EE49001: Control and Electronic System Design

Assignment-7: RLC Circuit Control

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State-Space Representation and Controllability

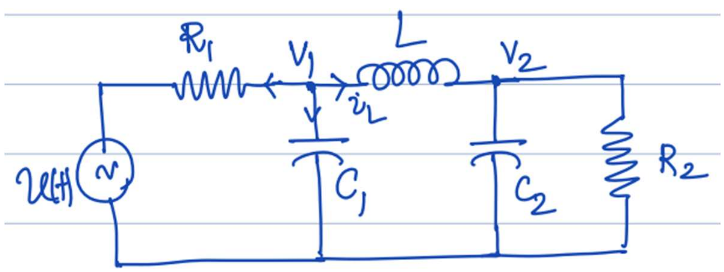

Fig. Circuit Diagram

We have the circuit parameters as follows: $R_1 = 1k\Omega, R_2 = 2k\Omega, C_1 = 1mF, C_2 = 4mF, L = 0.5H$. The states for the state space considered are: $v_1,\ v_2,\ i_L$.

The state space: $X = \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix}$

Therefore, we can write the equations as follows:

$$X(\dot{t}) = AX(t) + Bu(t)$$

Here, $A = \begin{bmatrix} 0 & \frac{1}{L} & \frac{-1}{L} \\ \frac{-1}{C_1} & \frac{-1}{C_1R_1} & 0 \\ \frac{-1}{C_2} & 0 & \frac{1}{C_2R_2} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{C_1R_1} \\ 0 \end{bmatrix}$

Controllability check is executed in MATLAB using the following snippet.

MATLAB Code

```
clear; close all; clc;


R1 = 1e3; C1 = 1e-3;

R2 = 2e3; C2 = 4e-3;
```

```
L = 0.5;


A = [
    -1/(R1*C1), 0, -1/C1;
    0, -1/(R2*C2), 1/C2;
    1/L, -1/L, 0;
    ];
B = [1/(R1*C1); 0; 0];
D = 0;


Co = ctrb(A,B);
disp("Rank of the controllability matrix is: "+rank(Co));
```

On executing the above code, we get the rank of the controllability matrix $= 3$. Hence, the system is controllable.

Estimation of Observability

We need to find if the system is observable for the following choice of outputs:

$$v_1 \coloneqq \text{voltage across } C_1$$

$$v_2 \coloneqq \text{voltage across } C_2$$

$$i_1 \coloneqq \text{current across } L$$

For different output states, we have different vectors C. We can write output y as:

$$y = CX + Du(t)$$

Since $D = 0$. We have matrix C for each case as follows:

$$C_1 = [0 \quad 1 \quad 0]$$
$$C_2 = [0 \quad 0 \quad 1]$$
$$C_3 = [1 \quad 0 \quad 0]$$

For all the three cases, the rank of the observability matrix is calculated by running the following script:

MATLAB Code

```
clear; close all; clc;
R1 = 1e3; C1 = 1e-3;
```

```
R2 = 2e3; C2 = 4e-3;
L = 0.5;

A = [
    -1/(R1*C1), 0, -1/C1;
    0, -1/(R2*C2), 1/C2;
    1/L, -1/L, 0;
    ];
B = [1/(R1*C1); 0; 0];
D = 0;

Co = ctrb(A,B);
disp("Rank of the controllability matrix is: "+rank(Co));
C11 = [1, 0,0]; Ob1 = obsv(A,C11);
C12 = [0, 1,0]; Ob2 = obsv(A,C12);
C13 = [1,-1,0]; Ob3 = obsv(A,C13);

disp("Rank of the observability matrix when output is v1: "+rank(Ob1));
disp("Rank of the observability matrix when output is v2: "+rank(Ob2));
disp("Rank of the observability matrix when output is iL: "+rank(Ob3));
```

From the MATLAB output, we find that the rank of observability matrix in all the three cases: 3. Therefore, we can infer that the system is observable with these output variables.

