

## Problem set 4

S520

**Upload your answers as ONE file (PDF preferred) through the Assignments tab on Canvas by 11:59 pm, Thursday 21st September.**

Trosset question numbers refer to the hardcover textbook. Show working (answers only will not get full credit.) You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources. You may (and sometimes have to) use R; include your R code where relevant.

1. Let  $X$  be a random variable with PDF

$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x < 20 \\ \frac{1}{60} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the CDF of  $X$ ,  $F(y)$ , for all  $y$ .
  - (b) Suppose that buses go past my stop at exactly twenty minutes past the hour and forty minutes past the hour (e.g. 9:20, 9:40) every hour. I arrive at my stop at a completely random time during the day. What is the expected value of the length of time I'll have to wait for a bus?
  - (c) Find  $y$  such that  $F(y) = 0.5$ . (That is, set the correct piece of  $F(y)$  equal to 0.5, and solve for  $y$ .) Is this larger than, smaller than, or the same as  $E(X)$ ?
2. Trosset chapter 5.6 exercise 2, parts (b) and (c).
  3. (Adapted from the Spring 2017 takehome.) The tempo (speed) of a piece of music is usually measured in beats per minute (BPM.) A study<sup>1</sup> found that the BPM in disco songs was approximately normal with mean 120 and standard deviation 20. (Treat BPM as a continuous random variable for the purpose of this question.) The best tempo for dancing is considered to be 115 to 135 BPM, while anything above 160 BPM is exhausting.

Suppose a DJ gets lazy and puts her MP3 player containing a large, representative collection of disco songs on shuffle.

- (a) Suppose the MP3 player randomly selects a disco song. What is the probability that the BPM is between 115 and 135?
- (b) Suppose the MP3 player randomly selects ten disco songs. What is the probability at least one of the ten songs has a BPM over 160?

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<sup>1</sup><https://github.com/nikhilunni/BPMFinder/>

- (c) Suppose the MP3 player randomly selects two disco songs. What is the probability the *average* BPM of these two songs is over 160? (Hint: What's the distribution of the sum of two independent normal random variables? How big does the sum have to be for the average to be over 160?)
4. Trosset chapter 5.6 exercise 8. Note: Remember that  $\text{Normal}(1, 9)$  denotes a *variance* of 9, not a SD.
5. (From the Spring 2017 midterm.) Let  $X$  be a continuous random variable with probability density function (PDF)

$$f(x) = \begin{cases} 0.1 & 0 \leq x < 1 \\ 0.2 & 1 \leq x < 2 \\ 0.4 & 2 \leq x < 3 \\ 0.3 & 3 \leq x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

To answer the following questions, it may help to draw a graph of this PDF.

- (a) Complete  $F(y)$ , the cumulative distribution function of  $X$ , by complete the expression below:

$$F(y) = \begin{cases} & y < 0 \\ & 0 \leq y < 1 \\ & 1 \leq y < 2 \\ & 2 \leq y < 3 \\ & 3 \leq y < 4 \\ & y \geq 4 \end{cases}$$

- (b) Is the median of  $X$  equal to 2, less than 2, or greater than 2? Explain why or calculate the median.
- (c) Is the expected value of  $X$  equal to 2, less than 2, or greater than 2? Explain why or calculate the expected value.
6. Trosset chapter 6.4 exercise 2.