EE49001: Control and Electronic System Design

Assignment-6: Observability in Discrete Time

Submitted By:

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Continuous and Discrete step response

Consider the linear dynamical system in state-space form.

$$\dot{x_c}(t) = Ax_c(t) + Bu_c(t)$$

$$y_c(t) = Cx_c(t) + Du_c(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The vector $x_c \in \mathbb{R}^4$ denotes the state variables, $u_c \in \mathbb{R}^2$ denotes the inputs, and $y_c \in \mathbb{R}^2$ denotes the output of the continuous-time plant.

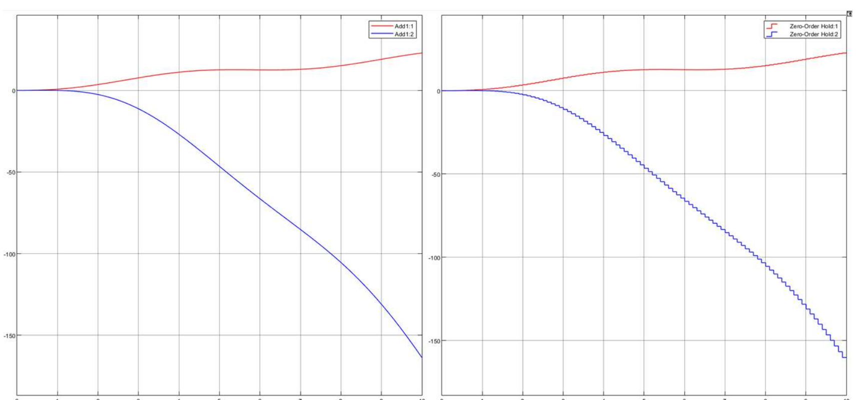

Fig: Continuous output and the Zero Order Hold Sampled output

Discretization of Continuous System

$$\dot{x}((k+1)T_s) = A_d x(kT_s) + B_d u(kT_s)$$

$$y((k+1)T_s) = C_d x(kT_s) + D_d u(kT_s)$$

For $k = 0, 1, 2, 3, \ldots..$

The continuous time dynamical system is discretized with the help of 'c2d' command and the following state space parameters are obtained.

```
Ad:
    1.0150    0.0998         0    0.0100
    0.2995    0.9950         0    0.1997
   -0.0010   -0.0100    1.0000    0.0993
   -0.0300   -0.1997         0    0.9800

Bd:
    0.0050    0.0003
    0.0998    0.0100
   -0.0003    0.0050
   -0.0100    0.0993

Cd:
     1     0     0     0
     0     0     1     0

Dd:
     0     0
     0     0
```

```
Controllability Matrix:
    0.0050    0.0003    0.0149    0.0023    0.0247    0.0063    0.0343    0.0122
    0.0998    0.0100    0.0988    0.0299    0.0969    0.0495    0.0939    0.0686
   -0.0003    0.0050   -0.0023    0.0148   -0.0063    0.0239   -0.0122    0.0321
   -0.0100    0.0993   -0.0299    0.0953   -0.0495    0.0874   -0.0686    0.0756

Rank of Controllability matrix: 4
On=bservability Matrix:
    1.0000         0         0         0
         0         0    1.0000         0
    1.0150    0.0998         0    0.0100
   -0.0010   -0.0100    1.0000    0.0993
    1.0598    0.1987         0    0.0399
   -0.0080   -0.0399    1.0000    0.1947
    1.1340    0.2955         0    0.0893
   -0.0269   -0.0893    1.0000    0.2821

Rank of Observability matrix: 4
```

The controllability matrix $\left[B_d \ \ A_d B_d \ \ A_d^2 B_d \ \ A_d^3 B_d\right]$ has a rank of 4 (from MATLAB), thus the system is controllable.

