.W/ Difference b/w Coherent and Incoherent Sources

<u>Two Beam Superpositing</u>:
Let us take two waves having diff. amplitude same frequency different phases propagating in same direction superpose

$$\psi_1 = A_1 \sin(kx - \omega t + \phi_1)$$
$$\psi_2 = A_2 \sin(kx - \omega t + \phi_2)$$

$$\psi = A_1 \sin(kx - \omega t + \phi_1) + A_2 \sin(kx - \omega t + \phi_2)$$
$$= A_1 [\sin(kx - \omega t) \cdot \cos\phi_1 + \cos(kx - \omega t) \sin\phi_1]$$
$$+ A_2 [\sin(kx - \omega t) \cdot \cos\phi_2 + \cos(kx - \omega t) \cdot \sin\phi_2]$$

$$= \sin(kx - \omega t)[A_1 \cos\phi_1 + A_2 \cos\phi_2] +$$
$$\cos(kx - \omega t)[A_1 \sin\phi_1 + A_2 \sin\phi_2]$$

Let, $A\cos\phi = A_1 \cos\phi_1 + A_2 \cos\phi_2$
$A\sin\phi = A_1 \sin\phi_1 + A_2 \sin\phi$
$$= \sin(kx - \omega t) A\cos\phi + \cos(kx - \omega t) A\sin\phi$$
$$= A\sin(kx - \omega t + \phi)$$

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$$\boxed{\psi = A\sin(kx - \omega t + \phi)}$$

Amplitude of the resultant wave.

Q. Show that the superposition of two harmonic waves produces a harmonic wave. then find out its resultant amplitude and phase.

$$I = A^2 = (A\cos\phi)^2 + (A\sin\phi)^2$$
$$= (A_1\cos\phi_1 + A_2\cos\phi_2)^2 + (A_1\sin\phi_1 + A_2\sin\phi_2)^2$$
$$= A_1^2\cos^2\phi_1 + A_2^2\cos^2\phi_2 + 2A_1A_2\cos\phi_1\cos\phi_2$$
$$+ A_1^2\sin^2\phi_1 + A_2^2\sin^2\phi_2 + 2A_1A_2\sin\phi_1\sin\phi_2$$
$$= A_1^2 + A_2^2 + 2A_1A_2(\cos\phi_1\cos\phi_2 + \sin\phi_1\sin\phi_2)$$

$$\boxed{I = A_1^2 + A_2^2 + \underbrace{2\cos(\phi_1 - \phi_2)A_1A_2}_{\text{Interference term}}}$$

($\phi_1 - \phi_2$ = Phase diff.)

$$\tan\phi = \frac{A\sin\phi}{A\cos\phi} = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2} \Rightarrow \phi = \tan^{-1}\left(\frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}\right)$$

Coherent Superposition:-

Case-I:-

Let $(\phi_1 - \phi_2) = 0, \pm 2\pi, \pm 4\pi, \ldots, \pm 2n\pi$.

$$I = A^2 = (A_1 + A_2)^2 \longrightarrow I_{max}.$$

Special Case:

If $A_1 = A_2 = a$

$$I = I_{max} = 4a^2$$

Case II:-

$$(\phi_1 - \phi_2) = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots \pm(2n+1)\pi$$

$$\cos(\phi_1 - \phi_2) = -1.$$

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Prove that the coherent superposition can produce an interference pattern.

$I = (A_1 - A_2)^2 = I_{min}$

<u>Special Case:</u> If $A_1 = A_2 = a$.

$I_{min} = 0$.

<u>Incoherent Superposition-</u>

Time avg. value $\langle \cos(\phi_1 - \phi_2) \rangle = 0$.

$$I = A^2 = A_1^2 + A_2^2$$
$$= I_1 + I_2$$

Q. Given $y_1 = a\sin(\omega t - kx)$ and $y_2 = a\cos(\omega t - kx)$

Q. If two waves are represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin(\omega t + \pi/3)$ interferes at a point. find the amplitude of the resultant wave. $\pi/3$

19/8/19 Q. Show that the resultant wave due to the superposition of two harmonic waves of same frequency diff. amplitude, diff. phases and travelling in same direction is also harmonic. Find the ~~result~~ amplitude and phase of resultant wave.

Q. Coherent superposition results an interference pattern show it by taking of diff. amplitude. diff. phases propagating in same direction in the

Multiple Beam Superposition.

Let us consider 'n' no. of waves having diff. amplitude same frequency diff. phase propogating in same direction superpose.

$$\psi_1 = A_1 \sin(kx - \omega t + \phi_1).$$
$$\psi_2 = A_2 \sin(kx - \omega t + \phi_2).$$
$$\vdots$$
$$\psi_n = A_n \sin(kx - \omega t + \phi_n).$$

After superposition,

The resultant wave.

$$\psi = \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n.$$
$$= \sum_{i=1}^{n} \psi_i$$

$$\psi = A_1 \sin(kx - \omega t + \phi_1) + A_2 \sin(kx - \omega t + \phi_2) \cdots + A_n \sin(kx - \omega t + \phi_n)$$

$$\psi = A \sin(kx - \omega t + \phi).$$

$$A \sin\phi = A_1 \sin\phi_1 + A_2 \sin\phi_2 + \cdots + A_n \sin\phi_n.$$
$$A \cos\phi = A_1 \cos\phi_1 + A_2 \cos\phi_2 + \cdots + A_n \cos\phi_n.$$

$$I = A^2 = (A\sin\phi)^2 + (A\cos\phi)^2$$
$$= \cancel{A_1^2 + A_2^2 + \cdots + A_n^2 + n\cos(\phi_1 - \phi_2) A_1 A_2 \cdots A_n}$$
$$= \sum_{i=1}^{n} A_i^2 + 2\sum_{i,j=1}^{n} A_i A_j \cos(\phi_i - \phi_j)$$

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For $n=2$.

$$I = A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2)$$

For maximum intensity, $I = A_1^2 + A_2^2 + 2A_1A_2$
$= (A_1 + A_2)^2$.

