EE69205: Signal Processing System Design
Indian Institute of Technology, Kharagpur

Bearing Fault Analysis using Signal Processing

Anirvan Krishna | Roll no. 21EE38002

1. Objective

Majority of problems in rotating machinery are caused by faulty gears, bearings etc. Failure in bearing is one of the primary causes of breakdown in rotating machines. Such breakdowns can lead to expensive shutdowns, drifts in production and even human casualties The objective of this experiment is to perform fault diagnosis of a rolling element bearing based on acceleration signals, especially in the presence of strong masking signals from other machine components. This will demonstrate how to apply envelope spectrum analysis to diagnose bearing faults.

During the course of this experiment, we'll be observing charecteristics of vibration signals to identify Inner Raceway Fault (IRF) and Outer Raceway Fault (ORF) in the rotatory components. We'll be comparing these signal characteristics with the baseline signal to observe their respective intricacies.

2. Observing the Signals: Time and Frequency Domain

In this section we will first observe the time and frequency domain characteristics of signals without fault, with IRF and with ORF. This observation will increase our understanding of the nature of the signal and will be beneficial in deciding the processing techniques to be used further.

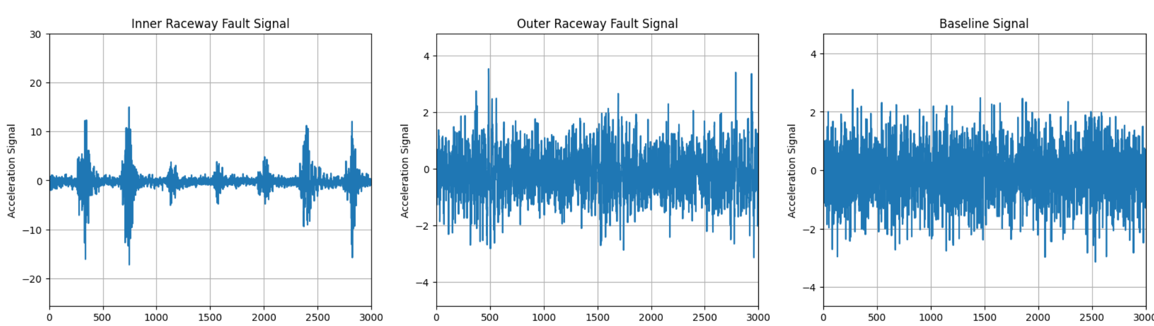

Fig. 1. Time Domain Representation of Signals

2.1. Spectral Analysis: Fast Fourier Transform—The Fast Fourier Transform (FFT) is an efficient algorithm for computing the Discrete Fourier Transform (DFT) of a sequence or signal. It is widely used in various signal processing applications, including frequency analysis, filtering, and spectral estimation.

The DFT of a sequence of N complex numbers $x[n]$ is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N} \tag{1}$$

where $X[k]$ represents the k-th frequency component of the signal.

The FFT algorithm exploits the symmetry properties of the DFT to reduce the number of computations required. It divides the input sequence into smaller sub-sequences and recursively computes their DFTs. This divide-and-conquer approach significantly reduces the computational complexity from $O(N^2)$ to $O(N \log N)$.

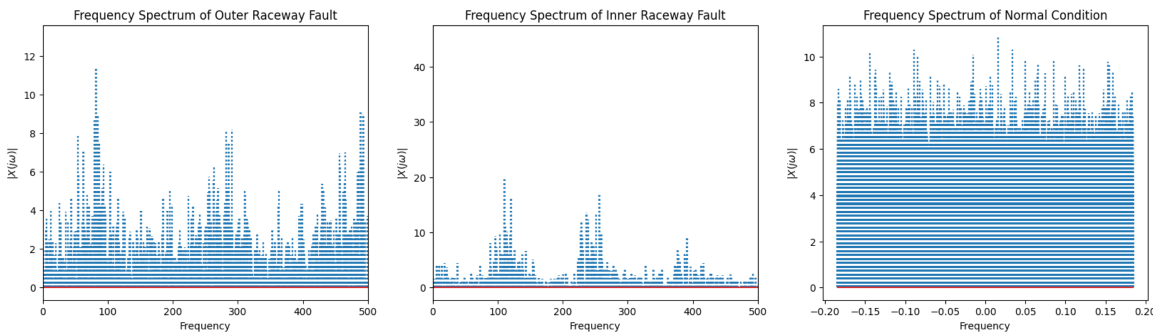

Fig. 2. Frequency Spectra of Signals using 4-point FFT

Observing the frequency spectra, we observe that the baseline case (signal with no fault) has a nearly flat spectrum i.e, all frequency components have nearly the same energy. Now, for observing the detailed understanding of the frequency contents of the faulty signals, let us analyze the spectrogram of the signals.

