EE49001: Control and Electronic System Design

Assignment-7: Transfer Function Estimation

Submitted By:

21EE30004: Anirvan Krishna | 21EE30001: Aditya Kumar

Simulink Model of RLC Series Circuit and Step Response

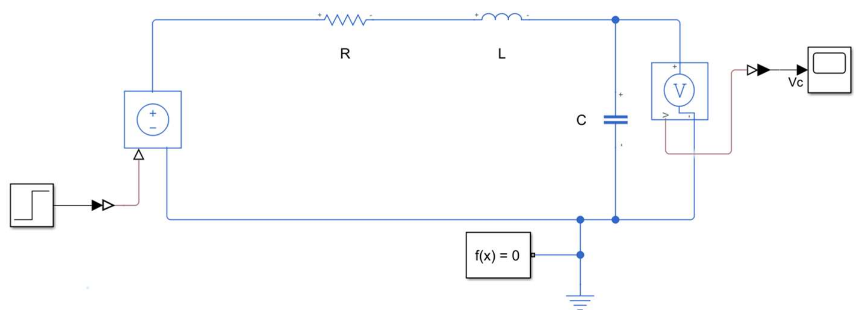

Fig. RLC Series Circuit Simulink Model

Here, we have: $\mathrm{R} = 3\Omega;\ \mathrm{L} = 2\mathrm{H};\ \mathrm{C} = 0.5\mathrm{F}$

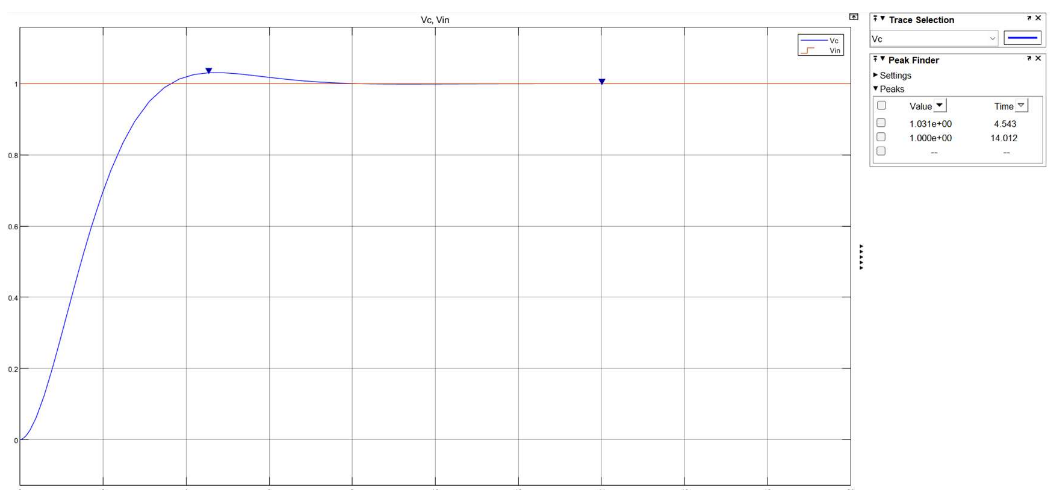

Fig. Input (V_{in}) and Output (V_c) vs. Time

From the system response, we can observe that the system is underdamped.

Transfer Function computation using Step Response

For a second order system, transfer function can be generalized as:

$$G(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Using properties of 2nd order system, we have:

Peak Time:

$$T_{peak} = \frac{\pi}{\omega_d}$$

% Overshoot:

$$\%MP = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$

Therefore, we have:

$$\zeta = \frac{ln\left(\frac{100}{\%MP}\right)}{\sqrt{\pi^2 + ln\left(\frac{100}{\%MP}\right)^2}}$$

