

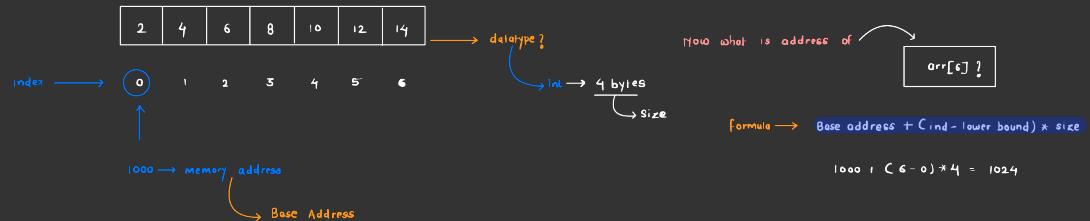
When to use array ?

Searching

Python has concept of dynamic array

No need to define size

Address of 1d array



2-d Array

Compose of rows and columns*



Address of 2d array



Linear Search

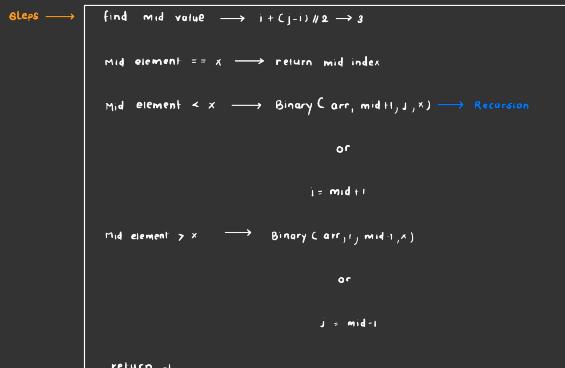


Binary Search



Small problem → Single element

Big Problem → More than one element



Search 2d matrix

Row wise sorted → left to right ↗

First integer of each row > last integer of previous row

	0	1	2	3	
0	1	3	5	7	$m \rightarrow \text{rows} \rightarrow 3$
1	16	11	16	20	$n \rightarrow \text{columns} \rightarrow 4$
2	25	30	34	60	3×4

target = 3

Brute force approach

traverse through all the rows and column

time complexity $\rightarrow O(m \times n)$

Optimized approach

Arrange above matrix in row major form

1	3	5	7	10	11	16	20	23	29	34	60
---	---	---	---	----	----	----	----	----	----	----	----

Now we

$$\text{mid} = \frac{0 + (n-1)}{2} = 5$$

$$\text{row} = \text{mid} / n_c = 5 / 4 = 1^*$$

$$\text{Column} = \text{mid} \% n_c = 5 \% 4 = 1^*$$

Now we know that $\text{mid} > \text{target} \rightarrow 11 > 3$

left = 0

right = mid - 1

1	3	5	7	10
---	---	---	---	----

$$(0+4) // 2 = 2 \quad \text{row} = 2 // 4 = 0 \quad \text{Column} = 2 \% 4 = 2$$

Now the time complexity $\rightarrow O(\log n)$

Ternary Search

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

Steps \rightarrow

$$\text{mid}_1 = \frac{l + (r-l)}{3} \quad \text{mid}_2 = \frac{r - (r-l)}{3} \longrightarrow \text{dividing array in three equal parts}$$

$$\text{mid}_1 = 0 + 9 // 3 = 3 \quad \text{mid}_2 = 9 - 9 // 3 = 6$$

$|target| = 5 \longrightarrow \text{Searching element}$

$\text{arr}[\text{mid}_1] = x \longrightarrow q \neq 5 \longrightarrow \text{false}$
if true \longrightarrow return mid_1

