

# gradient-domain image processing

October 25, 2022

```
[1]: import numpy as np
import matplotlib.pyplot as plt
import cv2
from scipy.interpolate import interpn
from scipy.signal import convolve2d
from tqdm import tqdm
```

```
/home/aramesh/anaconda3/envs/comp-photo/lib/python3.10/site-
packages/scipy/__init__.py:146: UserWarning: A NumPy version >=1.16.5 and
<1.23.0 is required for this version of SciPy (detected version 1.23.1
  warnings.warn(f"A NumPy version >={np_minversion} and <{np_maxversion}")
```

```
[2]: # load images
museum_amb = './data/museum/museum_ambient.png'
museum_flash = './data/museum/museum_flash.png'

museum_amb = cv2.imread(museum_amb,-1)[:,:,:3][:,:,:-1]
museum_flash = cv2.imread(museum_flash,-1)[:,:,:3][:,:,:-1]

# normalize to 0-1
# museum_amb_norm = (museum_amb - np.min(museum_amb)) / (np.max(museum_amb) -
↳ np.min(museum_amb))
# museum_flash_norm = (museum_flash - np.min(museum_flash)) / (np.
↳ max(museum_flash) - np.min(museum_flash))
museum_amb_norm = museum_amb / 255
museum_flash_norm = museum_flash / 255
```

```
[3]: def gradient(img) :

    temp = img[:,:].copy()
    row = np.zeros((1,temp.shape[1]))
    col = np.zeros((temp.shape[0]+2,1))
    temp2 = np.vstack((row,np.vstack((temp, row))))
    temp2 = np.hstack((col,np.hstack((temp2, col))))
    img = temp2.copy()
```

```

    # we remove the last row and the last column corresponding to img_x at the
    ↪end
    # we remove the last col and the last column corresponding to img_y at the
    ↪end

    img_x = np.diff(img,n=1,axis=1) # change along row (i.e. diff bw columns)
