

1.  $H_0$ : start position affects horse's chances  
 $H_1$ : start position doesn't affect horse's chances

Let's calculate our test statistic

$$\chi^2 = \frac{\sum (\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Let's tabulate the test statistic

O	P	E	(O-E)	(O-E) <sup>2</sup>	(O-E) <sup>2</sup> /E
29	0.125	18	11	121	6.72
19	0.125	18	1	1	0.05
18	0.125	18	0	0	0
25	0.125	18	7	49	2.72
17	0.125	18	-1	1	0.05
10	0.125	18	-8	64	3.55
15	0.125	18	-3	9	0.5
11	0.125	18	-7	49	2.72
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					16.33

Our test statistic  $\chi^2 = 16.33$

$$\text{D.O.F} = 8 - 1 = 7$$

$$P\text{-value} = 1 - \text{pchisq}(16.33, \text{df} = 7) = 0.022$$

$\therefore$  we reject the null hypothesis which means that the starting position doesn't affect the horse's chances

2. Tall - T Dwarf - D Cut - C Potato - P

We know that the probability for a recessive trait is  $\frac{1}{4}$ . Then, probability of a dominant trait will be  $\frac{3}{4}$ .

There are 4 possible combinations

$$E_1 = TC$$

$$P(E_1) = P(T) \times P(C) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$E_2 = TP$$

$$P(E_2) = P(T) \times P(P) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

$$E_3 = DC$$

$$P(E_3) = P(D) \times P(C) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{4}$$

$$E_4 = DP$$

$$P(E_4) = P(D) \times P(P) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

b.  $H_0$  : Observed and expected frequencies are similar  
 $H_1$  : Observed and expected frequencies are not similar

$$O_1 = 928, O_2 = 288, O_3 = 293, O_4 = 104$$

We are also given  $n = 1611$

$$\chi^2 = \frac{\sum (\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

We have the observed but we can calculate the expected value by  $np$ .

$$E_1 = \frac{9}{16} \times 1611 = 906.19$$

$$E_2 = \frac{3}{16} \times 1611 = 302.06$$

$$E_3 = \frac{3}{16} \times 1611 = 302.06$$

$$E_4 = \frac{1}{16} \times 1611 = 100.69$$

$$\chi^2 = \frac{(926 - 906.19)^2}{906.19} + \frac{(288 - 302.06)^2}{302.06} + \frac{(293 - 302.06)^2}{302.06} + \frac{(104 - 100.69)^2}{100.69}$$

$$= 0.433 + 0.654 + 0.271 + 0.108 = 1.466$$

$$\text{D.O.F} = \text{No. of categories} - 1 = 4 - 1 = 3$$

$$p\text{-value} = 1 - pchisq(1.466, df = 3) = 0.6901$$

We fail to reject the null hypothesis. ∴,

3.  $H_0$ : Sex ratio of sandflies does not vary with ground height.

$H_1$ : Sex ratio of sandflies varies with height of ground

Given:

	3ft	35ft	Total
Male	173	125	298
Female	150	73	223
Total	323	198	521

Expected =  $n \times p$

$$\begin{aligned} P(\text{male 3ft}) &= P(\text{Male}) \times P(3\text{ft}) \\ &= \frac{298}{521} \times \frac{323}{521} \end{aligned}$$

$$E(\text{male 3ft}) = 521 \times \frac{298}{521} \times \frac{323}{521} = 184.75$$

$$\begin{aligned} P(\text{male 35ft}) &= P(\text{male}) \times P(35\text{ft}) \\ &= \frac{298}{521} \times \frac{198}{521} \end{aligned}$$

$$E(\text{male 35ft}) = 521 \times \frac{298}{521} \times \frac{198}{521} = 113.25$$

$$E(\text{Female } 3\text{ft}) = \frac{323}{521} \times \frac{298}{521} \times 521 = 138.25$$

$$E(\text{Female } 35\text{ft}) = \frac{198}{521} \times \frac{223}{521} \times 521 = 84.75$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(173-138.25)^2}{138.25} + \frac{(150-138.25)^2}{138.25} + \frac{(125-113.25)^2}{113.25} + \frac{(73-84.75)^2}{84.75}$$

$$= 0.747 + 1.219 + 0.998 + 1.493 = 4.457$$

$$\begin{aligned} \text{D.O.F} &= (\text{no. of rows} - 1)(\text{no. of columns} - 1) \\ &= (2-1) \times (2-1) = 1 \end{aligned}$$

$$P\text{-value} = 1 - p\text{chisq}(4.457, df=1) = 0.03$$

∴ We reject the null hypothesis and say that the sex ratio of sandflies varies with height of the ground.

