PROBLEM SET - 3

1. (a) Out of the 7 trials, if we have to calculate the probability of getting exactly 5 heads, we need to do the following:

$$\underset{\frac{1}{2}}{\underline{H}}\ \underset{\frac{1}{2}}{\underline{H}}\ \underset{\frac{1}{2}}{\underline{H}}\ \underset{\frac{1}{2}}{\underline{H}}\ \underset{\frac{1}{2}}{\underline{H}}\ \underset{\frac{1}{2}}{\underline{T}}\ \underset{\frac{1}{2}}{\underline{T}}$$

Along with all the possible combinations:

$$^7C_5 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^2 = \underline{\underline{0.164}}$$

(b) PMF

We know the formula for P.M.F of binomial Random variable is:

$$^nC_x \times P^x \times Q^{n-x}$$

So, to calculate F(2), we need to find $P(X=0) + P(X=1) + P(X=2)$

$$P(X=0) = \frac{7!}{0!\,7!} \times (0.5)^0 \times (0.5)^7$$

$$= 0.0078$$

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$$P(X=1) = \frac{7!}{1!\,6!} \times (0.5)^1 \times (0.5)^6 = 0.054$$

$$P(X=2) = \frac{7!}{2!\,5!} \times 0.0078 = 0.1638$$

$$\Rightarrow 0.0078 + 0.054 + 0.1638 = 0.2256$$

(c) Expectedvalue for a Binomial random variable

$$= n.p = 7 \times 0.5 = 3.5$$

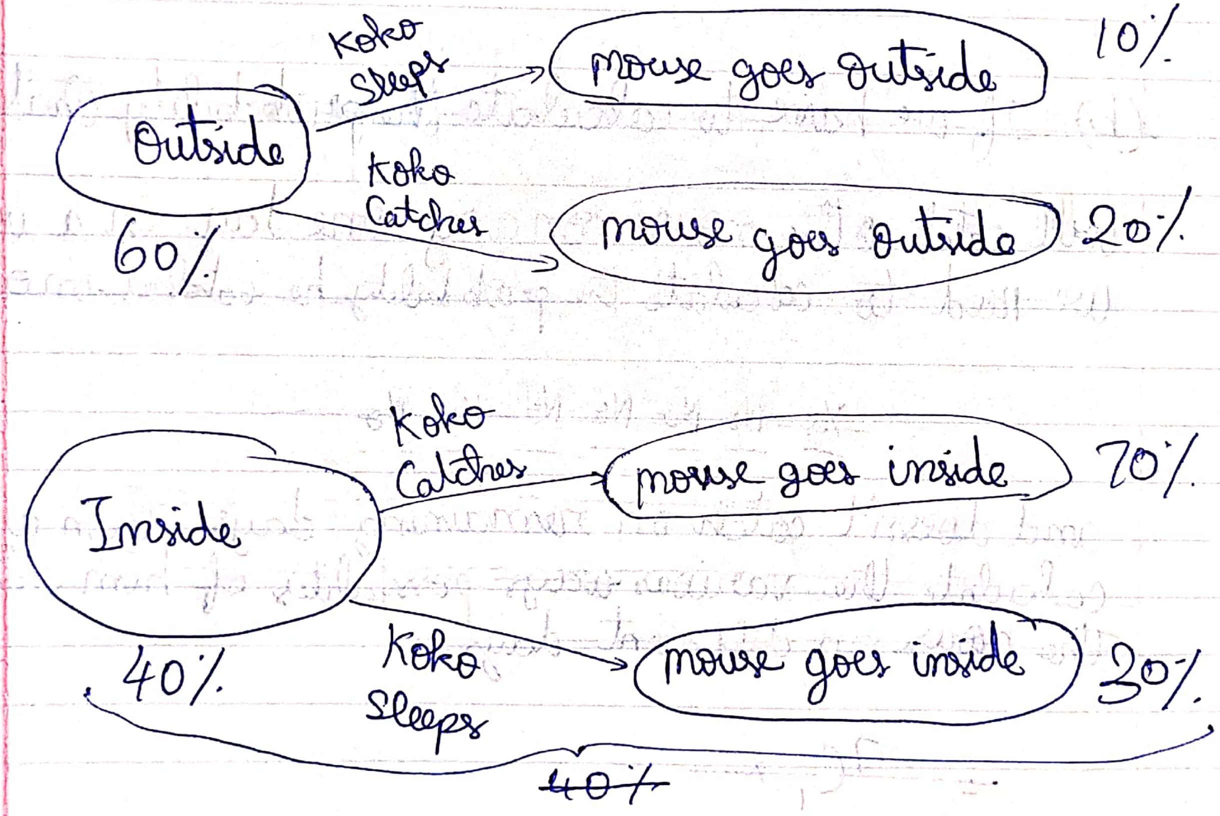
~~(d)~~ $\sigma = \sqrt{n \times p \times (1-p)}$ ~~or $\sqrt{3.5} = 1.32$~~

