Linear Layouts of Chordal Bipartite Graphs

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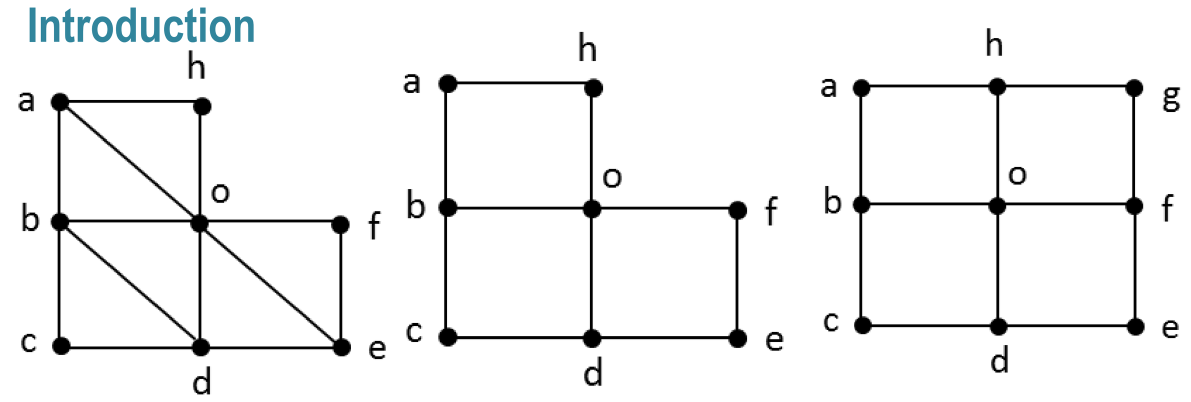
Applications

- Point Placement Problem.
- Importance in field of molecular biology.
- Modelled as graph. Absent edges need not be considered as constraint.
- NP-hard in even 2D.
- Variants in 1D studied.
- Triangulated and Weakly Triangulated Graphs are such variants.

Introduction



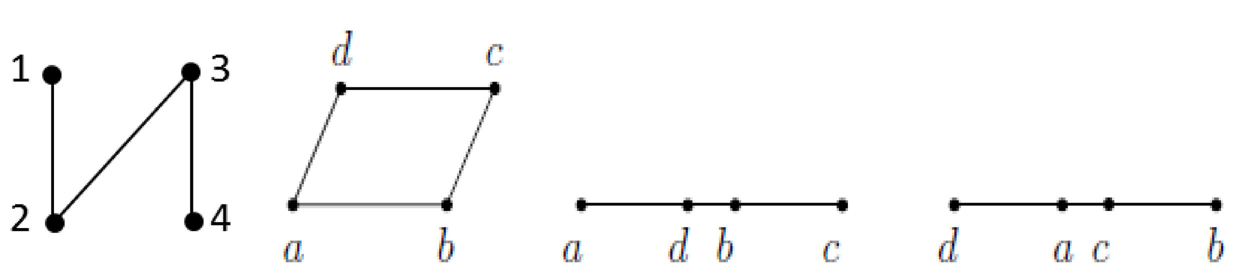
Triangulated Weakly Triangulated Neither

- Many Algorithms proposed for both graphs.
- Need for verification.
- How to generate ?
- Finding Linear Layouts.
- Already Solved for Triangulated Graphs.

Problem Statement

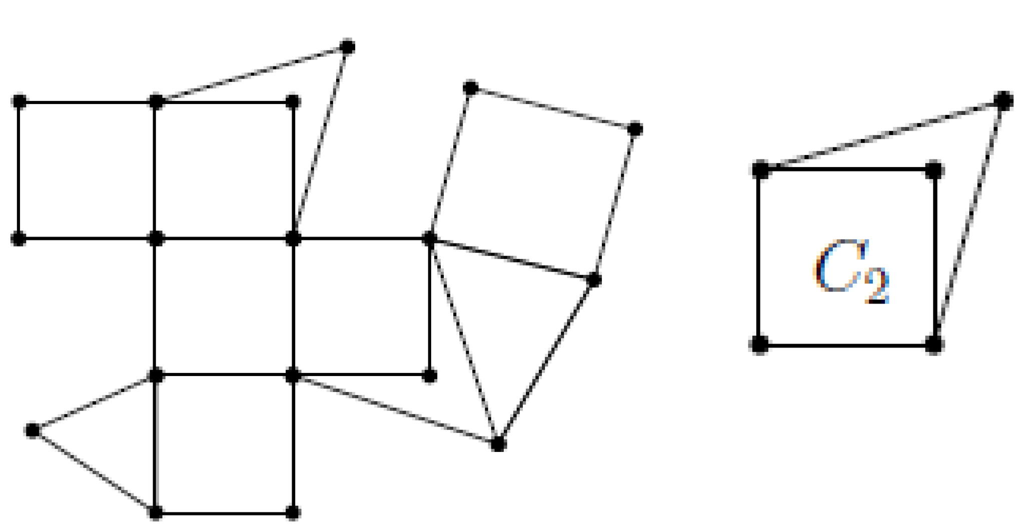
- Finding Linear Layouts of Chordal-Bipartite Graphs.

