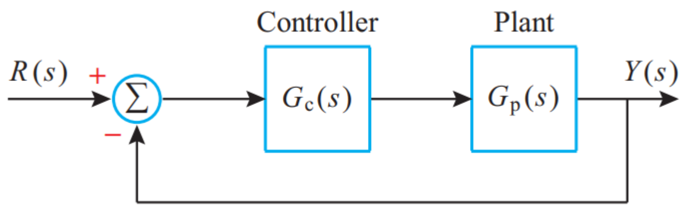
Assignment 3: Part I. Deadline: 30/01/2024, 4:55pm

Consider the following block diagram.



Our objective is to design the controller transfer function, $G_c(s)$, to meet frequency-domain design specifications.

1. Consider
$$G(s) = G_c(s)G_p(s) = \frac{6K}{(s+1)(s+2)(s+3)}.$$
   (a) Find K such that gain crossover frequency, $\omega_c = 2$rad/s.
      i. Use MATLAB to draw the Bode magnitude and phase plots of $G(s)$ at $K = 1$.
      ii. Find $\angle G(j\omega_c)$ from the plot.
      iii. Verify that phase margin $= \angle G(j2) + 180° > 0$. If not, conclude that the crossover frequency 2 rad/s cannot be achieved.
      iv. Mark the 2 rad/s frequency on the 0 dB line and determine whether or not the magnitude plot needs to move up or down so that it intersects the 0 dB line at 2 rad/s.
      v. Find the corresponding K, i.e., if the plot is to be raised by A dB, then $K = 10^{A/20}$.
   (b) Find K such that phase margin (PM) $= 60°$.
      i. Recall the Bode magnitude and phase plots of $G(s)$ at $K = 1$.
      ii. From the phase plot, find w_c at which
$$\angle G(j\omega_c) = PM - 180°.$$
      iii. Determine whether the magnitude plot needs to move up or down so that it intersects the 0 dB line at ω_c.

iv. Compute the corresponding K, i.e., if the plot is to be raised by A dB, then $K = 10^{A/20}$.

(c) Find K such that the position error constant, $K_p = 5$.

i. Recall the Bode magnitude plot of $G(s)$ at $K = 1$.

ii. At the low frequency end of the magnitude plot, where the plot is flat, mark the desired K_p in dB. Assuming that the desired K_p is higher than the one for $K = 1$, you need to raise the plot by A dB. Set $K = 10^{A/20}$.

(d) Find K such that

$$20 \log |G(j\omega)| \geq 10 \text{ dB for } \omega \in [0, 0.3] \text{ rad/s}.$$

i. Recall the Bode magnitude plot of $G(s)$ with $K = 1$.

ii. At $\omega = 0.3$, determine the increase in the magnitude plot (say, A dB) needed to bring $|G(j0.3)|$ to the desired level. Set $K = 10^{A/20}$.

iii. Explain why working with only $\omega = 0.3$ suffices.

(e) Draw the Bode magnitude and phase plots with the designed K's using MATLAB and demonstrate that the desired specifications in (b), (c) and (d) are met.

Assignment 3: Part II. Deadline: 06/02/2024, 4:55pm

2. Consider the plant transfer function

$$G_p(s) = \frac{10}{s(s+10)}.$$

Design a lead-lag compensator to have (i) phase margin (PM) $\approx 60^\circ$, (ii) crossover frequency $(\omega_c) \approx 20$ rad/s, and (iii) loop gain $\geq 100\,(40\,\text{dB})$ over the frequency band $[0, 0.5]$ rad/s.

(a) Anticipating that the Bode magnitude plot of $G_c(j\omega)G_p(j\omega)$ is non-increasing at low frequency, the requirement (iii) is equivalent to

$$|G_c(j0.5)G_p(j0.5)| \geq 100.$$

(b) Design a lead compensator

$$G_{c_1}(s) = \frac{K(Ts+1)}{\beta Ts + 1},\ K > 0,\ T > 0,\ 0 < \beta < 1$$

to meet the requirements (i) and (ii). Pick PM assuming that a lag compensator will be used to increase the low frequency gain.

i. Calculate the uncompensated phase margin,

$$\text{PM}_u = \angle G(j\omega_c) + 180^\circ.$$

ii. Choose

$$\phi_{\max} = \text{PM} - \text{PM}_u$$

to increase the phase at ω_c to the desired level.

iii. Calculate

$$\beta = \frac{1 - \sin\phi_{\max}}{1 + \sin\phi_{\max}}.$$

iv. Choose T such that

$$\omega_c = \frac{1}{T\sqrt{\beta}}.$$

v. Choose K such that

$$|G_c(j\omega_c)G_p(j\omega_c)| = 1.$$

(c) Design a lag compensator

$$G_{c_2}(s) = \frac{\alpha(T_1 s + 1)}{\alpha T_1 s + 1},\ T_1 > 0,\ \alpha > 1$$

to increase the loop gain at $\omega = 0.5$ rad/s by the factor, $F = 100/|G_c(j0.5)G_p(j0.5)|$.

i. Choose T_1 such that

$$0.1\omega_c \leq \frac{1}{T_1} \leq 0.2\omega_c.$$

ii. Choose $\alpha \geq F$. Adjust T_1, if needed.

(d) Set

$$G_c(s) = G_{c_1}(s)G_{c_2}(s).$$

(e) Draw the Bode magnitude and phase plots of the compensated system using MATLAB and demonstrate that the desired specifications are met.