For $n=3$.

$$I = A_1^2 + A_2^2 + A_3^2 + 2A_1A_2\cos(\phi_1-\phi_2) + 2A_2A_3\cos(\phi_2-\phi_3) + 2A_1A_3\cos(\phi_1-\phi_3).$$

[Special case. $4a^2 = (2a)^2$ [Coherent Superposition]]

For maximum intensity

$$I = A^2 = A_1^2 + A_2^2 + A_3^2 + 2A_1A_2 + 2A_2A_3 + 2A_1A_3.$$

Special case, $A_1 = A_2 = A_3 = a$.

$I = 9a^2 = (3a)^2$ [Coherent Superposition]

Incoherent superposition $\langle\cos(\phi_i - \phi_j)\rangle = 0$.

$n = 2 = I = A_1^2 + A_2^2$

Special case, $I = 2a^2$

$n = 3$, $I = A_1^2 + A_2^2 + A_3^2$

Special case, $I = 3a^2$.

For 'n' no. of waves-

$$I_{con} = (na)^2$$
$$I_{incon} = na^2$$

$$\boxed{I_{con} = n \cdot I_{incoh.}}$$

Coherent superposition with phases of successive waves suffering by a constant amount.

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$$1+x+x^2+x^3+\dots\dots+x^m = \frac{1-x^{m+1}}{1-x}.$$

$$\psi_1 = A\sin(kx-\omega t)$$
$$\psi_2 = A\sin(kx-\omega t+\theta)$$
$$\psi_3 = A\sin(kx-\omega t+2\theta)$$
$$\psi_4 = A\sin(kx-\omega t+3\theta)$$
$$\vdots$$
$$\psi_n = A\sin(kx-\omega t+(n-1)\theta)$$

$$\psi_1 = Ae^{i(kx-\omega t)}$$
$$\psi_2 = Ae^{i(kx-\omega t+\theta)}$$
$$\psi_3 = Ae^{i(kx-\omega t+2\theta)}.$$
$$\vdots$$
$$\psi_n = Ae^{i(kx-\omega t+(n-1)\theta)}.$$

After Superposition, the resultant wave,

$$\psi = \psi_1+\psi_2+\psi_3+\dots+\psi_n$$
$$= Ae^{i(kx-\omega t)} + Ae^{i(kx-\omega t+\theta)} + Ae^{i(kx-\omega t+2\theta)} + \dots$$
$$\dots + Ae^{i(kx-\omega t+(n-1)\theta)}.$$

$$= Ae^{i(kx-\omega t)}\left[1+e^{i\theta}+e^{2i\theta}+\dots\dots+e^{i(n-1)\theta}\right]$$

$$= Ae^{i(kx-\omega t)}\left[\frac{1-e^{in\theta}}{1-e^{i\theta}}\right]$$

$$\psi = Ae^{i(kx-\omega t)}\left[\frac{e^{in\theta}-1}{e^{i\theta}-1}\right]$$

$$= Ae^{i(kx-\omega t)}\frac{e^{in\theta/2}}{e^{i\theta/2}}\left[\frac{e^{in\theta/2}-e^{-in\theta/2}}{e^{i\theta/2}-e^{-i\theta/2}}\right]$$

$$e^{i\theta} = \cos\theta + i\sin\theta.$$
$$e^{-i\theta} = \cos\theta - i\sin\theta.$$

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$$\psi = A e^{i(kx-\omega t)} e^{i\frac{(n-1)}{2}\theta} \left[\frac{2i\sin n\theta/2}{2i\sin\theta/2}\right]$$

$$\boxed{\psi = \frac{A\sin n\theta/2}{\sin\theta/2} e^{i(kx-\omega t+(\frac{n-1}{2})\theta)}}$$

$$\boxed{\psi = R e^{i(kx-\omega t+\beta)}}$$

R → Amplitude of resultant wave

β → Phase.

$$\boxed{R = \frac{A\sin n\theta/2}{\sin\theta/2}}$$

21/8/19 Interference.

- The redistribution of intensities in the region of superposition is called as interference. It is of 2 types-

(i) Constructive Interference - Resultant amplitude.

(ii) Destructive Interference

Constructive Interference-

Resultant amplitude is the sum of individual amplitude. $A_1 + A_2$

Destructive Interference

Resultant amplitude is the diff. of individual amplitude $A_1 - A_2$

The methods for obtaining interference pattern (coherent sources)-

Two Methods-

(i) Division of wave front Ex- Young's Double slit Exp, Fresnel's Biprism.

(ii) Division of amplitude. Ex- Newton's Rings, Michelson's Interferometer.

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When the incident amplitude is divided into two parts by reflection, refraction or diffraction and transverse through diff. optical path and reunited, then they produce an interference pattern which is called as the interference by division of amplitude.

When the incident wavefront is divided into two parts by reflection, refraction or diffraction and transverse through diff optical path and reunited, then they produce an interference pattern which is called as the interference by division of wavefront

Conditions for Interference-

1. Conditions for sustained interference.
→ Coherent Sources.
→ Frequency must be equal/very nearly equal (wavelength)

2. Conditions for clear observation.
→ Background must be dark.
→ Distance b/w slit and screen is very large & distance b/w the slits must be very very small.

3. Conditions for good contract b/w dark and bright fringes.
→ Monochromatic source.
→ Amplitude must be equal/very nearly equal.
→ Slits must be narrow.

Newton's Ring-

When a plano convex lens is placed on a glass plate an air film is created with circular geometry, whose thickness gradually increases and becomes maximum at the edge of the lens. The thickness of the air film is zero at the same time.

Normal incidence of monochromatic light on the planoconvex lens will produce bright and dark concentric rings around the point of contact b/w the lens and the glass plate this phenomenon first explained by Newton so called as.