$$= \sqrt{7 \times 0.5 \times 0.5} = \sqrt{1.75}$$

$$= 1.322$$

(d) If we consider "All heads on 1st four tosses" as event A and "all tails on last four tosses" then event A will be dependent on B because for A to happen, B cannot hold true. ~~Similarly~~ Which means for 1st 4 to be heads, last 4 cannot be tails. Only if last 3 are tails or heads, can the first 4 be heads.

2.



(a) ~~I~~ Now we can calculate the probability of Koko catching the mouse Outside and Inside.

Let's take Outside first:

$$0.6 \times 0.2 = 0.12$$

Now, for inside, we are also given that Koko has a 40% chance of catching if mouse goes inside and Koko stays awake.

$= 0.4 \times 0.7 \times 0.4 = \underline{0.112}$

So, Koko is more likely to catch the mouse if he waits outside.

~~(b) If we have to calculate the probability that Koko will catch the mouse on any one day in a week, we need to calculate the probability he catches on one day~~

~~Yes No No No No No No~~

~~and doesn't catch on remaining days. Then we calculate the various ways possibilities of him catching the mouse on different days.~~

$\cancel{= {}^7C_1 \times}$

(b) If we have to calculate the probability that Koko will catch the mouse on any one day in a week, we need to calculate the probability he catches on one day and doesn't catch on remaining days. Then we calculate the various possibilities of him catching the mouse on different days.

$$= {}^7C_1 \times 0.12 \times (0.88)^6 = 0.39 = 39\%$$

3. We know that for a Binomial Distribution the PMF formula is $P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

(a) $P(X=0) = {}^{2}C_{0} (0.7)^{0} (0.3)^{2}$

$= \frac{2!}{0!\,2!} \times 0.3^{2} = \underline{0.09}$

$P(X=1) = {}^{2}C_{1} (0.7)^{1} (0.3)^{1}$

$= \frac{2!}{1!\,1!} (0.7)(0.3) = \underline{0.42}$

$P(X=2) = {}^{2}C_{2} (0.7)^{2} (0.3)^{0}$ ~~= 1~~

$\frac{2!}{0!\,2!} (0.49) = \underline{0.49}$

(b) The CDF will be the sum of all the probabilities before our upper limit

$$F(y) = \begin{cases} 0.09 & y < 1 \\ 0.51 & 1 \le y < 2 \\ 1 & y \ge 2 \end{cases}$$

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(C) We know the expected value can be calculated by the below formula:

$$\mu = E = n.p = 2.(0.7) = 1.4$$

5. Given,

$$f(x) = \begin{cases} \frac{1}{20} & 20 \le x < 40 \\ 0 & \text{Otherwise} \end{cases}$$

a. We ~~kow~~ know from the given information that for all values less than or equal to 40 and greater than 20, it is 1/20. And all other values it is 0. &

∴ it is never negative.