Simulink Implementation

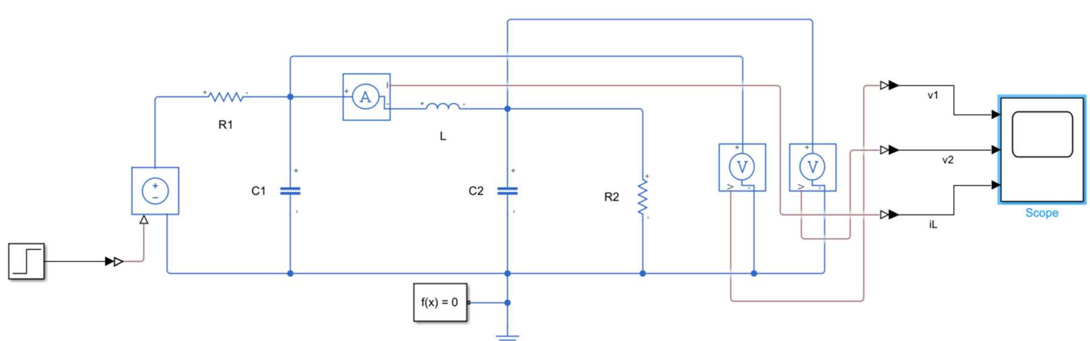

Fig: Simulink Model implementation

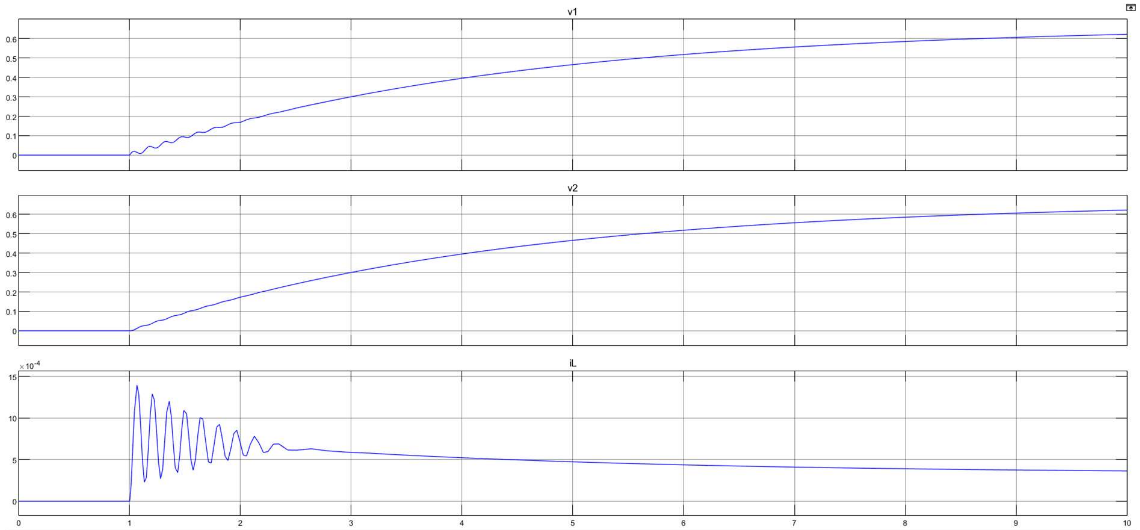

Fig: Output Voltages at steady state across L, C_1, C_2

Design of State-Feedback Controller

We are required to implement a state feedback controller to regulate the voltage $v_2 = 10V$ when the voltage across C_1 i.e. v_1 is known after being sampled at $1\ kHZ$.

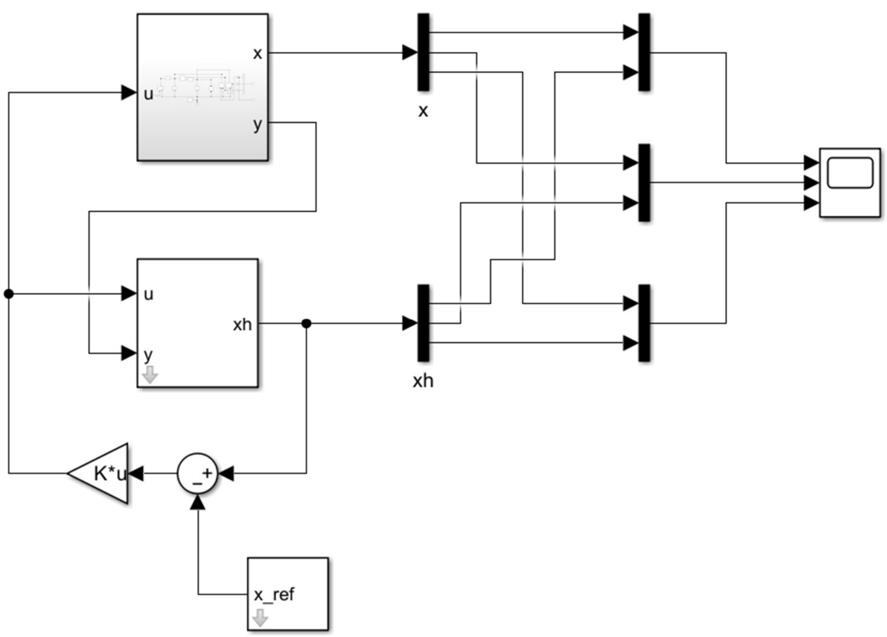

Fig. State Feedback Controller Design

Fig. Trajectory of the three states after application of state feedback controller

We observe that the voltage v_2 finally settles to the required value i.e. $10\ V$.

