21/10/19 Quantum Mechanics. (Module-III)

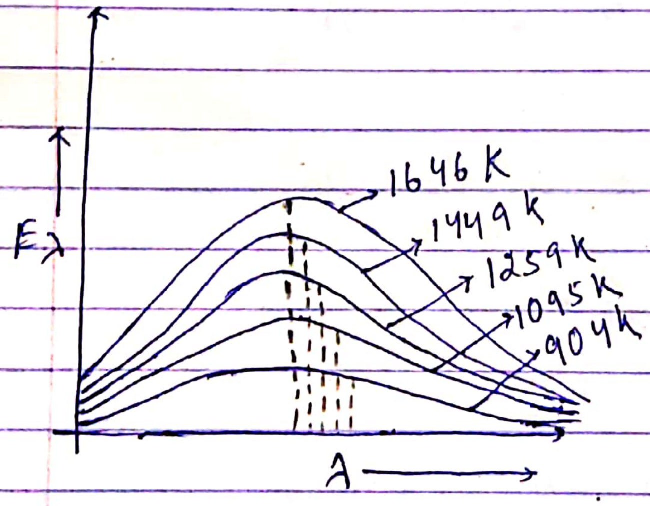
The quantum theory is developed which couldn't explain by the help of classical theory / classical physics.

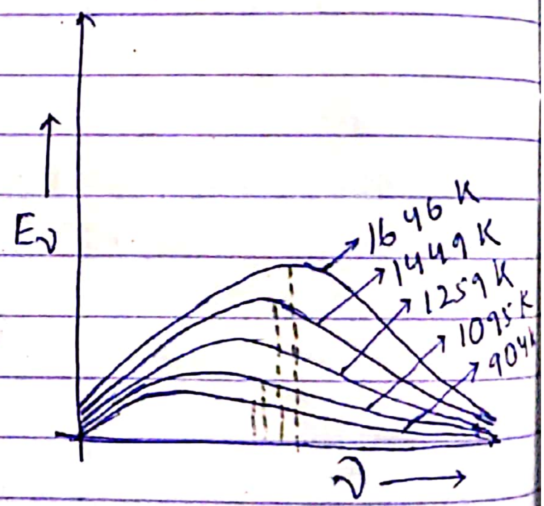
(i) Planck's theory of black body radiation (1900)
(ii) Einstein's theory of photoelectric effect (1905)
(iii) Bohr's theory of hydrogen spectrum (1913)
(iv) Compton's theory of X-rays scattering (1922)
(v) De-Broglie's hypothesis (1924)
(vi) Heisenberg's uncertainity principle (1927)
(v) S. ' wave eq^n (1928)

• **Planck's theory Of black body radiations-**

Experimental Observation of black body radiation

E_λ or E_ν → Radiant Energy density i.e. energy radiated per unit volume. [$\lambda = c/\nu$]





(i) At a given temp. the energy density has max. value corresponding to a value of frequency or wavelength.

(ii) The frequency corresponding to max. energy density increases with increase of temp.

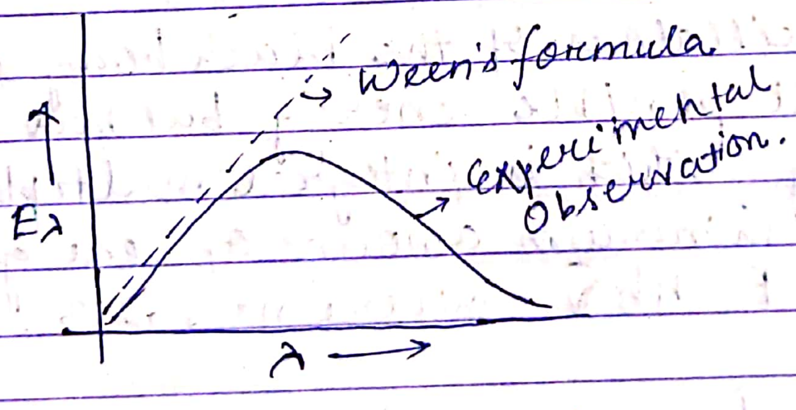
(iii) The energy density decreases to zero for both higher and lower value of frequency or wavelength.

(iv) The energy density corresponding to a given frequency or wavelength increases with increase of temp.

(v) The total energy radiated at any temp. is given by the area b/w the curve corresponding to that temp. and the horizontal axis.

