EE49001: Control and Electronic System Design

Assignment-3: Determination of DC Gain, Part:1

Submitted By:

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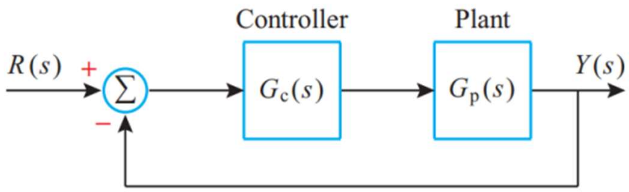


From the above block diagram

$$G(s) = G_c(s) \cdot G_p(s)$$
$$= \frac{6K}{(s+1)(s+2)(s+3)}$$

Design K such that gain crossover frequency is 2 rad/s

Theoretically

From the above-mentioned transfer function, the gain equation is

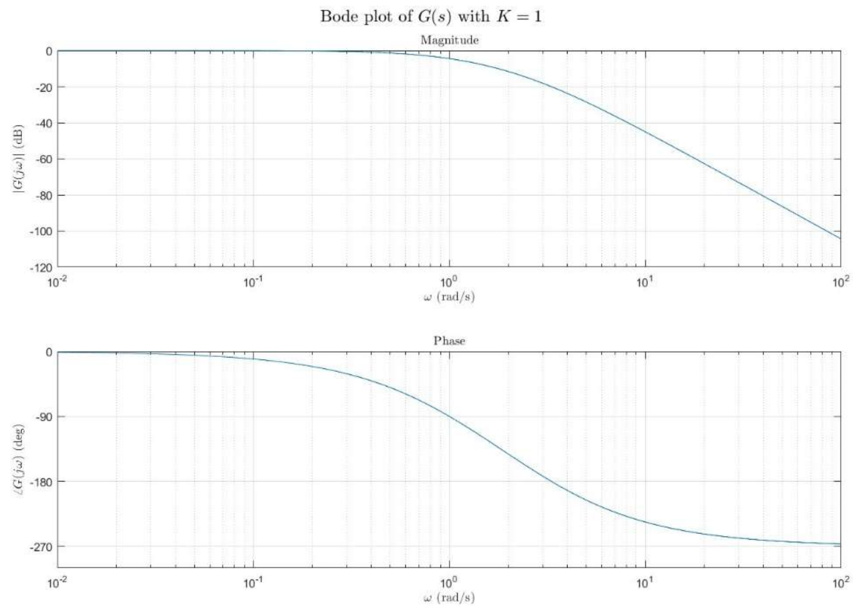
$$|G(s)| = \frac{K}{\sqrt{1+\left(\frac{\omega}{1}\right)^2} \cdot \sqrt{1+\left(\frac{\omega}{2}\right)^2} \cdot \sqrt{1+\left(\frac{\omega}{3}\right)^2}}$$

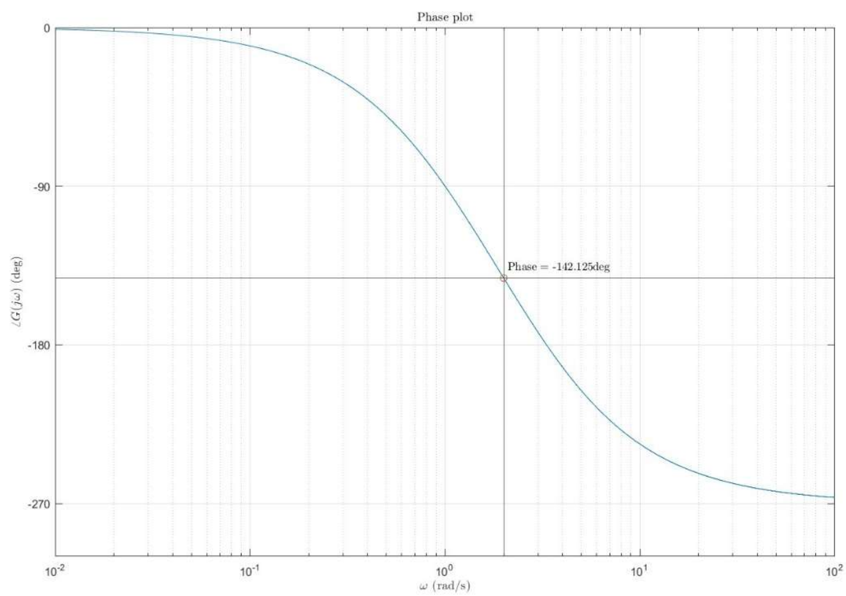
For gain crossover frequency to be ω_c to be 2 rad/s