$\text{arr}[\text{mid}_2] = x \longrightarrow \text{return mid}_2$

$x < \text{arr}[\text{mid}_1] \longrightarrow 5 < 3 \longrightarrow \text{false}$

right = $\text{mid}_1 + 1$

$x > \text{arr}[\text{mid}_2] \longrightarrow 5 < 7 \longrightarrow \text{true}$

left = $\text{mid}_2 + 1$

else

$\text{mid}_1 = \text{mid}_2 + 1$

Sorting Algorithm

20	10	12	17	27	42	39
----	----	----	----	----	----	----

Stable and non-stable sorting

Stable

non stable

Relative Order Should be maintained

$\{20, 10, 12, 17, 27, 42, 39\}$
dict → (4,2) (5,7) (4,3) (6,8) (7,10)
Hashing data structure

After sorting
(4,2)

(4,2) (5,7) (6,8) (7,10) → Relative order maintained
OR

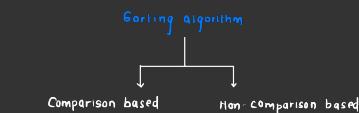
(4,3) (4,2) (5,7) (6,8) (7,10)

Quick Sort

Selection sort

heap sort

stable sorting



Comparison within elements

Selection Sort

Bubble sort

Insertion sort

Quick sort

merge sort

heap sort

divide & conquer

merge sort → operation of $d \times c$
Inplace and Outplace sorting algorithm
not using any extra place

Comparison - Bubble Sort

70	20	50	30	90	6	15
----	----	----	----	----	---	----

Steps: → 70 > 20

point to be noted:

if $n=7$ → then we need $n-1$ steps to sort array

70 > 50

Comparison

$$\rightarrow (n-1) + (n-2) + (n-3) + \dots + 3 + 2 + 1 \rightarrow \frac{n(n-1)}{2}$$

> 20 50 70

$$\text{Sum of } n \text{ natural number} \rightarrow \frac{n(n+1)}{2}$$

50 > 30

Time complexity → $O(n^2)$

> 20 30 50 70 90

90 > 50 50 > 30 30 > 20 20 > 15

70 > 50 50 > 30 30 > 20 20 > 15

> 50 30 20 15 10

90 > 70 70 > 50 50 > 30 30 > 20 20 > 15

70 > 50 50 > 30 30 > 20 20 > 15

Comparison Selection Sort

50	38	45	79	19	27	29
----	----	----	----	----	----	----

Concept → Take minimum index

Steps	① min → 0	② min - 1	③ min 2	④ min - 3	⑤ min - 4	⑥ min - 5
	50 > 38	28 < 45 38 > 79	45 < 79 45 < 50	79 > 50	50 < 79 50 > 45	79 > 50
Change min → 1	28 < 50 38 > 27	45 < 28	Change min → 4	Change min → 4	Change min → 6	
38 < 45 38 > 79	Change min → 5	Change min → 5	50 > 38	19, 21, 27, 45, 50, 79	19, 21, 27, 45, 50, 79	
38 > 19	19, 21, 45, 79, 50, 38, 29	38 > 29	Change min → 5			
Change min → 4		Change min → 6	19, 21, 29, 38, 50, 79, 45			
19, 38, 45, 79, 50, 27, 29		19, 21, 29, 74, 50, 38, 45				

Note → At every pass → one swap required

$$\text{Time Complexity} \rightarrow \# \text{ Comparison} \rightarrow \frac{n(n-1)}{2} = O(n^2)$$

$$\# \text{ Swap} \rightarrow (n-1)^{\text{th}} \text{ Swap} \rightarrow O(n)$$

$$\# \text{ Time complexity} \rightarrow O(n^2)$$

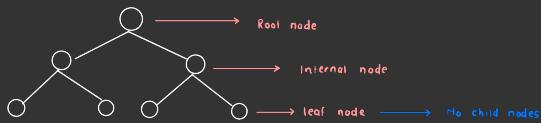
Comparison Insertion sort

75	90	100	95	85	50	100	110	7
----	----	-----	----	----	----	-----	-----	---

Steps →	75 90	Time Complexity →	# Comparison → 1 + 2 + 3 + ... + n-1
	75 90 100		$\frac{n(n-1)}{2} = O(n^2)$
	75 90 95 100		
	100 > 95		
	75 90 95 100 85		
	100 > 85 95 > 85 90 > 85		
	75 85 90 95 100 50		
	100 > 50 95 > 50 90 > 50 85 > 50		
	50 75 85 90 95 100 110 7		
	100 > 7 95 > 7 90 > 7 85 > 7		
	7 50 75 85 90 95 100 110	→ [final output]	