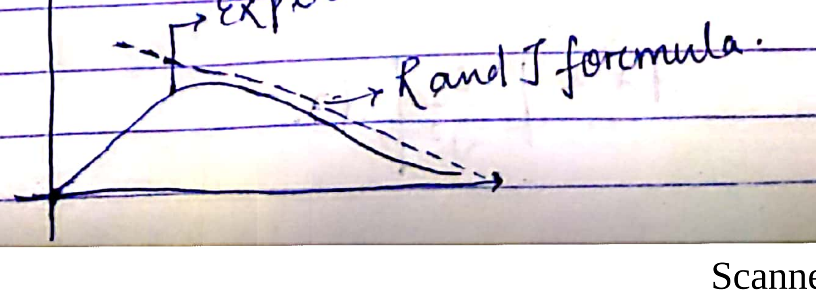
Ween's formula for Black body radiation-

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda KT} d\lambda.$$



Rayligh and Jean (1900)

$$E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda.$$



Planck's Quantum theory of Black Body Radiation -

In order to explain the experimental observ. distribution of energy planck in 1900 suggested some assumptions which is called as planck's quantum theory.

→ A chamber containing black body radiation contain some simple harmonic oscillators. of molecular dimensions which can vibrate. with all possible frequencies.

→ The classical principle of equipartition of energy is not applicable to the black body radiation.

→ The frequency of radiation emitted by an oscillator. is the same as that of the frequency of vibration.

→ The oscillators of the black body cannot have all possible energy but has discrete energy which is integral multiple of some minimum amount of energy.

$$E = nh\nu \quad \text{where } n = 1, 2, 3 \ldots, n \neq 0$$

Planck's formula of black body radiation.

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda \longrightarrow (1).$$

$$\lambda = c/\nu$$

$$d\lambda = \left| \frac{-c}{\nu^2} \right| d\lambda.$$

$$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/KT}-1} d\nu$$

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Planck's formula reduced to Ween's formula for low wavelength region.

For low wavelength.

λT is very small.

$e^{hc/\lambda KT} >>> 1$

$$e^{hc/\lambda KT} - 1 \simeq e^{hc/\lambda KT}.$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{e^{hc/\lambda KT}} d\lambda.$$

$$= \frac{8\pi hc}{\lambda^5} \times e^{-hc/\lambda KT}.$$

So, Planck's formula has a very good fitting to the experimental result at low wavelength region.

Planck's formula reduced to Rayleigh's formula for high wavelength region:

λT is very large.

$$e^{hc/\lambda KT} = 1 + \frac{hc}{\lambda KT} + \frac{1}{2!}\left(\frac{hc}{\lambda KT}\right)^2 + \ldots\ldots$$

$$= 1 + \frac{hc}{\lambda KT}.$$ (Neglecting higher order).

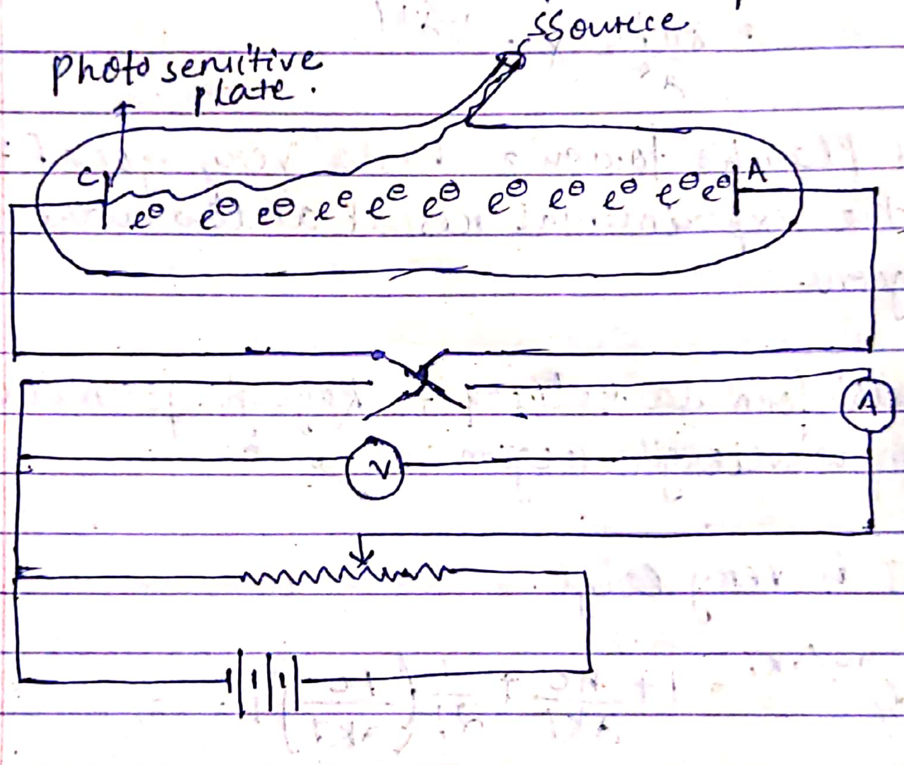
$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{\frac{hc}{\lambda KT}}$$