$$20\log|G(j\omega_c)| = 0$$
$$\Rightarrow |G(j\omega_c)| = 1$$
$$\Rightarrow K = \sqrt{1+2^2} \cdot \sqrt{1+1^2} \cdot \sqrt{1+\left(\frac{2}{3}\right)^2}$$
$$\Rightarrow K = \sqrt{\frac{130}{9}} \approx 3.8006$$

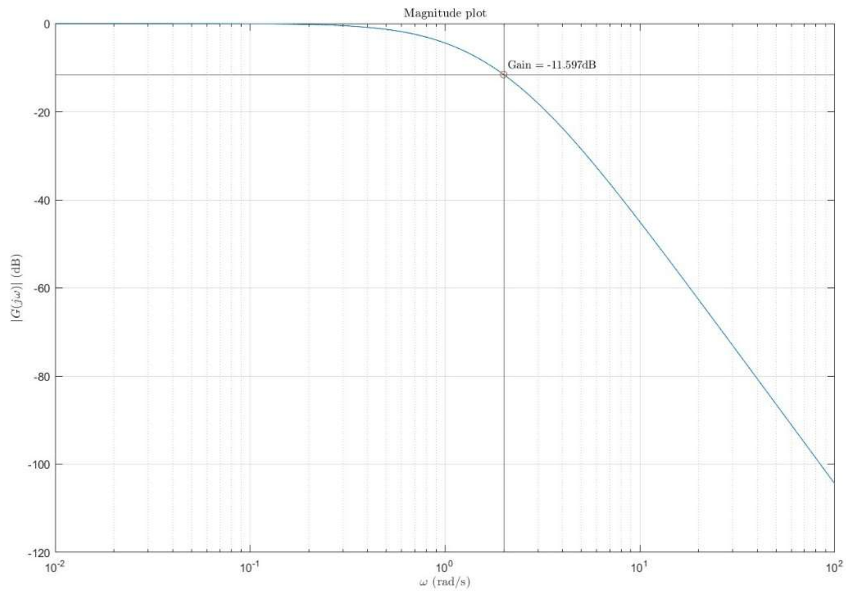
Design Steps

If $K = 1$ is used in the transfer function G the following Bode plot is obtained





From the phase plot of $G(s)$ it is evident that $\angle G(j\omega_c) = -142.125°$. Thus $\angle G(j2) + 180° > 0$ and it can be concluded that gain crossover frequency of 2 rad/s can be achieved.



As can be observed, $20\log|G(j2)| = -11.597$dB, thus to intersect the 0 dB line at $\omega = 2$rad/s the magnitude plot must be pulled down by 11.597dB.

Therefore, our required DC gain is

$$K = 10^{\frac{11.597}{20}} \approx 3.8005$$

Which is very close to our calculated value in the first step.

