How K-Means and the Elbow Method Work (Behind the Scenes)

# 🎯 Objective

This document explains how K-Means clustering and the Elbow Method work behind the scenes using a real numerical example.

# ⚙️ What is WCSS?

WCSS (Within-Cluster Sum of Squares) is the sum of squared distances between each data point and the centroid of its assigned cluster. In simpler terms, it's a measure of how tightly grouped the data points in a cluster are.

# 📊 Example Dataset (5 Points)

We use 5 points in 2D space:  
(1,2), (2,3), (6,6), (7,8), (9,10)

## Step 1: k = 1 (All in One Cluster)

- Centroid = Average of all 5 points = (5, 5.8)  
- For each point, calculate the squared distance to this centroid.  
- Add all these squared distances = WCSS for k = 1 (high value).

## Step 2: k = 2 (Split Into 2 Clusters)

- Cluster 1: (1,2), (2,3)  
- Cluster 2: (6,6), (7,8), (9,10)  
- Centroid 1 = (1.5, 2.5), Centroid 2 = (7.33, 8)  
- Now calculate squared distances of each point to their respective centroid.  
- Add these = WCSS for k = 2 (much lower value).

# ✅ What We Learn from This

- As k increases, WCSS decreases.  
- But after a certain point, the decrease slows down.  
- The 'Elbow Point' is where adding more clusters gives little improvement.  
- This is how the Elbow Method helps determine the optimal number of clusters.

# 🧠 Summary Table

k = 1 → High WCSS (all points in one big cluster)  
k = 2 → Lower WCSS (smaller, tighter clusters)  
k = 3+ → WCSS keeps dropping but less and less — that's the 'elbow'