$$= \frac{8\pi KT}{\lambda^4} d\lambda$$ → Rayleigh and Jean formula.

So Plank's formula has a very good fitting to the experimental result at high wavelength region.
So, it concludes this formula agrees for to the experimental result for all ranges of wavelengths

Photoelectric Effect:-

The emission of $e^{\ominus}$s from a metal plate when illuminated by a light radiation of suitable frequency or wavelength is called as photoelectric effect.
The emitted $e^{\ominus}$s are called as photoelectrons.



Experimental Observation -

→ Photoelectric effect is an instantaneous process. As soon as the radiation is incident on the metal plate, photo electrons are emitted. The time lag b/w them is 10^{-8} sec.

→ For a given frequency and given photo potential diff. b/w anode and photocathode, the photocurrent is directly proportional to the intensity of incident radiations.

→ The emitted photoelectrons do not have same kinetic energy that varies from zero to a maximum value.

∴ For a given photocathode and given potential difference the maximum kinetic energy depends upon the frequency of radiation but independent of intensity.

An increase in intensity increases the no. of photoelectrons but not the maximum kinetic energy.

→ For a given photocathode, the photoelectric effect can take place only if the frequency of radiation must have a minimum value called as threshold frequency, below which no photoelectrons are emitted. even the intensity is made very high.

→ If the anode made -ve and cathode made +ve then photocurrent decreases even the electrons has max. kinetic energy but could not reach the anode.

So, the minimum potential (Negative potential) to get zero photocurrent is called as stopping potential

→ For a given metal surface stopping potential is directly proportional to the frequency of incident radiation but independent of intensity.

Einstein's Explanation Of Photoelectric Effect -

Einstein extended Planck's idea to explain photoelectric effect. So, ~~we~~ he assumed that electromagnetic wave of frequency ν can be regarded as a stream of particles of energy '$h\nu$' each and this particle are called as photons.

The energy of photons is used in two parts -

(i) A part of it is used to free the electrons from the atom and make away from the metal surface. This energy is called as workfunction of the metal and denoted by "W_0"

(ii) The rest part of energy of photons is used in giving the kinetic energy to the electrons.

$$\therefore h\nu = W_0 + \frac{1}{2}mv^2 \quad \text{--- (1)}$$

where, $W_0 \rightarrow$ workfunction of metal surface

v - velocity of the photoelectron.

Eqn (1) is the Einstein's photoelectric eqn.

$W_0 = h\nu_0$

ν_0 - minimum frequency required for which photoelectric effect can take place i.e.

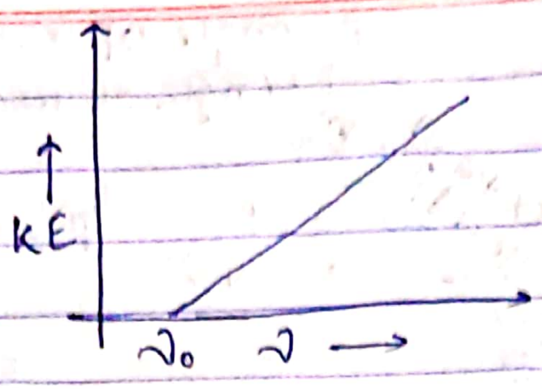
ν_0 - ~~thresh~~ threshold frequency.

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = h\nu - h\nu_0$$

$$KE = h(\nu - \nu_0) \quad \text{--- (2)}$$

$$\frac{1}{2}mv^2 \propto \nu \quad \text{and} \quad v^2 \propto \nu$$



Stopping Potential-

$\frac{1}{2}mv^2 = qV.$

q - Charge particle.

