My research goal is to develop novel theory and statistically principled methodologies for learning and understanding fundamental network structures in complex interactive systems. A key focus is characterizing statistical complexity, computational efficiency, estimation accuracy, and their trade-offs for network structure inference under realistic network constraints. My research interests lie at the intersection of high-dimensional statistics, graphical models, network theory, and optimization.

Large-scale networked systems are ubiquitous across diverse domains, ranging from physical systems such as power and transportation networks to conceptual networks such as brain and social interaction networks. A fundamental challenge in understanding these systems is that their **network structure** is **often unknown and needs to be estimated from data**. This task, commonly referred to as the *network structure learning problem*, extends beyond merely identifying the network's connectivity pattern—it also encompasses learning richer structures such as hierarchical organizations, recurring motifs like cycles, trees, and other higher-order patterns. This task is a critical step for effective modeling, management, and control of networked systems. While learning the network structure is crucial for several downstream tasks like resource allocation in power networks or analyzing the spread of rumors in social networks, it is an end goal for many scenarios such as understanding the human brain connectome. Despite remarkable advances in high-dimensional estimation and statistical inference, applying these methods to real-world networks such as biological, sensor, and other engineered systems—remains challenging. Domain-specific constraints like partial observability, network process constraints, and strict sample budgets often render existing methods sub-optimal or ineffective. Motivated by these challenges, my long-term research goal is to build the next generation of statistically principled methods and theory for large-scale network structure learning that is robust, interpretable, and adaptive to domain-specific constraints.

The task of network structure learning necessitates a systematic study of how information or signals distributed over a network interact through an underlying network process, and how these interactions can be harnessed to infer the network's structure. To formalize this idea, I introduce Network Stochastic Process (NSP)—a novel framework that underpins my research vision. A network stochastic process (NSP) is defined as a tuple  $(Y, X, \mathcal{N}, f)$ , where  $\mathcal{N} = \{\mathcal{V}, \mathcal{E}\}$  represents the network with the vertex set  $\mathcal{V}$  (objects) and the edge set  $\mathcal{E}$  (connections between objects). The component X denotes an exogenous multivariate stochastic process supported on the network, meaning that each node i is associated with the i-th coordinate of X. The observed process Y is given by  $Y = f(X, \mathcal{N}, \text{measurement noise})$ , where f is a function that maps  $(X, \mathcal{N}) \mapsto Y$  that encapsulates how the network structure and the underlying stochastic process together generate the observed data. While Y and Xrepresent the signals on the network, f characterizes the interactions between them, modeling simple linear interactions to highly complex dynamics. A canonical example of an NSP is a power network where the network structure  $\mathcal{N}$  is characterized by its graph Laplacian L, X represents current injected into the system, Y corresponds to the measurable node potentials, and f is a mapping such that  $Y = L^{-1}X$  (see, Section 1). While our goal is **to** estimate the network structure from observational data Y alone, this problem is fundamentally ill-posed with**out knowledge of** X **or additional structure**. This motivates the need for a broader organizing principle to frame the learning problem. In my work, I approach the network structure learning through the lens of learning simple network models from simple network processes. This perspective is grounded in the principle of parsimony—the insight that simple models and processes can not only explain complex data effectively but are also easier to estimate, leading to gains in statistical, computational efficiency, and interpretability. The incorporation of parsimony into the learning framework forms the core of my research vision and provides a unifying theme across my past, future work, and long-term goals. I will provide an overview of my prior research, demonstrating how this vision shapes my approach to tackling challenges at the intersection of statistics, computation, and learning.

## 1 Overview of my research

Learning Networks that Obey Conservation laws [1–4]: Conservation laws regulate edge flows in several networked systems like power grids (electric current), pipelines (fluid flow), transportation networks (traffic flow),

and even brain networks (information flow in the brain). The network process of such systems is described by steady-state equations (also called balance equations) given by X = LY, where X is the random vector of exogenous inputs (e.g., current injections in power networks), Y represents a vector of node observations (e.g., node potentials) and L is the Laplacian that encodes connectivity structure of the network. From the network stochastic process (NSP) framework, this relationship simplifies to  $Y = L^{-1}X$ . In [1], we address the challenge of estimating the matrix L from n independent and identically distributed (i.i.d.) samples of observational data Y when the latent inputs X are unobservable but with known covariance structure. We propose an estimator  $\hat{L}$  that recovers the network structure (ie., recovers the support of L and is close to L in matrix norm) in the high dimensional regime where the network size far exceeds the number of samples. Furthermore, we give a precise characterization of the statistical complexity for the estimator to achieve this task. In [2], we extend this framework to the more challenging setting where the noise injections  $X_t$  follow a wide-sense stationary (WSS) stochastic process. Notwithstanding the challenges posed by temporal dependencies in the data and the intractability of the estimator from [1], we introduce a novel estimator that achieves analogous results as that of [1]. Our approach is further validated through extensive experimental studies on real-world networks, including power, water, and brain networks. While our earlier works focused on static networks, we also explore dynamic networks that evolve over time, such as changing friendship ties in social networks. In [3,4], we tackled the differential network analysis problem: estimating the sparse difference matrix  $\Delta^* = L_2^* - L_1^*$  from i.i.d. node potential observations at two distinct time points. The proposed estimator directly learns the sparse edge changes without estimating the individual networks separately, providing strong theoretical guarantees similar to those in our previous works.

