EE49001: Control and Electronic System Design

Assignment-9: Blood Glucose Level Regulation

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System Modelling

The glucose regulation system dynamics is modelled as

$$\dot{x}_1 = f_1 \coloneqq -ax_1 + b\gamma$$
$$\dot{x}_2 = f_2 \coloneqq -(c + x_1)x_2 + d$$
$$y = g_1 \coloneqq x_2$$

Here, $y(t)$ denotes the level of glucose in the blood and γ denotes the insulin concentration

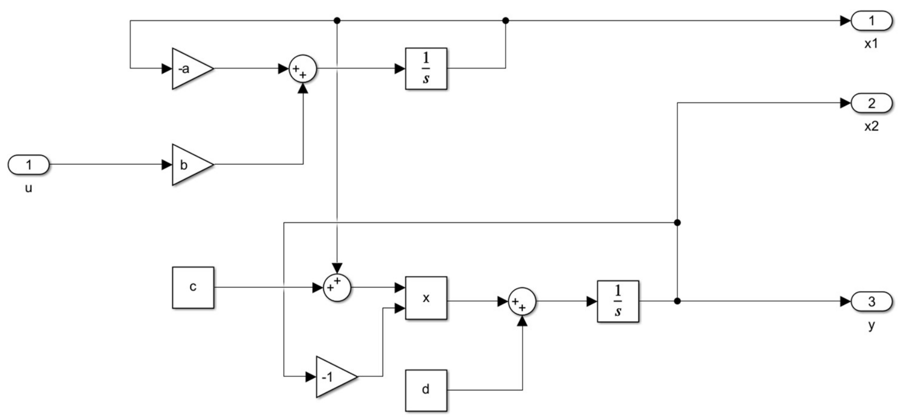

Fig: Non-Linear Dynamics as a block diagram in Simulink

Equilibrium Point Calculation

We know that for an equilibrium point $(\bar{x}_1, \bar{x}_2, \bar{\gamma}), (\dot{x}_1, \dot{x}_2)_{(\bar{x}_1, \bar{x}_2, \bar{\gamma})} = (0,0)$ and it is given that $\gamma = \bar{\gamma} > 0$. Thus,

$$-a\bar{x}_1 + b\bar{\gamma} = 0$$
$$-(c + \bar{x}_1)\bar{x}_2 + d = 0$$

Therefore,

$$\boxed{\bar{x}_1 = \frac{b\bar{\gamma}}{a}} \quad \boxed{\bar{x}_2 = \frac{d}{c + \bar{x}_1} = \frac{ad}{ca + b\bar{\gamma}}}$$

System Linearization

The above dynamics when linearized around the equilibrium points results in the following system (in state space form)

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{\gamma}$$
$$\tilde{y} = C\tilde{x}$$

Where,

$$\tilde{x} = \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \end{bmatrix} \text{ and } \tilde{\gamma} = \tilde{u} = \gamma - \bar{\gamma}$$

$A = \left[\frac{\partial(f_1, f_2)}{\partial(x_1, x_2)}\right]_{(\bar{x}_1,\bar{x}_2,\bar{\gamma})} = \begin{bmatrix} -a & 0 \\ -\bar{x}_2 & -(c+\bar{x}_1) \end{bmatrix}$	$B = \left[\frac{\partial(f_1, f_2)}{\partial \gamma}\right]_{(\bar{x}_1,\bar{x}_2,\bar{\gamma})} = \begin{bmatrix} b \\ 0 \end{bmatrix}$
$C = \left[\frac{\partial g}{\partial(x_1, x_2)}\right]_{(\bar{x}_1,\bar{x}_2,\bar{\gamma})} = \begin{bmatrix} 0 & 1 \end{bmatrix}$	$D = 0$

The corresponding transfer function will be

$$\boxed{\begin{aligned}\frac{\tilde{Y}}{\tilde{\Gamma}} &= \mathrm{C}(sI_2 - A)^{-1}\mathrm{B} \\ &= \frac{-b\bar{x}_2}{(s+c+\bar{x}_1)(s+a)}\end{aligned}}$$

Here, $\mathrm{c} = 0$, therefore:

$$\boxed{\frac{\tilde{Y}}{\tilde{\Gamma}} = \frac{-b\bar{x}_2}{(s+\bar{x}_1)(s+a)}}$$