2.2. Spectrogram Analysis: Short-time Fourier Transform—By applying the STFT, we can obtain a spectrogram, which is a 2D representation of the signal's frequency content over time. The magnitude of the STFT represents the signal's energy at different frequencies and time instants, while the phase provides information about the signal's phase relationships. The STFT is defined as follows:

$$X(t,\omega) = \int_{-\infty}^{\infty} x(\tau) \cdot w(t-\tau) \cdot e^{-j\omega\tau} d\tau \tag{2}$$

where $x(t)$ is the input signal, $w(t)$ is a window function, ω is the angular frequency, and $X(t,\omega)$ represents the time-frequency representation of the signal.

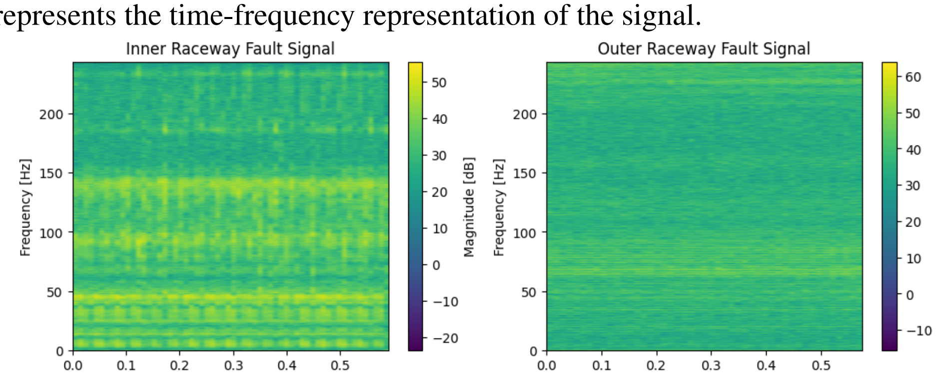

Fig. 3. Spectrogram of IRF and ORF Signals

3. Signal Filtering: Bandpass FIR Filter

A bandpass filter is a type of filter that allows a specific range of frequencies, known as the passband, to pass through while attenuating frequencies outside the passband. FIR bandpass filters are designed to selectively filter out unwanted frequencies and retain the desired frequency range.

The general equation for an FIR filter can be represented as:

$$y[n] = \sum_{k=0}^{N-1} h[k] \cdot x[n-k] \tag{3}$$

where:

- $y[n]$ is the output sample at time index n
- $x[n-k]$ is the input sample at time index $n-k$
- $h[k]$ is the filter coefficient at index k
- N is the filter order

To design a bandpass filter, we need to determine the filter coefficients $h[k]$ that define the desired frequency response. The frequency response of an FIR filter can be obtained by taking the Discrete Fourier Transform (DFT) of the filter coefficients. Once the cutoff frequencies are determined, the filter coefficients can be calculated using various filter design techniques, such as the window method.