Design K such that Phase Margin is 60deg

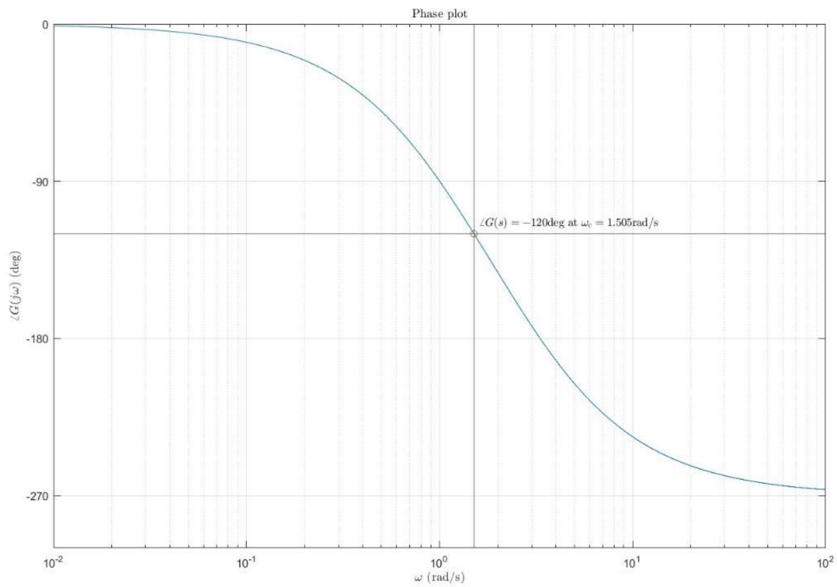
Theoretically

$$\angle G(j\omega) = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3} = PM - 180$$
$$\Rightarrow \tan^{-1}\omega_c + \tan^{-1}\frac{\omega_c}{2} + \tan^{-1}\frac{\omega_c}{3} = 120°$$
$$\Rightarrow \omega_c \approx 1.505\text{rad/s}$$

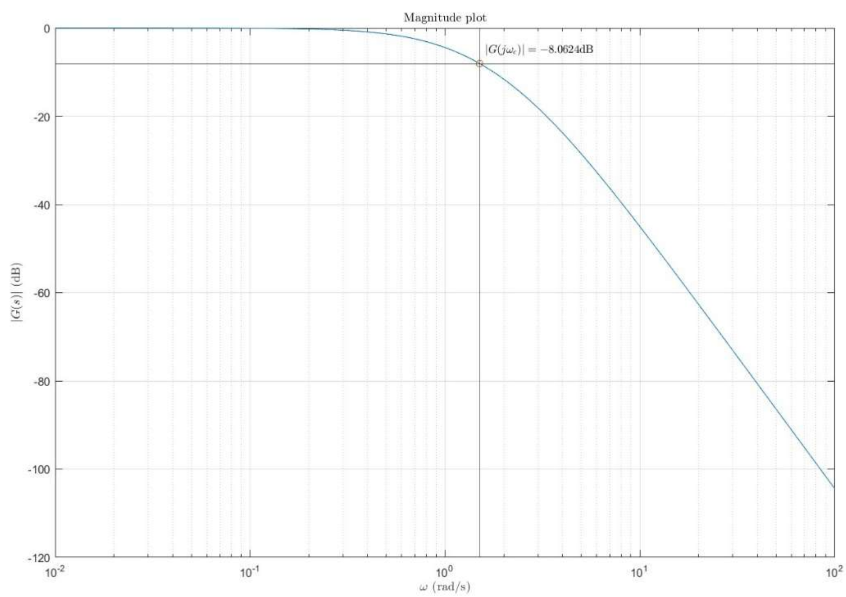
Therefore,

$$|G(j\omega_c)| = 1$$
$$\Rightarrow k = \sqrt{1 + 1.505^2} \cdot \sqrt{1 + \left(\frac{1.505}{2}\right)^2} \cdot \sqrt{1 + \left(\frac{1.505}{3}\right)^2}$$
$$\Rightarrow k \approx 2.53$$

Design steps



From the phase plot it can be seen that $\omega_c = 1.505$ for $\angle G(j\omega_c) = -120°$



As can be observed from the magnitude plot that the magnitude plot needs to be pulled down by 8.0624dB so that it intersects 0dB line at ω_c.

