EE49001: Control and Electronic System Design

Assignment-4: State Space

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Linear Dynamic System

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Dy(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Fig. State Space Matrices

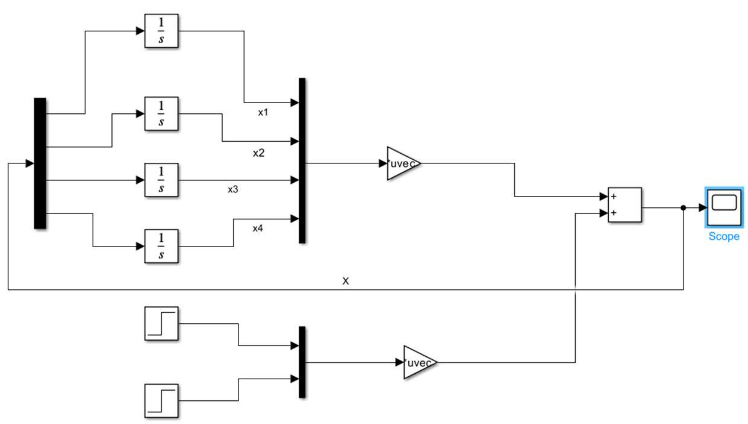

Fig. Simulink Model for the State Space

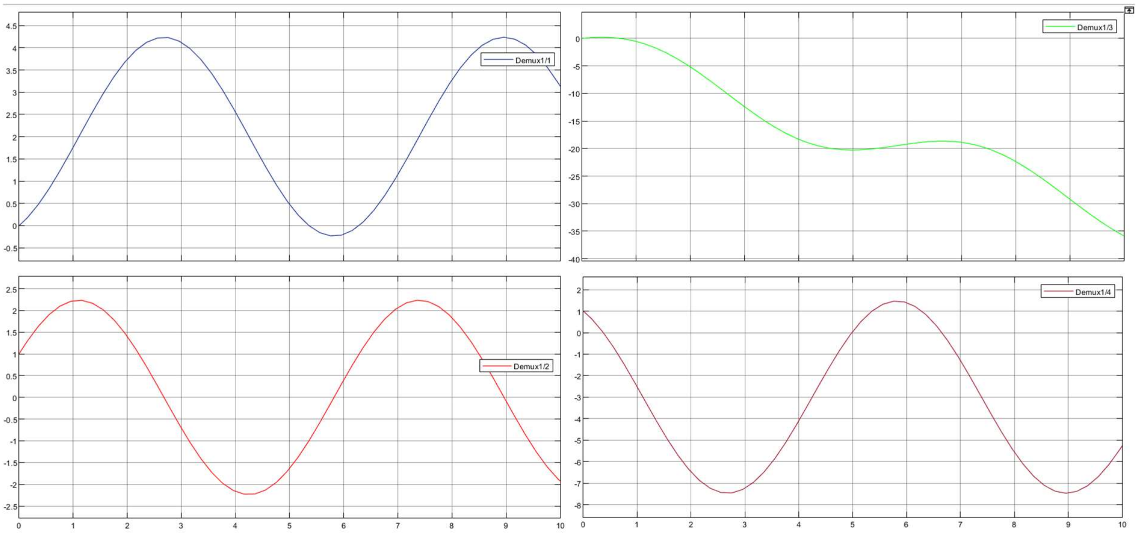

Fig. $\dot{x}(t)$ for Step Input $u(t)$

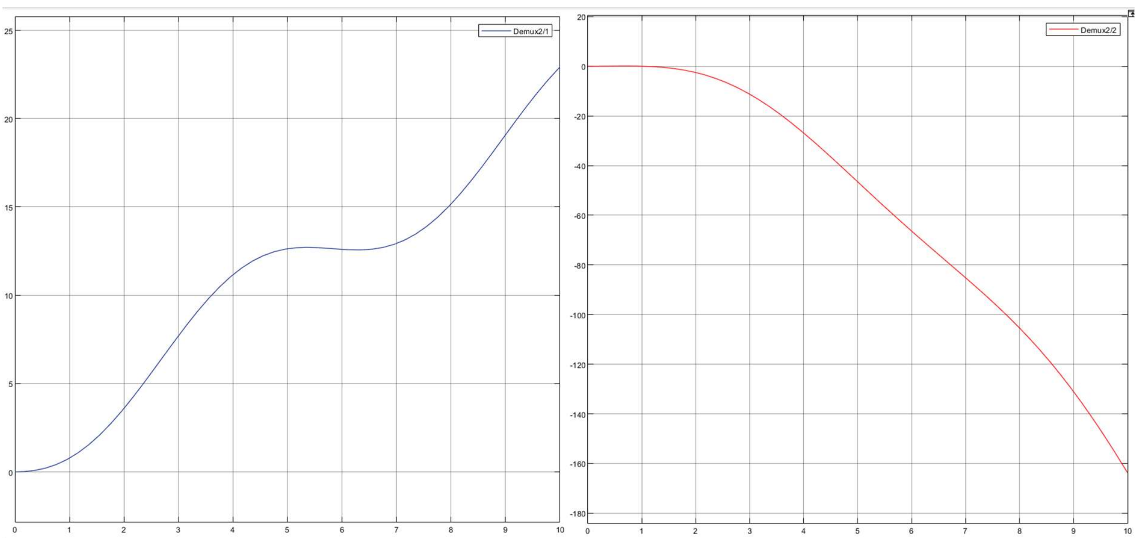

Fig. System Output $y(t)$

Finding Transfer Function

We have 4 states, 2 inputs and 2 outputs. Therefore, we have 4 transfer functions. We can obtain the transfer functions corresponding to each input and output pair using MATLAB.

$$G_i(s) = \frac{Y_i(s)}{X_i(s)}$$

We have the following transfer functions:

$\frac{s}{s^3+s}$	$\frac{-2s}{s^4+s^2}$
$\frac{2}{s^3+s}$	$\frac{s^2-3}{s^4+s^2}$

The eigen values of Matrix A are as follows:

```
e =

   0.0000 + 0.0000i
   0.0000 + 1.0000i
   0.0000 - 1.0000i
   0.0000 + 0.0000i
```

Therefore, the eigen values of the matrix A coincide with the poles of the system transfer functions. Hence, the system is unstable.

Controllability

For Controllability, we have $x \in R^4$. Therefore, the rank of matrix C needs to be 3 for controllability.

$$C = [B, AB, A^2B, A^3B]$$

We have Matrix-C as:

```
c =

     0     0     1     0     0     2    -1     0
     1     0     0     2    -1     0     0    -2
     0     0     0     1    -2     0     0    -4
     0     1    -2     0     0    -4     2     0
```

Therefore, $\mathrm{Rank}(C) = 4$. Hence, the system is controllable. We can find the rank using MATLAB as:

```
c = [B, A*B, A^2*B, A^3*B];
rank(c);
```

