Assignment 2: Part I. Deadline: 16/01/2024, 4:55pm

A pendulum consisting of a mass-less rod of length $\ell = 0.36m$ and a bob of mass $m = 0.26kg$ is attached to a cart of mass $M = 2.4kg$ moving on a rail. Let the position of the cart on the rail be denoted by x and the angle of the rod with respect to the upward vertical by θ. The cart is propelled by a horizontal force of u along the rail.

1. Represent the nonlinear dynamics of the plant in the form $\dot{z} = f(z, u)$ where z denotes the states and u denotes the input.
2. Determine the equilibrium points of the above dynamical system.
3. Linearize the above nonlinear dynamics around the unstable equilibrium point which has $\theta = 0$, and obtain the linearized dynamics of the form: $\dot{z} = Az + Bu$ for suitable matrices A and B. Determine the dimensions of A and B.
4. Obtain the transfer functions $\frac{\theta(s)}{U(s)}$ and $\frac{X(s)}{U(s)}$ assuming θ to be close to zero.

The pendulum is to be balanced in the inverted position with the cart being within the rail by applying a suitable u.

5. Explain if the pendulum can be balanced using only θ or only x feedback. In other words, determine if the system is controllable when the input u is equal to either θ or x.

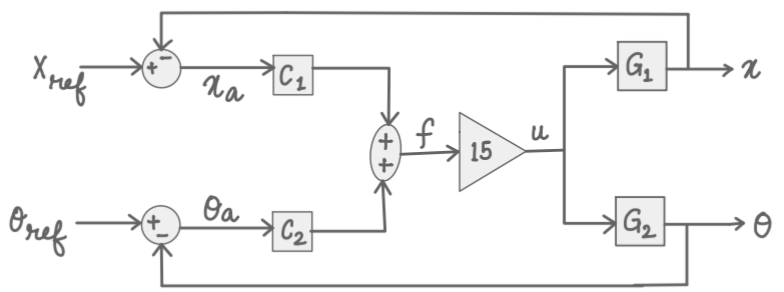
Open Loop Response:

6. Create a model of the nonlinear dynamical system in SIMULINK.
7. Create a model of the linearized system in SIMULINK.
8. Plot the step response of both systems and note the difference.

Assignment 2: Part II. Deadline: 23/01/2024, 4:55pm

We want to design a controller with both θ and x feedback such that the following specifications are met: settling time ≤ 5s and phase margin $\geq 40\,\mathrm{deg}$.

1. Consider the following control configuration with θ and x feedback. Design the controllers $C_1 = \frac{a_1 s + a_0}{s + q_0}$ and $C_2 = \frac{p_1 s + p_0}{s + q_0}$ following the steps described



   below.

   (a) Write G_1 and G_2 based on your calculations in Part I.

   (b) Find characteristic equation of the closed-loop system. Notice that the equation is of order 5.

   (c) Find poles of a second order system whose phase margin is $60\,\mathrm{deg}$ and settling time is 5s.

   (d) We should design the controller such that the desired characteristic equation of the closed-loop system has two poles with values obtained in 1c, and the real parts of the remaining three poles at least 10 times the real part of the two poles obtained in 1c.

   (e) Rewrite the characteristic equation and solve for a_0, a_1, p_0, p_1 and q_0.

2. Check using MATLAB-SIMULINK if the linearized system meets the desired specifications with the designed controller.

3. Find gain margin and maximum overshoot from the responses obtained from your MATLAB model of the linearized system.

4. Apply the designed controllers on the actual nonlinear system in MATLAB-SIMULINK and verify if the desired specifications are met. Comment on your observation.