Heap data structure*

↳ Complete binary tree



Binary tree → # Child nodes 0 1 2

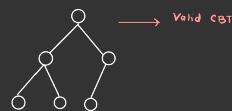
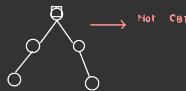
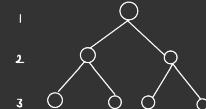
Almost node 2

full binary tree → Every node has 2 child

apart from leaf nodes

Complete binary tree → After completion of first level than only move towards the next level

After completion of left side node than only go for right side node



Properties of full binary tree

> nodes → n level → k

Now $n = 63$, $k = ?$

$$k=3, n=?$$

$$n = 2^{k-1}$$

$$n = 2^k - 1 \rightarrow 2^3 - 1 = 7$$

$$2^k = n + 1$$

$$k = \log_2(n+1) \rightarrow \log_2(63+1) \rightarrow \log_2^6$$

> $\frac{n}{2}$ → # Number of leaf nodes

$$k = 6$$

let assume $n = 7$

$$\frac{7}{2} \rightarrow 4 \text{ leaf node}$$

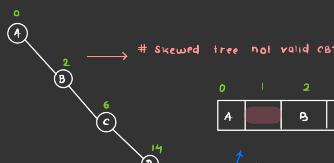
$\frac{7-4}{2} \rightarrow 1 \text{ non leaf node}$