And to check if area under $f(x)$ is 1, we can integrate the area b/w 20 & 40.

$$= \int_{20}^{40} \frac{1}{20}.dx = \left[\frac{1}{20}.x\right]_{20}^{40} = \frac{1}{20}[40-20]$$

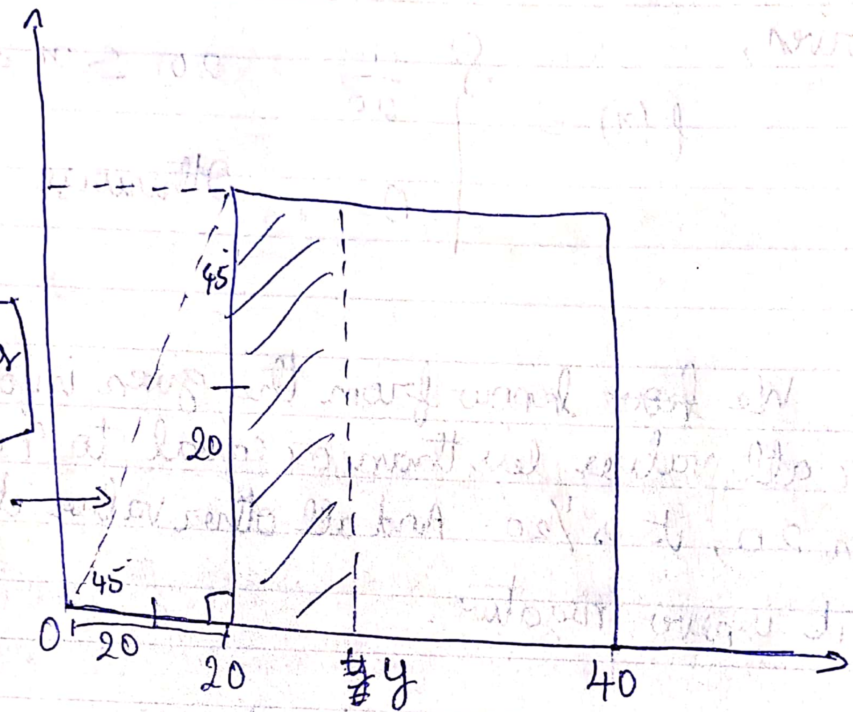
$$= \frac{40}{20} - \frac{20}{20} = \frac{20}{20} = 1$$

∴ Area under $f(x)$ is found to be 1 through integration

b.

To calculate the CDF now. For $y < 20$, we know it will be 0 because of the PDF given. ~~Similarly for $y \geq 40$, it will be 1~~. But for CDF b/w 20 & 40 i.e, $20 \le y < 40$, will be the area under the P.D.F but left of y.

This will be isosceles triangle because the hypotenuse will divide the angles equally. And in isosceles triangle, 2 sides are equal



This can be calculated as base $= \dfrac{y-20}{20}$

$= \dfrac{y}{20} - 1$

$$\text{C.D.F} = F(y) = \begin{cases} 0 & y < 20 \\ \dfrac{y}{20} - 1 & 20 \le y < 40 \\ 1 & y \ge 40 \end{cases}$$

$\boxed{\dfrac{y}{20} - 1 \quad \dfrac{40}{20} - 1 = 2 - 1 = 1}$

(c) $P(30 \le X < 50) = P(X \le 50) - P(X \le 30)$

$= F(50) - P(30)$

$= 1 - \left[\dfrac{30}{20} - 1\right]$

$= 1 - 0.5 = 0.5$

(d). $E(x) = \mu = \int_a^b x \cdot f(x) \cdot dx = \frac{a+b}{2}$

After ~~done~~ integrating the L.H.S we get the above value.

