Problem set 3

S520

Upload your answers as ONE file (PDF preferred) through the Assignments tab on Canvas by 11:59 pm, Thursday 14th September.

Trosset question numbers refer to the hardcover textbook. Show working. You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources. Some questions are best solved with R; if you use R, include your code.

1. (From the Spring 2017 in-class midterm. You might want to do this question by hand, rather than use R, as practice for your midterm.) I toss a coin seven times, independently. The coin is fair, so the probability of getting a head is 0.5 for each toss. Let X be a binomial random variable representing the number of heads in the seven tosses.

   (a) What is the probability I get exactly five heads?

   (b) Find $F(2)$, the cumulative distribution function of X at 2.

   (c) Find the expected value and standard deviation of X.

   (d) Are the events "I get all heads on the first four tosses" and "I get all tails on the last four tosses" independent? Explain why or why not.

2. Trosset chapter 4.5 exercise 7. Note: You don't have to submit the tree diagram, though it would be a good idea to draw it for yourself to help understand the problem.

3. According to Pew Research, 70% of Generation Z Americans (those aged 13 to 21 in 2018) believe "government should do more to solve problems." I select two Generation Z Americans at random. Let X be a discrete random variable representing the number of the people I selected who believe "government should do more to solve problems." Since the number of Generation Z Americans is very large, X will have very close to a binomial distribution.

   (a) Find $P(X = 0)$, $P(X = 1)$, and $P(X = 2)$.

   (b) Write down an expression for $F(y)$, the cumulative distribution function (CDF) of X, for all y-values from $-\infty$ to ∞.

   (c) Find the expected value of X.

4. Trosset chapter 4.5 exercise 14.

5. Let X be a uniform random variable with probability density function (PDF)

$$f(x) = \begin{cases} \frac{1}{20} & 20 \le x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Verify that f is a PDF. That is, check that $f(x)$ is always non-negative and that the area under $f(x)$ is 1.

   (b) Find the CDF of X, $F(y)$, for all y.

   (c) Find $P(30 \le X < 50)$.

   (d) Find the expected value of X.

   (e) Find the variance and standard deviation of X. (Hint: You can use calculus if you wish, and it's good practice, but it'll be easier to use the formula for the variance and SD of a uniform random variable.)

6. I go to work five days this week, Monday to Friday. I model the time (in minutes) it takes me to get to work each day as having a Uniform$(20, 40)$ distribution.[1] Assume the time I take one day is independent of all the other times.

   (a) What's the probability it takes me less than 25 minutes to get to work on Monday?

   (b) What's the probability that on at least two days out of five, it takes me less than 25 minutes to get to work?

   (c) Let Y be the *sum* of the times it takes me to get to work over all five days. What are the expected value, variance, and standard deviation of Y? (Hint: The properties of sums of independent discrete random variables in Trosset chapter 4.3 also hold for independent continuous random variables.)

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[1]The time mostly depends on whether I see interesting mushrooms to take photos of while walking.