

1) Dataset

class A :  $\{[0.5, 0.5]^T, [0.75, 0.25]^T, [0.25, 0.75]^T, [1, 1]^T, [2, 2]^T\}$

class B :  $\{[-2, -2]^T, [-0.25, -0.25]^T, [-1.5, -2.5]^T, [-3, -2]^T, [-2, -3]^T\}$

$$q = [0, 0]^T$$

distances of  $q$  from each of the 10 points. (Euclidean distances)

A) 1)  $[0.5, 0.5]^T \rightarrow 0.707$

A) 2)  $[0.75, 0.25]^T \rightarrow 0.790$

A) 3)  $[0.25, 0.75]^T \rightarrow 0.790$

A) 4)  $[1, 1]^T \rightarrow 1.414$

A) 5)  $[2, 2]^T \rightarrow 2.828$

B) 6)  $[-2, -2]^T \rightarrow 2.828$

B) 7)  $[-0.25, -0.25]^T \rightarrow 0.354$

B) 8)  $[-1.5, -2.5]^T \rightarrow 2.915$

B) 9)  $[-3, -2]^T \rightarrow 3.6$

B) 10)  $[-2, -3]^T \rightarrow 3.6$

If  $K=1$

Using KNN, point  $q[0, 0]^T$  is closest to  $[-0.25, -0.25]^T$

$$\text{distance} = 0.354$$

$\therefore q$  belongs to class (B)

If  $K=3$

using KNN, point  $q[0, 0]^T$  is closest to  $[-0.25, -0.25]^T$  (dis = 0.354),

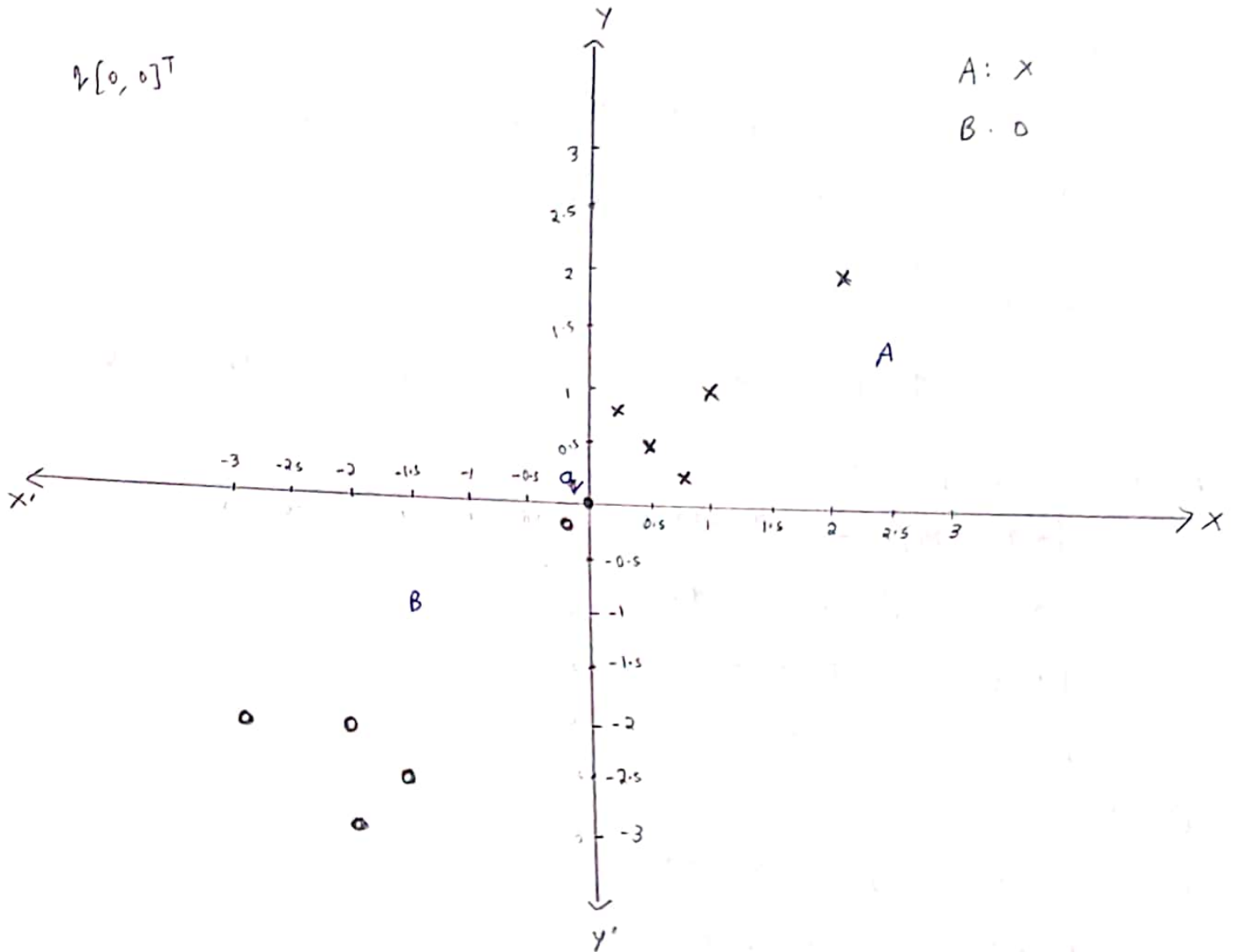
$[0.5, 0.5]^T$  (dis = 0.707),  $[0.75, 0.25]^T$  (dis = 0.790). Since two points belong to class A,  $q$  belongs to class (A)