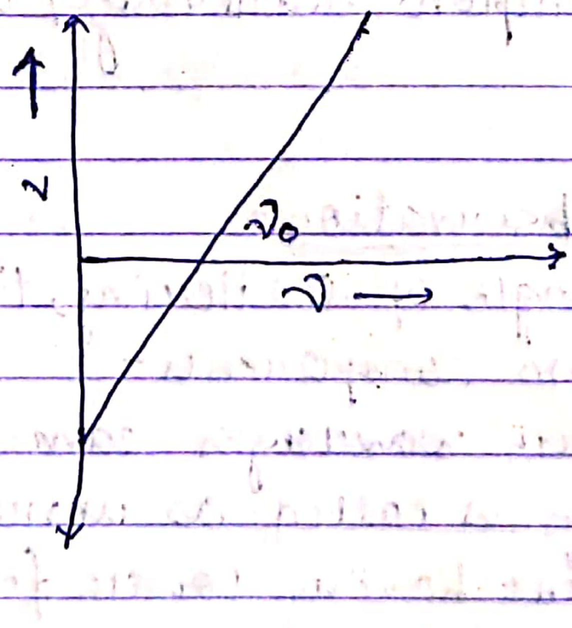
V - negative potential given in b/w cathode & anode.

$KE = \frac{1}{2}mv^2 = eV$

So, eqⁿ (2) can be written as

$eV = \frac{\phi}{\phi} h (\nu - \nu_0)$

So, $\boxed{V \propto \nu}$



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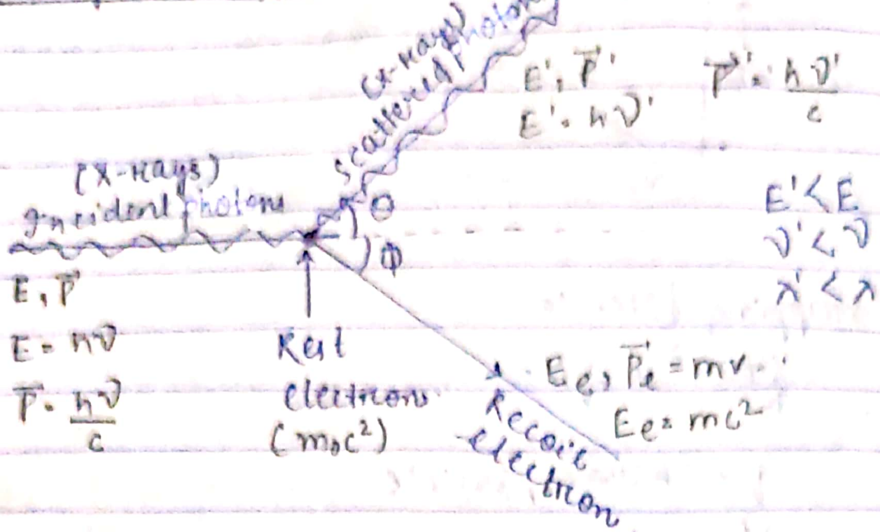
Compton's Effect:

θ - Angle of scattering
ϕ - Angle of recoil electron



When a monochromatic beam of high frequency radiation such as X-rays collides with the rest electron, a part of energy given to that electron. Due to this e^0 will gain K.E. and scattering photon (X-rays) will have lower energy. So lower frequency or greater wavelength than the incident one which is called as Compton's Scattering / Compton's Effect.

Experimental Observation:

→ For a given angle of scattering, the scattered X-rays have two components -

(i) A component with wavelength same as that of the incident X-rays is called as unmodified component.

(ii) Another component having lower frequency and greater wavelength is called as modified component or Compton's Component.

→ The increase in wavelength in Compton's component is called as Compton's shift which depends upon the angle of scattering.

→ The Compton's shift is independent of the wavelength of the incident radiation.

Classical Theory failed to explain experimental observations by considering the radiations (X-rays) as wave nature.

Compton's Explaination-
An adequate explaination of this effect was provided by Compton in 1992 on the basis of Planck's quantum theory of radiation According to that quantum concept the radiation is constituted by energy packets called photon having energy $h\nu$.
The incident rays with frequency 'ν' are regarded as a stream of particles with energy '$h\nu$'.