Findings



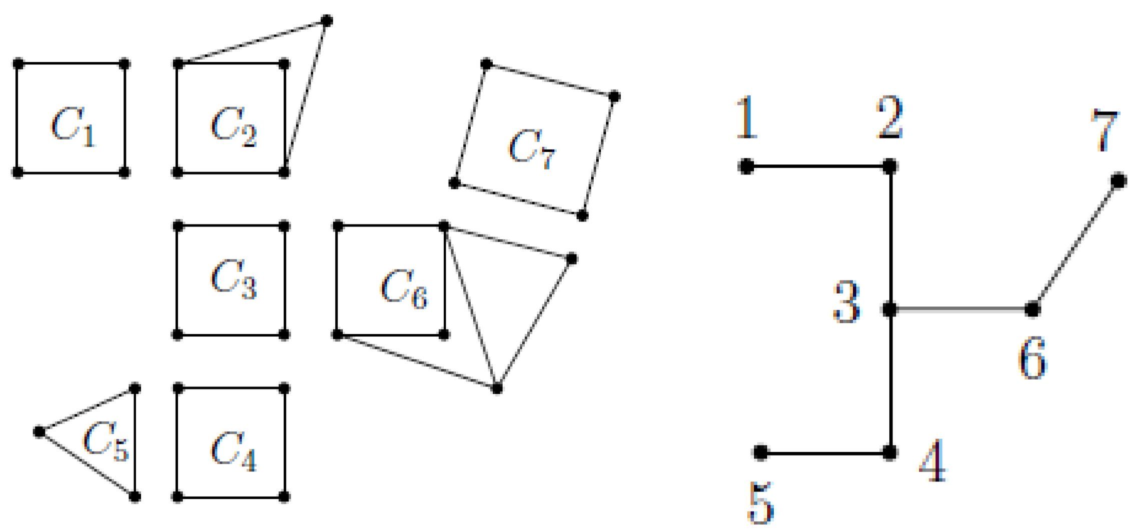
P4 Parallelogram: Not Line Rigid

- Peripheral Edge Order Property.
- Weakly Triangulated Graphs have this property.
- Rigid: valid edge assignment & unique linear layout.



Maximally Line Rigid

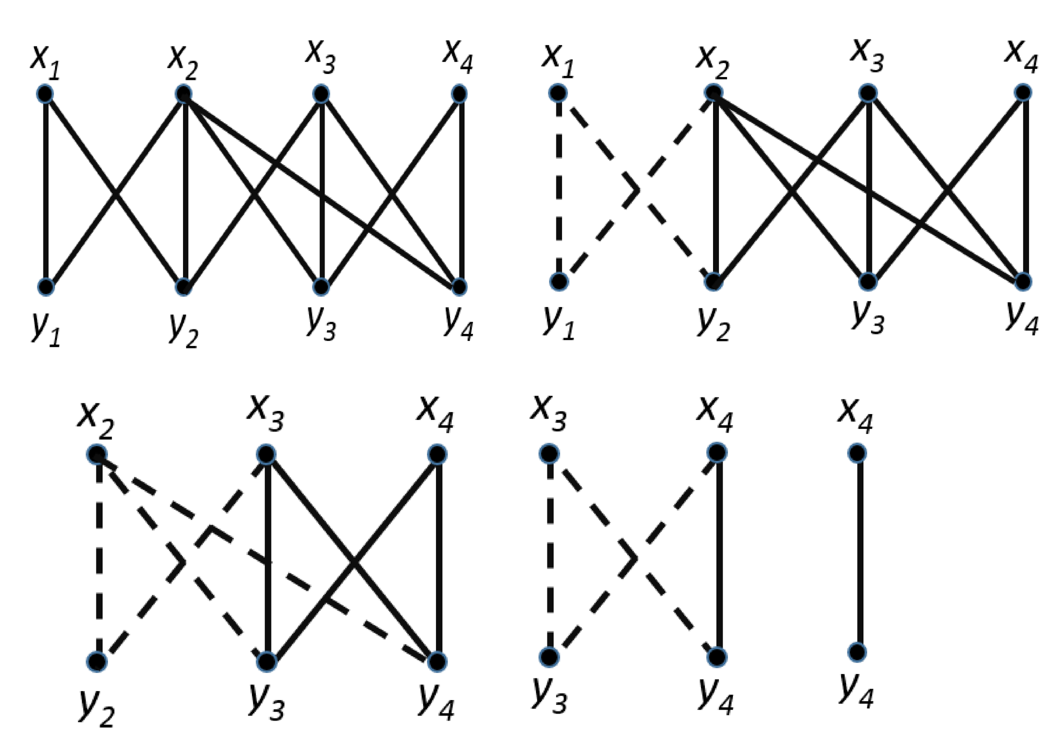
- Maximally Line Rigid: No Super Graph is Line Rigid.