d.  $H_0$ : Response to treatment does not vary by histological type of Hodgkin's disease

$H_1$ : Response to treatment varies by histological type of Hodgkin's disease

Expected:	positive	Partial	None
LP	$\frac{314 \times 104}{538} = 60.698$	$\frac{98 \times 104}{538} = 18.94$	$\frac{126 \times 104}{538} = 24.356$
NS	$\frac{314 \times 96}{538} = 56.029$	$\frac{98 \times 96}{538} = 17.49$	$\frac{126 \times 96}{538} = 22.48$
MC	$\frac{314 \times 266}{538} = 155.25$	$\frac{98 \times 266}{538} = 48.45$	$\frac{126 \times 266}{538} = 62.29$
LD	$\frac{314 \times 72}{538} = 42.022$	$\frac{98 \times 72}{538} = 13.11$	$\frac{126 \times 72}{538} = 16.86$

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(74-60.7)^2}{60.7} + \frac{(18-18.94)^2}{18.94} + \frac{(12-24.36)^2}{24.36} + \frac{(68-56.03)^2}{56.03} + \frac{(16-17.49)^2}{17.49} + \frac{(12-22.48)^2}{22.48}$$



$$\begin{aligned}
 & + \frac{(154 - 155.25)^2}{155.25} + \frac{(54 - 48.45)^2}{48.45} + \frac{(58 - 62.3)^2}{62.3} + \\
 & \frac{(18 - 42.02)^2}{42.02} + \frac{(10 - 13.12)^2}{13.12} + \frac{(44 - 16.86)^2}{16.86} = 75.89
 \end{aligned}$$

$$df = (r-1)(c-1) = (4-1)(3-1) = 6$$

$$p\text{-value} = 1 - pchisq(75.89, df = 6) = 2.52 \times 10^{-14}$$

∴ We reject  $H_0$  and say there is a difference in the response to the treatment varies by histological type of Hodgkin's disease.

5.  $H_0$ : There's no association b/w anger and heart disease

$H_1$ : There's association b/w anger and heart disease

Observed

Anger/ Heart disease	Present	Absent	Total
Low	53	$3110 - 53 = 3057$	3110
Moderate	110	$4731 - 110 = 4621$	4731
High	27	$633 - 27 = 606$	633
Total	190	8284	8474

Expected

Anger/Heart  
Disease

Present

Absent

$$\frac{3110 \times 190}{8474} = 70$$

$$\frac{3110 \times 8284}{8474} = 3040$$

Low

$$\frac{4731 \times 190}{8474} = 106$$

$$\frac{4731 \times 8284}{8474} = 4625$$

Moderate

High

$$\frac{633 \times 190}{8474} = 14$$

$$\frac{633 \times 8284}{8474} = 619$$

$$\chi^2 = \frac{\sum (O-E)^2}{E} = \frac{(53-70)^2}{70} + \frac{(3057-3040)^2}{3040}$$

$$+ \frac{(110-106)^2}{106} + \frac{(4621-4625)^2}{4625} + \frac{(27-14)^2}{14}$$

$$+ \frac{(606-619)^2}{619}$$

$$= 4.28 + 0.095 + 0.151 + 0.003 + 12.07 + 0.273$$

$$= 16.72$$

$$df = (r-1)(c-1) = (3-1)(2-1) = 2 \times 1 = 2$$

$$p\text{-value} = 1 - p\chi^2_{df=2}(16.72) = 0.0002$$

∴ We reject the null hypothesis and we say there is association b/w anger & heart disease.

(b) We cannot prove causality just by a statistical model. It only helps quantify the relationship but doesn't necessarily control for every possible confounding variable in our problem. So it's highly likely that there are other factors that are affecting this relationship. Just because there is a correlation b/w 2 aspects like anger and heart disease it isn't

to say definitely that there is a causality.  
More quality data will be required to  
make a better conclusion on the causality.