Assignment 10: Deadline: 02/04/2024, 4:55pm

1. Consider the system

$$x(t+1) = \begin{pmatrix} 1.5 & 0 \\ 1 & -1.5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t). \tag{1}$$

   (a) Show that the system is open-loop unstable.

   (b) Show that the system is controllable.

   (c) Let $T = 10$. Use MATLAB script to generate $(x(t))_{t=0}^{T}$ for different choices of i) $x(0)$ whose elements belong to the interval $[-10, +10]$ and ii) $(u(t))_{t=0}^{T-1}$ belonging to the interval $[-5, +5]$.

   (d) Note down all combinations of $x(0)$ and $u(t)_{t=0}^{T-1}$ for which you observe satisfaction of the following:

$$\begin{pmatrix} -10 \\ 10 \end{pmatrix} \leq x(t) \leq \begin{pmatrix} +10 \\ +10 \end{pmatrix} \text{ for all } t = 0, 1, \ldots, T. \tag{2}$$

2. Consider the setting where system (1) is subject to external disturbances.

$$x(t+1) = \begin{pmatrix} 1.5 & 0 \\ 1 & -1.5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} w(t). \tag{3}$$

   Here, w denotes an external disturbance. Use the combinations of $x(0)$ and $u(t)_{t=0}^{T-1}$ in 1.(d) and find the largest range of w for which (2) holds. Start with $w(t) \in [-0.5 + 0.5]$ for all $t = 0, 1, \ldots, T-1$ and keep increasing/decreasing (as applicable) the size of the interval.

3. Consider the setting where system (1) is subject to parametric uncertainties.

$$x(t+1) = \begin{pmatrix} 1.5 + w^p(t) & 0 \\ 1 & -1.5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t). \tag{4}$$

   Here, w^p denotes the parametric uncertainty, possibly caused by inaccurate modelling of the system or variation in the system over time. Use the combinations of $x(0)$ and $u(t)_{t=0}^{T-1}$ in 1.(d) and find the largest range of w^p for which (2) holds. Start with $w^p(t) \in [-0.5 + 0.5]$ for all $t = 0, 1, \ldots, T-1$ and keep increasing/decreasing (as applicable) the size of the interval.

4. Consider the setting where system (1) is subject to both external disturbance and parametric uncertainties.

$$x(t+1) = \begin{pmatrix} 1.5 + w^p(t) & 0 \\ 1 & -1.5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} w(t). \qquad (5)$$

   Use the combinations of $x(0)$ and $u(t)_{t=0}^{T-1}$ in 1.(d) and find the largest range of w^p and w for which (2) holds. Follow the procedure used in 2. and 3. above.

5. Discuss your observation on the effects of external disturbance and parametric uncertainty on a system behaviour.

6. Can you give example of a practical system where magnitude of system state needs to be restricted (á la condition (2))?