Heap ↗ minheap
maxheap ↗ p tree

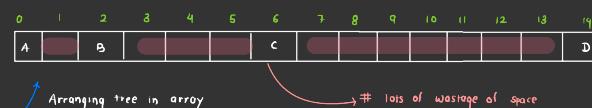


0	1	2	3	4	5
A	B	C	D	E	F

Arranging tree in array



Skewed tree not valid CBT



Arranging tree in array

lots of wastage of space

less wastage of space → Almost complete binary tree

Minheap *

parent node < child node

Root node → Contains smallest element

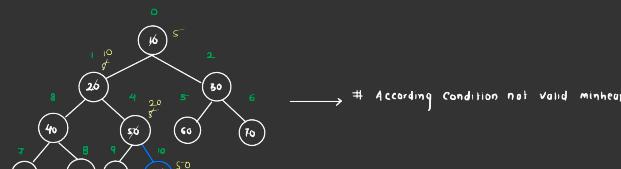
Maxheap *

parent node ≥ child node

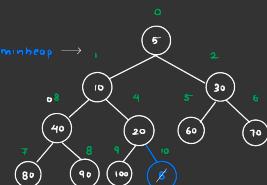
Root node → Contains largest element

Insertion in minheap

let suppose want to insert 50 in minheap



After inserting minheap →



n = 2^K - 1 → Complete binary tree

Comparison in minheap → log n

n = 2^K

Swap → log n

log_2(n+1) = log_2 2^K

Time complexity → O(log n) → for insertion

K = log_2(n+1) → # levels

Deletion in min or maxheap

deletion always start from min index

And last element 110 will place at 0th position

110 > 20 and 30 → but will take smallest one

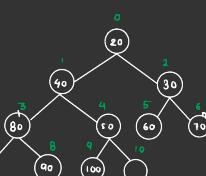
Swap

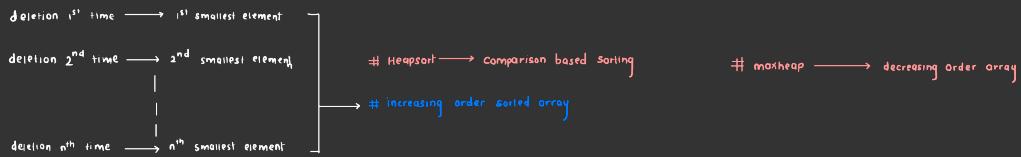
110 > 40 and 50

Swap

110 > 60 and 70

Swap

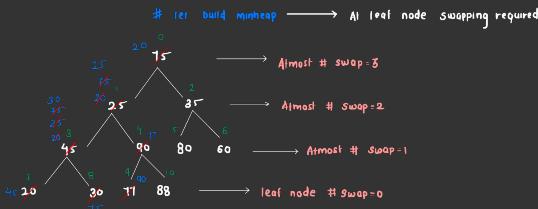




Overall time Complexity \longrightarrow O(n log n) \longrightarrow Heapsort

Build Heap

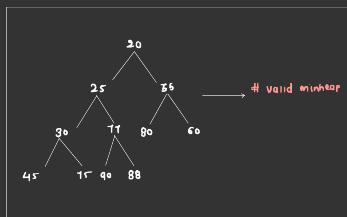
75, 25, 35, 45, 90, 80, 60, 20, 30, 77, 88 \longrightarrow Building heap



Steps

Create Complete binary tree

Swap element for creating min or max heap.



Time Complexity \longrightarrow O(n)

Comparison Heapsort

1) Build heap \longrightarrow minheap or maxheap O(n)

Overall time Complexity \longrightarrow O(n) + O(n log n)

2) delete all the elements and store deleted element O(n log n)

(# O(n log n))

Practice Question

[1, 1, 1, 2, 2, 3] \longrightarrow k=2 \longrightarrow Top 2 frequent element?

1 \longrightarrow 3 # frequencies

2 \longrightarrow 2

3 \longrightarrow 1
↓
key value

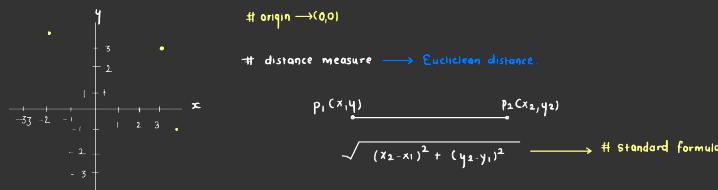
Expected output \longrightarrow [1, 2]

Counter in python \longrightarrow Count the frequency

Coding \longrightarrow

`heapq.nlargest(K, count.keys())`

points $\rightarrow [3,3, (-2,-1), -2,4]$ $k=2 \longrightarrow$ # closest points } # Output $\longrightarrow [3,3, -2,4]$



In our question we have measure from origin $\longrightarrow (x_1, y_1) = 0$

$$\sqrt{x_1^2 + y_1^2}$$

Create minheap \longrightarrow Distance, points

\curvearrowright # final output

Applying \longrightarrow delete operation on minheap \longrightarrow # k times

\curvearrowright Top K minimum distances

Recursion

\curvearrowright functions calls itself directly or indirectly

Example \longrightarrow factorial

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$n! = n \times (n-1)! \longrightarrow \text{# formula}$$

Base condition \longrightarrow Code will terminate at condition.

fact(n): # pseudo Code

```
if n=0 or n=1  $\longrightarrow$  Return 1      # Small problem (Base Condition)
else    n>1  $\longrightarrow$  Recursive Call  $\longrightarrow$  Return n * fact(n-1)
```

