Problem Set - 6 ANIRUDH PENMATCHA

1. (a) - There are 59 white balls
   - Each week 5 balls are drawn without replacement
   - Most frequent no. 23

The probability of getting a 23 will be equal to $\frac{5}{59}$ because it could be a one off case that 23 was frequent We don't have sufficient information to determine will be $>$ or $<$ $\frac{5}{59}$.

(b) False, because the first 6 games that they won Could be a part of $\frac{81}{162}$ probability from the past. At least, it's too early to say anything.

Another way to look at this is the probability of them winning is half $= (0.5)$. For 6 games $= (0.5)^6$. But for remaining games out of the total $= (0.5)^{156}$ is too low. Hence the chances of them winning is very low.

(c) True, for now that will still be the best prediction because we don't have enough information yet to determine whether they will win more than 81 games.

(d) False, the Central Limit Theorem says nothing about the population mean. It only talks about the sample mean.

(e) True, because in this case we are dealing with sample income and so there is a decent possibility for the sample to be normally distributed. However, it's possible for the data skewed to any side depending on how the sample was taken.

2. Please refer to the submitted .Rmd file

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4. For a fair coin, we know that the probability of getting a heads is 0.5.

Now we are told that in the first set of 100 tosses we get 60 heads and we are asked if in the next set of 100 tosses we will get 40 heads because of the law of averages.

Law of averages states that sample proportions will approach their true probabilities on the long run. The true probability here is 0.5. So if our trials were large enough, then it would be safe enough to say that we will get heads in half the no. of trials.

However, we know that we are doing only 100 tosses which is not nearly large enough to even conclusively say that we will get half of the outcomes as heads. We may get 40, more than 40, or even less than 40. Thus the law of averages here doesn't really help us in implying whether we will get 40 heads or not in the next set of 100 tosses.

5. Given,

two AAAA batteries needed for laser pointer

1 pair lasts 5 hours $\pm$ 30 mins

avg = 5 $\quad \sigma = 0.5$ hrs

20 packs of two AAAA batteries

Let's say X is a random variable that represents the no. of hours a pair of batteries last.

So we will have 20 different values possible for our random variable X. $X_1, X_2, X_3 \dots X_{20} = S$

The mean for S would be

$$E(S) = E(X_1 + X_2 + X_3 + \dots + X_{20})$$
$$= E(X_1) + E(X_2) + \dots E(X_{20})$$
$$\Rightarrow \mu + \mu + \dots \mu$$
$$= 20\mu = 20 \times 5 = \boxed{100}$$

$$\text{var}(S) = \text{var}(X_1 + X_2 \dots + X_{20})$$
$$= \text{var}(X_1) + \text{var}(X_2) + \dots \text{var}(X_{20})$$

$$= \sigma^2 + \sigma^2 + \ldots \sigma^2 = 20\sigma^2$$

$$= 20 \times (0.5^2)$$

$$var(S) = \boxed{5}$$

$$\sigma = \sqrt{5} = \boxed{2.236}$$

The required probability can be calculated by using the central limit theorem :

$$P(S > 105) = P\left(\frac{S - \mu_S}{\sigma_S} > \frac{105-100}{2.236}\right)$$

$$= P(z > 2.24)$$

$$= 1 - P(z \leq 2.24)$$

We can use the pnorm function in R to calculate $P(z \leq 2.24)$

Code: pnorm(2.24,0,1)

Output: 0.9874545

$$= 1 - 0.9875 = \boxed{0.0125}$$