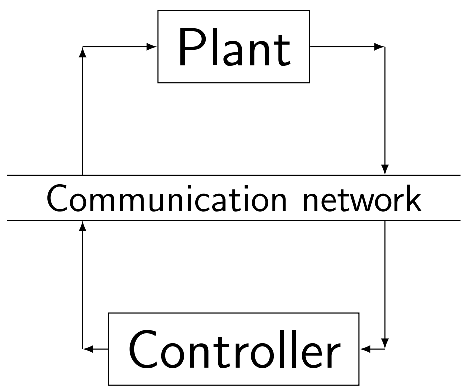
Assignment 5: Deadline: 27/02/2024, 4:55pm

Note: We will use MATLAB script(s) and NOT SIMULINK model(s) in this assignment.

Section I:

Networked Control Systems (NCSs) are spatially distributed systems where plants and controllers are geographically apart and exchange information over a shared band-limited communication network. Examples include remote surgery, unmanned aerial vehicles, autonomous vehicles, transportation networks, power networks, etc.



While the presence of a communication network results in widespread applications of control systems, it introduces new challenges to a control engineer.
In this exercise we will first demonstrate how communication network may affect system theoretic properties of a plant.

1. Consider an NCS whose plant is a discrete-time linear system

$$x(t+1) = \underbrace{\begin{pmatrix} -2 & -13 & 9 \\ -5 & -10 & 9 \\ -10 & -11 & 12 \end{pmatrix}}_{A} x(t) + \underbrace{\begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}}_{B} u(t), \;\; x(0) = \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}, \;\; t \in \mathbb{N}_0, \tag{1}$$

   and the controller is state-feedback

$$u(t) = \underbrace{\begin{pmatrix} 2.2225 & -10.44 & 5.5944 \end{pmatrix}}_{K} x(t), \;\; t \in \mathbb{N}_0. \tag{2}$$

Write a MATLAB script to plot the state trajectory of system (1)-(2) until $t = 15$ units of time, i.e., $\big(x(t)\big)_{t=0}^{15}$.

2. Suppose that the communication channels from the controller to the plant are prone to data losses. As a result, control input u may be lost in the network intermittently. Let $\kappa : \mathbb{N}_0 \to \{0,1\}$ denote the data loss signal, defined as follows: if $\kappa(t) = 0$, then the control input $u(t)$ is lost in the network at time t and if $\kappa(t) = 1$, then the control input $u(t)$ is received by the plant at time t, $t \in \mathbb{N}_0$.

   (a) Consider the following data loss signal:

$$\begin{array}{rrrr} \kappa(0) = 0, & \kappa(1) = 0, & \kappa(2) = 1, & \kappa(3) = 0, \\ \kappa(4) = 0, & \kappa(5) = 0, & \kappa(6) = 1, & \kappa(7) = 0, \\ \kappa(8) = 1, & \kappa(9) = 0, & \kappa(10) = 0, & \kappa(11) = 1, \\ \kappa(12) = 0, & \kappa(13) = 0, & \kappa(14) = 0, & \ldots. \end{array}$$

   Write a MATLAB script to plot the state trajectory of system (1)-(2) until $t = 15$ units of time, i.e., $\big(x(t)\big)_{t=0}^{15}$, under the given data loss signal, $\big(\kappa(t)\big)_{t=0}^{14}$.

   (b) Compare your observation between ideal communication and communication under data losses from the controller to the plant.

   (c) Suppose that at every instant of time t, a data loss occurs with a probability p. In other words,

$$\kappa(t) = \begin{cases} 0 \text{ with probability } p, \\ 1 \text{ with probability } 1-p, \end{cases} \qquad t \in \mathbb{N}_0.$$

   Vary p from 0.1 to 0.9 with a step size 0.1. Generate a data loss signal, $\big(\kappa(t)\big)_{t=0}^{100}$, for each choice of p, and plot the corresponding $\big(x(t)\big)_{t=0}^{100}$. Use a MATLAB script for this purpose.

   (d) Give the range of p for which you observe $x(15) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

For analyzing system theoretic properties of and designing controllers for NCSs under network induced limitations and uncertainties, a switched system model is widely used. Such systems switch between multiple dynamics over time. For

example,
Let $A_1 = A$ and $A_2 = A + BK$. The dynamics of (1)-(2) can be modelled as the following switched system:

$$x(t+1) = A_{\sigma(t)}x(t), \;\; x(0) = x_0, \;\; t \in \mathbb{N}_0, \tag{3}$$

where

$$\sigma(t) = \begin{cases} 1, & \text{when } \kappa(t) = 0, \\ 2, & \text{when } \kappa(t) = 1. \end{cases}$$

We call A_1, A_2 as subsystems and $\sigma : \mathbb{N}_0 \to \{1, 2\}$ as a switching signal. Notice that (3) is a time-varying system.

Section II:

Beyond NCSs, switched systems, as a mathematical tool, finds a wide range of applications in power systems and power electronics, automotive control, aircraft and air traffic control, etc. In the remainder of this exercise we will familiarize ourselves with an interesting feature of a switched system.

3. Consider a switched system

$$x(t+1) = A_{\sigma(t)}x(t), \; x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \; t \in \mathbb{N}_0, \tag{4}$$

where $A_1 = \begin{pmatrix} 0.47 & 0.12 \\ -3.90 & 0.19 \end{pmatrix}$, $A_2 = \begin{pmatrix} -0.03 & 0.78 \\ 0.60 & 0.47 \end{pmatrix}$ and $\sigma : \mathbb{N}_0 \to \{1, 2\}$.

(a) Let

$$\begin{array}{cccc} \sigma(0) = 1, & \sigma(1) = 1, & \sigma(2) = 2, & \sigma(3) = 2, \\ \sigma(4) = 2, & \sigma(5) = 2, & \sigma(6) = 1, & \sigma(7) = 1, \\ \sigma(8) = 2, & \sigma(9) = 2, & \sigma(10) = 2, & \sigma(11) = 2, \\ \sigma(12) = 1, & \sigma(13) = 1, & \sigma(14) = 2, & \ldots\ldots\ldots \end{array}$$

Write a MATLAB script to plot the trajectory of the switched system (4) up to $t = 15$ units of time, i.e., $\big(x(t)\big)_{t=0}^{15}$.

(b) Repeat (a) with

$$\begin{array}{cccc} \sigma(0) = 1, & \sigma(1) = 1, & \sigma(2) = 1, & \sigma(3) = 2, \end{array}$$

$$
\begin{array}{llll}
\sigma(4) = 2, & \sigma(5) = 1, & \sigma(6) = 1, & \sigma(7) = 1, \\
\sigma(8) = 2, & \sigma(9) = 2, & \sigma(10) = 1, & \sigma(11) = 1, \\
\sigma(12) = 1, & \sigma(13) = 2, & \sigma(14) = 2, & \ldots\ldots\ldots
\end{array}
$$

(c) Based on experiments (a) and (b) comment on the following: "Switching between stable subsystems arbitrarily leads to a stable switched system".