Also, the observability matrix $\left[C_d \ \ C_d A_d \ \ C_d A_d^2 \ \ C_d A_d^3\right]^T$ has a rank of 4 (from MATLAB), thus the system is observable

Implementation of Luenburger observer for discrete-time system

A Luenburger Observer for discrete-time model can be given by:

$$\hat{x}_b\big((k+1)T_s\big) = A_d \ \hat{x}_b(kT_s) + B_d u(kT_s) + L\big(y_c(kT_s) - C_d \ \hat{x}_b(kT_s) - D_d u(kT_s)\big)$$

We can implement a Luenburger model in SIMULINK as follows:

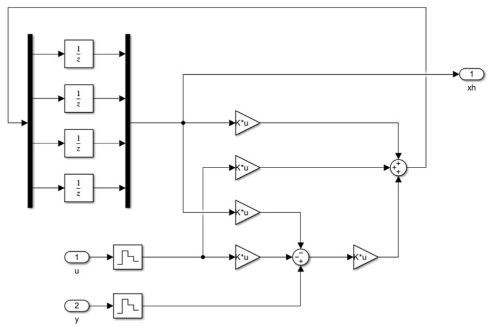

Fig. Luenburger observer model

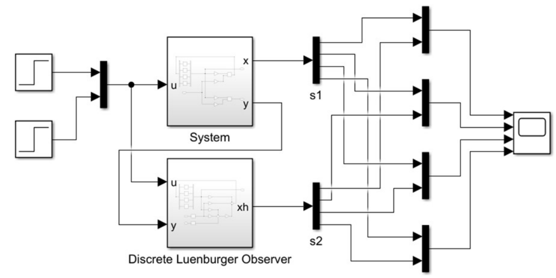

Fig. Discrete system with observer model

Luenburger gain

For the eigenvalues of $A_d - L_d C_d$ to located at $p = (0.05, 0.06, 0.07, 0.08)$, we can use the place command in MATLAB, which results the following

$$L_d = \begin{bmatrix} 1.8728 & 0.1916 \\ 8.7740 & 2.7853 \\ -0.2077 & 1.8572 \\ -2.9411 & 8.3885 \end{bmatrix}$$

Therefore, the desired values of poles are close to zero, such that the Luenburger Gain has high values and the observer tracks the states faster.

State trajectories with unit step inputs

The system is excited with a step input and the response is fed into the discrete Luenburger Observer along with the inputs. The following state trajectories are obtained.

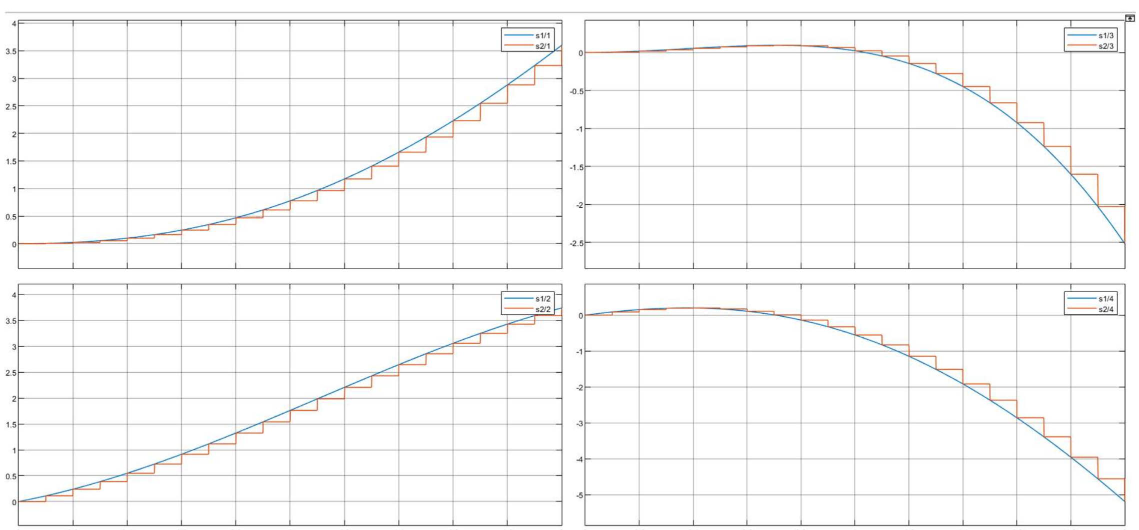

Fig. Actual state $x_i(t)$ and observed state $\mathrm{x_i}\left((k+1)T_s\right)$ for $T_s = 0.1s$