Before Collision-
The energy of the incident photon,

$$E = h\nu$$

Momentum of the incident photon,

$$\vec{p} = h\nu/c$$

Energy of the rest $e^{\ominus}$, $= m_0 c^2$
Momentum of the rest electron $= 0$

After Collision-
Energy of the scattered photon,

$$E' = h\nu'$$

Momentum of the scattered photom.

$$\vec{p}' = h\nu'/c$$

Energy of the recoil $e^{\ominus} = mc^2 = E_e$.
Momentum of the recoil $e^{\ominus}$,

$$\vec{P_e} = mv$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$\Rightarrow m^2 c^2 - m^2 v^2 = m_0^2 c^2$$

Multiplying c^2 both sides.

$$\Rightarrow m^2 c^4 - m^2 c^2 v^2 = m_0^2 c^4$$

$$\Rightarrow E_e^2 = P_e^2 c^2 + m_0^2 c^4 \quad \text{——— (1)}$$

Applying law Of Conservation Of Energy.

$$E + m_0 c^2 = E' + E_e$$

$$E + m_0 c^2 = E' + \sqrt{P_e^2 c^2 + m_0^2 c^4}$$

$$\Rightarrow E + m_0 c^2 - E' = \sqrt{P_e^2 c^2 + m_0^2 c^4} \quad \text{——— (2)}$$

Applying law Of Conservation Of Momentum

$$\vec{P} = \vec{P}' + \vec{P_e}$$

$$\vec{P_e} = \vec{P} - \vec{P}'$$

$$\Rightarrow \vec{P_e}^2 = \vec{P_e} \cdot \vec{P_e} = (\vec{P} - \vec{P}') \cdot (\vec{P} - \vec{P}')$$

$$P_e^2 = P^2 + P'^2 - 2PP'\cos\theta \quad \text{— (3)}$$

Squaring eqn (2) both the sides.

$$(E - E')^2 + \cancel{m_0^2 c^4} + 2(E - E')m_0 c^2 = P_e^2 c^2 + \cancel{m_0^2 c^4} \quad \text{— (4)}$$

Substitute the value of P_e from eq^n 3 in eq^n 4

$$(E-E')^2 + 2(E-E')m_0c^2 = p^2c^2 + p'^2c^2 - 2pp'c^2\cos\theta.$$

$$\Rightarrow (E-E')^2 + 2(E-E')m_0c^2 = E^2 + E'^2 - 2EE'\cos\theta.$$

Add and substract $2EE'$ in RHS

$$(E-E')^2 + 2(E-E')m_0c^2 = E^2 + E'^2 - 2EE'\cos\theta + 2EE' - 2EE'$$

$$\Rightarrow \cancel{(E-E')^2} + 2(E-E')m_0c^2 = \cancel{(E-E')^2} + 2EE' - 2EE'\cos\theta.$$

$$\Rightarrow \cancel{2}(E-E')m_0c^2 = \cancel{2}EE'(1-\cos\theta).$$

$$\Rightarrow \frac{E}{EE'} - \frac{E'}{EE'} = \frac{1-\cos\theta}{m_0c^2}.$$

$$\Rightarrow \frac{1}{E'} - \frac{1}{E} = \frac{1-\cos\theta}{m_0c^2}$$

$$E = h\nu = \frac{hc}{\lambda}, \quad E' = h\nu' = \frac{hc}{\lambda'}$$

$$\Rightarrow \frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1-\cos\theta}{m_0c^2}$$

$$\Rightarrow \frac{\lambda'}{h} - \frac{\lambda}{h} = \frac{1-\cos\theta}{m_0c}$$

$$\lambda' - \lambda = \frac{h(1-\cos\theta)}{m_0c}$$

$$\Rightarrow \boxed{\Delta\lambda = \lambda' - \lambda = \frac{h(1-\cos\theta)}{m_0c}}$$

$$\boxed{\Delta\lambda = \lambda_c(1-\cos\theta)}$$

Compton's shift

λ_c - Compton's wavelength.

$$= \frac{h}{m_0c}$$

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The compton's shift '$\Delta\lambda$' only depends upon the angle of scattering 'θ' and independent of the wavelength of the incident radiation.

Different cases -

Case I - $\theta = 0$

$$\Delta\lambda = 0$$

i.e. $\lambda' = \lambda$ i.e. no Compton's shift

Case II - $\theta = \pi/2$

$$\Delta\lambda = \lambda_c$$
$$\lambda' - \lambda = \lambda_c$$
$$\lambda' = \lambda_c + \lambda$$

Case III - $\theta = \pi$

$$\Delta\lambda = 2\lambda_c$$

which is the maximum value of Compton's shift

Q.1. Find the Compton's shift for X-rays of wavelength 1.5 A° scattered by 60°. Also find the wavelength of the scattered X-rays.