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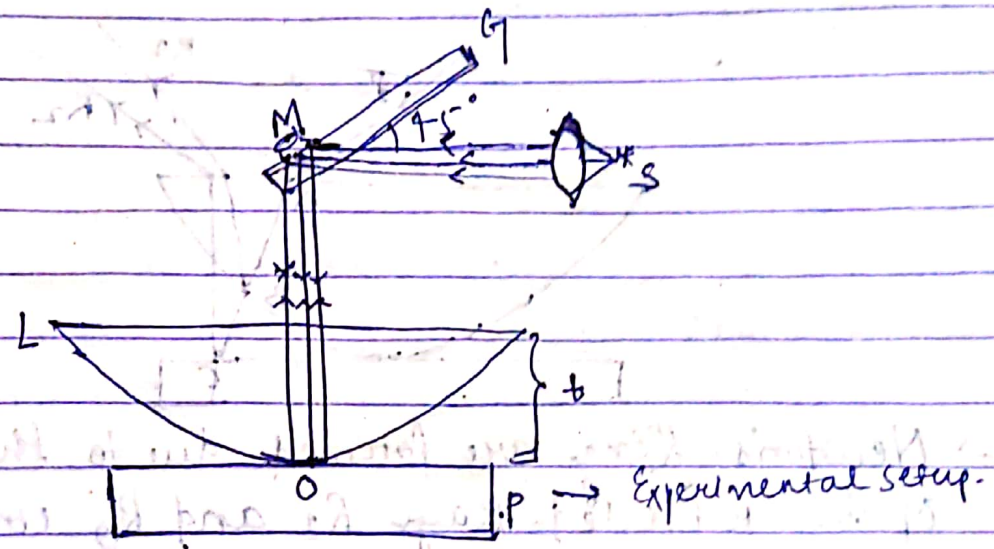
Newton's Ring.



S = Monochromatic source.
M = Travelling microscope.
G - Glass plate held at an angle 45°
t - thickness of air film.
P - Glass plate.
L - Plano convex lens having large radius of curvature.

→ S be the monochromatic source from which light rendered parallel and fall on a glass plate G. which is held at an angle 45°; so that light incident on the plano convex lens normally.

→ The light passing through the plano convex lens and reflected from upper and lower surface of the air film formed in b/w plano convex lens and glass plate (P)

→ The reflected rays interfere and forms the fringe pattern which are circular in nature and viewed through the microscope.

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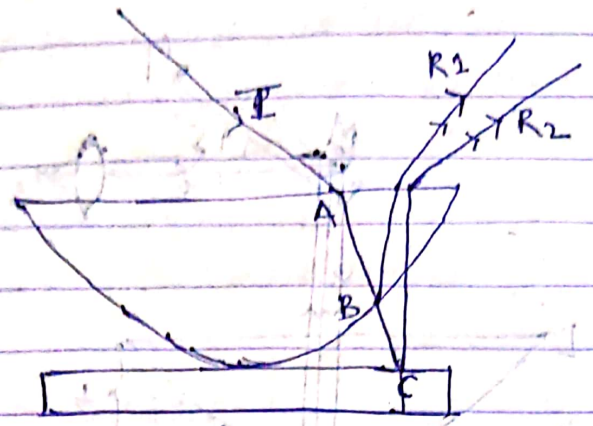
Explaination Of Fringe Formation -



→ Newton's Ring are formed due to the interference of two reflected rays R_1 and R_2 coming from top and bottom surface of the air film formed b/w the lens and the glass plate.

→ The incident ray 'I' entered into the lens at the point 'A', then at point 'B' a part of it is reflected back in form of ray R_1 and rest part is transmitted through the air film and again reflected back to the same medium in form of ray R_2.

→ So R_1 and R_2 are coherent as produced from the same incident ray 'I'.

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For reflected rays,

the path difference is $2\mu t\cos r + \lambda/2$

r - angle of refraction.

transmitted ray, there is no path difference.

In our case,

$\mu = 1$ (air film).

$r = 0$. $\cos r = 1$.

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The path difference b/w R_2 and R_1.

$$\boxed{\Delta = 2t + \lambda/2} \quad \text{--- (1)}$$

At centre, $t = 0$.

$$\boxed{\Delta = \lambda/2}$$

Centre is Dark.

<u>Position of Bright Fringe.</u>

$$\Delta = n\lambda. \qquad n = 1, 2, 3 \ldots\ldots$$

$$2t + \lambda/2 = n\lambda.$$

$$\Rightarrow \boxed{2t = (2n-1)\lambda/2} \quad \text{--- (2)}$$

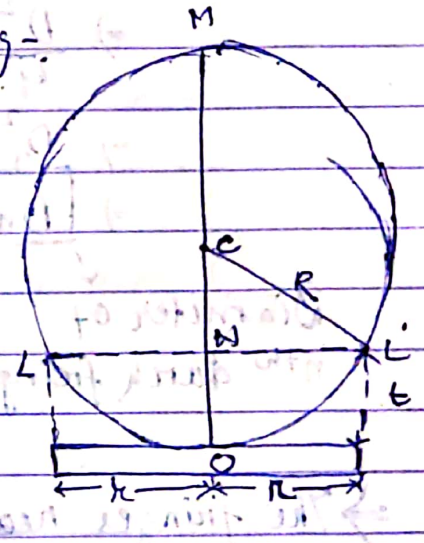
Position of Dark Fringe.

$$\Delta = (2n+1)\lambda/2$$

$$2t + \lambda/2 = (2n+1)\lambda/2$$

$$\Rightarrow \boxed{2t = n\lambda} \quad \text{--- (3)} \quad n = 1, 2, 3 \ldots\ldots$$

<u>Diameter Of Newton's Ring:-</u>



Use Property Of Circle.

$$MN \times NO = LN \times L'N.$$

$$(2R - t) \times t = r \times r.$$

$$2Rt - t^2 = r^2.$$

$t <<< R$

$$2Rt = r^2$$

t^2 can be neglected.

$$\boxed{2t = \frac{r^2}{R}} \quad \text{--- (4)}$$

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Compare eq^n (4) and (2) (Bright Fringe)

$$\frac{r^2}{R} = (2n-1)\lambda/2$$

$$r^2 = \frac{R\lambda}{2}(2n-1).$$

$$\Rightarrow \frac{D^2}{4} = \frac{R\lambda}{2}(2n-1)$$

$$\Rightarrow D^2 = 2R\lambda(2n-1)$$

$$\boxed{D_n = \sqrt{2R\lambda(2n-1)}} \quad n = 1, 2, 3 \ldots$$

Diameter of nth Bright Fringe.

$$D_n \propto \sqrt{2n-1}$$

Compare eq^n (4) and (3)

$$\frac{r^2}{R} = n\lambda$$

$$\Rightarrow r^2 = Rn\lambda$$

$$\Rightarrow \frac{D^2}{4} = Rn\lambda$$

$$\Rightarrow D_n^2 = 4Rn\lambda$$

$$\Rightarrow \boxed{D_n = 2\sqrt{Rn\lambda}}$$

Diameter of nth dark fringe

$$D_n \propto \sqrt{n}$$

$\Rightarrow$ The fringes near to the centre is more wider than the fringes away from the centre

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$2t = n\lambda$ (air)
$2\mu t = n\lambda$ (liquid).

