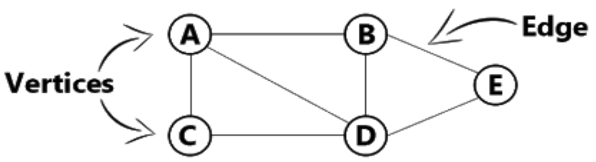
Graph

- Like trees, graph is also a non-linear data structure

- A **graph** G= (V, E) consists of 2 sets:
    - A set V, of **vertices** (or nodes) and
    - A set E, called set of **edges** (or arcs)

Example:



Graph G1 above contains 5 vertices and 7 edges:

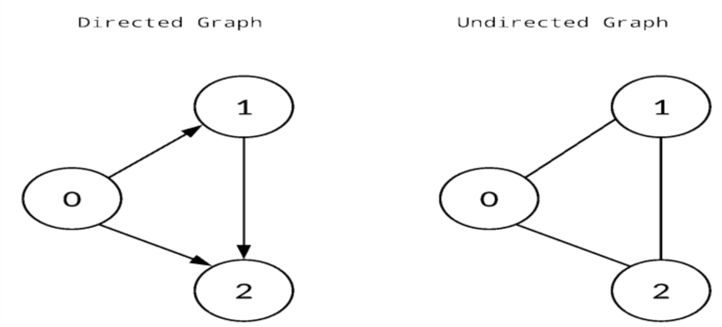
V={A, B, C, D, E}

E={(A,B), (A,C), (A, D), (B,D),(B,E), (C,D),(D,E)}

- Edges of the graph can be directed or undirected

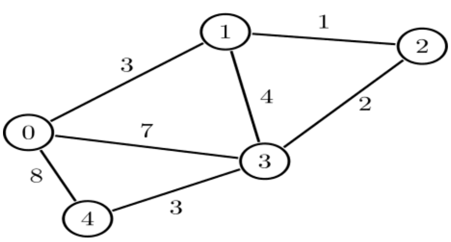
Digraph: If all edges of the graph are directed it is called a directed graph or diagraph.

Undirected graph: If all edges of the graph are undirected it is called undirected graph.



Weighted graph: A graph is called a weighted graph if all the edges are labelled with some weights.

Example:

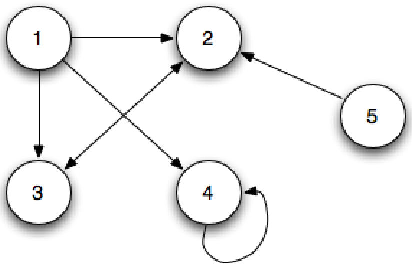


Adjacent Vertices: A vertex v_i is adjacent to (or neighbour of) another vertex v_j if there is an edge from v_i to v_j .

Example: vertex 1 is adjacent to vertex 2, 3, and 4 in below figure.

Self loop: If there is an edge whose start and end vertices are same i.e. (v_i , v_i) then it is called a self loop or loop.

Example: The edge (4, 4)



Degree: Total number of edges connected to a vertex.

Indegree: Total number of incoming edges connected to a vertex is called the indegree of that vertex.

Example: indegree of vertex 2 =3 in above figure

Outdegree: Total number of outgoing edges connected to a vertex is called the outdegree of that vertex.

Example: outdegree of vertex 5 =1 in above figure

Path: Sequence of vertices in which each pair of successive vertices is connected by an edge

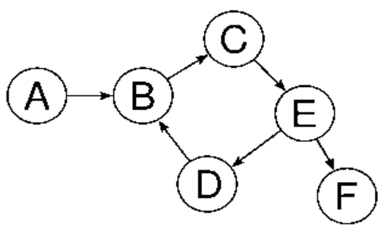
Example: path to reach from vertex 0 to 3 in below figure:

Path-1: 0-1-3,

Path-2: 0-2-3

Cycle: A path that starts and ends on the same vertex

Example:



Graph contains cycle: B-C-E-D-B

<u>Representation of Graphs:</u>

Graphs can be represented using 2 approaches:

1. Sequential or Matrix representation
2. Linked list representation

1. Matrix Representation:

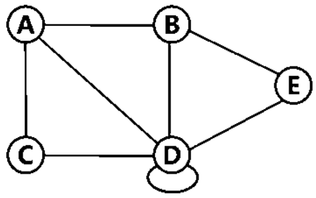
There are 2 commonly used matrices to represent graphs:

A. Adjacency Matrix
B. Incident Matrix

A. **Adjacency Matrix:**

- In this representation, the graph is represented using a matrix of size:
  total number of vertices X *total number of vertices*.
- i.e a graph with 5 vertices is represented using a matrix of size 5 X 5.
- In this matrix, both rows and columns represent vertices.
- This matrix is filled with either 1 or 0.
- Here, 1 represents that there is an edge from row vertex to column vertex and
  0 represents that there is no edge from row vertex to column vertex.

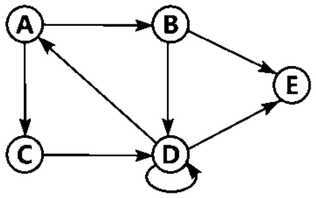
Example-1: undirected graph representation



$$\begin{array}{cc} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

For any edge (A,B) in undirected graph, matrix[A][B]=1, also matrix[B][A]=1

Example-2: directed graph representation



$$\begin{array}{cc} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

For any edge (A,B) in directed graph, matrix[A][B]=1, but matrix[B][A]=0

For self loop for the edge(D,D), matrix[D][D]=1

B. Incidence Matrix

- In this representation, the graph is represented using a matrix of size:

  total number of vertices X *total number of edges.*

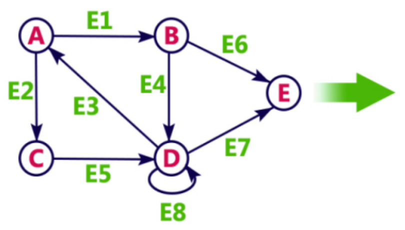
- i.e a graph with 5 vertices and 8 edges is represented using a matrix of size 5 X 8.
- In this matrix, rows represent vertices and columns represents edges.
- This matrix is filled with 0 or 1 or -1.
- Here,

  0 represents that the row edge is not connected to column vertex,

  1 represents that the row edge is connected as the outgoing edge to column vertex and

  -1 represents that the row edge is connected as the incoming edge to column vertex.

Example:



	E1	E2	E3	E4	E5	E6	E7	E8
A	1	1	-1	0	0	0	0	0
B	-1	0	0	1	0	1	0	0
C	0	-1	0	0	1	0	0	0
D	0	0	1	-1	-1	0	1	1
E	0	0	0	0	0	-1	-1	0

For vertex A:

outgoing edges are: E1, E2 => matrix[A][E1] and matrix[A][E2]=1

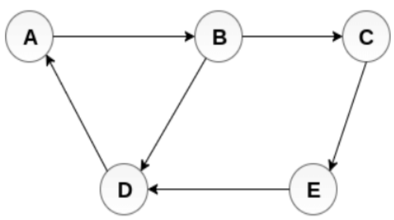
incoming edge: E3 => matrix[A][E3]=-1

all other entries for Ath row in matrix are 0

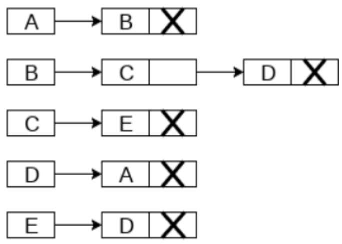
2. Linked Representation:

An adjacency list is maintained for each node present in the graph which stores the node value and a pointer to the next adjacent node to the respective node. If all the adjacent nodes are traversed then store the NULL in the pointer field of last node of the list.

Example:



Directed Graph



Adjacency List