NOTE -  $[0.25, 0.75]^T$  is another point at dis 0.790 in class A again.

$$r[0, 0]^T$$

A: x

B: o



Graph for KNN (class A, class B)

	B	C	D	E	F	G	H
	Name	Age	Height	Role	Batting avg	Bowling avg	No of matches
1	Virat Kohli	30	175	1	59.4	166.25	236
2	Rohit Sharma	32	170	1	48.92	64.38	215
3	Mayank Agarwal	28	175	1	NA	NA	NA
4	Kuldeep Yadav	24	168	2	12.62	23.97	51
5	Mohammed Shami	28	178	2	7.39	24.76	67
6	Jasprit Bumrah	25	178	2	3.8	21.88	58
7	Bhuvaneshwar Kumar	29	175	2	14.58	34.98	111
8	Yuzvendra Chahal	29	168	2	7.8	26.36	49
9	Rishab Pant	21	170	3	26.12	160	9
10	Lokesh Rahul	27	180	3	39.11	160	23
11	MS Dhoni	38	175	3	50.58	31	350
12	Dinesh Karthik	34	170	3	30.21	160	94
13	Hardik Pandya	25	183	4	29.91	40.65	54
14	Kedar Jadhav	34	165	4	43.24	35.96	65
15	Ravindra Jadeja	30	173	4	30.61	35.9	153
	Kumar Sangakkara	41	178	1	41.99	160	404
	David Warner	32	170	1	45.36	160	116
	AB De Villiers	35	178	3	53.5	28.86	228

2) 1) Player similar to Sangakara Using KNN

Representing Sangakara in the same feature space and applying KNN we can write distance (euclidean dist.) by the formula. (Shown in data set)

$$\text{dis from other players} = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + (x_3 - x'_3)^2 + (x_4 - x'_4)^2 + (x_5 - x'_5)^2 + (x_6 - x'_6)^2} \rightarrow \textcircled{1}$$

( $x_1, x_2, \dots, x_6$  is Sangakara's feature,  $x'_1, x'_2, \dots, x'_6$  is the other player's features)

distances obtained are :-

✓ Kohli	169.3	Jasprit B	374.8	MS Dhoni	140.1
Rohit S	212.2	Bhuvneshwar T	319.4	Dinesh K	310.6
Mayank A	NA (did not play odi)	Yuzvendra Chahal	381.1	Hardik P	370.3
Kuldeep Y	379.9	Rishab P	345.1	Kedar J	361.2
Mohammed S	365.0	Lokesh P	351.2	Ravindra J	280.5

MS Dhoni with KNN algo seems to be the shortest distance / closest / similar to Sangakara.

NOTE : Same formula of euclidean distance  $\textcircled{1}$  was used in next 2 questions. Bowling avg was considered a high value of 160 if not given.

2) Player similar to David Warner. Representing David in the same feature space and applying the above formula, we have respective distances as :-

Kohli	121.0	Jasprit B	155.8	MS Dhoni	267.3
Rohit S	137.6	Bhuvneshwar T	128.9	Dinesh K	26.8
Mayank A	NA (did not play odi)	Yuzvendra Chahal	156.1	Hardik P	136.2
Kuldeep Y	154.4	Rishab P	109.2	Kedar J	134.2
Mohammed S	149.0	Lokesh R	93.8	Ravindra J	130.4

Dinesh Karthik not Rohit Sharma seems to be similar to David Warner with the shortest distance of 26.8



### 3) Testing AB De Villiers (Player of choice)

Assigning the same feature space for AB De Villiers and computing distance from other players for KNN, we have,

Kohli	137.8	Jasprit B	177.5	MS Dhoni	122.1
<u>Rohit S</u>	<u>39.0</u>	Bhuvaneshwar T	123.6	Dinesh K	184.1
Mayank A	NA (didn't play ODI)	Yuzvendra Chahal	185.1	Himanshu P	176.3
Kuldeep Y	182.3	Rishabh P	257.2	Kedar J	163.9
Mohammed S	167.6	Lokesh R	243.9	Ravindra T	79.0

Rohit Sharma is most similar to AB De Villiers with the smallest distance of 39.0.

