**EE49001: Control and Electronic System Design**

Assignment-2: Inverted Pendulum, Part:1

Submitted By:

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# A pendulum consisting of a mass-less rod of length and a bob of mass is attached to a cart of mass moving on a rail. Let the position of the cart on the rail be denoted by and the angle of the rod with respect to the upward vertical by . The cart is propelled by a horizontal force of along the rail.

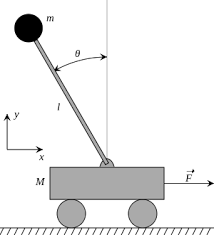


Fig: Diagram for Inverted Pendulum Setup

For the following system, we have 4 states: , where:

Translational velocity of the cart along direction

Angular velocity of the rod, Horizontal force on the cart

## Represent the nonlinear dynamics of the plant in the form where denotes the states and denotes the input

We can, using Newton’s Laws of Motion, say that:

Considering the state space,

## Determine the equilibrium points of the above dynamical system.

The system is at equilibrium, means and . It means, . Putting these values in the non-linear dynamics’ equations, we get: . For equilibrium we have . That is: .

## Linearize the above nonlinear dynamics around the unstable equilibrium point which has , and obtain the linearized dynamics of the form: for suitable matrices and Determine the dimensions of and .

For Linearization of the given state equations in the neighbourhood of i.e. where , we can assume the small angle approximations to be valid. Therefore, we have and .

Hence, we can write:

For small , assuming

These are the linearized dynamics equations in the neighbourhood of

The 4 nonlinear equations can be linearised in the form , where the matrices are defined as follows:

is the unstable equilibrium point,

Dimensionally, and

is the unstable equilibrium point

## Obtain the transfer functions and assuming θ to be close to zero.

Taking Laplace Transform we get

Where,

On solving and simplifying we get,

Numerical Values of transfer function:

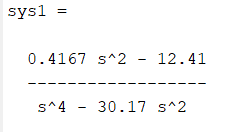
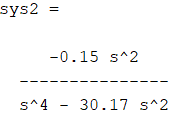
 

Fig: Transfer function Fig: Transfer function

## The pendulum is to be balanced in the inverted position with the cart being within the rail by applying a suitable . Explain if the pendulum can be balanced using only or only feedback. In other words, determine if the system is controllable when the input u is equal to either or .

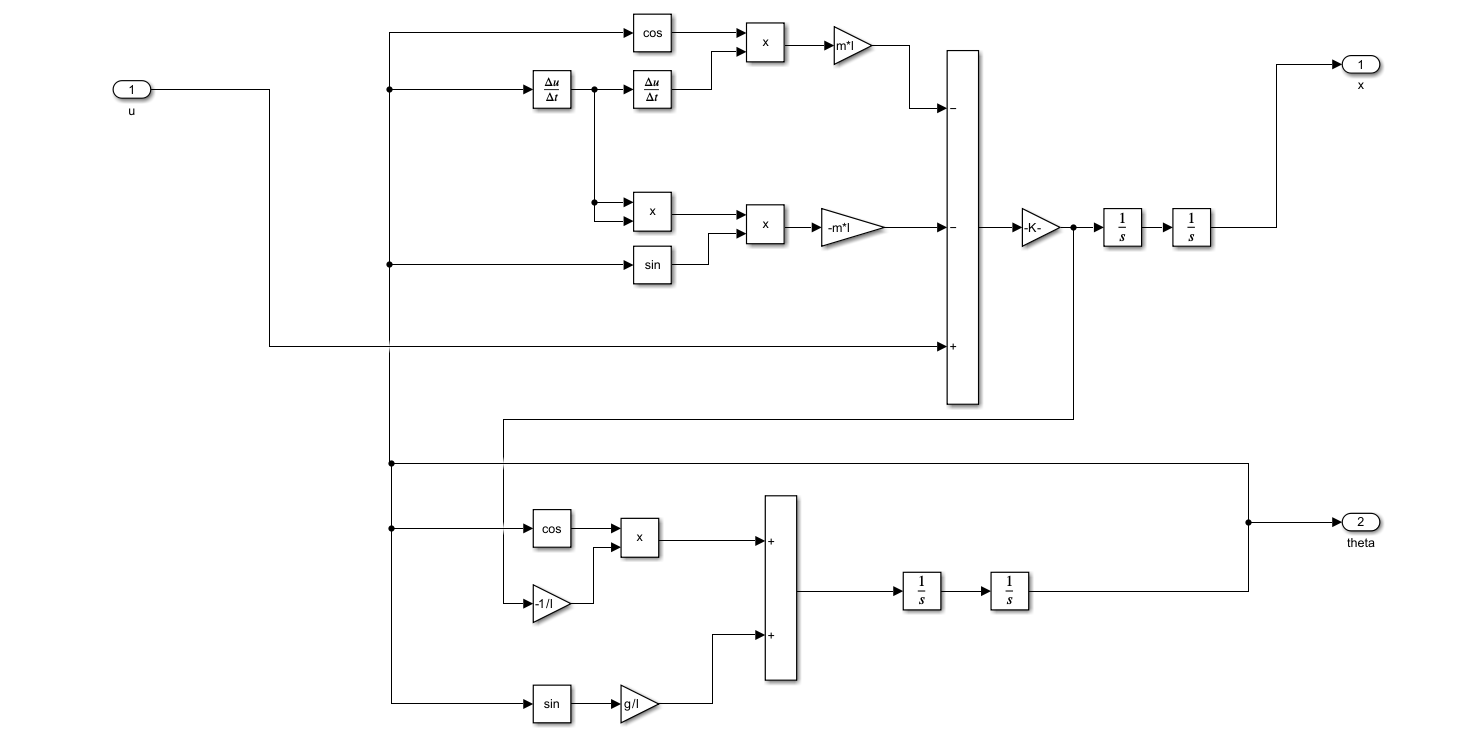
For : There is a pole-zero cancellation for the pole at