Therefore, our required DC gain is

$$K = 10^{\frac{8.0624}{20}} \approx 2.528$$

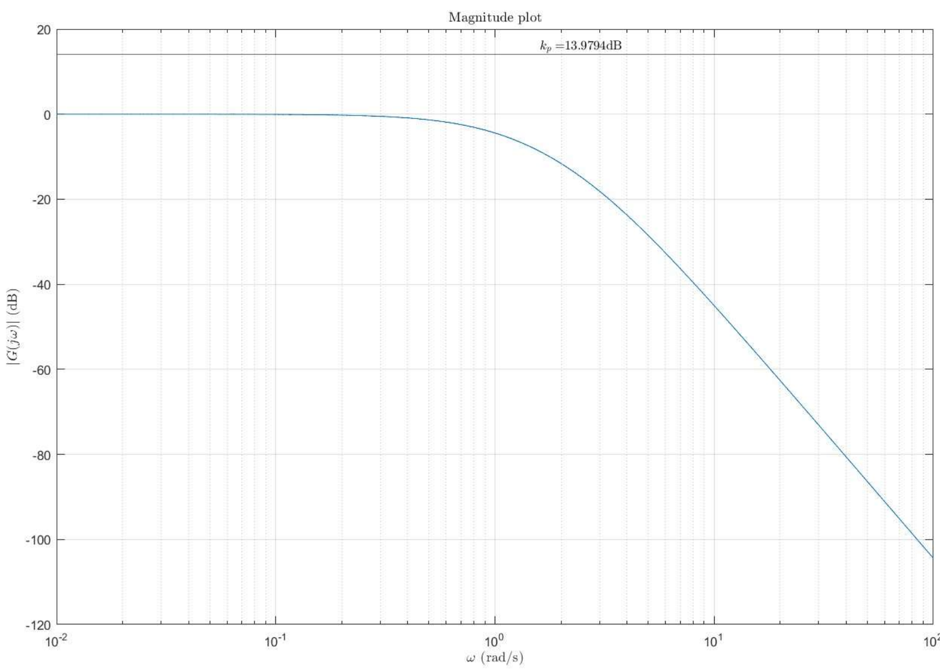
Design K such that Position Error Constant is 5

Theoretically

For a unity feedback system, as is given, the position error constant can be defined as

$$\begin{aligned} k_p &= \lim_{s\to 0} G(s) \\ &= \lim_{s\to 0} \frac{6K}{(s+1)(s+2)(s+3)} = 5 \\ \Rightarrow K &= 5 \end{aligned}$$

Design Steps



Therefore, from the above magnitude plot the it can be observed that at the low frequency end, where the plot is flat, it is lower than the desired k_p by 13.9794dB (since the magnitude plot is flat at 0dB). Thus, the plot needs to be shifted up by 13.9794dB. Hence our required DC gain will be

$$\begin{aligned} K &= 10^{\frac{13.9794}{20}} \\ &\approx 5 \end{aligned}$$

Design K such that $20\log|G(s)| \geq 10$dB for $\omega \in [0,0.3]$

Theoretically

Since it is sufficient to make $20\log|G(j0.3)| \geq 10$ for the said condition to hold. Therefore,

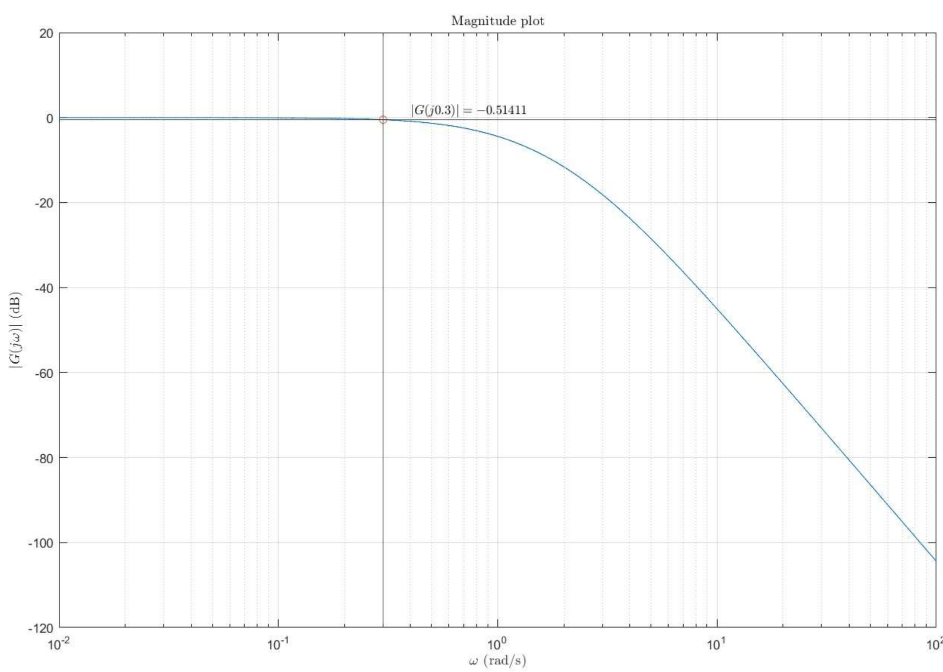
$$|G(j0.3)| = \sqrt{10}$$

$$\Rightarrow \frac{K}{\sqrt{1+\left(\frac{0.3}{1}\right)^2}\cdot\sqrt{1+\left(\frac{0.3}{2}\right)^2}\cdot\sqrt{1+\left(\frac{0.3}{3}\right)^2}} = \sqrt{10}$$