Return result

Recurrence Relation

$$T(n) = \begin{cases} 1 & n=1 \\ T(n-1) + C & n>1 \end{cases}$$

$T(n) = T(n-1) + C \longrightarrow$ # Substitution method

$$put \longrightarrow n=n-1$$

$$= T(n-1) + C + C$$

$$= T(n-2) + 2C$$

$$= T(n-3) + 3C$$

\downarrow # k times \longrightarrow n-k+1 k = n-1

$$T(n) = T(n-k) + kC$$

$$= T(0) + nC - C$$

$$T(n) = O(n) \longrightarrow \text{# Time Complexity}$$

Fibonacci Series

0	1	1	2	3	5	8	13	21	34	→	# series
0	1	2	3	4	5	6	7	8	9		n=10

Pseudo Code

```
if n <= 1 → return n # Base condition
else → return fib(n-1) + fib(n-2)
```

Count number of ways to reach upstairs
One or two steps at a time

Steps → n=1 → Only one way to reach upstairs

n=2 → Two ways → (1,1) (2)

n=3 → Three ways # (1,2), (1,1,1), (2,1)

n=4 → Five ways # (1,1,1,1), (2,2), (1,2,1), (2,1,1), (1,1,2)

Fibonacci Series

Pseudo Code

Helper method { if n <= 1 → Return n

else → Return Cn(n-1) + Cn(n-2)

Another function for counting ways to reach

Return helper(STL)

Final Output

When n → Very large → we can optimize time complexity using dp

Time Complexity → $O(2^n)$ # Exponential time complexity

Divide & Conquer

Big problems → divide and conquer

Small problems → Return Solution

4 Recursion → Function calling itself

Recursive tree

Recurrence relation

1) Substitution

2) Recursive

3) Master theorem

Pseudo Code

```
divideAndconquer(arr, p, q)
```

```
if small → arr, p, q :
```

Return Solution

p → lower index = 0

q → Higher index = len(arr) - 1

else:

mid = divide(arr, p, q)

Concept of Recursion → Conquering part

Return combine(Recursion) → combine part → not always required

$$T(n) = 2T\left(\frac{n}{2}\right) + C \quad \longrightarrow \quad T(n) = aT\left(\frac{n}{b}\right) + f(n) \quad \# \text{ Master theorem}$$

Binary Search Recurrence Relation

$$T(n) = T(n/2) + C$$

QuickSort Recurrence Relation

$$T(n) = 2T\left(\frac{n}{2}\right) + c$$



308

318

Best case $\longrightarrow (n-1) \cdot 1 = \Theta(n)$

Best Case $\longrightarrow (n-1) \cdot 2 = O(n)$

Average case $\longrightarrow \frac{n-1}{2} + \frac{n+1}{2}$

- Only one comparison
- Iterating through all

Small problem \longrightarrow $n=1$ or $n=2$ # Single Comparison

Return n # max and minima

Big problem \longrightarrow Recursive Tree

Binary Search

> Sorted array

2	4	8	12	20	25	60	70
0	1	2	3	4	5	6	7

X=50 \longrightarrow Target element

logic and pseudo code

Small problem \longrightarrow Single element

X=50

if arr[0] == X \longrightarrow Return i

Otherwise Return -1

Big problem \longrightarrow elements more than 1 or 2

$$\text{mid} = (\text{i} + \text{j}) / 2 = (0 + 7) / 2 = 3$$

if arr[mid] == X \longrightarrow Return mid

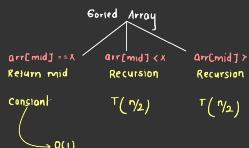
elif arr[mid] < X

will go to \longrightarrow Right side of array \longrightarrow Recursion

else:

will go to \longrightarrow Left side of array \longrightarrow Recursion

Recurrence Relation and time complexity



$$T(n) = \begin{cases} 1 & n=1 \\ T(n/2) + c & n>1 \end{cases} \longrightarrow \text{# Recurrence Relation}$$