From the plot, we have maximum overshoot: 1.031

Therefore, $\%MP = 3.1\%$

Putting these values in the equation we get: $\zeta = 0.7417$

We have the peak time: $T_{peak} \approx 4.543$ seconds

Therefore, $\omega_n = \frac{\pi}{T_{peak}\sqrt{1-\zeta^2}} = 1.028$ rad/s

Therefore, the transfer function comes out to be:

$$G(s) = \frac{1.056}{s^2 + 1.52s + 1.056}$$

Transfer Function computation using Bode Plot

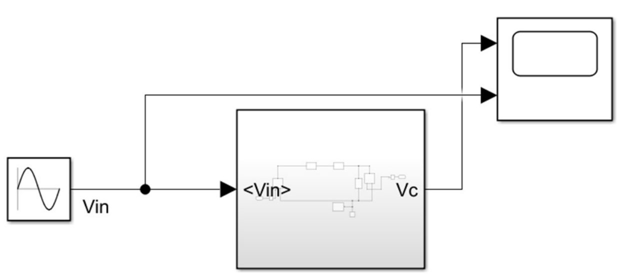

Fig. Application of Sinusoidal Input to the system

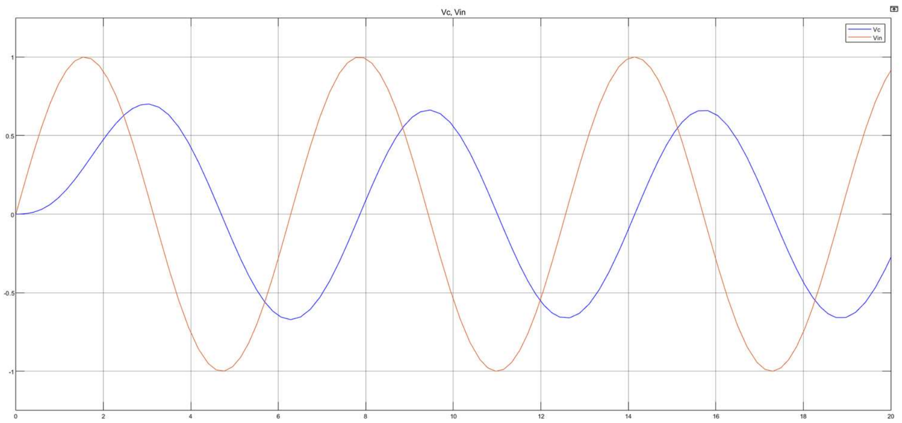

Fig. System response corresponding to a sinusoidal input

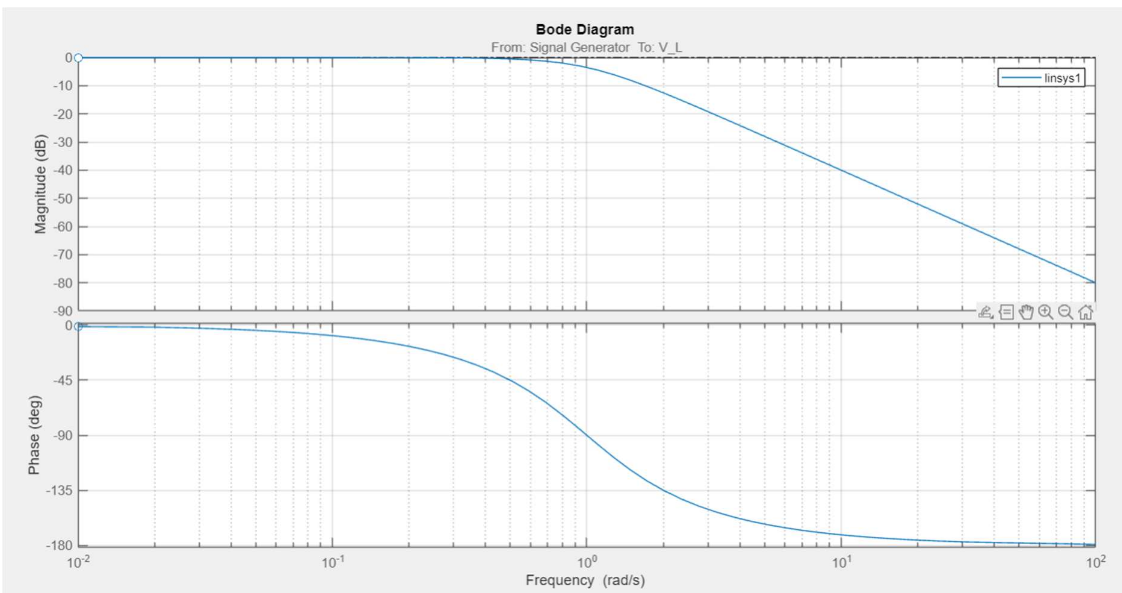

Fig: Bode Output response with sinusoidal Input

Transfer Function obtained by analysing bode plot:

(i) At first, we look for ω_n , it is the frequency at which the phase response of the output is equal to-90^0

$$\omega_n = 1 \text{ rad/sec}$$

(ii) Second, we get the resonant frequency of the system

$$\omega_r = 0.33 \text{ rad/sec.}$$

Resonant frequency and damping ratio of the system are related as with $\omega_n = 1$ rad/sec.

$$\zeta = \sqrt{\frac{1-\omega_r^2}{2}} = 0.6675.$$

We get the transfer function:-

$$G(s) = \frac{1}{s^2 + 1.335s + 1}$$