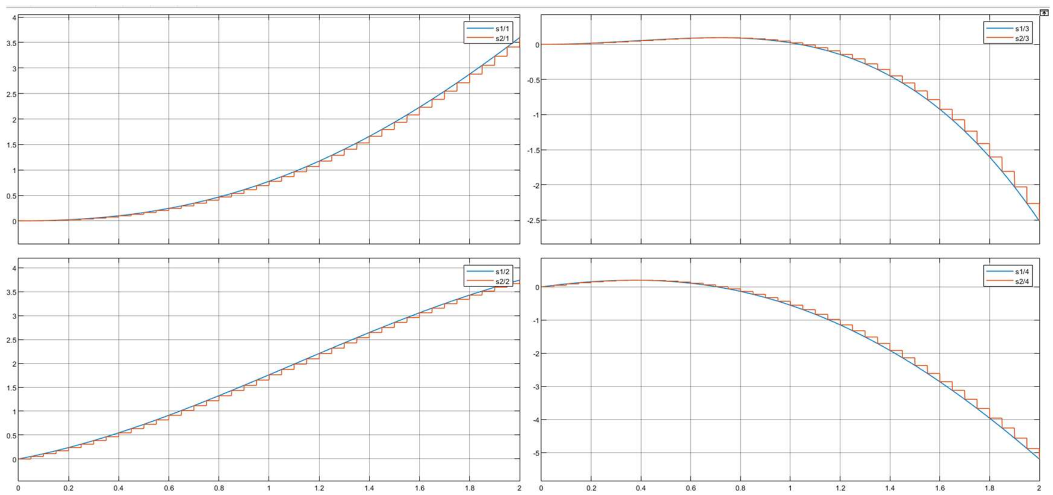

Fig. Actual state $x_i(t)$ and observed state $\mathrm{x_i}\big((k+1)T_s\big)$ for $T_s = 0.05s$

Here, we run the simulation for two different values of sampling time, $T_s = 0.1s, 0.05s$. We observe that as the sampling time decreases, the observer becomes more accurate at observing the states of the system. This is because, as the sampling time reduces, the system becomes closer to a continuous system.

Observer-based state-feedback controller

For the eigenvalues of $A_d - B_d K_d$ to be at $10p$, we can use the `place` command in MATLAB, which results the following.

$$K_d = \begin{bmatrix} 14.9469 & 6.6389 & 2.3139 & 2.1953 \\ 4.6160 & -0.5582 & 10.9474 & 6.2894 \end{bmatrix}$$

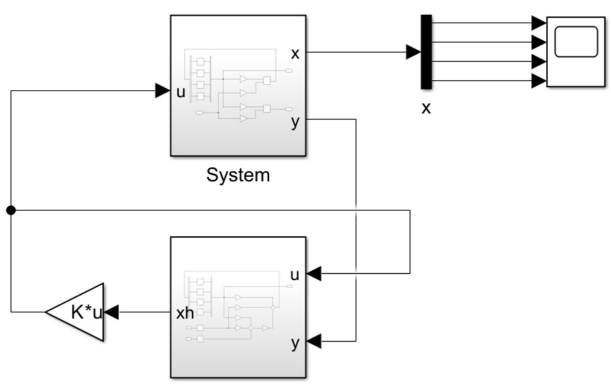

Fig. Observer-based state feedback controller in Simulink

The system is simulated with initial state of the continuous time plant be $x_c(0) = [10, 20, -10, -20]$ and for the discrete model be $\hat{x}(0) = 0$.

