Home Work 4

1) Bag 1, 4 Mango 2 Apples $P(\text{Mango}) = 2/3$
Bag 2, 4 apples 2 Mangoes $P(\text{Mango}) = 1/3$

Biased coin $P(\text{Heads}) = 0.6 = 3/5$ $P(\text{tails}) = 0.4 = 2/5$

heads ⇒ pick from bag 1
tails ⇒ pick from bag 2

$P(\text{Mango}) = P(\text{Mango from bag 1}) + P(\text{mango from bag 2})$

$$P(\text{mango from bag 2}) = \frac{P(\text{mango from bag 2})}{P(\text{mango})}$$

(By Baye's theorem)

$$P(\text{mango}) = \frac{3}{5} \times \frac{2}{3} + \frac{2}{5} + \frac{1}{3}$$

$$P(\text{mango from bag 2}) = \frac{\frac{2}{5} \times \frac{1}{3}}{\frac{3}{5} \times \frac{2}{3} + \frac{2}{5} + \frac{1}{3}}$$

$$= \frac{2}{15} \times \frac{15}{8} = \frac{1}{4}$$

∴ Answer = 0.25 or 1/4

2) A, B are similar if there exist a non-singular matrix P such that

$$B = P^{-1} A P$$

First we prove A, B have same characteristic polynomial. Then the result follows since eigenvalues are got by the characteristic polynomial.

Let $P_A(\lambda)$ and $P_B(\lambda)$ be characteristic polynomials of A and B respectively.

$$P_B(\lambda) = \det(B - \lambda I) = \det(P^{-1} A P - \lambda I)$$

$$= \det(P^{-1}(A - \lambda I) P) \quad \text{since } P^{-1}P = I$$

$$= \det(P^{-1}) \det(A - \lambda I) \det(P)$$

Now $\det(P^{-1}) = \det(P)^{-1}$

$\therefore \det(P^{-1}) \det(P) = 1$

$\det(B - \lambda I) \Rightarrow \det(A - \lambda I)$

or $P_B(\lambda) = P_A(\lambda) \Rightarrow$ eigen values will be same for A and B

Now $B = P^{-1}AP \Longleftrightarrow PBP^{-1} = A$

If $Av = \lambda v$ then $PBP^{-1}v = \lambda v$

$\Rightarrow BP^{-1}v = \lambda P^{-1}v.$

So if v is an eigenvector of A, with eigenvalue λ, then

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$P^{-1}v$ is an eigenvector of B with the same eigenvalue.

So every eigenvalue of A is an eigenvalue of B and since you can interchange the roles of A and B, every eigenvalue of B is an eigenvalue of A too.

Here A, B have same eigenvalues.

Geometrically, v, $P^{-1}v$ are the same vector. They are written in different coordinate systems.

Actually A, B have same eigenvectors too but they are in different basis. (coordinate systems)

3) Exp for classes.

1) $\mu_A = [1, 2, 3]$

$\mu_B = [4, 5, 6]$

Σ covariance $(A = B)$ $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Multivariate normal distribution :-

$P(x \mid \mu, \Sigma) =$

$$\frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}} e^{-1/2 \left[(x-\mu)^T \Sigma^{-1} (x-\mu) \right]}$$

2) $\mu_A = [1, 3, 5]$

$\mu_B = [2, 4, 6]$

Σ covariance $(A = B)$ $= \begin{bmatrix} 2.716 & 1.716 & 1.672 \\ 1.716 & 1.304 & 0.983 \\ 1.672 & 0.483 & 1.287 \end{bmatrix}$