    img_y = np.diff(img,n=1,axis=0) # change along column (i.e. diff bw rows)
#     print(img_x.shape, img_y.shape)
#     img_x, img_y = img_x[:-1,:], img_y[:, :-1]
#     print(img_x.shape, img_y.shape)
    return img_x, img_y # vector field

def divergence(u,v) :
    u_x = np.diff(u,n=1,axis=1) # change along row (i.e. diff bw columns)
    v_y = np.diff(v,n=1,axis=0) # change along column (i.e. diff bw rows)
    u_x, v_y = u_x[1:-1,:], v_y[:, 1:-1]

    out = u_x + v_y
    return out # scalar field

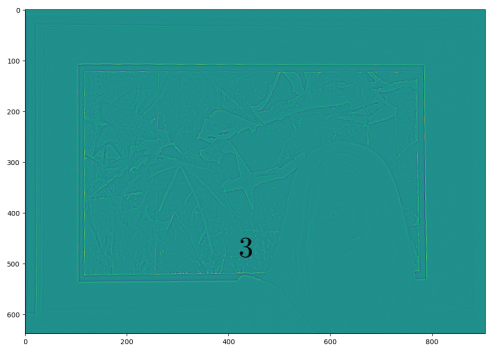
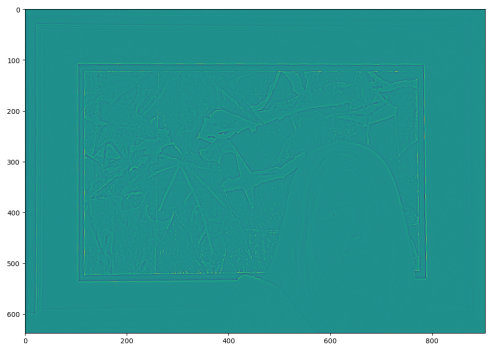
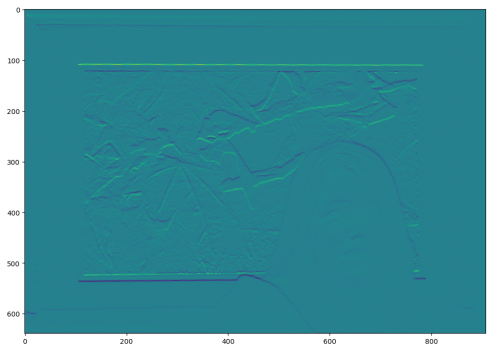
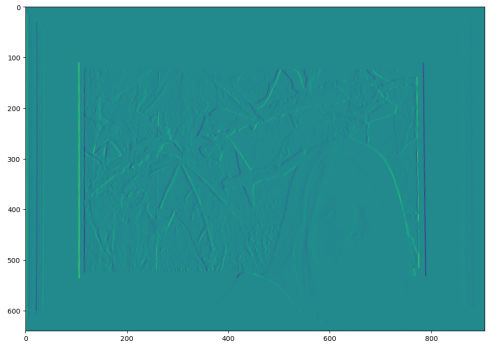
def laplacian(img) :
    kernel = np.array([[0,1,0],[1,-4,1],[0,1,0]])
    out = convolve2d(img, kernel, mode='same', boundary='fill', fillvalue=0)
    return out # scalar field

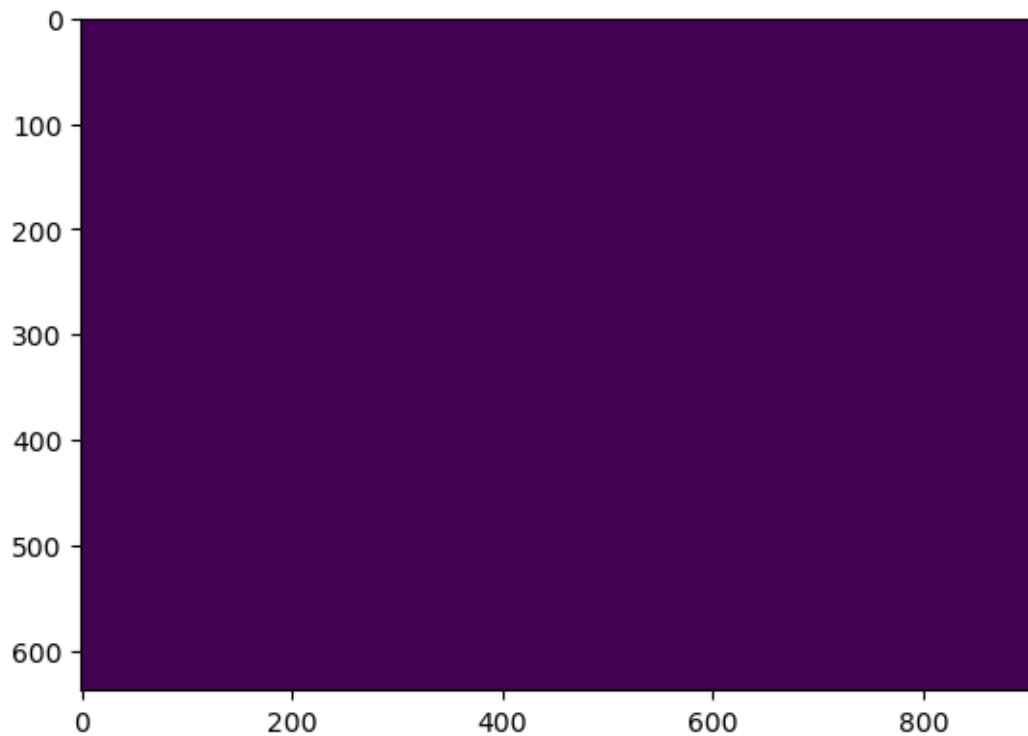
# testing functions above

museum_amb_norm_gradient_x,museum_amb_norm_gradient_y =
    ↪gradient(museum_amb_norm[:, :, 0])
museum_amb_norm_lhs =
    ↪divergence(museum_amb_norm_gradient_x,museum_amb_norm_gradient_y )
museum_amb_norm_rhs = laplacian(museum_amb_norm[:, :, 0])

fig, ax = plt.subplots(5, figsize=(50,50))
ax[0].imshow(museum_amb_norm[:, :, 0])
ax[1].imshow(museum_amb_norm_gradient_x)
ax[2].imshow(museum_amb_norm_gradient_y)
ax[3].imshow(museum_amb_norm_lhs)
ax[4].imshow(museum_amb_norm_rhs)
plt.show()
plt.imshow(np.around(museum_amb_norm_lhs,5) == np.around(museum_amb_norm_rhs,5))
plt.show()
print("np.allclose(museum_amb_norm_lhs, museum_amb_norm_rhs) = ",np.
    ↪allclose(museum_amb_norm_lhs, museum_amb_norm_rhs))