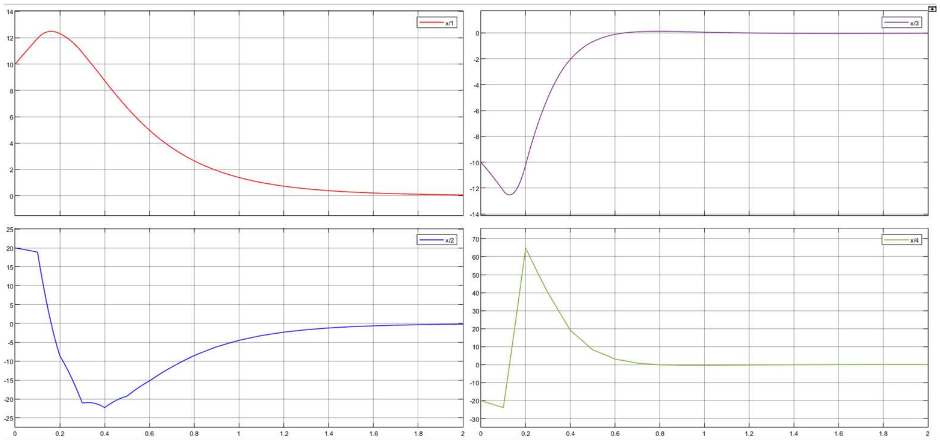

Fig. Trajectory of the state $x_i(t)$ for $T_s = 0.1s$

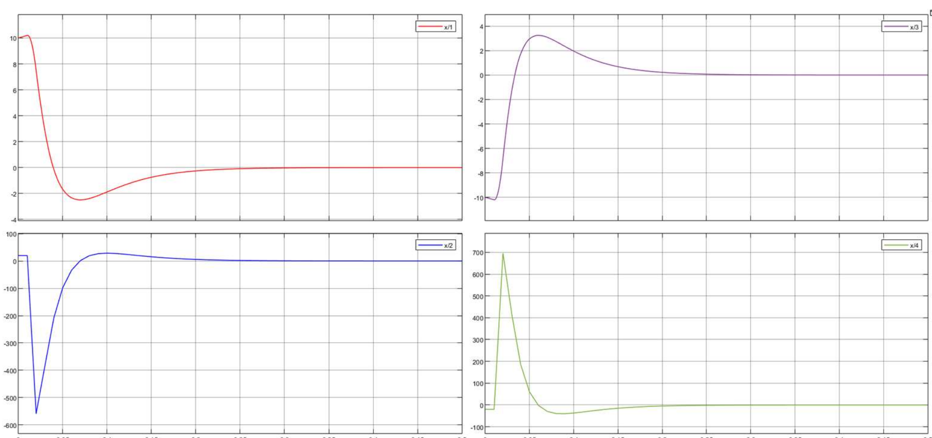

Fig. Trajectory of state $x_i(t)$ for $T_s = 0.01s$

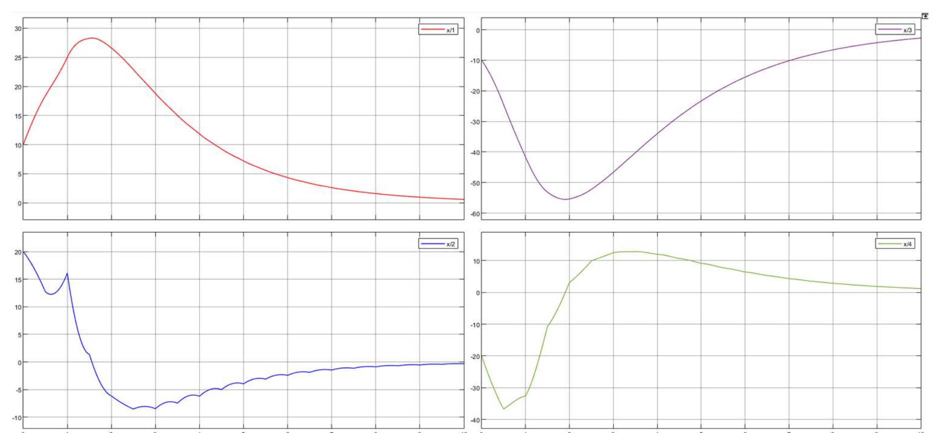

Fig. Trajectory of the state $x_i(t)$ for $T_s = 0.5s$

The simulation has been carried out with three sample times, $T_s = 0.1s, 0.01s, 0.5s$. We observe that the curves corresponding to the predicted states gets smoother as the sampling time is reduced. This is because, reduction in sampling time brings the system behaviour closer to a continuous model.