$$\Rightarrow K = \sqrt{10}\cdot\sqrt{1.09}\cdot\sqrt{1.0225}\cdot\sqrt{1.01}$$

$$K \approx 3.3551$$

Design Steps



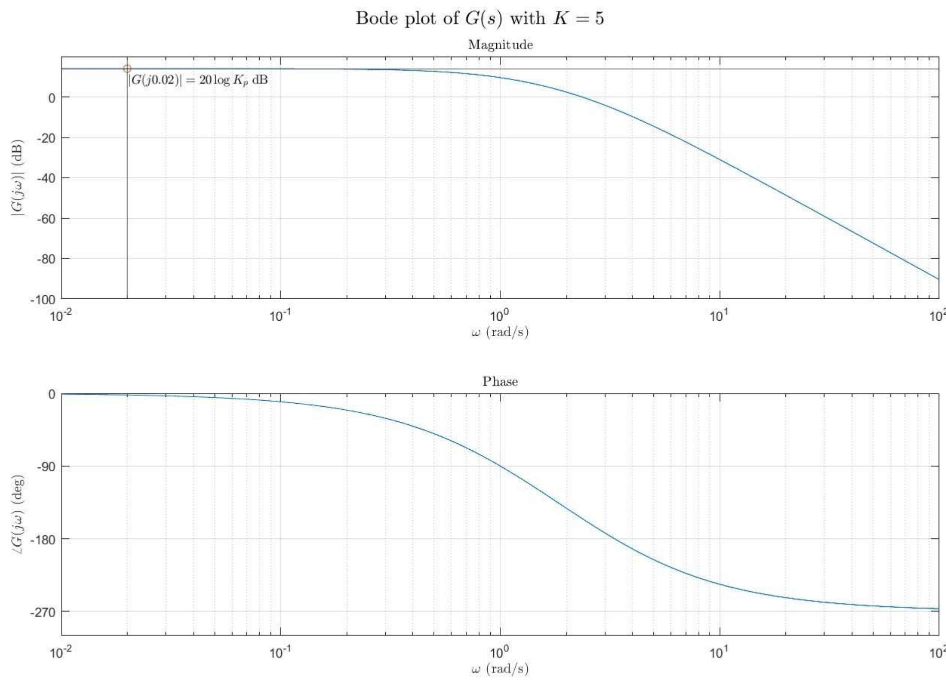
Since $|G(j0.3)| = -0.514$, thus to bring $|G(j0.3)|$ to 10dB, we need to shift the magnitude plot up by $10-(-0.51411) = 10.51411$. Therefore, our required DC gain is