Trajectory of the States

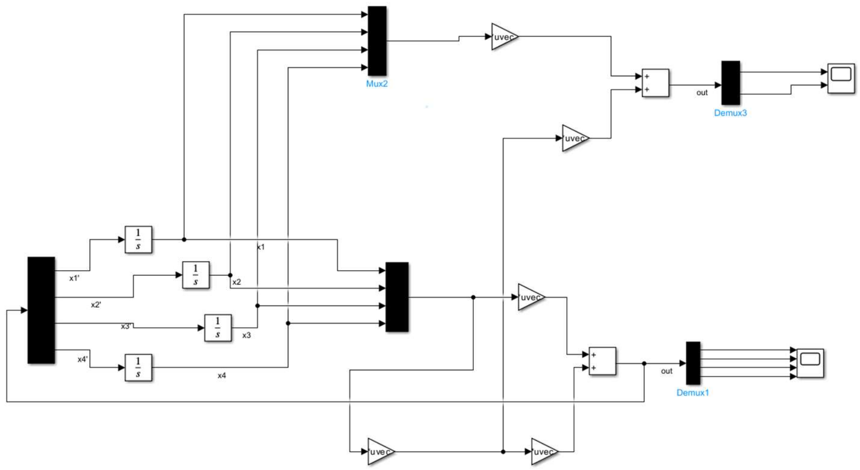

Fig. Substituting $u(t) = -Kx(t)$

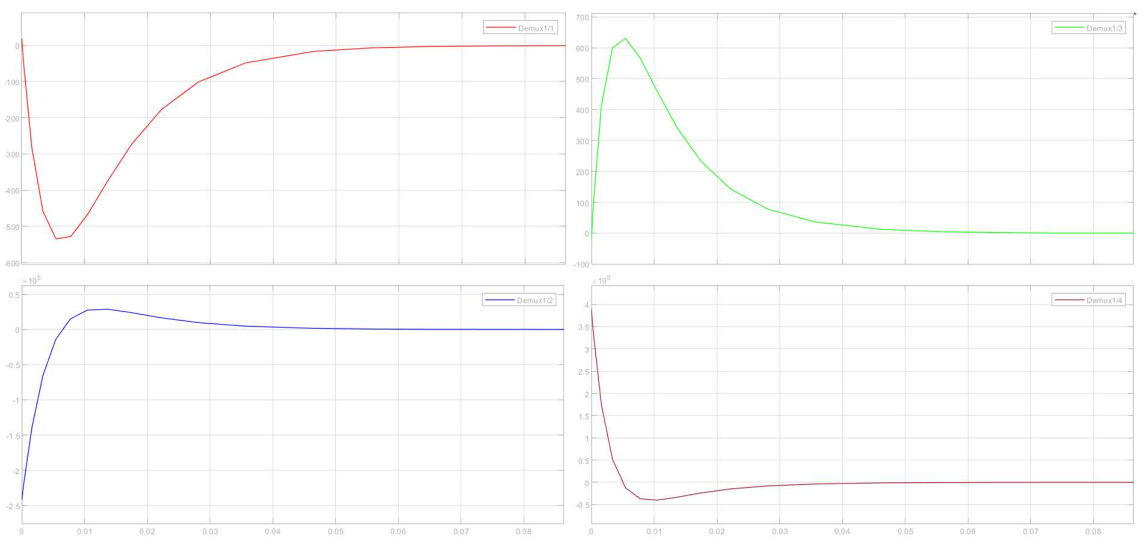

Fig. Trajectory of states corresponding to poles $[-100, -200, -300, -400]$

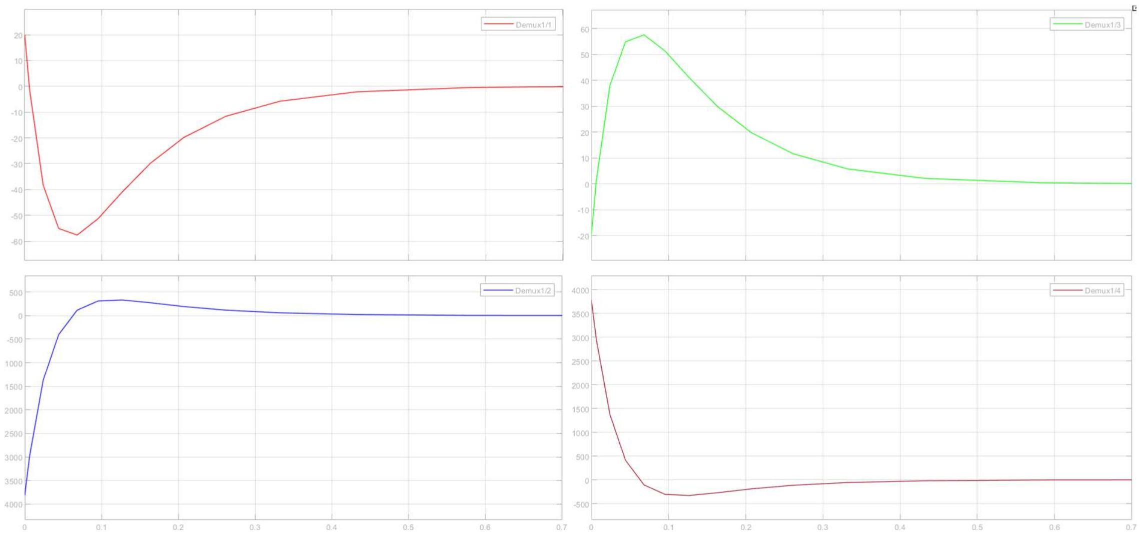

Fig. Trajectory of states corresponding to poles $[-10, -20, -30, -40]$

With poles much left to the origin, it indicates that the system is starting with high initial energy So, it settles down fast but it has high overshoot.