Estimator for estimating the value of R_1

We need to estimate the value of R_1 using the control input and the three estimators i.e. v_1, v_2, i_L.

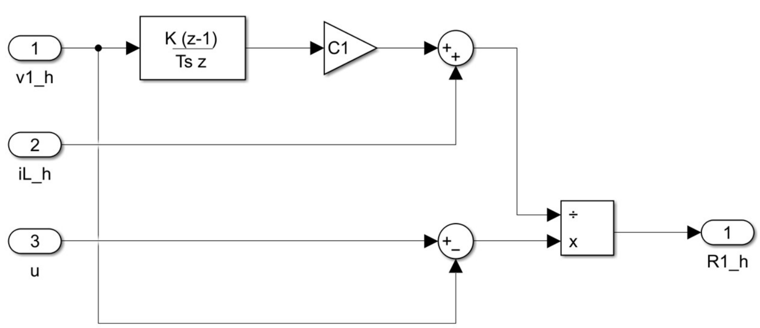

Fig. Estimator Subsystem

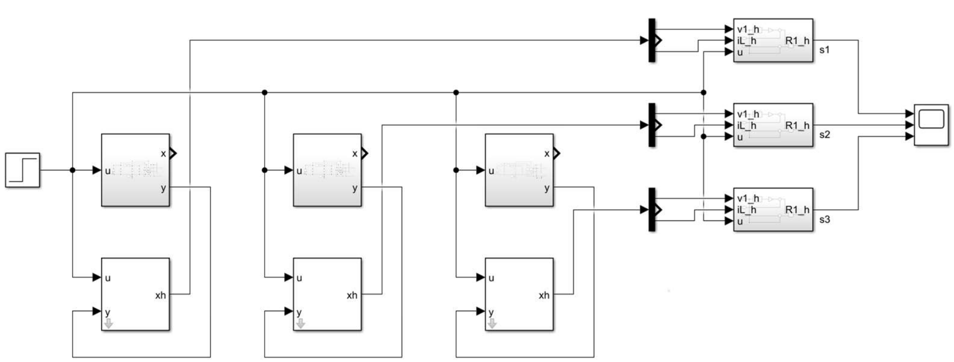

Fig. Estimator Model with different observer for observing different outputs

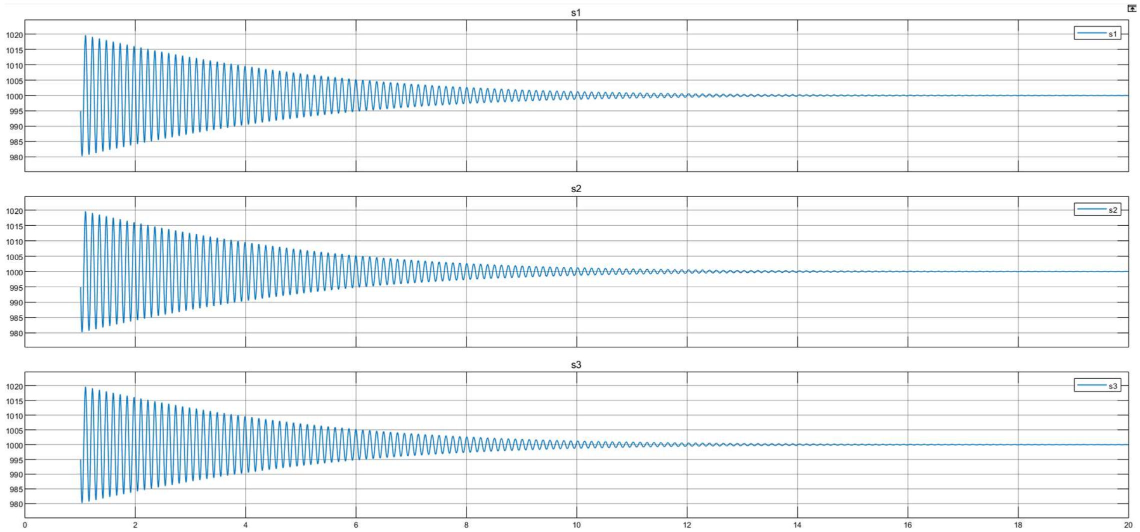

Fig. Estimated value of R_1 from different observed quantities

In all the three cases, i.e. by observing the states v_1, v_2, i_L, we can see the variation in the estimated value of R_1. As evident from the three plots attached above, the value of R_1 oscillates and finally settles to its actual value i.e. $1\ K\Omega$.