Poles, Zeros and Stability Estimation from Experimental Values

Experimentally determined values are as follows:

$\bar{x}_2 = 100$	$a = \bar{x}_1 = \frac{1}{33}$	$\frac{b\bar{x}_2}{a^2} = 3.3$

Therefore, the transfer function:

$$\frac{\tilde{Y}}{\tilde{\Gamma}} = \frac{-b\bar{x}_2}{(s+\bar{x}_1)(s+a)} = \frac{-b\bar{x}_2}{\left(s+\frac{1}{33}\right)\left(s+\frac{1}{33}\right)}$$

$$\therefore\ Poles = \left(-\frac{1}{33}, -\frac{1}{33}\right);\ Zeros\ =\ None$$

Since, both the poles are in the Left Half Plane: The linearized system is stable.

Variable Insulin Concentration

Now the insulin concentration is evolving in response to an external input u as

$$\dot{\gamma} = -f\gamma + gu$$

Therefore, new state equations are

$$\begin{aligned} \dot{x}_1 &= f_1 := -ax + b\gamma \\ \dot{x}_2 &= f_2 := -(c + x_1)x_2 + d \\ \gamma &= f_3 := -f\gamma + gu \\ y &= x_2 \end{aligned}$$

Now the state variables are $x = [x_1 \quad x_2 \quad \gamma]$ and u is the input. If the above system is linearised as

$$\dot{\tilde{x}} = A_1\tilde{x} + B_1\tilde{u}$$
$$\tilde{y} = C_1\tilde{x}$$
$$\overline{\gamma} = \frac{gu}{f}$$

Where,

$$\tilde{x} = \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \\ \gamma - \bar{\gamma} \end{bmatrix} \text{ and } \tilde{u} = u - \bar{u}$$

$A_1 = \left[\frac{\partial(f_1, f_2, f_3)}{\partial(x_1, x_2, \gamma)}\right]_{(\bar{x}_1, \bar{x}_2, \bar{\gamma}, \bar{u})} = \begin{bmatrix} -a & 0 & b \\ -\bar{x}_2 & -(c + \bar{x}_1) & 0 \\ 0 & 0 & -f \end{bmatrix}_{(\bar{x}_1, \bar{x}_2, \bar{\gamma}, \bar{u})} = \begin{bmatrix} -\frac{1}{33} & 0 & \frac{1}{33000} \\ -100 & -\frac{1}{33} & 0 \\ 0 & 0 & -\frac{1}{5} \end{bmatrix}$	$B_1 = \left[\frac{\partial(f_1, f_2, f_3)}{\partial u}\right]_{(\bar{x}_1, \bar{x}_2, \bar{\gamma}, \bar{u})} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}_{(\bar{x}_1, \bar{x}_2, \bar{\gamma}, \bar{u})} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{5} \end{bmatrix}$
$C_1 = [0 \quad 1 \quad 0]$	$D_1 = 0$

Thus, the transfer function can be calculated as

$$\boxed{\begin{aligned} \frac{\tilde{Y}}{\tilde{U}} &= C_1(sI_3 - A_1)^{-1}B_1 \\ &= \frac{-0.0006061}{s^3 + 0.2606s^2 + 0.01304s + 0.0001837} \end{aligned}}$$

Thus, the poles and zeros of the above transfer function are

$$p = \text{poles} = \left(-\frac{1}{5}, -\frac{1}{33}, -\frac{1}{33}\right)$$
$$z = \text{zeros} = \text{no zeros}$$

Again, since all the poles are on the left-half of s-plane, the linearized model of the system is stable.

Design of Feedback Controller

The given PID feedback controller if characterised by

$$u(t) = K_p(y(t) - \bar{y}) + K_pT_d\frac{dy(t)}{dt} + \frac{K_p}{T_i}\int_0^t (y(\tau) - \bar{y})d\tau$$

Thus, the controller transfer function is

$$G_c = K_p\left(1 + T_ds + \frac{1}{T_is}\right)$$

The controller and the plant are connected in feedback as characterised above and is modelled below.

