1. (e) An event is considered dependent if $P(B|A) \neq P(B)$. In this case, because we're talking about movies that were shot in the 1900's (20th century), not all movies are made in colour. Therefore, the probability for a movie of the genre "Western" being picked from the collection of movies shot in colour will be lesser than $P(B)$. Hence, A and B here are dependent events. If we look at it $P(A|B)$, they still work out to be dependent events.
(f) These events are also going to be dependent. However we look at it, $P(A|B)$ or $P(B|A)$, these events outcomes will be dependent on the other. For example, with $P(A|B)$, the probability that a person attends college of william and mary given that they graduated from high school in Virginia will not be equal to $P(A)$. Therefore, dependent events.
(g) Finally, for this question as well, the events are going to be dependent. Not all Ph.D. holders are female. There are Ph.D. holders who are male as well. So $P(A|B)$ or $P(B|A)$ work out to dependent events. For example, $P(B|A)$ is the probability that a person is female given that they also earned a Ph.D. before 1950, will not equal to $P(B)$. $P(B)$ can be 0.5 but $P(B|A)$ will be lower than that.

2. (a) Let us assume that the U.S. consists of only 100 people. Then $P(A|B)$ will be 85% of $100 = (85/100 *100)/100 = 85/100 = 0.85$. And $P(A^c|B)$ means complement of A given B. Complement of A here will be that the person is female. So that will be the remaining people struck by lightning which is 15% of $100 = (15/100 * 100)/100 = 0.15$.
(b) $P(A|B) \neq P(A)$ because $P(A)$ is the probability of the person being male. $P(A|B)$ is the person being a male given that they are also struck by lightning. $P(A|B)$ will be lower than the probability that the person is male $P(A)$. Hence, A and B are dependent events.
(c) Because masculine characteristics are such they don't care about how they are and they're surroundings are. But feminine characteristics are such that they like things to be a certain way with themselves and their surroundings. Which is why women care more about their appearance than men. Otherwise men would also be doing makeup. These are just characteristics of masculinity and femininity. Not necessarily it has to do with their gender. Meaning, it's even possible for a man to be like a woman in how they and their surroundings are. This could be a potential explanation as to why more men would be outside when it's raining. Indian culture speaks in a lot of depth about masculinity and femininity. It's said that everyone should have a balance between their masculinity and femininity. In India, this is symbolically represented as "Ardhanari".

3. (a) Contingency Table

		Positive	Negative
~~354~~ [0.428%]	Has Cancer	82.6%	17.4%
~~9559~~	No Cancer [99.572%]	9.6%	90.4 %

~~⇒ P(Has Cancer) + P(tested positive)~~

~~0.826 × 100 +~~

Let's say there 100,000 women in total

Then 0.428% of 100,000 will be 428
and 82.6% of that will be 354 approx.

Probability of that will be $\frac{354}{100,000}$

$\simeq 0.00354$

3.

(b) P (Woman tests +ve)

$$(0.428\% \text{ of } 100{,}000)\,82.6\% + (99.572\% \text{ of } 100{,}000)\,9.6\%$$

$$= 354 + 9559 = 9913$$

Probability of 1 person being picked from here will $\frac{9913}{100{,}000} = 0.09913$

(C) ~~We know from previous question that there 9913 people who tested +ve.~~

The probability that a person has breast cancer given that they tested positive is

$$= \frac{82.6\% \text{ of } 428}{82.6\% \text{ of } 428 + 9.6\% \text{ of } 99{,}572}$$

$$= \frac{354}{9912.44} = 0.0357127 \simeq 0.0357$$

(d) Let us take the sample from the population as 99,999. In this case, it is mostly safe to say that the probabilities will remain the same. And also assuming that 1 person left out doesn't have breast cancer.

P(at least 1 has cancer among the 10) = P(1 - None have cancer) = 1- (99572/99999) = 1 - 0.996 = 0.004.