But when the poles are shifted towards the origin i.e. $[-10, -20, -30, -40]$ and further $[-1, -2, -3, -4]$the system starts with lesser initial energy. So, the system oscillates and settles with more delay and lesser overshoot.

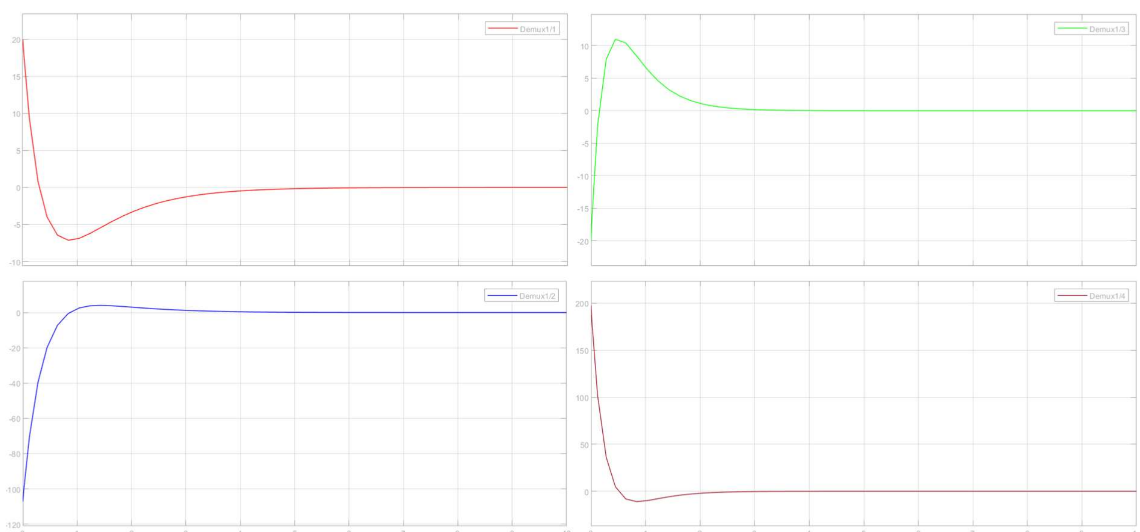

Fig. Trajectory Corresponding to poles $[-1, -2, -3, -4]$

Observability

To determine the observability of a system, we build the observability matrix O.

$$O = [C, CA, CA^2, CA^3]^T$$

Therefore, Observability matrix is as follows:

```
observable_matrix =

     1     0     0     0
     0     0     1     0
     0     1     0     0
     0     0     0     1
     3     0     0     2
     0    -2     0     0
     0    -1     0     0
    -6     0     0    -4
```

The rank of the observability matrix $= 4$. Therefore, the system is observable.

Gain Matrix (L) Calculation for given Poles

For the given pair (A, C) and set of poles $(-10, -20, -30, -40)$, we have the Matrix L as follows.

```
p1= [-10, -20, -30, -40];
L1= place(A',C',p1);
L = L1';
```

So, we get L matrix as:

```
L =

   54.7506   10.8748
  685.7670  312.6156
    6.7248   45.2494
  107.7831  408.9316
```