$$K = 10^{\frac{10.51411}{20}}$$
$$\approx 3.354$$

Desired Plots

Position Error Constant is 5

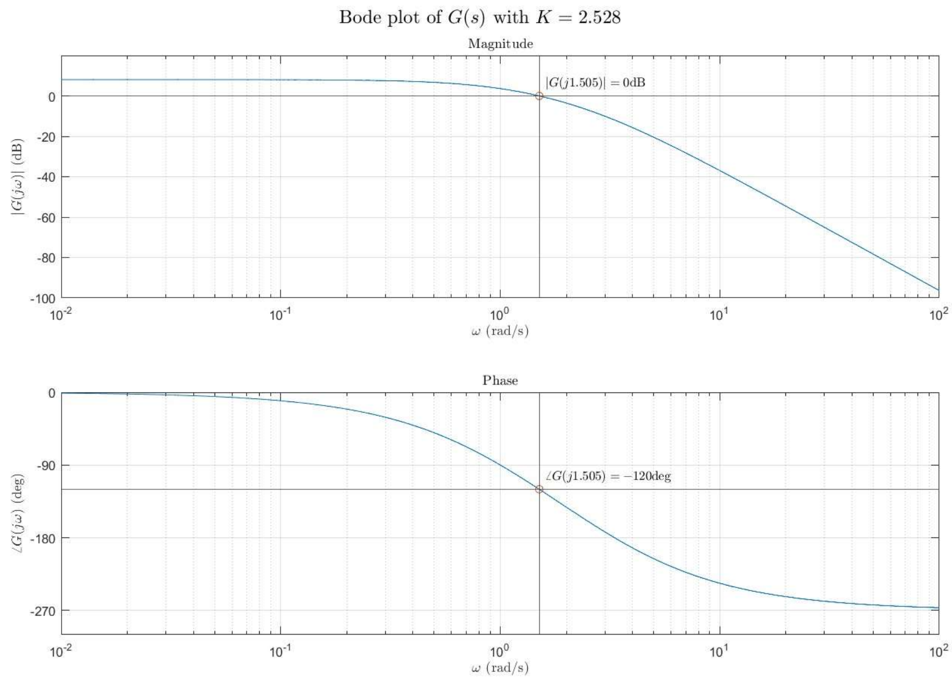
K is set to 5 as obtained before in our calculation and the bode plot of $G(s)$ is obtained.



As can be observed the gain at lower frequencies (assumed 0.02) is around 13.979 with is approximately equal to $20 \log 5$. Thus, it can be concluded that the position error constant K_p is 5.

Phase Margin is 60deg

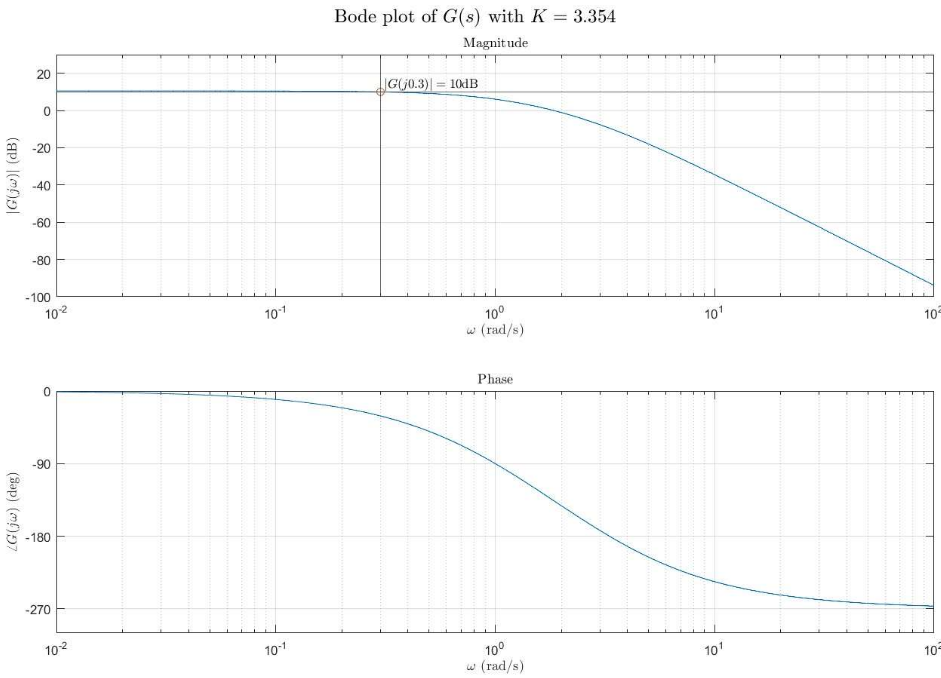
K is set to 2.528 as obtained before in our calculation and the bode plot of $G(s)$ is obtained.



As can be observed from the above bode plot, the gain crossover frequency is 1.505 rad/s and correspondingly $\angle G(j1.505) = -120$deg. Thus, the phase margin will be $-120 + 180 = 60°$ Therefore, design of K is just as expected.

Gain at 0.3 is greater than 10dB

K is set to 3.354 as obtained before our calculation and the bode plot of $G(s)$ is obtained.



As can be observed that the gain at 0.3 rad/s is 10 dB which is consistent to our design goals.