$a = 20 \quad b = 40$

$= \frac{20+40}{2} = 30$

(e) Var (x) can be calculate by integrating

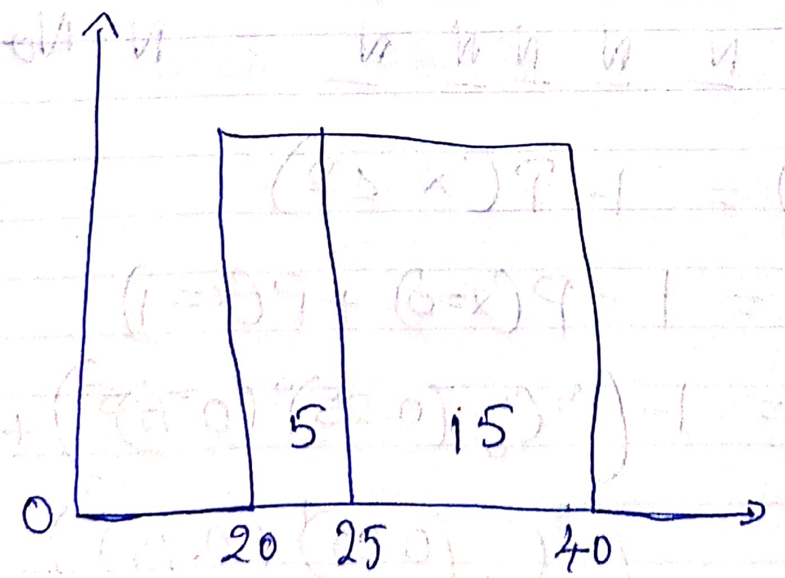
$$\int_a^b (x-\mu)^2 \cdot f(x) \cdot dx = \frac{(b-a)^2}{12} = \frac{(40-20)^2}{12}$$

$$= \frac{400}{12} = 33.\overline{3}$$

$$S.D(x) = \cancel{\frac{b-a}{\sqrt{12}}} \sqrt{var} = \sqrt{\frac{(b-a)^2}{12}} = \frac{b-a}{\sqrt{12}} = \frac{20}{\sqrt{12}}$$

$$= 5.77$$

6.



(a) The probability that will take her less than 25 minutes on Monday :

$$\frac{\text{Distance b/w } 25 \text{ to } 20}{\text{Distance b/w } 40 \text{ to } 20}$$

Because we are only interested in the left portion of 25.

So, $\frac{5}{20} = \frac{1}{4} = 0.25$

(b) Now, probability that she takes <25 minutes on at least 2 days is complement of she takes <25 minutes on 0 days and 1 day

$\underline{Y}$ $\underline{N}$ $\underline{N}$ $\underline{N}$ $\underline{N}$ Y - Yes

$\underline{N}$ $\underline{N}$ $\underline{N}$ $\underline{N}$ $\underline{N}$ N - No

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - P(X=0) + P(X=1)$$
$$= 1 - \left({}^5C_0 (0.25)^0 (0.75)^5\right) + \left({}^5C_1 (0.25)^1 (0.75)^4\right)$$

$= 1 - ((0.75)^5) + (5 \times 0.25 \times 0.316)$

$= 1 - (0.23) + 0.395 = 1 - 0.625 = \underline{\underline{0.375}}$

(c) Assuming $X_n = 1,2,3,4,5$ are discrete values
we know that sum of all expected values will be our total

$E(n) = \frac{a+b}{2}$

$\cancel{E\,E}\ EX_1 = 30 \quad EX_2 = 30 \quad EX_3 = 30$

$Ex_4 = 30 \quad Ex_5 = 30$

Total = 150

Variance can also be added likewise

$Var_1 = 33.33 \ \cancel{E}\ Var_{2,3,4,5} = 33.33$

Total = 166.66

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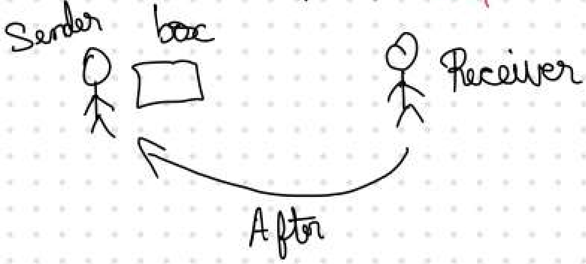
$\downarrow$

ESP Experiment



4.
$$P \qquad Q$$
$$\frac{1}{5} \qquad \frac{4}{5}$$
$$0.2$$

25 trials

(a) $E = n \times p = 25 \times \frac{1}{5} = 5 = 0.2$

(b) $P(X > 7) = 1 - P(X \leq 7)$

Using Pbinom function in R

pbinom (7, 25, 1/5)

$= 1 - 0.8908772$

$= 0.1091 = 10.91\%$

(c) 21

$1 \rightarrow 20$

$= 1 - P(\text{None get ESP})$

$= 1 - (0.8909)^{20}$

$= 1 - 0.0992$

$= 0.9008 = 90.08\%$