Analytical Derivation of the Transfer Function

For an RLC Series circuit, total impedance, in Laplace domain can be written as:

$$\mathrm{Z(s) = R + sL} + \frac{1}{sC}$$

$$V_c(s) = \frac{V_{in}(s)}{Z} \cdot \frac{1}{sC} = \frac{V_{in}}{LCs^2 + RCs + 1}$$

Therefore, the transfer function $\mathrm{G}(s)$ is given as:

$$\mathrm{G}(s) = \frac{V_c(s)}{V_{in}(s)} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{s^2 + 1.5s + 1}$$

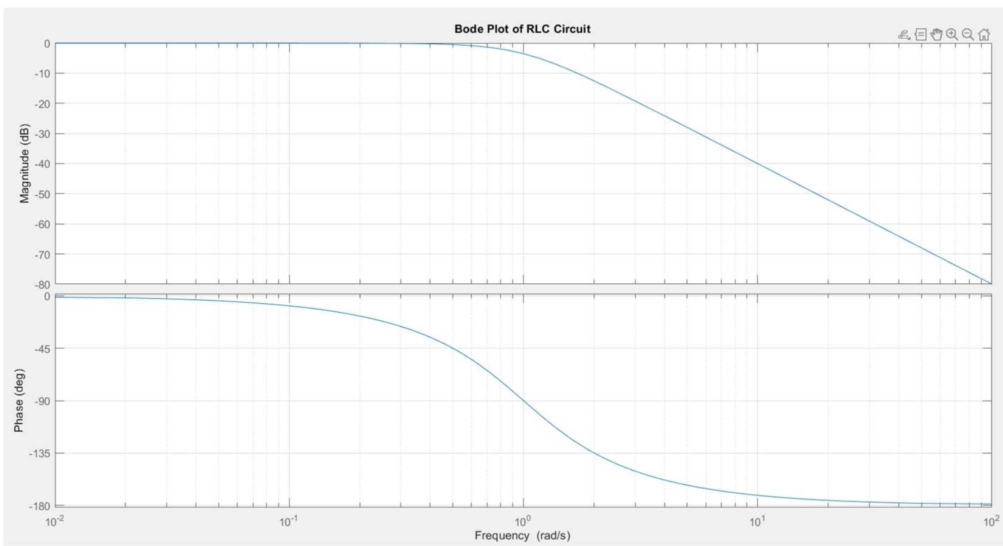

Fig. Bode Plot corresponding to the Analytical Transfer Function

Comparing the Transfer functions obtained by three methods

We have found the transfer function by three methods:

1. Using step response and properties of 2nd order systems
2. Using Bode Plot i.e., Frequency response
3. Analytically using properties of RLC Circuits

In all the three cases, the transfer functions are numerically very close to each other with very slight deviation from the analytical function appearing in Step Response and Bode Plot obtained Transfer functions

Discretization of continuous-time Transfer Function

The continuous-time transfer function $G(s)$ can be discretized into discrete-time transfer function $H(z)$.