4. (a) $P(X \leq 2) = F(2) = (2+1)/4 = 3/4 = 0.75$
(b) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.75 = 1/4 = 0.25$
(c) $P(0.5 < X \leq 2.5) = P(X \leq 2.5) - P(X \leq 0.5) = F(2.5) - F(0.5) = (2.5 + 1)/4 - 0.5/2 = 3.5/4 - 1/4 = 2.5/4$
(d) $P(X = 1) = P(X \leq 1) - P(X < 1) = F(1) - F(0.999\ldots) = 2/4 - 0.999\ldots/2 = 1/2 - 1/2 = 0$
(e) First we can try equating 0.6 to y/2 and check if q is between $0 \leq y < 1$. $y/2 = 0.6 \Rightarrow y = 1.2$. But 1.2 is not between $0 \leq y < 1$. Next we can try $(y+1)/4$ and check if q is in between $1 \leq y < 3$. $(y+1)/4 = 0.6 \Rightarrow y+1 = 2.4 \Rightarrow y = 2.4-1 = 1.4$. And 1.4 is in between $1 \leq y < 3$. Therefore, $q = 1.4$.

PLEASE SCROLL DOWN FOR QUESTIONS 5 & 6

5. (a) $f(x) = \begin{cases} \frac{(7-x)}{20} & x \in \{1,2,3,4,5\} \\ 0 & x \in \{6\} \end{cases}$

x	1	2	3	4	5	6
$f(x)$	$\frac{6}{20}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	0

(b) $F(y) = \begin{cases} 0 & y < 1 \\ \frac{6}{20} & 1 \le y < 2 \\ \frac{5}{20} & 2 \le y < 3 \\ \frac{4}{20} & 3 \le y < 4 \\ \frac{3}{20} & 4 \le y < 5 \\ \frac{2}{20} & 5 \le y < 6 \\ & y \geq 6 \end{cases}$

(c) $\left(1 \times \frac{6}{20}\right) + \left(2 \times \frac{5}{20}\right) + \left(3 \times \frac{4}{20}\right) + \left(4 \times \frac{3}{20}\right) +$
$\left(5 \times \frac{2}{20}\right) + (6 \times 0)$

5.

$$= \frac{6}{20} + \frac{10}{20} + \frac{12}{20} + \frac{12}{20} + \frac{10}{20}$$

$$E(x) = \mu = \frac{50}{20} = 2.5$$

(d) $$\left((1-2.5)^2 \times \frac{6}{20}\right) + \left((2-2.5)^2 \times \frac{5}{20}\right) +$$

$$\left((3-2.5)^2 \times \frac{4}{20}\right) + \left((4-2.5)^2 \times \frac{3}{20}\right) +$$

$$\left((5-2.5)^2 \times \frac{2}{20}\right)$$

$$= 0.675 + 0.0625 + 0.05 + 0.3375 + 0.625$$

$$= 1.75$$

(e) $\sqrt{1.75} = 1.322$

6. (a) $f(x) = \begin{cases} \frac{4}{10} & x=1 \\ \frac{1}{10} & x=2 \\ \frac{2}{10} & x=5 \\ \frac{3}{10} & x=10 \end{cases}$

(b) $F(y) = \begin{cases} 0 & y<1 \\ \frac{4}{10} & 1\leq y<2 \\ \frac{5}{10} & 2\leq y<5 \\ \frac{7}{10} & 5\leq y<10 \\ 1 & y\geq 10 \end{cases}$

6.

(c) $\left(1\times\frac{4}{10}\right)+\left(2\times\frac{1}{10}\right)+\left(5\times\frac{2}{10}\right)+\left(10\times\frac{3}{10}\right)$

$\frac{4}{10}+\frac{2}{10}+\frac{10}{10}+\frac{30}{10}=\frac{46}{10}=4.6$

(d) $\left((1-4.6)^2\times\frac{4}{10}\right)+\left((2-4.6)^2\times\frac{1}{10}\right)+\left((5-4.6)^2\times\frac{2}{10}\right)$

$+\left((10-4.6)^2\times\frac{3}{10}\right)$

$= 5.184+0.676+0.032+8.748$

$= 14.64$

(e) $\sqrt{14.64} = 3.83$