**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: μ = 45, σ = 8

If X is the amount of time taken to complete the repair on a customer car. To finish in one hour, we must have X <= 50.

Pr(X>50)

Normal table is calculated by:

Pr(X>50) = 1 – Pr(X<=50)

Z = (X - μ)/σ = (X-45)/8

Thus, the question can be answered by using normal table:

Pr(X<=50) = Pr(Z <=(50-45)/8) = Pr(Z<+0.625) = 73.4%

100-73.4 = 26.6%

26.6/100 = 0.266

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Mean = 38

Sd = 6

Z score = (value – mean)/SD

Z score for 44 = (44-38)/6 = 1 = 84.13%

People aboce 44 age = 100 – 84.13 = 15.87%

i.e. 63 out of 400

If Z score for 3 = (38-38)/6 = 0 = 50%

Hence people between 38 and 44 age = 84.13 -50 = 34.13%

i.e. 137 out of 400

So more employees at the processing center are older than 44 than between 38 and 44 is FALSE.

Z score for 30 = (30-38)/6 = -1.33 = 9.15%

i.e. 36 out of 400

So A training program for employees under the age 30 at the center would be expected to attract about 36 employees TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Probability of getting value between a and ab is 0.99

Probability of going wrong or outside a and b is 0.01 (i.e. 1-0.99 = 0.01)

Probability towards left from a = -0.005

Probability towards right from b = +0.005

Z = (X - μ) / σ

Z score of 0.005 = -2.57

Z \* σ + μ = X

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?
5. Ans: **Rs. 603.68**
6. **Rs. 476.33**
7. **First Division**