Q.2. X-rays of 1A° are scattered from a carbon block find the wavelength of the scattered beam in a direction making 90° with incident beam. How much K.E is imparted to the recoil $e^{\ominus}$?

Q.3. X-rays of wavelength 1.2 A° under the Compton's scattering due to $e^{\ominus}$s. What is max. possible value of Compton's shift if the Compton's wavelength of the $e^{\ominus}$ is 0.0242 A°.

Q.4. Calculate the velocity of photon $e^{\ominus}$ if the workfunction of the target material is 1.24eV and the wavelength of incident rays is 4.36×10^{-7} m. What is the retarding potential necessary to stop the emission of this electron?

Q.5. Electrons are emitted with zero velocity from a certain metal surface when it is exposed to a radiation of 6800 A°. Calculate the threshold frequency and workfunction of the metal.

Q.6. The workfunction of the aluminium is 4.2 eV. Calculate the K.E of the fastest and slowest $e^{\ominus}$, the stopping potential. and the cut-off wavelength when light of wavelength 2000 A° falls on the clean aluminium surface.

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Distinguish b/w the photoelectric effect and Compton's Scattering.

Photoelectric Effect.	Compton Scattering.

Pair Production -

Impt Dual Nature Of Radiation (de-Broglie hypothesis).

Matter Waves.

$$\boxed{\lambda = \frac{h}{p}}$$

λ - wavelength

p - momentum, (mv).

Looking to the discuss given by photoelectric effect, Compton's Scattering and Pair production Louis de-Broglie proposed that all material particles are associated with waves called as matter waves or de-Broglie's wave. According to his hypothesis the wavelength λ of the matter wave associated with a moving particle of linear momentum 'p' is given by

$$\boxed{\lambda = \frac{h}{p}}$$

where h - planck's constant.

Calculation of de-Broglie's Wavelength-.

1. Free Particle - $KE = \frac{1}{2}mv^2$

~~Tota~~ No potential Energy.

Total Energy = KE

$$KE = \frac{p^2}{2m}.$$

$$p = \sqrt{2mE}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

2. For any particle-

Let the particle moves under potential 'V'

Total Energy = E + V. T- total energy

E = T - V V- Potential energy.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2m(T-V)}}}$$

3. For a charge particle.-

Let the charge particle having charge 'q' accelerated with the potential difference 'V".

KE of the charge particle = qV.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}.$$

Particular Case-

For electron

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}} A^\circ$$

4. For thermal particle.

KE of the thermal particle = $\frac{3}{2}KT$. T - Temp. in Kelvin
K - Boltz's constant

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}.$$

Q.1 Calculate the de-Broglie wavelength associated with a $e^{\ominus}$ subjected to the potential diff of 10 V.

Q.2. A ball of mass 0.1 g. has a speed of 300 m/s. Calculate the de-Broglie wavelength associated with.

Heisenberg's Uncertainity Principle -
Acc. to this principle. it is not possible to measure simultaneously the position and momentum of particle with 100% accuracy.

$$\Delta x \cdot \Delta p \geq \hbar/2 \qquad \hbar = \frac{h}{2\pi}$$

$$\Delta t \cdot \Delta E \geq \hbar/2.$$

Properties -
i) Non existance of electron in the nucleus.
ii) Ground state energy of harmonic oscillator is non zero.
(iii) Ground state energy of H-atom is non-zero.

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Proofs:

(i) Let us assume the $e^{\ominus}$ exist in the nucleus

dimension of nucleus $\approx 10^{-14}$ m.

$\Delta x \simeq 10^{-14}$ m.

Use Heisenberg's Uncertainity principle.

$$\Delta x \cdot \Delta p \simeq \frac{\hbar}{2}$$

$$\hbar = \frac{h}{2\pi} = 1.052 \times 10^{-34}$$

$$\Delta p = \frac{\hbar}{2\Delta x}$$

$$= \frac{1.052 \times 10^{-34}}{2 \times 10^{-14}} = 5.3 \times 10^{-21} \text{ kg m/s}$$

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$ [Crompton's Scattering]

$m_0 = 9.1 \times 10^{-31}$ kg.

$c = 3 \times 10^8$ m/s.