Newton's Ring formed by Transmitted light.

$\Delta = 2t$.

Bright fringe:

$$\boxed{2t = n\lambda} \quad -(5)$$

Centre $t = 0$.
$\Delta = 0$.
$\downarrow$
Bright centre.

Compare eqⁿ (5) and (4).

$$\frac{r^2}{R} = n\lambda$$

$$\boxed{D_n = \sqrt{4Rn\lambda}} \rightarrow \text{Diameter of } n^{th} \text{ Bright Fringe.}$$

Dark Fringe:-

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad -(6)$$

compare eqⁿ (5) and (4)

$$\frac{r^2}{R} = (2n-1)\lambda/2$$

$$\Rightarrow r^2 = R(2n-1)\lambda/2$$

$$\Rightarrow \boxed{D_n = \sqrt{2R\lambda(2n-1)}} \rightarrow \text{Diameter of } n^{th} \text{ Dark Fringe}$$

The fringe pattern formed in transmitted light and reflected light are complimentary to each other.

Determination Of Refractive Index of a liquid by Newton's Ring Method-

$$2\mu t = n\lambda$$

$$2t = \frac{n\lambda}{\mu}$$

$$\Rightarrow \frac{r^2}{R} = \frac{n\lambda}{\mu}.$$

$$\Rightarrow r^2 = \frac{Rn\lambda}{\mu} \Rightarrow \frac{D^2}{4} = \frac{Rn\lambda}{\mu}.$$

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$$D_{n\ell}^2 = \frac{4Rn\lambda}{\mu} \text{ (liquid)}$$

$$D_n^2 = 4Rn\lambda \text{ (air)}.$$

$$\boxed{\mu = \frac{D_n^2}{D_{n\ell}^2}}$$

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Determination of wave length of light by Newton's ring exp. –

$D_n^2 = 4Rn\lambda \rightarrow$ Dark Fringe. $\quad D_n^2 = 2R\lambda(2n-1)$ ↳ Bright Fringe.

$D_{n+p}^2 = 4R(n+p)\lambda$ ↳ Dark Fringe, $\quad D_{n+p}^2 = 2R\lambda[2(n+p)-1]$ ↳ Bright Fringe.

↳ air medium.

$$D_{n+p}^2 - D_n^2 = 4Rp\lambda.$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4Rp}}$$

↓ Dark.

$$D_{n+p}^2 - D_n^2 = 4Rp\lambda.$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4Rp}}$$

For liquid medium.

$$\boxed{D_{n+p}^2 - D_n^2 = \frac{4Rp\lambda}{\mu}}$$

$$\boxed{\mu = \frac{D_{n+p}^2 - D_n^2}{D_{n+p}'^2 - D_n'^2}}$$

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Why the Newton's Ring are circular?
(Why the fringe formation is circular?)

$2t = n\lambda$ (for dark fringe).

$n = 1, 2, 3, 4 \ldots\ldots$

t = constant, λ - constant.

thickness of the air ~~fringe~~ film is constant for any particular order.

So, the locus of the points for any constant value of t is circular. So, fringe formation is circular in nature w.r.t the point of contact b/w the plano convex lens and the glass plate.

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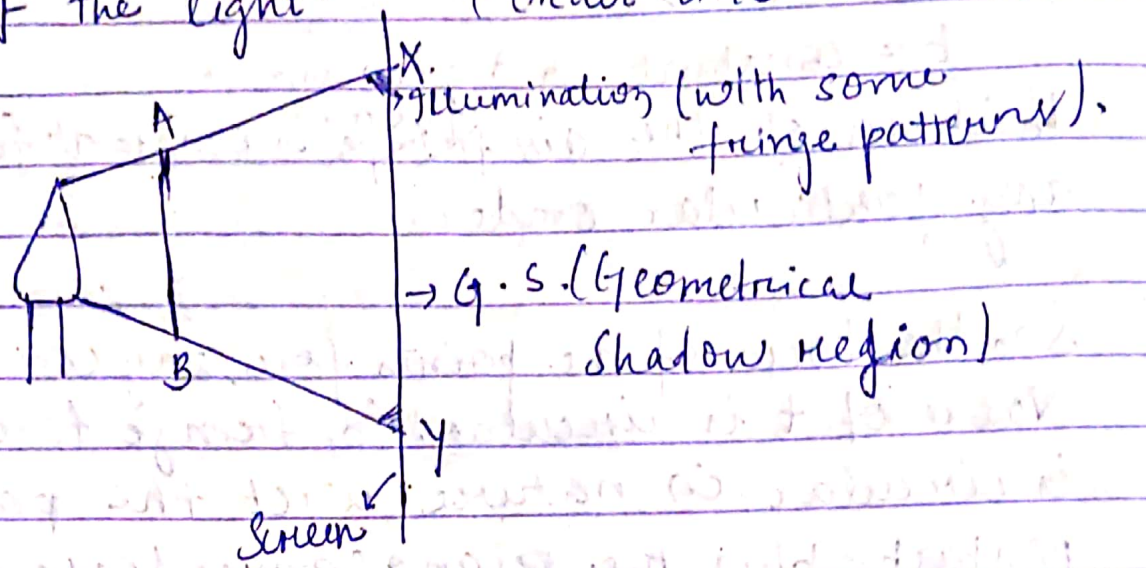
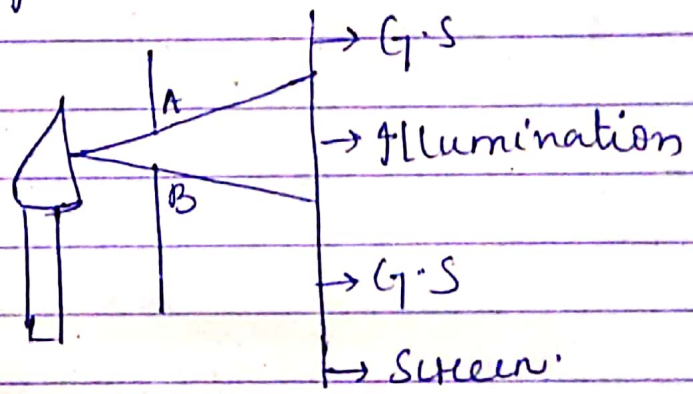
Diffraction

The bending of light at the corner of apperture (may be an obstracle / slit) is called as diffraction. When the dimension of the apperture is comparable to the wavelength of the light. (order is 10^{-4} cm)



Light bends at the corner region.