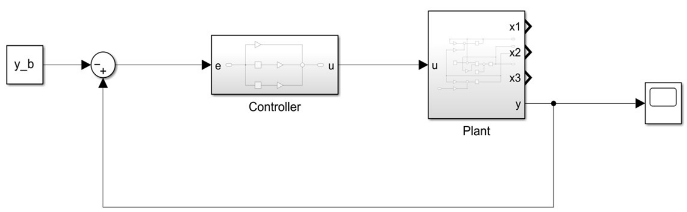

Fig. Feedback Controller

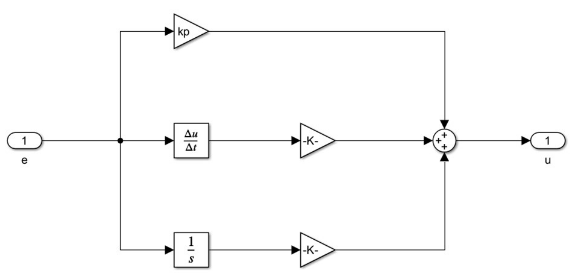

Fig. PID Controller System

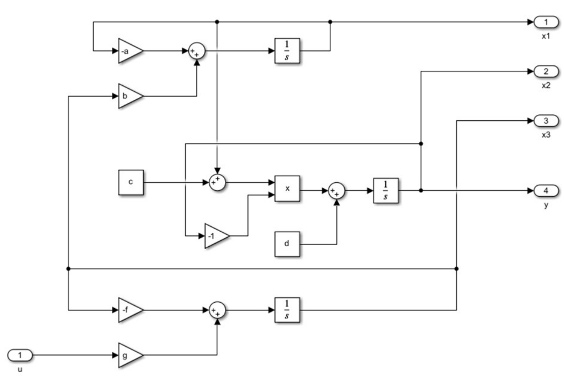

Fig. Block diagram of non-linear plant model

Root Locus

With Derivative Action:

Root Locus for system with $T_d = 38$ and $T_i = 100$ w.r.t K_p

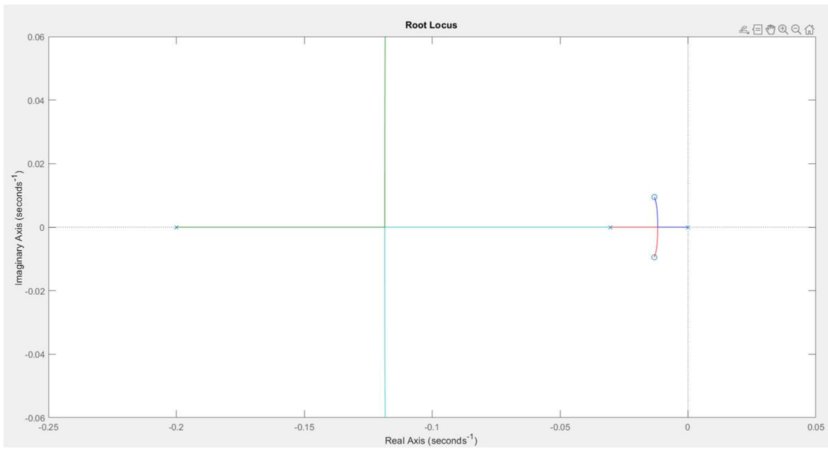

Fig. Root Locus of the closed-loop system with derivative action

As can be observed from the plot the locus never enters the right half plane as we increase the DC gain, thus the proportional gain can be increased unbounded.

Thus,

$$\boxed{K_p > 0}$$

Without Derivative Action:

$T_d = 0$ and $T_i = 100$ w.r.t K_p

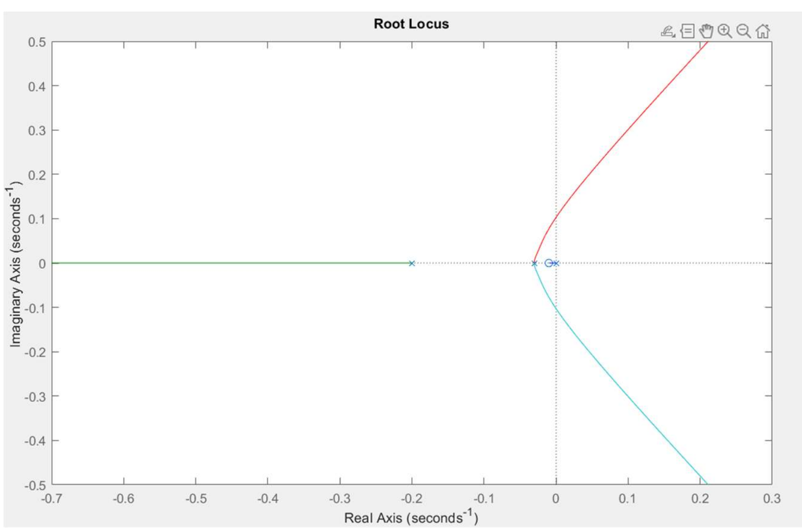

Fig. Root Locus of the closed-loop system without derivative action

As can be observed the root locus plot is entering into the right half plane and the value at the crossover region if 4.26 (from plot). Thus, range of the proportional gain is,

$$\boxed{K_p \leq 4.26}$$

Luenburger Observer

A Luenberger observer is introduced to track the state variables of the plant, with proportional gain set to $K_p = 0.17$.

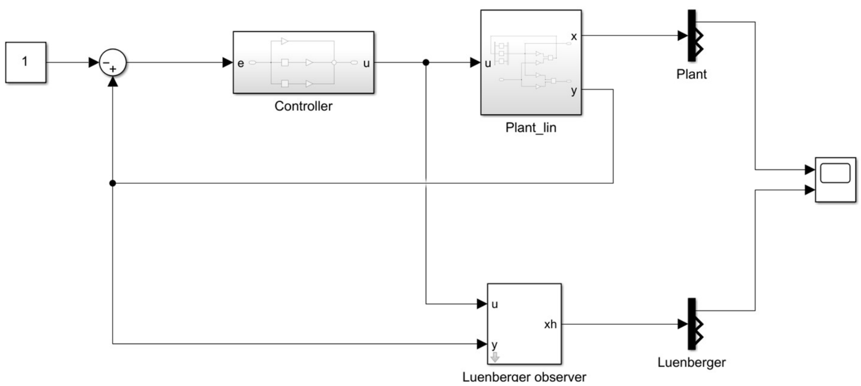

Fig. Implementation of Luenburger Observer

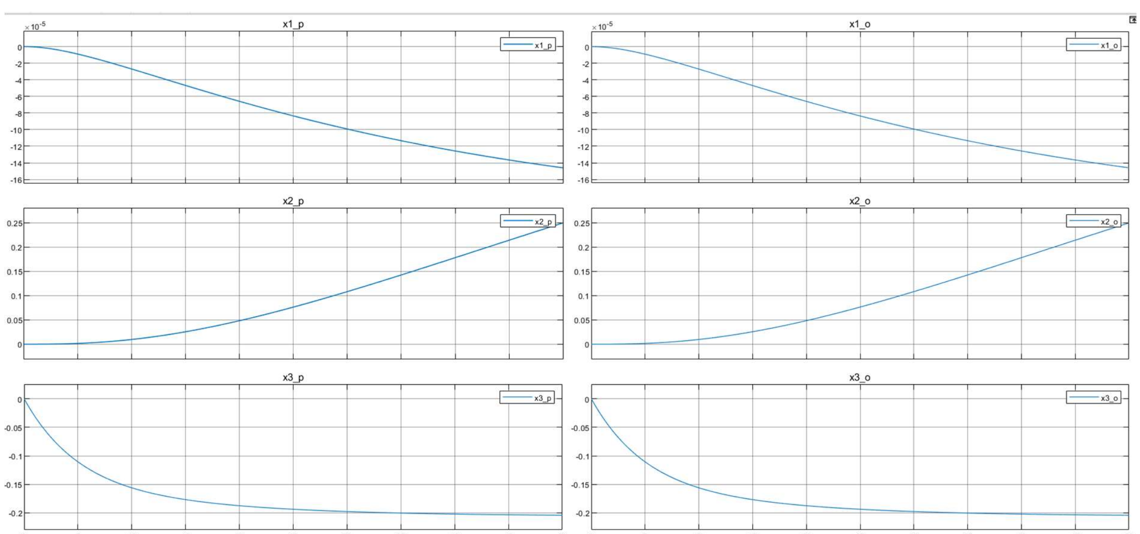

Fig. Luenberger Observer Output

As we can observe, the actual plant states and the states observed by the observer are very close. Therefore, we can conclude that the Luenberger observer is working well.