↳ The results of KNN are not meaningful here as parameters such as age, height, no of matches etc are all on different scales.

We can define a weighted distance function to improve similarity.

↳ Using weights for standardization/normalization

In the euclidean distance formula, we define  $w_i$  such that in

$$d_{x,b} = \left( \sum_{i=1}^n (w_i (x_i - b_i)^2) \right)^{1/2}$$

$w_i$  is the reciprocal of each measurement's variance. Hereby all measurements will be on same scale (say normalized from 0 to 1)

↳ Using weights for importance

(more imp ones)

We can assign higher weights to certain properties to ensure ~~that~~ they have more contribution in the weight func.

By using these techniques the result of the above KNN we applied can be improved.

3)  $\hookrightarrow x_1 = [1, 2, 3]^T \quad x_2 = [0, 3, 4]^T, \quad x_3 = [2, 4, 4]^T$

Find a  $w$  such that  $w^T x_1 < 0 \quad w^T x_2 > 0 \quad w^T x_3 > 0$

3 eq obtained are, considering  $w^T = [w_1, w_2, w_3]^T$

$$w_1 + 2w_2 + 3w_3 < 0 \quad (1)$$

$$3w_2 + 4w_3 > 0 \quad (2)$$

$$2w_1 + 4w_2 + 4w_3 > 0 \quad (3)$$

Solving,  $(3) - (2), \quad 2w_1 + w_2 > 0 \quad 2w_1 > -w_2 \quad \text{or} \quad w_1 > \frac{-w_2}{2}$

We can also write  $4w_1 > -2w_2 \rightarrow (4)$

adding  $(1) + (4), \quad w_1 + 3w_3 < 4w_1$

$$3w_3 < 3w_1 \quad \text{or} \quad w_3 < w_1 \quad \text{or} \quad \boxed{4w_3 < 4w_1}$$

By  $(2), \quad \boxed{4w_3 > -3w_2}$

Combining we have,  $-3w_2 < 4w_3 < 4w_1$

dividing by 4,  $\boxed{\frac{-3}{4}w_2 < w_3 < w_1} \quad *$

$\therefore$  Let  $w_1 = 0.2, \quad w_2 = 2.05, \quad w_3 = -1.5$

So  ~~$w^T = [0.2, 2.05, -1.5]$~~   $w^T = [0.2, 2.05, -1.5]$

~~$(\text{Satisfies the inequality } \frac{-3}{4} \times 2.05 < -1.5 < 0.2)$~~

So  $w^T = [0.2, 2.05, -1.5] \quad w^T = [0.2, 2.05, -1.5]$

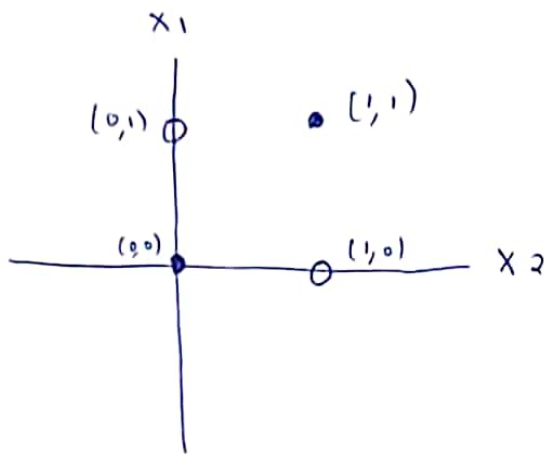
satisfies the inequality obtained too,  $\frac{-3}{4} \times 2.05 < -1.5 < 0.2$

ie  $-1.5375 < -1.5 < 0.2$

A set of  $w$  such  $w^T = [0.2, 2.05, -1.5]$  satisfies the above eqs

b) Four points  $x_1, x_2, x_3, x_4$  in 4D  
 No  $w$  should keep  $x_1, x_2$  on one side,  $x_3, x_4$  on the other.

Let us consider the linear inseparable pattern of logical XOR function. The 4 points cannot be separated by a single line ( $w$ ) such that 2 points  $(0,0)$  and  $(1,1)$  are on one side and  $(1,0), (0,1)$  on the other.



$x_1$	$x_2$	$y$
0	0	0
0	1	1
1	0	1
1	1	0

$$y = x_1 \oplus x_2$$

Similarly extending this to 4D, let us say the 4 points are

$$\begin{pmatrix} x_1 & 0, 0, 0, 0 \\ x_4 & 0, 1, 0, 0 \\ x_3 & 1, 0, 0, 0 \\ x_2 & 1, 1, 0, 0 \end{pmatrix}$$

We could not / prove no line separates this in 2D, hence no plane ( $w$ ) can keep  $x_1$  and  $x_2$  on one side and  $x_3$  and  $x_4$  on the other with these set of points in 4D  
 $(0, 0, 0, 0)$      $(0, 1, 0, 0)$      $(1, 0, 0, 0)$      $(1, 1, 0, 0)$