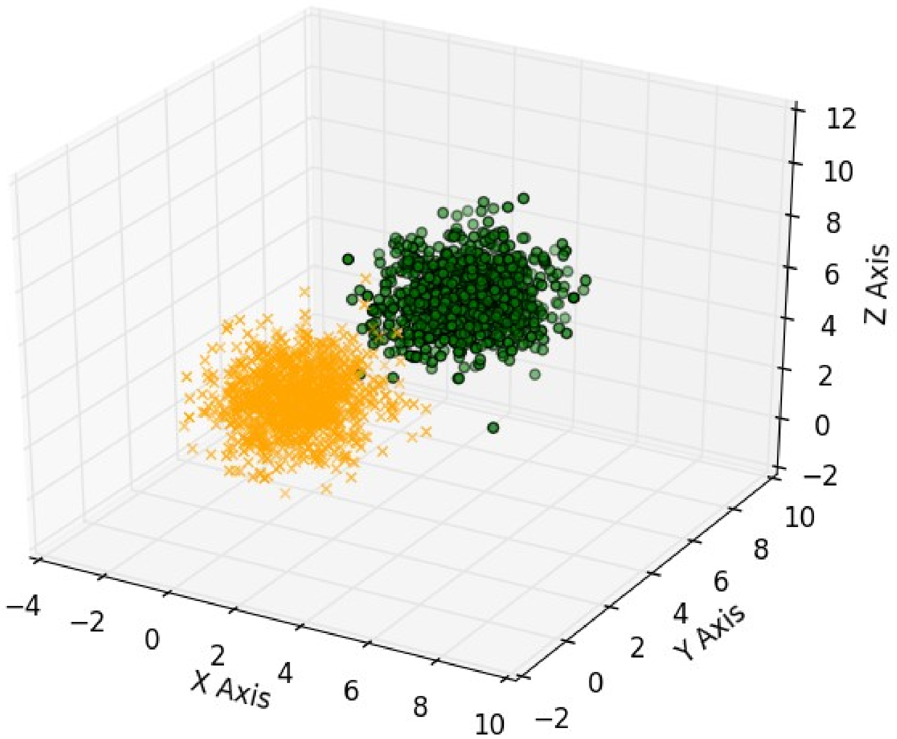
3) $\mu_A = \mu_B = [1, 2, 1]$

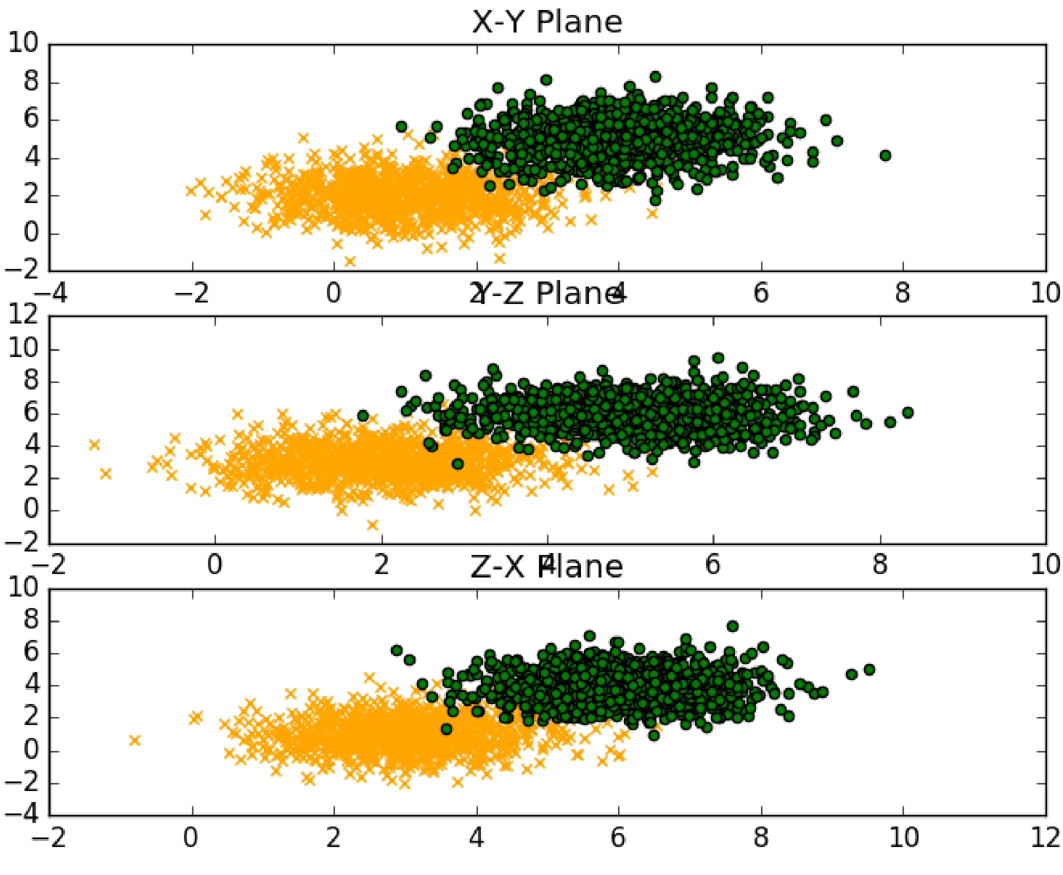
Σ covariance A $= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Σ covariance B $= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Plots for 1,2 and 3

1)





