Assignment 9: Deadline: 26/03/2024, 4:55pm

1. Our goal is to regulate the level of glucose in the blood, denoted by $y(t)$. Let γ denote the insulin concentration. The overall dynamics can be modeled as:

$$\begin{aligned} \dot{x}_1 &= -ax_1 + b\gamma, \\ \dot{x}_2 &= -(c + x_1)x_2 + d, \\ y &= x_2, \end{aligned}$$

   where a, b, c, d are constants. Represent the above dynamics as a block diagram in Simulink.

2. Calculate the unique equilibrium point $(\bar{x}_1, \bar{x}_2)$ of the above dynamics when $\gamma = \bar{\gamma} > 0$ is a constant input.

3. Linearize the above dynamics around $(\bar{x}_1, \bar{x}_2, \bar{\gamma})$ express the linearized dynamics both in the state-space form and in the form of transfer function. Recall that the state, input, and output of the linearized system are denoted by $\tilde{x}, \tilde{u}, \tilde{y}$ with $\tilde{x} = x - \bar{x}, \tilde{u} = u - \bar{u}, \tilde{y} = y - \bar{y}$.

4. Past work suggested that the following values are a good experimental fit:

$$\bar{x}_2 = 100, \quad a = c + \bar{x}_1 = \frac{1}{33}, \quad \frac{b\bar{x}_2}{a^2} = 3.3$$

   in the appropriate units. Determine the equilibrium, find the poles and zeros of the linearized dynamics, and determine the stability of the linearized dynamics.

5. Suppose that the insulin concentration does not change instantaneously. Rather, it evolves as

$$\dot{\gamma} = -f\gamma + gu,$$

   where u is the external input. Find the transfer function from $\tilde{u}$ to $\tilde{y}$. Determine the poles and zeros of the above transfer function if $f = \frac{1}{5}$ and $g = \frac{1}{5}$ in the appropriate units.

6. Consider the feedback controller

$$u(t) = K_p(y(t) - \bar{y}) + K_p T_d \frac{dy(t)}{dt} + \frac{K_p}{T_i} \int_0^t (y(\tau) - \bar{y}) d\tau.$$

Draw complete block diagram of the closed-loop system with integrators and signals $\bar{y}, y, u$.

7. Let $T_d = 38, T_i = 100$. Draw the root locus of the linearized plant with the above controller with respect to parameter K_p, and determine the range of values of K_p for which the closed-loop system is stable.

8. If $T_d = 0$, then determine the range of values of K_p for which the closed-loop system is stable.

9. Design a Luenberger observer for the linearized dynamics to estimate x_1 when the above control input is applied for $K_p = 0.17$, and check the performance of the controller and estimator for the nonlinear plant.