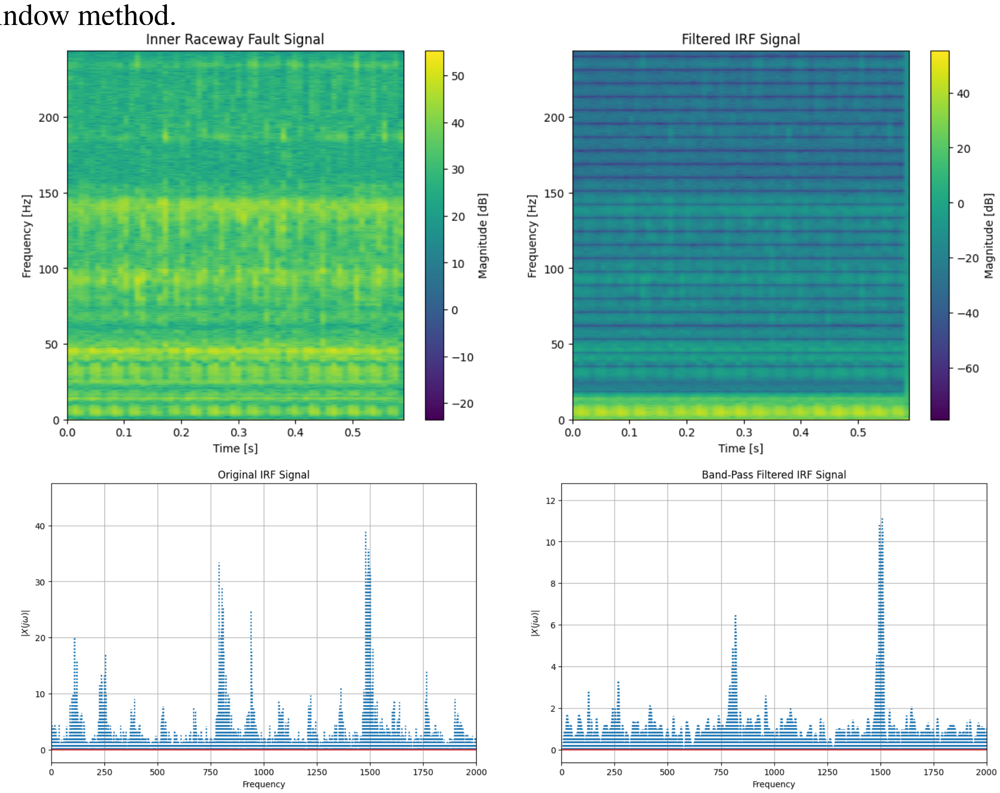

Fig. 4. Spectrogram and Spectrum of Filtered IRF Signal

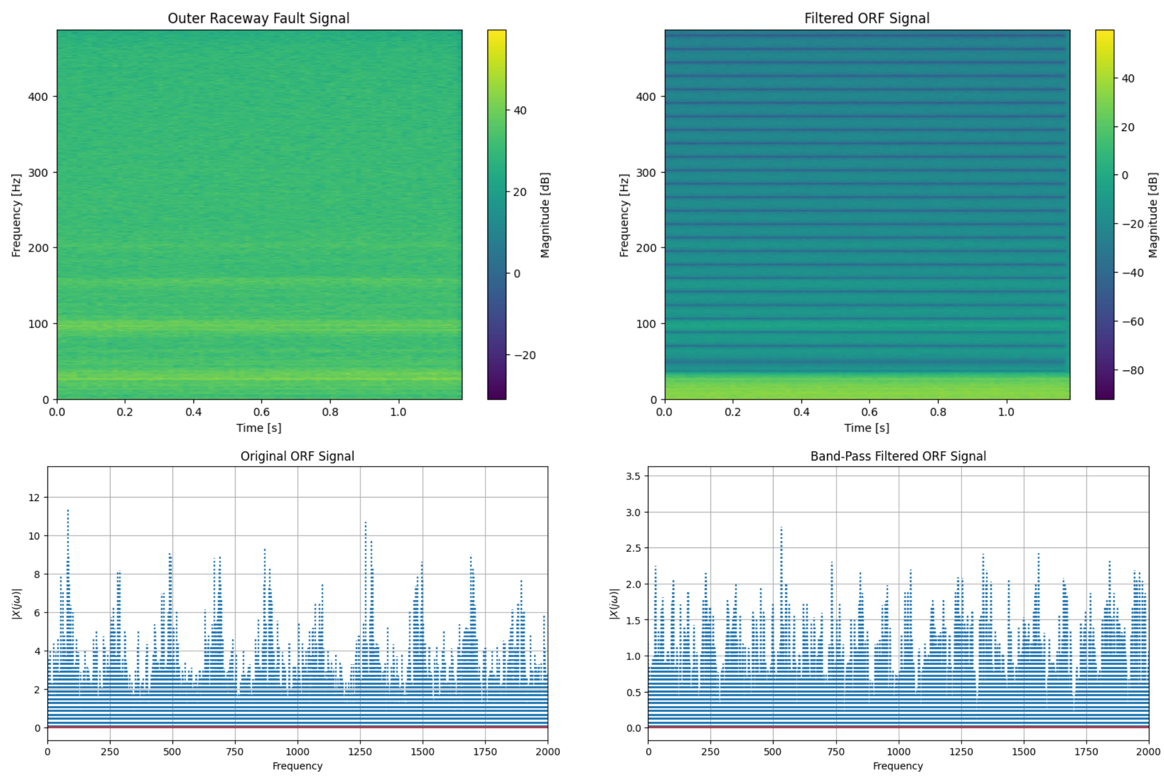

Fig. 5. Spectrogram and Spectrum of Filtered ORF Signal

4. Envelope Formation: Hilbert Transform

4.1. Hilbert Transform—The Hilbert transform of a real-valued signal $x(t)$ is a fundamental tool in signal processing, particularly in the analysis of analytic signals. The Hilbert transform $\mathscr{H}\{x(t)\}$ of $x(t)$ is defined as:

$$\mathscr{H}\{x(t)\} = \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau \tag{4}$$

The Hilbert transform shifts the phase of each frequency component of the signal by $-90°$, resulting in a new signal $\hat{x}(t)$, which is the imaginary part of the corresponding analytic signal.

