Assignment 8: Deadline: 19/03/2024, 4:55pm

Consider an R-L-C series circuit with $R = 3\Omega$, $L = 2$H and $C = 0.5$F.

1. Create a simulink model of the system. Consider the voltage across the capacitor, $v_C(t)$, as the output.
2. Apply a step input to the system and compute the transfer function of the system from the output response. Notice that the system under consideration is underdamped.
   Hint: (a) The DC gain can be computed from the ratio of the steady state step response to the magnitude of the step input. (b) The damping ratio can be calculated from the percent overshoot. (c) The natural frequency of can be found from the damped natural frequency which can be measured off the plot of the step response and the damping ratio which is calculated above.
3. Apply a sinusoidal input to the system and obtain the steady state output response. Create Bode magnitude and phase plots from the obtained data. Estimate the transafer function of the system under consideration by analyzing the Bode plot.
   Hint: You have done transfer function identification from Bode Plot in EE31009 Bode plot practice problem sheet.
4. Derive the transfer function of the circuit analytically.
5. Compare the transfer functions obtained in 2, 3 and 4 above, and comment on your observation.
6. Discretize the transfer function obtained in 4 with ZOH and sampling time $T_s = 0.1$s. You may use the `c2d` function in MATLAB.
7. Modify your simulink model to incorporate the discretization. Apply a square wave input and collect the output data for the time horizon $\{0, 1, 2, 3, 4\}$.
8. Recall that given $Hx = z$, the least squares estimation method allows us to choose $\hat{x}$ that minimizes $\|H\hat{x} - z\|_2$ as $\hat{x} = \left(H^\top H\right)^{-1} H^\top z$. Write a MATLAB script that uses least squares method to identify the discretized transfer function from the collected data.

Hint: Notice that the discretized transfer function is of the form $G(z) = \frac{Y(z)}{U(z)} = \frac{b_m z^{-m}+...+b_1 z^{-1}+b_0}{a_n z^{-n}+...+a_1 z^{-1}+a_0}$. Apply inverse z-transform and combine the (u_i, y_i), $i = 0, 1, 2, 3, 4$ relation in vector-matrix form.

9. Compare and comment on your results in 6 and 7 above.