Master's theorem

$$\log_b^0 = \log_2^1 = \text{constant} \quad f(n) = O(n^k \log^p n), k=0, p=0$$

$\log_b^0 = K \longrightarrow$ case 2

$O(C \log n) \longrightarrow O(\log n)$ # time complexity

Two pointer approach

A sorted array \longrightarrow 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 \longrightarrow Index

target = 90

Output: \longrightarrow arr[0] + arr[1] = target
 $\quad\quad\quad$ arr[1] + arr[2] = 90 + 50 = 90

Pseudo Code for two pointer approach

a=20, target-b=90-20=70 \longrightarrow present or not? in array

Searching over other elements

Linear Search $\bigcirc(n^2)$ Binary Search $\bigcirc(n \log n)$

Brute force Approach

Code

for i=0 to n-1

 if arr[i] + arr[b] == target \longrightarrow Return i, b

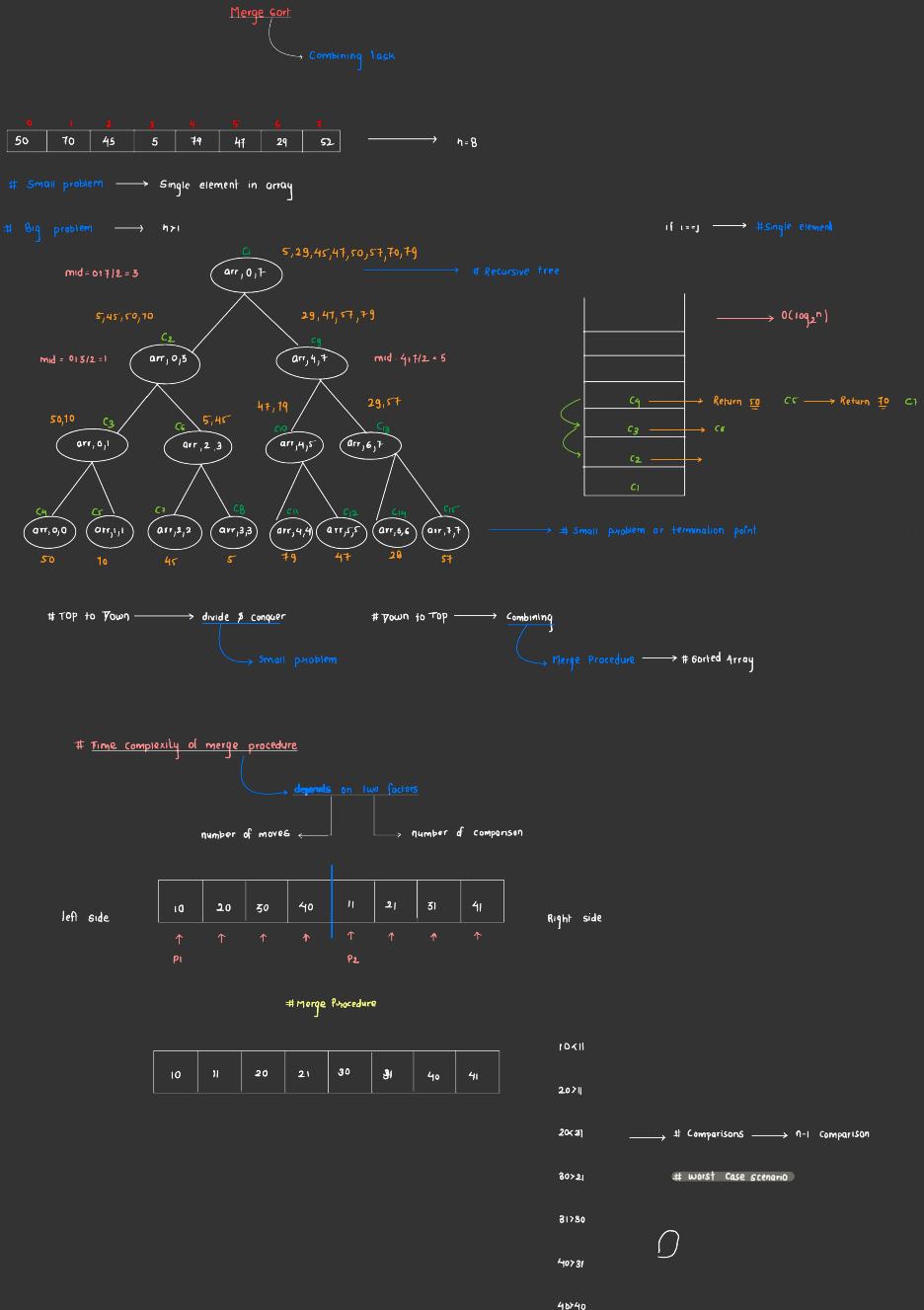
 elif arr[i] + arr[b] > target
 $\quad\quad\quad$ b = b-1