Discretization can be done by running the following code snippet:

MATLAB Code

```
%% RLC Circuit Parameters
R = 3; C = 0.5; L = 2;
sys = tf([1],[L*C, R*C, 1]);

%% Sample and Zero-order Hold
Ts = 0.1;
H = c2d(sys, Ts);
```

The discretized transfer function, obtained from this code output:

$$H(z) = \frac{0.004755\, z + 0.004523}{z^2 - 1.851\, z + 0.8607}$$

Incorporating Discretization into Simulink Block Diagram

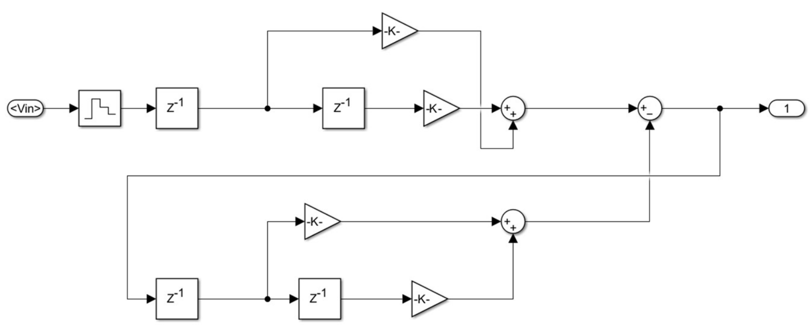

Fig. Discretized modification of the subsystem

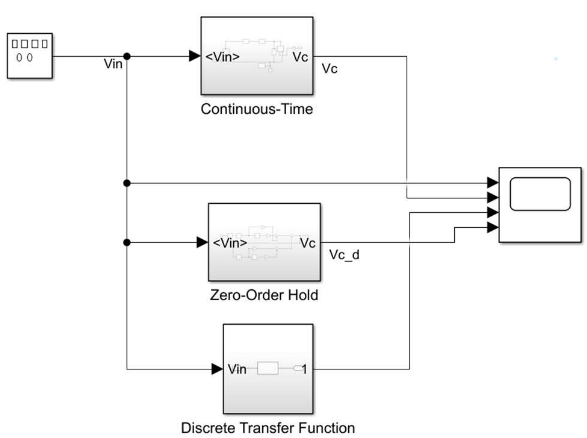

Fig. SIMULINK representation of discrete models

The output data form the time horizon $\{0,1,2,3,4\}$ is:

$$\{0, -0.0047, -0.0180, -0.0386, -0.06530\}$$

Comparing Discrete Transfer Functions

We have implemented both the discretized transfer function and the discrete system designed. We obtain both the waveforms to be perfectly overlapping. Hence both transfer functions are identical.

Least Square Estimation

We can compute the least squares by running this piece of code:

MATLAB Code

```
r = out.rd.Data;
u = out.ud.Data;
w = 5;

z = r(w:end);
m = height(r);
h1 = zeros(m-w+1, w);
h2 = zeros(m-w+1, w);

for idx = w:m
    h1(idx-w+1, :) = r(idx-w+1:idx)';
    h2(idx-w+1, :) = u(idx-w+1:idx)';
end
H = [h1,h2];

x = (H' * H) \ H' * z;
disp(x);
```

And $\hat{x}$ is found out using least square estimation method as

$$\hat{x} = (H^T H)^{-1} H^T z$$

$\hat{x} =$

```
 0.0948
-0.9057
 0.4491
 1.7017
-0.3599
-0.0005
 0.0032
 0.0100
 0.0065
-0.0000
```