```





```
np.allclose(museum_amb_norm_lhs, museum_amb_norm_rhs) = True
```

```
[35]: # Differentiate then re-integrate an image

# Poisson Solver

def conjugate_gradient_descent(I_init, B, I_star_boundary, eps, N, D) :

    I_star = B * I_init + (1-B) * I_star_boundary
    r = B * (D - laplacian(I_star))
    d = r.copy()
    delta_new = np.sum(r*r) # <r,r>
    n = 0

    while np.sqrt(np.sum(r*r)) > eps and n < N :

        if n % 200 == 0 : print(n,np.sqrt(np.sum(r*r)))
        q = laplacian(d)
        eta = delta_new / np.sum(d*q) # n = delta_new / <d,q>

        I_star = I_star + B * (eta * d)
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        r = B * (r - eta * q)

        delta_old = delta_new.copy()
        delta_new = np.sum(r*r)

        beta = delta_new / delta_old
        d = r + beta * d

        n = n + 1

    return I_star

def poisson_solver(lap_img, img, N = 1000, eps = 0.001) :

    I_init = np.zeros(img.shape)

    boundary_size = 1
    B = np.ones(img.shape)
    B[0:boundary_size,:], B[-boundary_size:-1,:], B[:,0:boundary_size], B[:,
    ↪-boundary_size:-1] = 0,0,0,0

    I_star_boundary = np.zeros(img.shape)
    I_star_boundary[0:boundary_size,:] = img[0:boundary_size,:].copy()
    I_star_boundary[-boundary_size:-1,:] = img[-boundary_size:-1,:].copy()
    I_star_boundary[:,0:boundary_size] = img[:,0:boundary_size].copy()
    I_star_boundary[:, -boundary_size:-1] = img[:, -boundary_size:-1].copy()

    I_star = conjugate_gradient_descent(I_init, B, I_star_boundary, eps, N,
    ↪lap_img)

    return I_star

# Laplacian (Divergence of gradient field)

# lap_img_r = laplacian(museum_amb_norm[:, :, 0])
# lap_img_g = laplacian(museum_amb_norm[:, :, 1])
# lap_img_b = laplacian(museum_amb_norm[:, :, 2])

# I_star_r = poisson_solver(lap_img_r, museum_amb_norm[:, :, 0])
# I_star_g = poisson_solver(lap_img_g, museum_amb_norm[:, :, 1])
# I_star_b = poisson_solver(lap_img_b, museum_amb_norm[:, :, 2])

# I_star = np.concatenate((np.expand_dims(I_star_r, 2), np.
    ↪stack((I_star_g, I_star_b), axis=-1)), axis=2)

# fig, ax = plt.subplots(2, figsize = (20, 20))

```

```
# ax[0].imshow(museum_amb_norm)
# ax[1].imshow(I_star)

# plt.show()
```