H.W Distinguish b/w interference & diffraction.

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Types of Diffraction-
(i) Fresnel's diffraction
(ii) Fraunhofer's diffraction.

Fresnel's	Fraunhofer's
→ The distance of the slit ~~and~~ from the source & the screen is finite.	→ The distance of the slit from the source & the screen is infinite.
→ No lens used.	→ Lens are used one in b/w the source & slit & another in b/w the slit & screen.
→ The incident wavefront are either spherical or cylindrical	→ The incident wavefront is plane.

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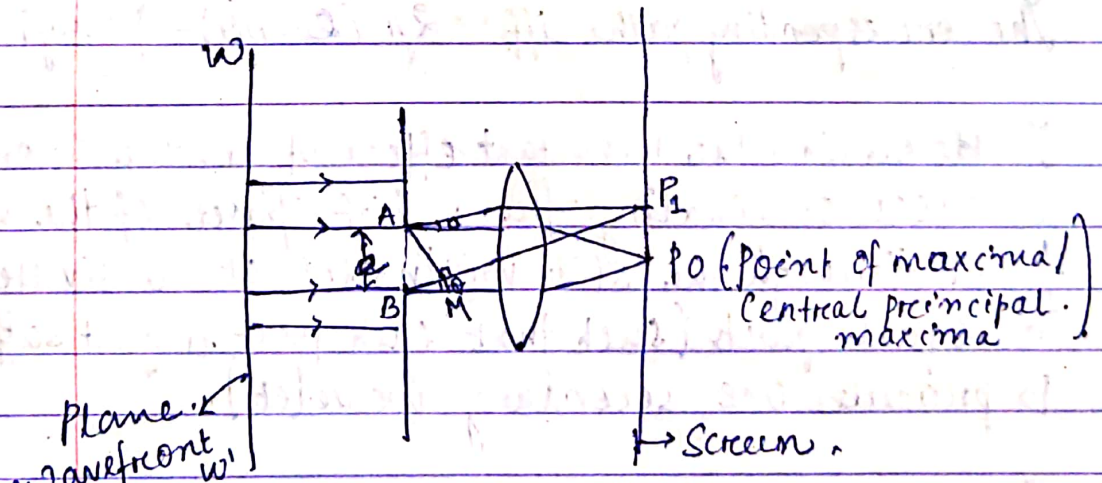
Fraunhofer's Single slit diffraction.

→ AB be a single slit of width a.
→ Plane wavefront.



Path diff., $BM = a \sin\theta$

→ Let AB be a single slit of width 'a'.
→ WW' be a plane wavefront originating from a monochromatic source of wavelength 'λ' propagating normal to the slit be incident on it

→ After diffraction, the diffracted rays are focused by using a convex lens and the screen is placed at the focal plane of the convex lens.

→ When the wavefront reaches through the width of the slit, the secondary wavelets travelling normal to the slit are brought to focus by the lens on the screen at point P_0. So, P_0 be a point of maxima or central principal maxima.

→ In order to consider all the secondary wavelets originated and propagated in different directions, consider the secondary wavelets in the direction of 'θ' are brought to focus by the lens at the point P_1 on the screen as shown in figure.

→ To know the point P_1 is a point of maxima/minima we have to calculate the path difference.

→ From fig the path difference, $BM = e\sin\theta$

The corresponding path diff. $= \frac{2\pi}{\lambda}(e\sin\theta) = \phi$ (say)

To determine the resultant effect of all the secondary wavelets originating from diff. parts of the wave front through the slit width, let it be divided into 'n' equal parts (such that each part is just sufficient to produce one secondary wavelet).

The phase difference b/w any two secondary wavelet originated from successive parts of the wavefront in the direction of 'θ' $= \frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right) = \delta$ (say).

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As all the secondary wavelets originated from the same wavefront so all the are having equal amplitude and let it be 'a'

Here we are having 'n' no. of secondary wavelets each having amplitude 'a' and successive phase difference 'd'.
So, their resultant amplitude at the point P_2

$$\boxed{R = \frac{a \sin nd/2}{\sin d/2}}$$

$$R = \frac{a \sin \cancel{\alpha}\left(\frac{1}{\cancel{2}}\frac{1}{\cancel{n}}\frac{\cancel{2}\pi}{\lambda} e\sin\theta\right)}{\sin\left(\frac{1}{2}\frac{1}{n}\frac{2\pi}{\lambda} e\sin\theta\right)}$$

$$R = \frac{a \sin \frac{\pi}{\lambda} e\sin\theta}{\sin\left(\frac{1}{n}\frac{\pi}{\lambda} e\sin\theta\right)}$$

Let $\frac{\pi}{\lambda} e\sin\theta = \alpha$

$$R = \frac{a \sin\alpha}{\sin(\alpha/n)}$$

α is small but 'n' is large.

$$\sin\alpha/n = \alpha/n$$

$$R = a\frac{\sin\alpha}{\alpha/n}$$

$$R = an\frac{\sin\alpha}{\alpha}$$

Let $an = A$.

(From multiple beam prod superposition)

Resultant amplitude. $\boxed{R = A\frac{\sin\alpha}{\alpha}}$

Resultant intensity. $\boxed{I = A^2\frac{\sin^2\alpha}{\alpha^2}}$

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$I \to I_{max}$ for $\alpha \to 0$.

$$\frac{\sin\alpha}{\alpha} = 1$$

$$\boxed{I = I_{max} = A^2}$$

Position.

$\alpha = 0$.

$\frac{\pi}{\lambda} e\sin\theta = 0$. $\quad \sin\theta = 0$

$\theta = 0$.

i.e. P_0. (Central principal maxima).