- Rigidity Tree Property.
- A node is quadrilateral or maximally rigid sub graph.
- Tow nodes share an edge if and only if their components share an edge.
- Two Structural constraints.
- No two maximally rigid components share an edge.
- Degree of node corresponding to quadrilateral <= 4 (hinge edges not allowed).

Findings

- A bipartite graph is called Chordal Bipartite if it does not contain any chordless cycle of length > 4.
- An edge e = xy of a bipartite graph H = (X,Y,E) is **bisimplicial** if the subgraph induced by vertices of N(x) and N(y) is a complete bipartite subgraph of H.



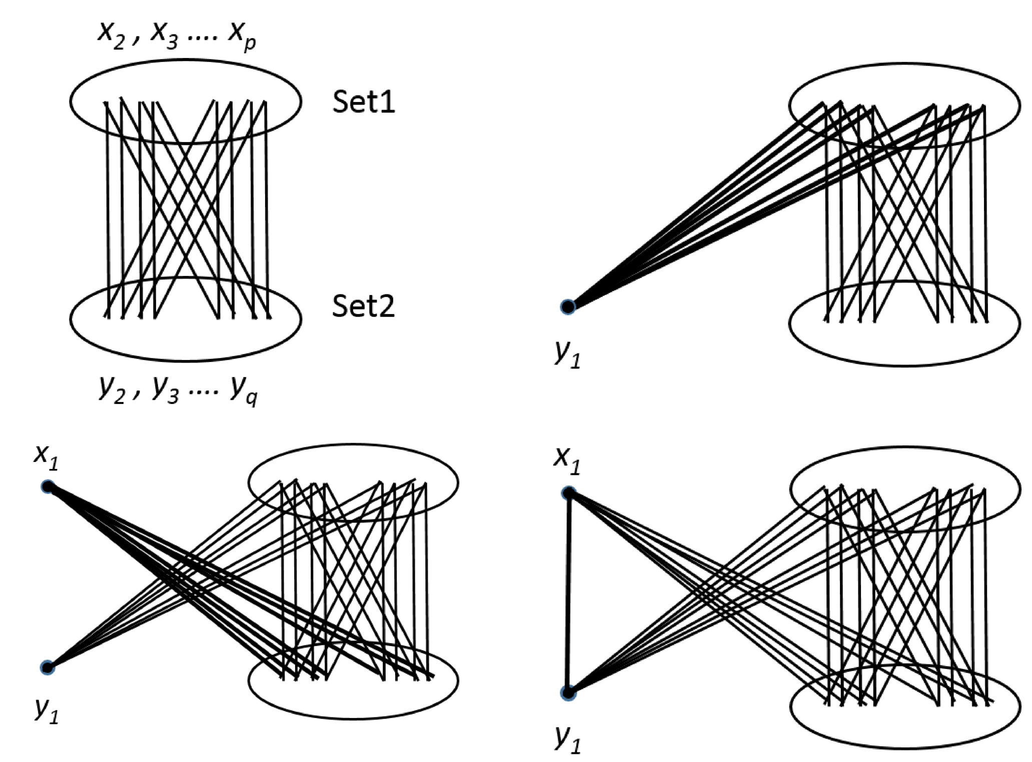
- A set of graphs $G_0, G_1, \ldots, G_k$ where G_0=G and G_i is obtained from G_{i-1} by removing a bisimplicial edge e=xy and the edges incident on x and y and finally G_k is an empty graph. An ordering of edges σ =[$e_0, e_1, \ldots, e_k$] is called **Perfect Edge without Vertex Elimination Ordering** if each edge e_i in σ is a bisimplicial edge for the graph G_{i-1}.