: Though, theoretically, there is no pole-zero cancellation, but pole and zero are very close. Therefore, for all practical purposes, pole-zero cancellation can be considered for the system.

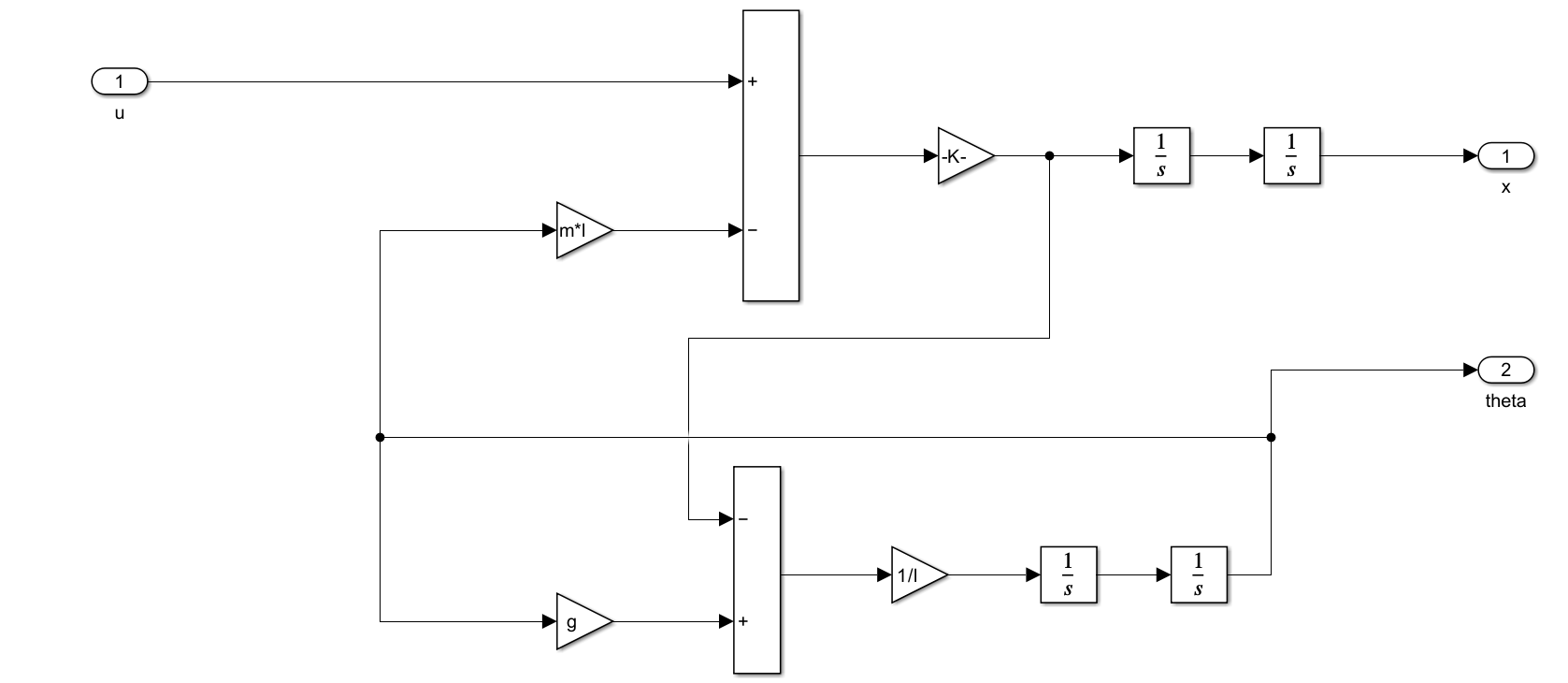
Though it is theoretically possible to control the system by only providing feedback of , it is not practically possible.