Learning Gaussian Graphical Models from Glauber dynamics [5]: Glauber dynamics (or Gibbs sampler) is a stochastic process where variables are sequentially updated based on their local statistics—an instance of parsimonious NSP. This process naturally emerges in various settings, from opinion formation in social networks to stock-price dynamics in financial markets. In [5], we developed the first algorithm for Gaussian graphical model selection from Glauber dynamics data. Our key contribution is characterizing the minimum observation time needed to estimate the graph structure. Notably, our algorithm is not only the first of its kind but is also nearly minimax optimal for a broad class of graphical models.

Integrating parsimonious assumptions into our framework naturally reduces statistical complexity—shaving off a factor of  $O(d^2)$  for [1,3,4] and  $O(d^3)$  for [2], where d is the maximum degree of the network. For [5], we show that learning a graphical model from complex processes like Glauber dynamics is nearly as efficient as learning a standard Gaussian graphical model. These results demonstrate how parsimony drives gains in statistical efficiency and theoretical simplicity.

## 2 Future Research Agenda

As highlighted in this statement, my research is characterized by a principled approach to integrating latent structures inherent in various statistical problems to achieve significant gains in statistical efficiency and interpretability. **Building on this foundation, I aim to extend this research vision to tackle critical applications in diverse domains, from neuroscience to computational chemistry and biology**. Below, I outline a few research directions that advance my research trajectory toward this goal.

Q Learning Graphs from Multimodal Data: While my research so far has primarily focused on learning undirected networks from i.i.d. and time series data—real-world data structures are far more complex. Recent engineering breakthroughs have enabled the acquisition of rich multimodal data, offering new opportunities to understand complex systems better. For instance, simultaneously collecting electroencephalogram (EEG) and functional magnetic resonance imaging (fMRI) data from the same set of subjects provides complementary time series data that capture distinct information attributes. Learning graphs—whether undirected or directed—from such multimodal data is a critical yet underexplored problem. While multimodal data holds significant promises for addressing data scarcity, it introduces several challenges. For instance, multimodal data exhibit high correlation across modes—rendering traditional graph learning methods inadequate. Additionally, data noise and sampling

inconsistencies—where some modes are densely sampled while others are sparse—can lead to dissimilar and inaccurate graphs when each modality is analyzed independently. Estimation challenges are further amplified in high-dimensional regimes where the ambient dimension far exceeds the sample size. To address these challenges, I aim to develop a systematic framework that models all data modalities as functional data (e.g., multivariate Gaussian processes) originating from a shared latent space. This framework will enable the estimation of edge sets in the latent space. Moreover, this approach also aligns closely with causal representation learning where the focus is on identifying causal relationships in latent spaces using high-level observational data such as images and videos. Developing a statistical theory for these frameworks is an open problem, and my research will focus on bridging this gap.

- Statistical Theory of Higher-Order Graph Representations: Building on their success as universal approximators, Graph Neural Networks (GNNs) have increasingly been used to learn undirected or causal graphs. Beyond graph generation, GNNs demonstrate exceptional performance in complex combinatorial tasks such as counting specific substructures or motifs in graph-structured data—a critical capability for applications in computational chemistry and biology. Notably, graph generation aligns closely with learning a function *f* in the Network Stochastic Process (NSP) framework proposed in this statement. Despite the strong empirical performance of GNNs in these unsupervised tasks, a rigorous theoretical understanding is still lacking. My future research goals aim to develop a non-parametric function approximation theory for GNNs, establishing a principled framework that enhances their applicability and theoretical understanding in high-dimensional and complex data environments.
- Structured Dimensionality Reduction of Large Graphs: While my research has largely focused on the statistical estimation of high-dimensional graphs, graph inference is equally important for enhancing interpretability. High-dimensional graphs are prevalent in various domains like neuroscience and genomics and current graph dimensionality reduction methods preserve structural metrics like Gromov-Wasserstein distance but neglect the semantic meaning of nodes and edges. This calls for a structured dimensionality reduction approach that maintains both graph distances and semantic properties, particularly conditional independence relationships in graphical models. Key questions include the feasibility of preserving conditional independence during dimensionality reduction, the nature of such mappings (linear vs nonlinear), and fundamental limits on dimensionality reduction. These questions have direct applications in neuroscience, where brain connectomes exhibit high dimensionality yet a natural hierarchical structure. Developing methods to harness this structure could provide new insights into the intricate relationship between structural and functional connectivity, advancing our understanding of the human brain connectome.

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