[38]: *# Creating fused gradient field*

```
def fuse_gradient_field(img_amb_norm, img_flash_norm, sigma, tau_s) :

    a_grad_x, a_grad_y = gradient(img_amb_norm)
    phi_dash_grad_x, phi_dash_grad_y = gradient(img_flash_norm)

    a_grad_x_org = a_grad_x.copy()
    a_grad_y_org = a_grad_y.copy()
    phi_dash_grad_x_org = phi_dash_grad_x.copy()
    phi_dash_grad_y_org = phi_dash_grad_y.copy()

    a_grad_x, phi_dash_grad_x = a_grad_x[1:,:], phi_dash_grad_x[1:,:]
    a_grad_y, phi_dash_grad_y = a_grad_y[:,1:], phi_dash_grad_y[:,1:]

    M_num = phi_dash_grad_x * a_grad_x + phi_dash_grad_y * a_grad_y
    M_den = np.sqrt(phi_dash_grad_x**2 + phi_dash_grad_y**2) * np.
↪sqrt(a_grad_x**2 + a_grad_y**2)
    M = M_num / (M_den+0.00001)
#     plt.imshow(M)
#     plt.show()

#     sigma = 40
#     tau_s = 0.9
    ws = np.zeros(a_grad_x.shape)
    ws[:-1,:-1] = np.tanh(sigma * (img_flash_norm-tau_s))
    ws = (ws - np.min(ws)) / (np.max(ws) - np.min(ws))
#     print(ws.shape, a_grad_x.shape, M.shape, phi_dash_grad_x.shape, a_grad_y.
↪shape, phi_dash_grad_y.shape)
    phi_star_grad_x = np.zeros(a_grad_x_org.shape)
    phi_star_grad_y = np.zeros(a_grad_y_org.shape)
    phi_star_grad_x[1:,:] = ws * a_grad_x + (1-ws) * (M * phi_dash_grad_x +
↪(1-M) * a_grad_x)
    phi_star_grad_y[:,1:] = ws * a_grad_y + (1-ws) * (M * phi_dash_grad_y +
↪(1-M) * a_grad_y)

    div = divergence(phi_star_grad_x,phi_star_grad_y)

    return div

sigma = 40
```

```

tau = 0.7
fused_image_r_div = fuse_gradient_field(museum_amb_norm[:, :, 0], ↵
    ↵museum_flash_norm[:, :, 0], sigma, tau)
fused_image_g_div = fuse_gradient_field(museum_amb_norm[:, :, 1], ↵
    ↵museum_flash_norm[:, :, 1], sigma, tau)
fused_image_b_div = fuse_gradient_field(museum_amb_norm[:, :, 2], ↵
    ↵museum_flash_norm[:, :, 2], sigma, tau)

I_star_r = poisson_solver(fused_image_r_div, museum_amb_norm[:, :, 0], N=5000, ↵
    ↵eps=0.0001)
I_star_g = poisson_solver(fused_image_g_div, museum_amb_norm[:, :, 1], N=5000, ↵
    ↵eps=0.0001)
I_star_b = poisson_solver(fused_image_b_div, museum_amb_norm[:, :, 2], N=5000, ↵
    ↵eps=0.0001)

I_star = np.concatenate((np.expand_dims(I_star_r, 2), np.
    ↵stack((I_star_g, I_star_b), axis=-1)), axis=2)

fig, ax = plt.subplots(2, figsize = (20, 20))

ax[0].imshow(museum_amb_norm)
ax[1].imshow(I_star)

plt.show()

