Assignment 6: Deadline: 05/03/2024, 4:55pm

Consider the linear dynamical system in state-space form

$$\dot{x}_c(t) = Ax_c(t) + Bu_c(t), \tag{1}$$
$$y_c(t) = Cx_c(t) + Du_c(t), \tag{2}$$

where $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

The vector $x_c \in \mathbb{R}^4$ denotes the state variables, $u_c \in \mathbb{R}^2$ denotes the inputs, and $y_c \in \mathbb{R}^2$ denotes the output of the continuous-time plant.

In Assignment 4, we designed the observer and the controller for this continuous time system. However, in practice, the measurements are often available in discrete time by sampling the continuous output. In this assignment, we will learn how to do state estimation and controller design using the sampled data.

1. Obtain the sampled output using "zero order hold" block with sampling time $T_s = 0.1$ second. Plot both the continuous output and the sampled output in the same figure.

2. Discretize the above continuous-time dynamical system using Matlab command "c2d," and obtain the matrices A_d, B_d, C_d and D_d. These matrices define the discrete-time evolution of the dynamics given by

$$x((k+1)T_s) = A_d x(kT_s) + B_d u(kT_s), \tag{3}$$
$$y(kT_s) = C_d x(kT_s) + D_d u(kT_s), \tag{4}$$

   for $k = 0, 1, 2, \ldots$. Check the controllability and observability of (A_d, B_d) and (A_d, C_d), respectively. The rank condition is same as the continuous case.

3. Implement Luenberger observer for the discrete-time model given by:

$$\widehat{x}((k+1)T_s) = A_d\widehat{x}(kT_s) + B_d u(kT_s) + L(y_c(kT_s) - C_d\widehat{x}(kT_s) - D_d u(kT_s)).$$

   To this end, create a model in Simulink with four discrete delay blocks for the states and use matrix multiplication option in the gain blocks, and mux/demux blocks to convert scalar signals into vectors. Same input is applied to both the continuous-time plant and the discrete-time copy of the plant using zero order hold blocks.

4. Find a gain matrix $L_d \in \mathbb{R}^{n_y \times n_x}$ such that the eigenvalues of $A_d - L_d C_d$ are located at $p = (0.05, 0.06, 0.07, 0.08)$. What should be the desired values of the poles in this case? Can we choose eigenvalues to be less than -1?

5. Excite the system with step inputs for both the input channels and plot the true state $x_c(t)$ and estimated state $x((k+1)T_s)$ for each of the states. There should be four figures, and each figure with two curves/plots.

6. Find K_d such that the eigenvalues of $A_d - B_d K_d$ are at $10p$. Implement the above designed observer-based state-feedback controller in Simulink and plot the trajectory of the states. Let the initial state of the continuous time plant be $x_c(0) = [10, 20, -10, -20]$ and for the discrete model be $x(0) = [0, 0, 0, 0]$.

7. Repeat the above for different sampling times $T_s = 0.01$ and 0.5 seconds. Explain the difference.