Minima -

$I \to I_{min} = 0$. $\quad \sin\alpha = 0$.

$\alpha \neq 0$

$\Rightarrow \alpha = \pm n\pi \quad n \neq 0 \quad n = 1, 2, 3 \ldots$

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$\frac{\pi}{\lambda} e\sin\theta = n\pi$

$\boxed{e\sin\theta = n\lambda}$ $\quad n \neq 0$, $n = 1, 2, 3, 4 \ldots$

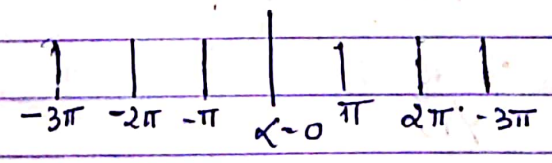
$\to$ Eq^n for minima in single slit.

$$\boxed{\theta = \sin^{-1}\frac{n\lambda}{e}}$$

$\alpha = 0 \to$ Principal maxima.

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \to$ minima.

(Not complete dark $\to$ Distinct dark)



Having one principal maxima at $\alpha = 0$ we have distinct minimas at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \ldots$

So, the question of secondary maxima and the condition for secondary maxima can be

Obtained.

$$\frac{dI}{d\alpha} = 0.$$

Already used this condition.

$$\frac{dI}{d\alpha} \; A^2 \, 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0.$$

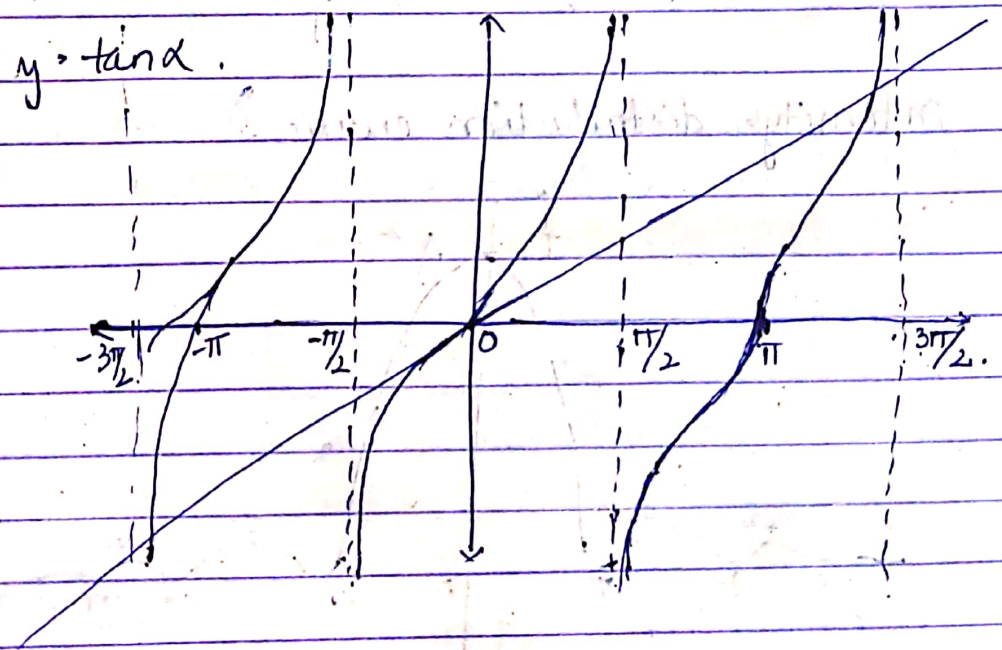
$A \neq 0$

$$\Rightarrow \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0.$$

$$\Rightarrow \alpha\cos\alpha = \sin\alpha.$$

$$\boxed{\alpha = \tan\alpha}.$$

By drawing the curve after taking $y = \alpha$. and $y = \tan\alpha$, the point of intersection of both the curves will give the position of secondary maxima.



The position of secondary maximas are

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$$

$$I = \left(A\frac{\sin\alpha}{\alpha}\right)^2$$

$\alpha = 0 \rightarrow$ Principal maxima

$\alpha = \pm\pi, \pm 2\pi \ldots\ldots \pm n\pi \rightarrow$ minimas

$\alpha = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2 \ldots\ldots \pm (n+1)\pi/2$

↳ Position of secondary maximas.

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I_{pm} = Intensity of principal maxima

Intensity of Secondary maxima -

I_1 - Intensity of first secondary maxima.

$$I_1 = \frac{A^2 \sin^2 3\pi/2}{\left(\frac{3\pi}{2}\right)^2} = \frac{4A^2}{9\pi^2} \simeq \frac{A^2}{22} = \frac{I_{pm}}{22}$$

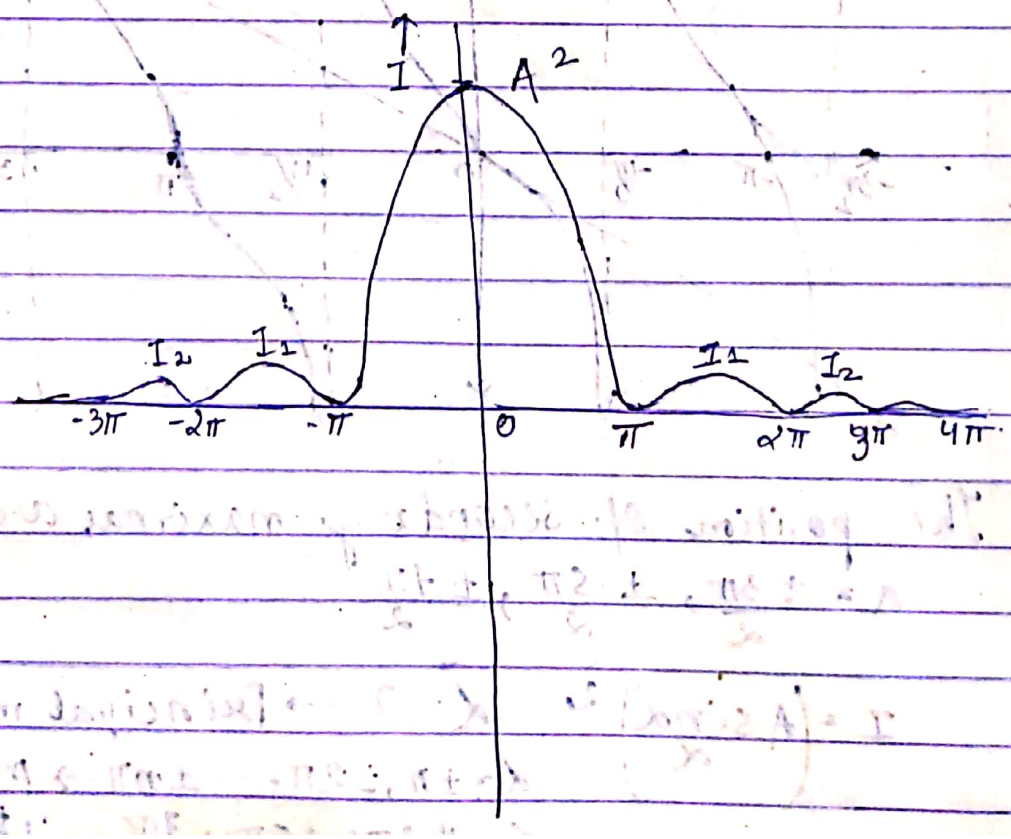
I_2 - Intensity of second secondary maxima.