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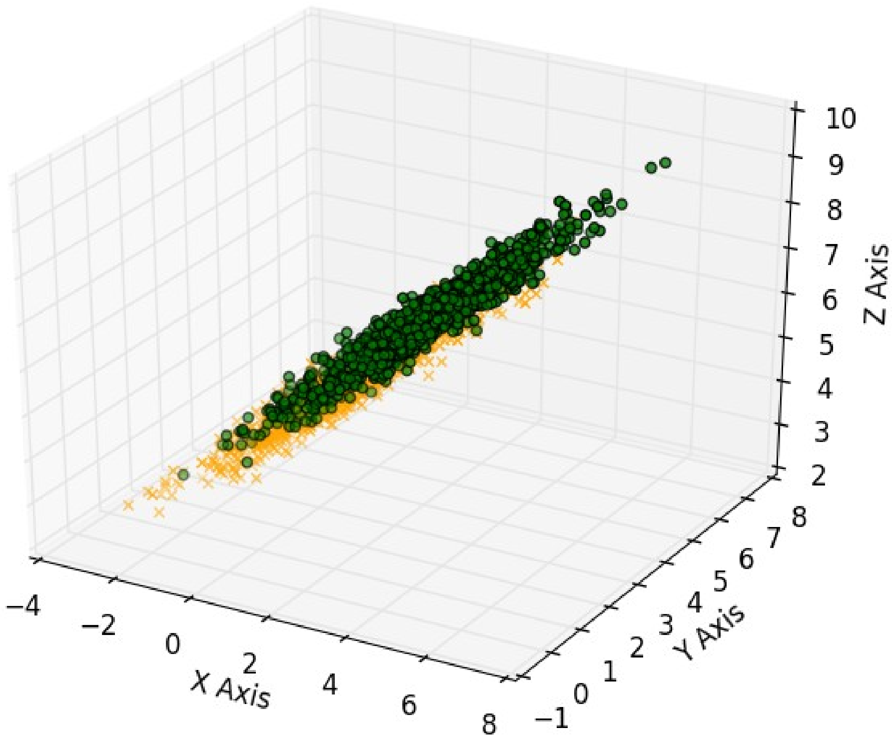