$\Delta p = p$.

$$E = \sqrt{(9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4 + (5.3 \times 10^{-21})^2 (3 \times 10^8)^2}$$

$E = 1.6 \times 10^{-12}$ J $\simeq 10$ MeV.

This means if the $e^{\ominus}$ exist inside the nucleus then ~~the~~ its energy must be in the order of 10 MeV

But the $e^{\ominus}$s emitted from the radio active β-decay has K.E about 1 MeV which is much smaller than the predicted value by uncertainity principle. So, $e^{\ominus}$s do not exist inside the nucleus.

$m \Delta v = 5.3 \times 10^{-21}$ kg m/s.

$$\Delta v = \frac{5.3 \times 10^{-21}}{9.1 \times 10^{-31}}$$

Uncertainity in velocity.

(ii) The energy of the 1-D harmonic oscillator is $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$ ——— ①.

Use Heisenberg's Uncertainty Principle,

$$\Delta x \cdot \Delta p \simeq \frac{\hbar}{2}$$

Let $\Delta x \approx x$

$\Delta p \approx p$.

$$\Rightarrow x \cdot p \approx \frac{\hbar}{2}$$

$$p = \frac{\hbar}{2x}$$

Substitute the values.

$$E = \frac{\hbar^2}{8mx^2} + \frac{1}{2}m\omega^2x^2 \quad ——— ②.$$

As the ground state energy is minimum for $x = x_0$, so we can write

$$\left.\frac{\partial E}{\partial x}\right|_{x=x_0} = 0.$$

$$\frac{\partial E}{\partial x} = \frac{-2\hbar^2}{8mx^3} + \frac{2}{2}m\omega^2x$$

$$\left.\frac{\partial E}{\partial x}\right|_{x=x_0} = \frac{-2\hbar^2}{8mx_0^3} + m\omega^2x_0 = 0.$$

$$x_0^4 = \frac{\hbar^2}{4m^2\omega^2}$$

$$\boxed{x_0^2 = \frac{\hbar}{2m\omega}}$$

$$E_0 = \frac{\hbar\omega}{4} + \frac{1}{2} m \omega^2 \times \frac{\hbar}{2m\omega}$$

Energy at ground state

$$= \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4} = \frac{\hbar\omega}{2}$$

(iii) The energy of H-atom,

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}.$$

r - radius of the orbit of the atom.

Use H.U.P.

$$\Delta x \cdot \Delta p \simeq \hbar$$

$$\Delta x \approx r$$

$$\Delta p \approx p.$$

$$p = \frac{\hbar}{r}.$$

Substitute the value.

$$E = \frac{\hbar^2}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\left.\frac{\partial E}{\partial r}\right|_{r=r_0} = 0$$

$$\frac{\partial E}{\partial r} = -\frac{2\hbar^2}{2mr^3} + \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\left.\frac{\partial E}{\partial r}\right]_{r=r_0} = 0$$

$$\Rightarrow \frac{e^2}{4\pi\epsilon_0 \cancel{r_0^2}} = \frac{\cancel{2}\hbar^2}{\cancel{2}m r_0^{\cancel{3}}}$$

$$\Rightarrow r_0 = \frac{\hbar^2 4\pi\epsilon_0}{e^2 m}$$

$$E_0 = \frac{\hbar^2 \times e^4 m^2}{2m\hbar^4 16\pi^2\epsilon_0^2} + \frac{e^2 \times e^4 m^2}{4\pi\epsilon_0 \hbar^4 16\pi^2\epsilon_0^2}$$

$$E_0 = \frac{1}{\hbar^4 16\pi^2\epsilon_0^2}\left[\frac{\hbar^2 e^4 m^2}{2m} + \frac{e^6 m^2}{4\pi\epsilon_0}\right]$$

Q.1. An $e^{\ominus}$ and a proton are accelerated through the same potential if their masses are m_e & m_p respectively then calculate the ratio of de-Broglie wavelength.

Q.2. Estimate the de-Broglie wavelength whose Energy is 45 eV.

Q.3. An $e^{\ominus}$ is accelerated by potential of V volt has a de-Broglie wavelength 'λ' if the $e^{\ominus}$ is accelerated by again by p.d of 4V find its de-Broglie wavelength.

Q.4 A proton and α-particle are accelerated by same p.d find the ratio of their de-Broglie wavelength

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Transition from Deterministic to Probabilistic -
(From book or google it)

In quantum mechanics every