Literature Survey

According to Kloks and Kratsch the perfect edge elimination ordering can easily be calculated from the doubly lexical ordering of the bipartite adjacency matrix in O(min(m log n, n^2)). Ryuhei Uehara showed that we can compute this ordering in O(m+n) time.

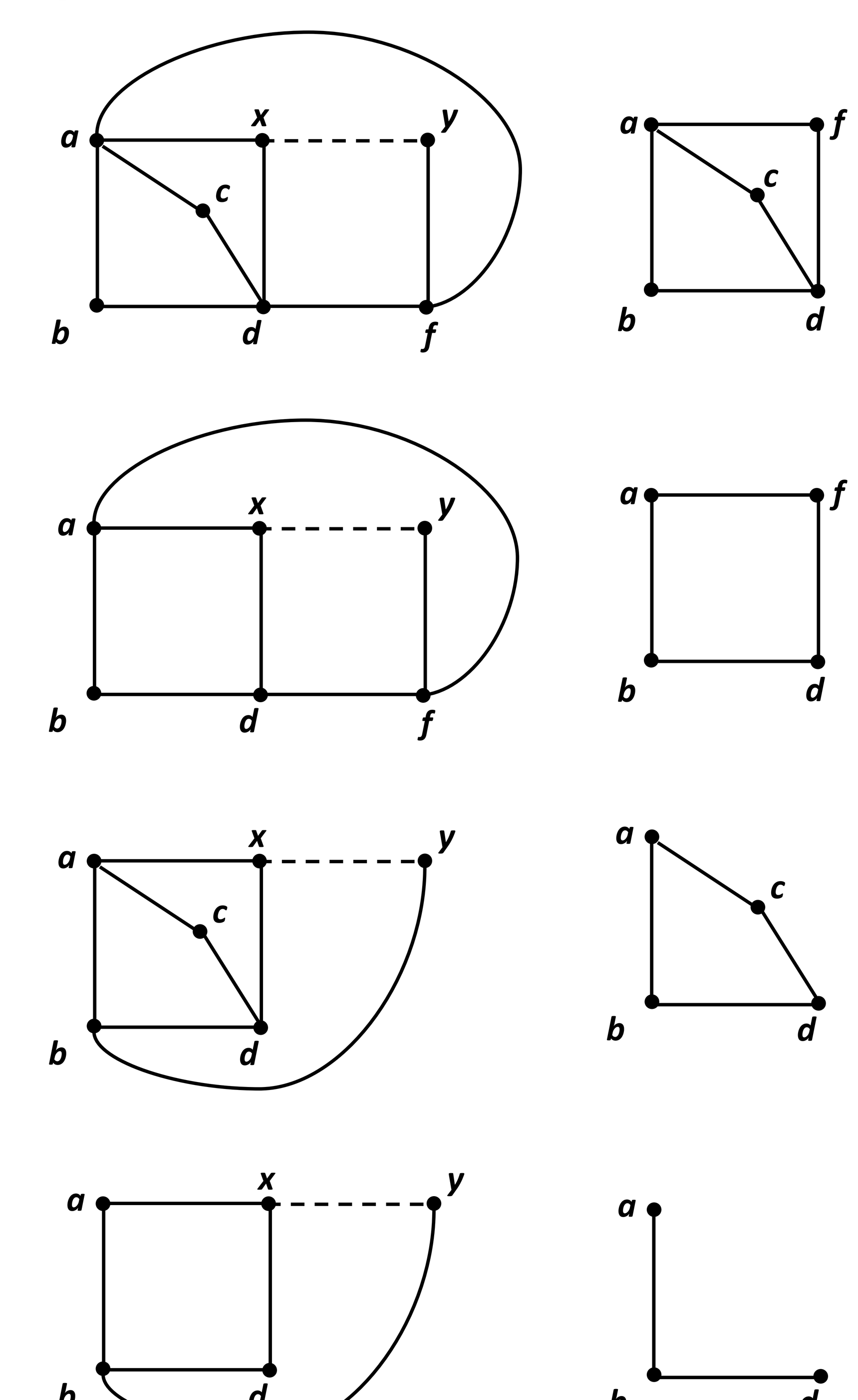
Finding Peripheral Edge Ordering

- We can get the Peripheral Edge Order from Perfect Edge Elimination Scheme using an O(m) algorithm.
- Add every edge in the vertex elimination order after adding all the edges in the neighbor hood of the vertices of the added edge. Doing this produces a valid peripheral edge ordering.