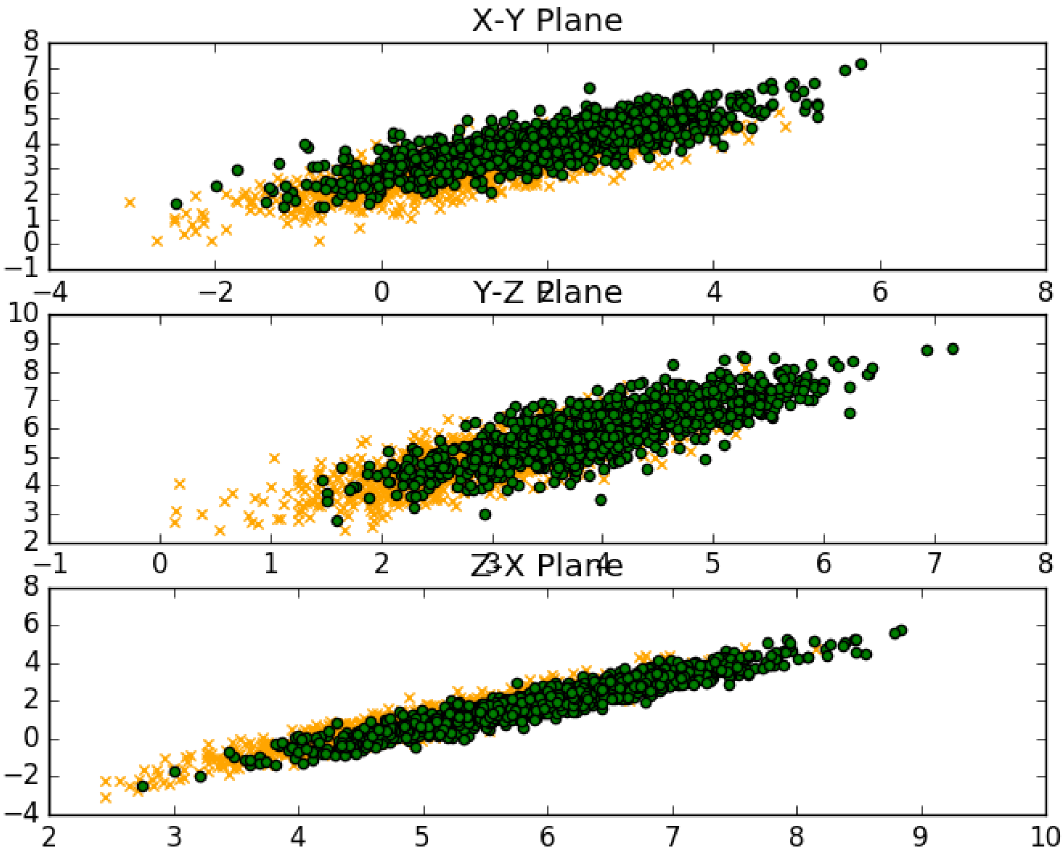

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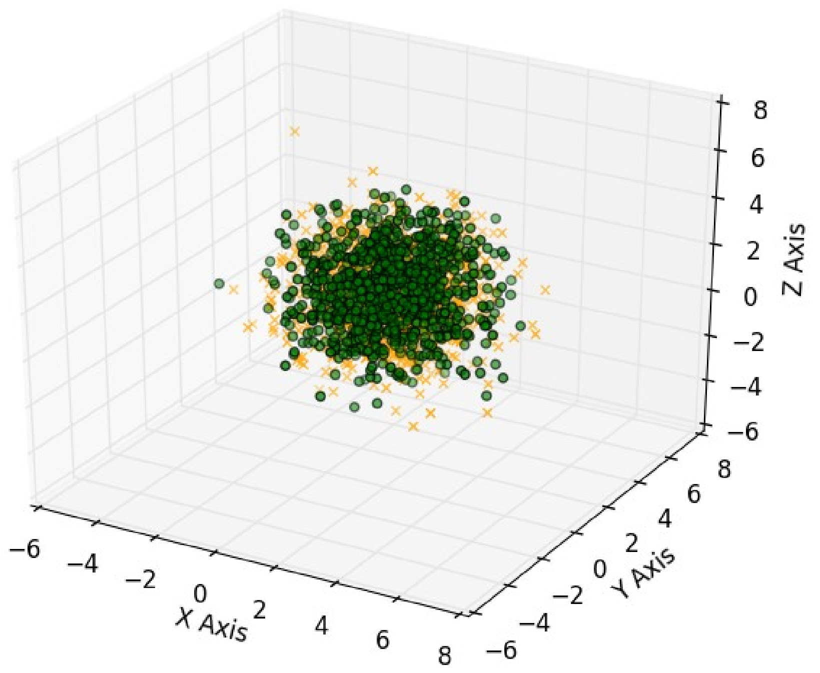

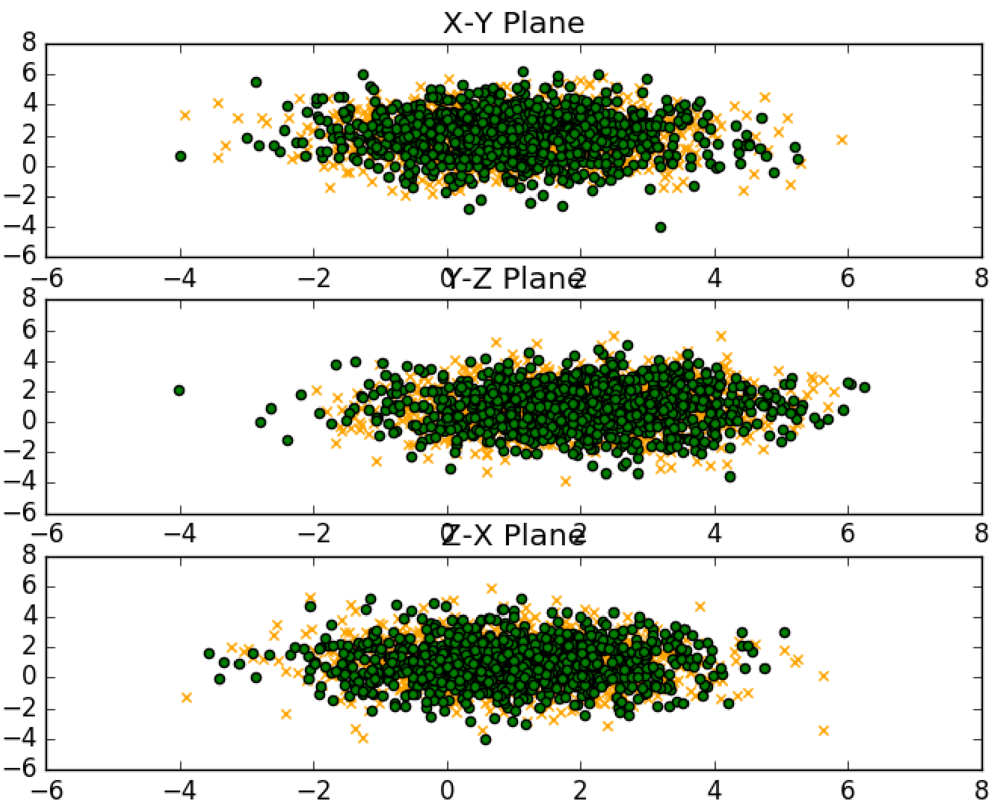
X-Y Plane
Y-Z Plane
Z-X Plane
8
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
8