Observability System

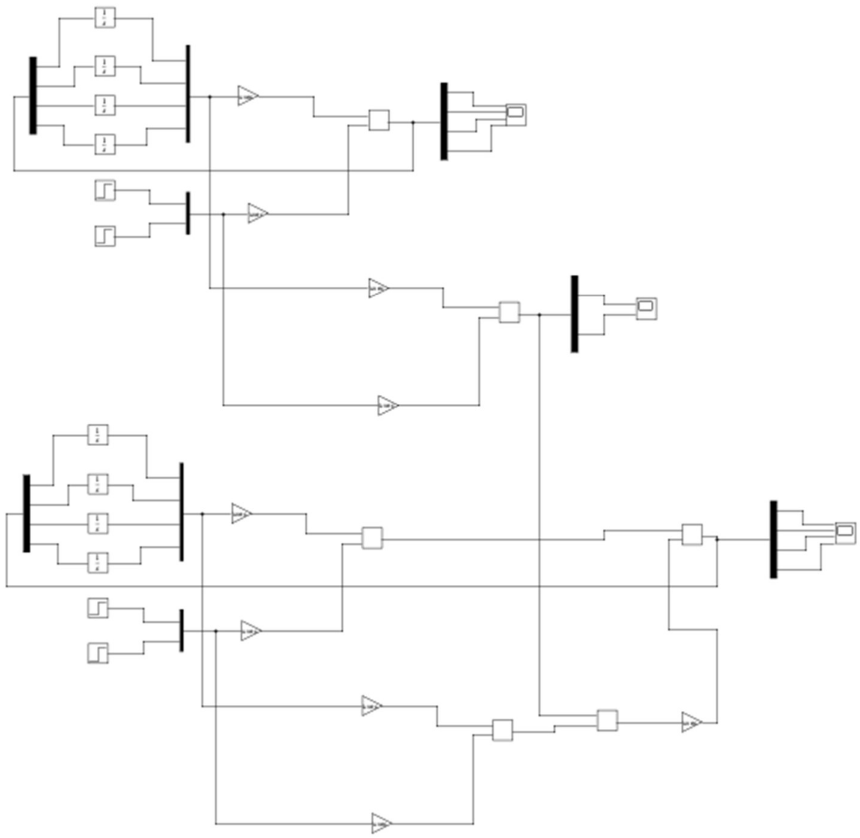

Fig. System Diagram for Observability System

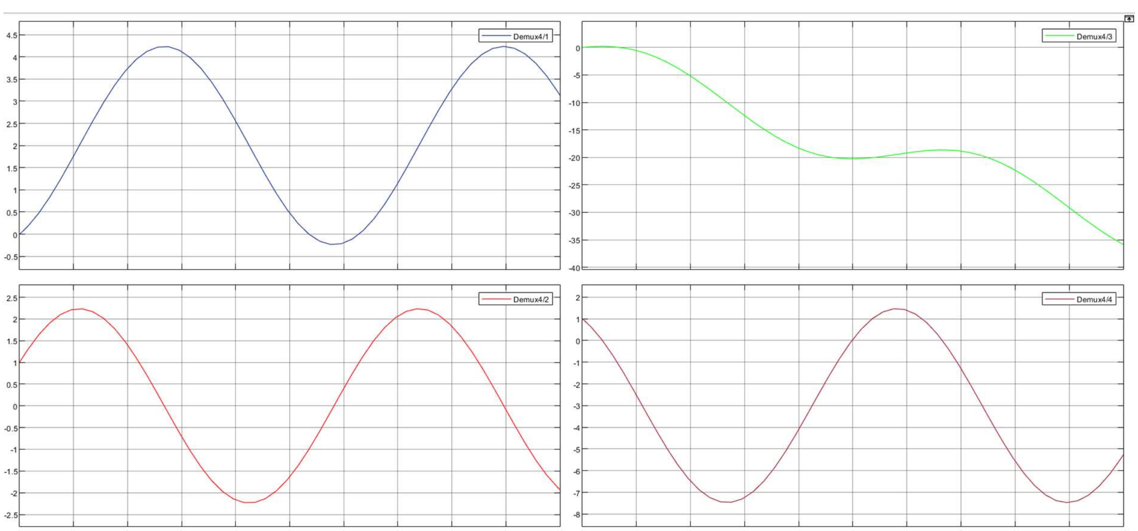

Fig. State Output $\hat{x}(t)$