Hence, the system is not controllable by providing feedback of only or

# Create a model of the nonlinear dynamical system in SIMULINK



# Create a model of the linearized system in SIMULINK.



# Plots for and

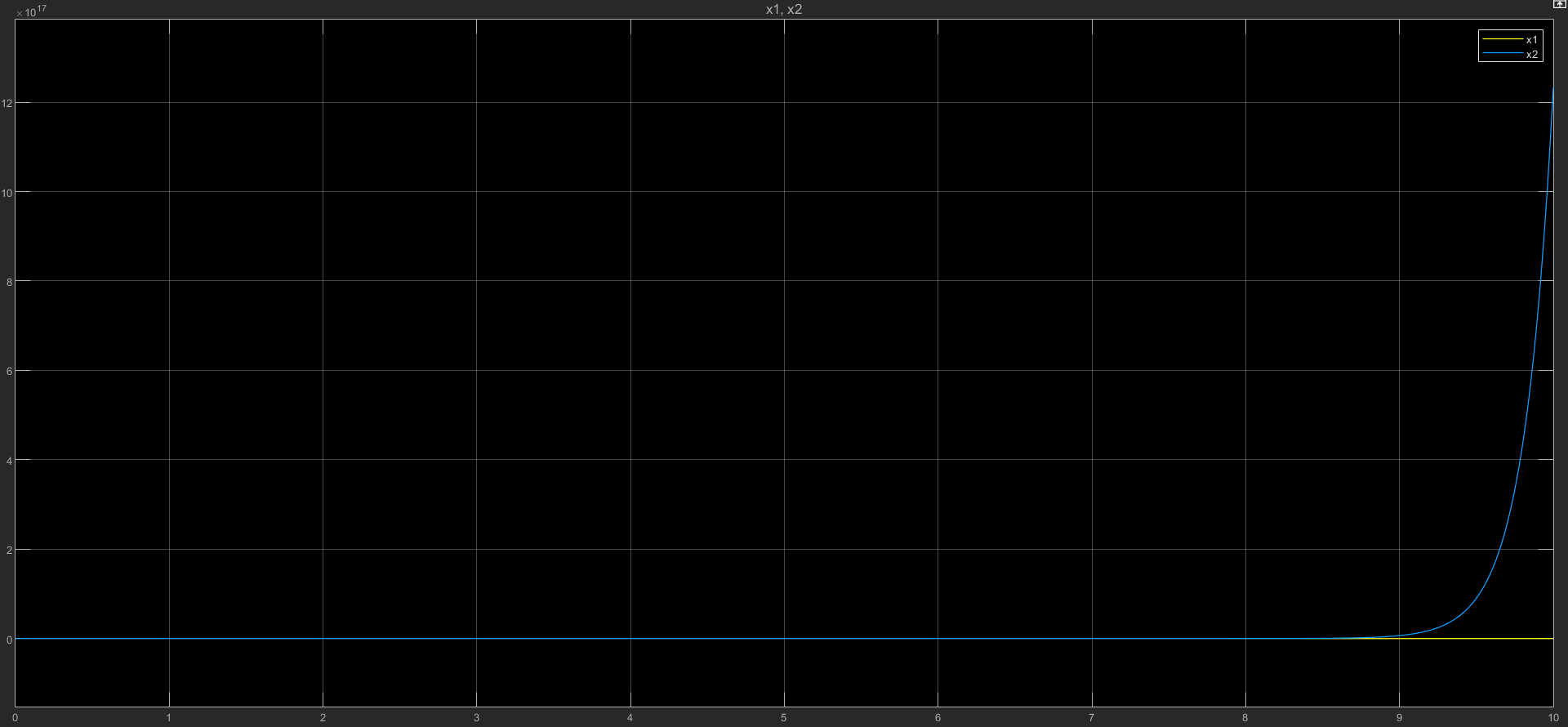


Fig. Comparison between (non-linear) and (linear)