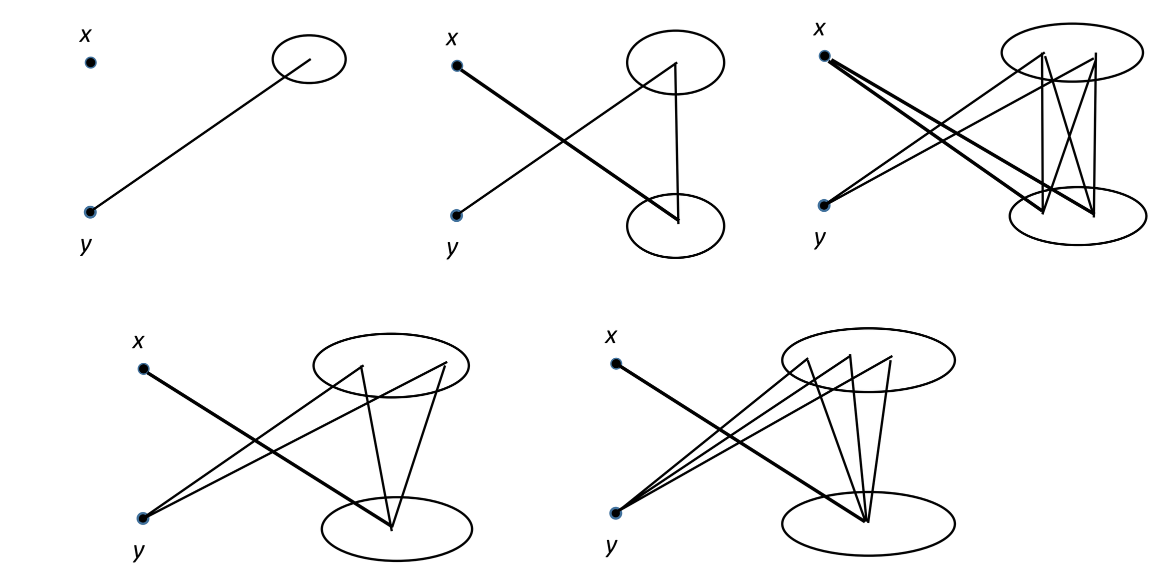
Theorem

- If we consider that the addition of a peripheral edge that is not bisimplicial during the reconstruction of the original graph doesn't change the structure of the Rigidity Tree, the final Rigidity Tree structure does not change.



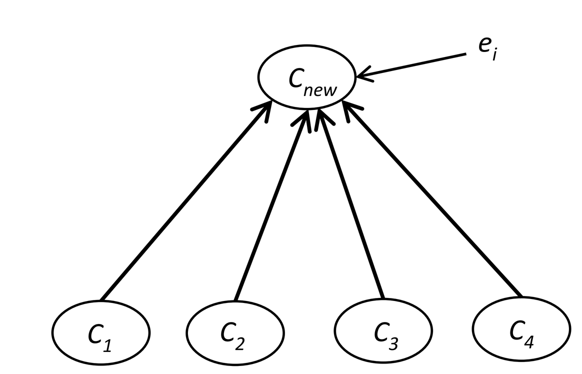
Finding 3-length paths efficiently

- $ING_{x,y} \subset K_{1,1}$, e_i is a dangling edge.
- $ING_{x,y} = K_{1,1}$, a non-rigid quadrilateral is formed.
- $ING_{x,y} = K_{1,2}$, $K_{2,3}$ is formed.
- $ING_{x,y} \supset K_{1,2}$, e_i becomes part of a rigid component



Union Find

The addition of an edge can change the rigidity of a component from non-rigid to rigid. In worst case each edge in the graph might have to go through the changes and thus in Asish Mukhopadhyay et al this took O(m^2) time. The Union-Find Data Structure can be used so that the total time complexity is O(1) .



Complexity analysis

In our case, the merging can only occur when we encounter a peripheral edge which is also bisimplicial. The upper bound on number of bisimplicial edges is O(n) and each such merging takes O(1) time thanks to the Union-Find Data Structure so the total time complexity is O(n) in our case using a different data structure. So, we can find the number of linear layout for a given chordal bipartite graphs without any articulation points and hinge edges in O(m + n) time.

References

- S. V. Rao, Ashish Mukhopadhyay. Linear layouts of weakly triangulated graphs. 2014.
- D.Kratsch T.Kloks. Computing a perfect edge without vertex elimination ordering of a chordal bipartite graph. 1995
- Ryuhei Uehara. Linear time algorithms on chordal bipartite and strongly chordal graphs,2002.

Future Work

- Future work includes extending the idea for general chordal bipartite graphs that can contain hinge edges and articulation points.