$$I_2 = \frac{A^2 \sin^2 (5\pi/2)}{(5\pi/2)^2} = \frac{4A^2}{25\pi^2} \simeq \frac{A^2}{62} = \frac{I_{pm}}{62}$$

I_3 → Intensity of third secondary maxima.

$$I_3 = \frac{A^2 \sin^2 (7\pi/2)}{(7\pi/2)^2} = \frac{A^2}{\frac{49\pi^2}{4}} \simeq \frac{A^2}{128} \simeq \frac{I_{pm}}{128}$$

Intensity distribution curve.



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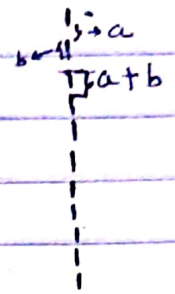
Plane diffraction grating-

It is an arrangement of combination of 'n' no. of slits having equal width and separated by equal opaque space.



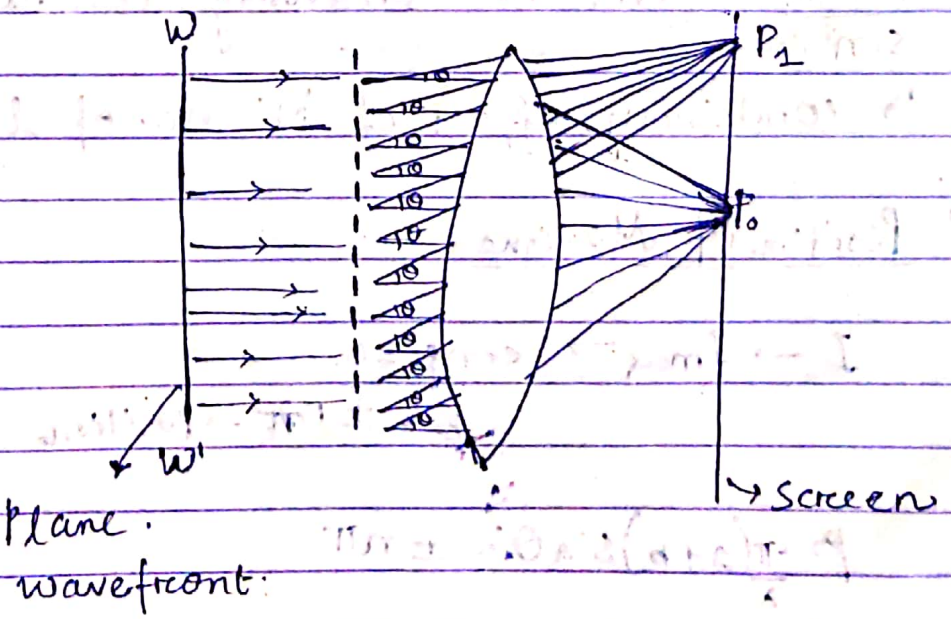
Gap b/w. any opaque space - b.

a → width of transparency (width).
b → width of opacity
(a+b) → Grating element.

Grating element is defined as no. of lines drawn in one cm.

5000 → lines in 1 cm to get diffraction in mercury light. Gap b/w the lines must be same

N = no. of lines drawn. $= L/N$.
L = width of the grating in cm.
1 inch = 2.5 cm.



Amplitude $= \left(\frac{A\sin\alpha}{\alpha}\right)$ $\quad \delta = 2\beta$.
N

The path diff. is $= (a+b)\sin\theta$.

The corresponding phase diff $= \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$ (say)

$$R = \frac{\left(\frac{A\sin\alpha}{\alpha}\right)\sin N\cdot\frac{2\beta}{2}}{\sin\frac{2\beta}{2}}$$

$$R = \frac{\left(\frac{A\sin\alpha}{\alpha}\right)\sin N\beta}{\sin\beta}$$

(For maxima & minima last term is considered.

$$\boxed{I = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta}} \quad \text{—— (1)}$$

In eq^n (1) $\left(\frac{A\sin\alpha}{\alpha}\right)^2$ gives the intensity contribution due to individual slits and $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the intensity contribution due to combining effect of 'N' no. of slits.

9/9/19 <u>Position Of Maxima</u>-

$I \to I_{max} \to \sin\beta = 0$.

$\beta = \pm n\pi \to$ position

$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$

* $\boxed{(a+b)\sin\theta = \pm n\lambda} \to$ Eq^n of principal maxima in grating

$n = 0, 1, 2, 3 \ldots$

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Position Of Minima-

$I \rightarrow I_{min} \Rightarrow \sin N\beta = 0$, but $\sin\beta \neq 0$.

$$N\beta = \pm m\pi$$

$m = 1, 2, 3, 4 \ldots\ldots (N-1)$ ~~N~~

$(N+1), (N+2), \ldots\ldots (2N-1)$ ~~2N~~.

$(2N+1), (2N+2) \ldots\ldots (3N-1)$ ~~3N~~.

→ In b/w 2 principal maxima there are $(N-1)$ minimas are there.