```

```

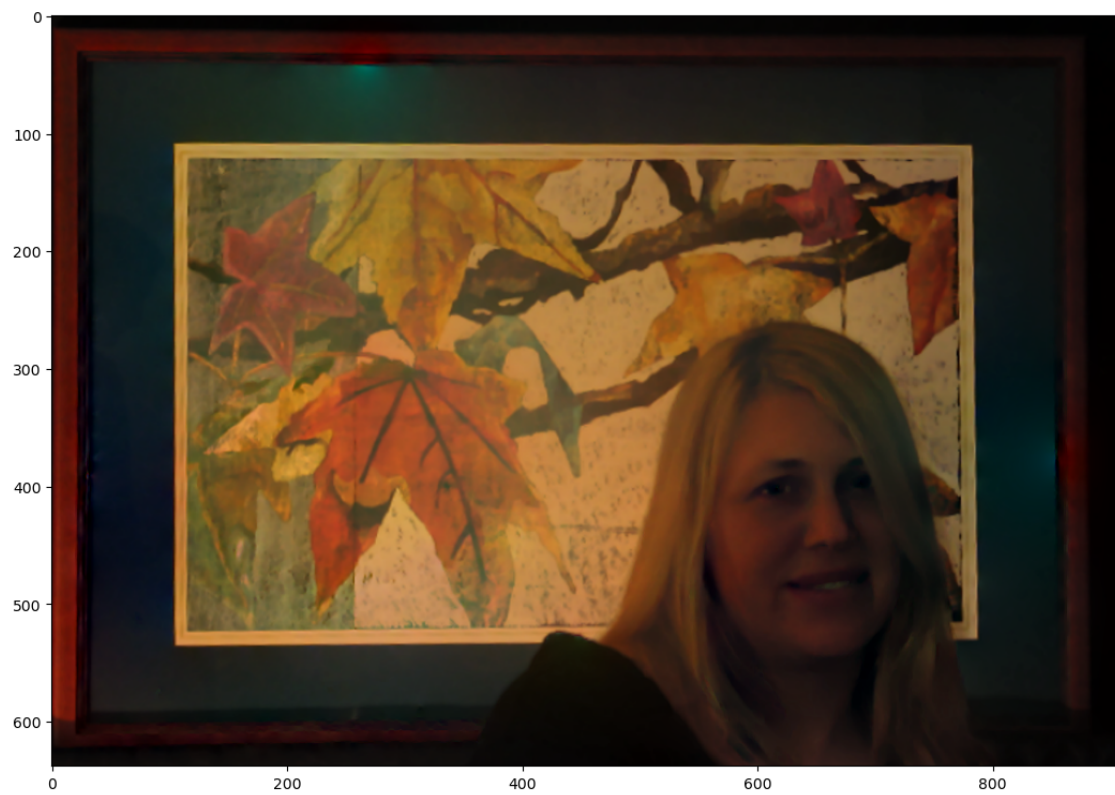
0 22.2254092831278
200 0.12270651688354539
400 0.10631794992179094
600 0.03653641182678293
800 0.015282583806361586
1000 0.005541985751994589
1200 0.001544945398920914
1400 0.00021016430601727298
0 15.58808710040839
200 0.10334098486855126
400 0.07923207086238547
600 0.026450632438439727
800 0.011515362061476098
1000 0.004136915146855653
1200 0.0013647960453020457
1400 0.00024004845182718466
0 11.611914397060135
200 0.06346618294283832
400 0.04498522244146792
600 0.017759121943766203
800 0.006093216765906235
1000 0.0020492192482132214

```

```
1200 0.0008824832813596229  
1400 0.0001425353718196643
```

Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).





```
[39]: plt.imsave('outputs/outputs_report_part2/boundary_condition_variation/
↳flashremoved_sigma{}_tau{}_boundary_ambflashmean.png'.format(sigma,tau),np.
↳clip(I_star,0,1))
plt.imsave('outputs/outputs_report_part2/boundary_condition_variation/
↳flashremoved_sigma{}_tau{}_scaled125percent_boundary_ambflashmean.png'.
↳format(sigma,tau),np.clip(I_star*1.25,0,1))
```

```
[190]: plt.imshow(np.clip(I_star*1.25,0,1))
```

```
[190]: <matplotlib.image.AxesImage at 0x7f426cc4e200>
```



```
[145]: museum_amb_norm_gradient_x.shape
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```
[145]: (640, 906)
```

```
[146]: museum_amb_norm_gradient_y.shape
```

```
[146]: (639, 907)
```

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[ ]:
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