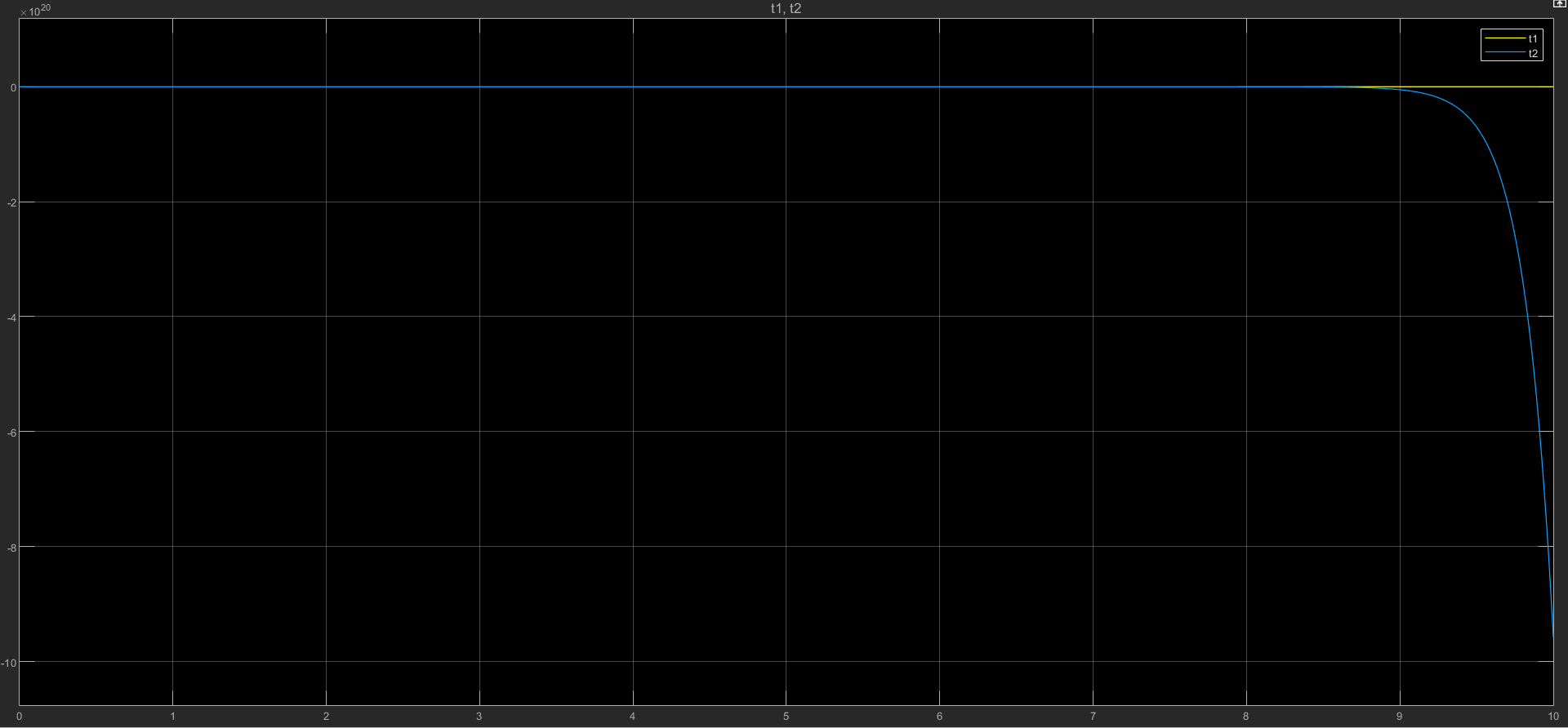


Fig. Comparison between (non-linear) and (linear)