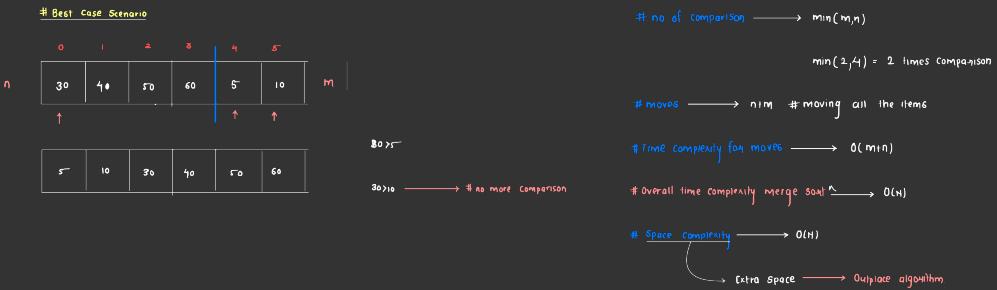
\longrightarrow decrement of b from 200 to 195

 else
 $\quad\quad\quad$ i = i+1 \longrightarrow increment of i

Return -1, -1

\curvearrowright No such kind of pairs





Pseudo code for mergesort

```
mergesort(Carr, i, j)
    if i == j  $\longrightarrow$  # small problem
        return Carr
    else
        mid = i + (j - i) // 2
        mergesort(Carr, i, mid)
        mergesort(Carr, mid + 1, j)
        mergeProcedure(Carr, i, mid, mid + 1, j)
    return Carr
```

Recurrence Relation

$$T(n) = 2T(\frac{n}{2}) + n$$

By Master's theorem

$$a=2, b=2, K=1, P=0$$

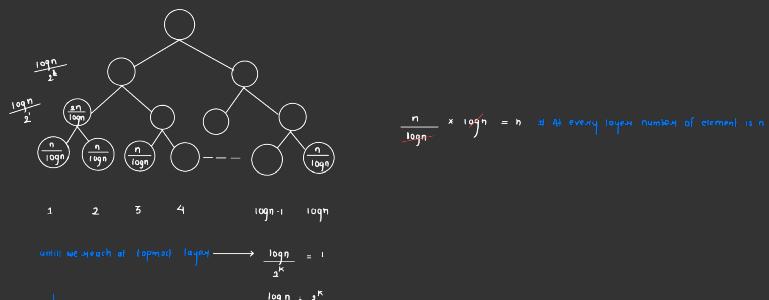
$$\log_2 a = \log_2 2 = 1 \longrightarrow \text{Case no. 2}$$

$$\log_2^a n = K$$

$$O(n \log n) \longrightarrow \# \text{ Time complexity}$$

Problems Related to merge sort

i. log₂n sorted subarray is each of subarray is of size $n/\log n$



Taking \log_2 on both side

$$\log_2(\log n) = K \longrightarrow \# \text{ number of levels}$$

Time complexity $\longrightarrow O(n \log(\log n))$