→ In b/w 2 principal maxima there are $(N-2)$ secondary maxima

That is why minima is not perfectly dark.

Different Parameters-

(i) Maximum no. of principal maxima-

Eq^n for principal maxima,

$$\boxed{(a+b)\sin\theta = \pm n\lambda}$$

$(a+b) \rightarrow$ grating element $\rightarrow$ const.

$\lambda \rightarrow$ Constant (wavelength of monochromatic light).

$n \rightarrow$ Order of principal maximum.

$n \rightarrow n_{max}$ when $\sin\theta = 1$.

$$(a+b) = n_{max}\lambda$$

$$\Rightarrow \boxed{n_{max} = \frac{(a+b)}{\lambda}}$$

Q. If a monochromatic Light of λ. 6000 A° incident normally on a plane diffraction grating having grating element. 6×10^{-5} cm, then find the max. no order of principal maxima observed.

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$1 \mu = 10^{-10}$ cm.

$n_{max} = \frac{6000 \times 10^{-10}}{6 \times 10^{-4}} = \frac{6 \times 10^{-5}}{6 \times 10^{-8} \times 10^{3}} = 1$

How many orders of principal maxima is visible if a light of 5000A incident normally on a grating having 2602 lines per inch.

$G \rightarrow 2602 \times 2.5$

$n_{max} = \frac{6505}{4000 \times 10^{-10}} = \frac{1626.22 \times 10^{7}}{4000 \times 10^{-10}} = 240.8$

$1 \rightarrow 2.5$

$\frac{2.5}{2602}$

2602×2.5

1301×5.204

5×5

120.4

$\frac{2602 \times 10}{25}$

120.4×2

240.8

$(a+b) = \frac{2.5 \times 10^{8}}{2602 \times 5000} = 19.2$

(ii) Absent Spectra / Missing Spectra -

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$$I = \left(\frac{A \sin\alpha}{\alpha}\right)^2 \left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$$

The principal maxima occurs when it satisfies the maxima for a single slit as well as for the 'N' no. of slits.

When the condition for principle maxima due to N no. of slits and the condition for minima due to single slits are satisfied simultaneously for the same angle θ then the principal maxima due to grating for that particular order will be absent or missed which is called as absent spectra / missing spectra.

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$(a+b)\sin\theta = n\lambda$ — (I) → Eq^n for principal maxima due to grating $n = 0, 1, 2, 3 \ldots$

$a\sin\theta = m\lambda$ — (II) → Eq^n for minima due to single slit $m = 1, 2, 3, 4$.

Dividing eq^n (I) and (2).

$$\boxed{\frac{a+b}{a} = \frac{n}{m}}$$

↳ Eq^n for absent spectra.

Example - If $a = b$.

$$\frac{n}{m} = 2 \Rightarrow n = 2m$$

$= 2, 4, 6, 8$.

↳ These orders will be missed.

Ex. $b = 2a \Rightarrow a = b/2$.

$$\frac{3a}{a} = n/m$$

$n = 3m$.

$= 3, 6, 9, 12$ ↳ These order will be missed.

Dispersive power Of Grating

It is defined as the ratio of variation of angle of diffraction with the variation of wavelength.

$\frac{d\theta}{d\lambda}$ → dispersive power.

$(a+b)\sin\theta = n\lambda$.

$\Rightarrow (a+b)\cos\theta\, d\theta = n \cdot d\lambda$.

$$\Rightarrow \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

It depends upon, Order of

$$\frac{d\theta}{d\lambda} \propto n \quad \text{and} \quad \frac{d\theta}{d\lambda} \propto \frac{1}{\cos\theta}$$

$$\propto \frac{1}{(a+b)}$$

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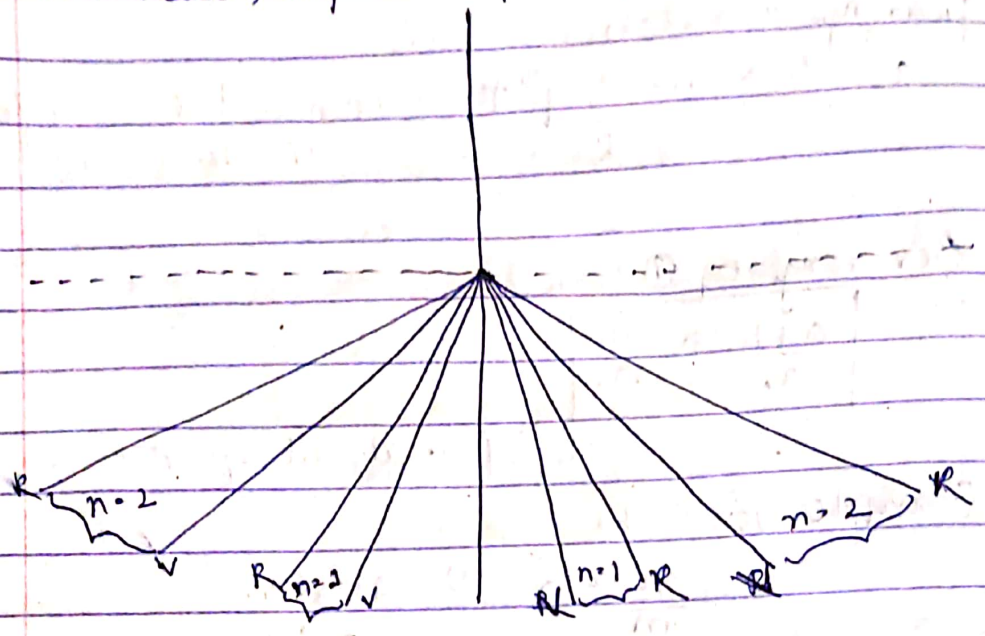
Formation Of Multiple Spectra-



When a white light is passed through a grating then multiple spectra or colour spectra are formed i.e. from violet to red.

Characteristics of Multiple Spectra-

(i) Most of the intensities go through zeroth order and rest will be distributed among other orders.
So, the spectra becomes less intense towards higher order.

(ii) Spectra of diff. orders are situated symmetric of both the sides of zeroth order image.

(iii) Spectral colours are in the order from violet to red.

(iv) Spectral lines are almost straight and sharp.

(v) For higher order, the spectral lines are more and more dispersed.

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