4.2. Analytic Signal—An analytic signal $z(t)$ is a complex-valued signal composed of the original signal $x(t)$ and its Hilbert transform $\hat{x}(t)$. It is given by:

$$z(t) = x(t) + j\hat{x}(t) \tag{5}$$

where $j = \sqrt{-1}$. The analytic signal removes the negative frequency components of $x(t)$ and provides a representation suitable for envelope detection.

4.3. Signal Envelope Formation—The envelope of a signal represents the smooth curve outlining the extremes of the oscillations of the signal. For an analytic signal $z(t) = x(t) + j\hat{x}(t)$, the envelope $E(t)$ is defined as the magnitude of the analytic signal:

$$E(t) = |z(t)| = \sqrt{x(t)^2 + \hat{x}(t)^2} \tag{6}$$

The envelope $E(t)$ gives a measure of the instantaneous amplitude of the signal $x(t)$.

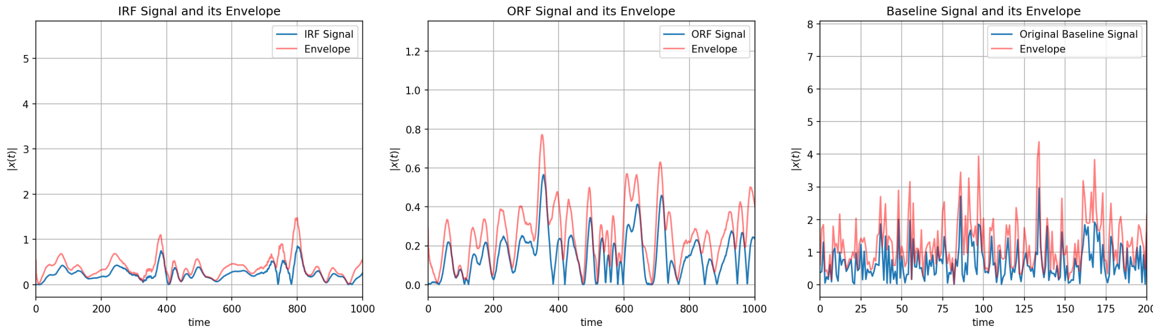

Fig. 6. Envelope formation for the Signals

5. One-Sided Power Spectrum Computation

The power spectrum of a signal provides insight into the distribution of signal power across various frequency components. For real-valued signals, the power spectrum can be simplified by using a one-sided representation, which only considers the positive frequencies.

Given a real-valued discrete-time signal $x[n]$, the power spectral density (PSD) is computed using the Discrete Fourier Transform (DFT). The two-sided power spectrum is:

$$P_{xx}(f_k) = \frac{|X(f_k)|^2}{N} \tag{7}$$

where $X(f_k)$ is the DFT of $x[n]$, N is the number of samples, and f_k represents the frequency components.

For the one-sided power spectrum, the symmetry of the DFT for real-valued signals allows us to focus on the positive frequencies $f_k \geq 0$, and the power for these frequencies is doubled (except at $f = 0$ and $f = \frac{f_s}{2}$, the Nyquist frequency):

$$P_{xx}(f_k) = \frac{2|X(f_k)|^2}{N}, \quad \text{for } 0 < f_k < \frac{f_s}{2} \tag{8}$$

At $f = 0$ and $f = \frac{f_s}{2}$, the power remains unchanged:

$$P_{xx}(f_0) = \frac{|X(f_0)|^2}{N}, \quad P_{xx}\left(\frac{f_s}{2}\right) = \frac{|X\left(\frac{f_s}{2}\right)|^2}{N} \tag{9}$$

Thus, the one-sided power spectrum represents the distribution of signal power over the positive frequency components, which is commonly used in practical signal analysis.

In this experiment, One-sided Power Spectrum is of a crucial importance. While analyzing the faults in rotatory systems, we define two critical frequencies: Ball-pass Frequency Outer Race (BPFO) and Ball-pass Frequency Inner Race (BPFI). During the specific conditions i.e, IRF and ORF, the major energy components of the signal are expected to be present in the harmonics of BPFI and BPFO respectively. For fault analysis in Ball bearings, the approximate value of BPFI is 118.75 Hz and BPFO is 81.125 Hz.

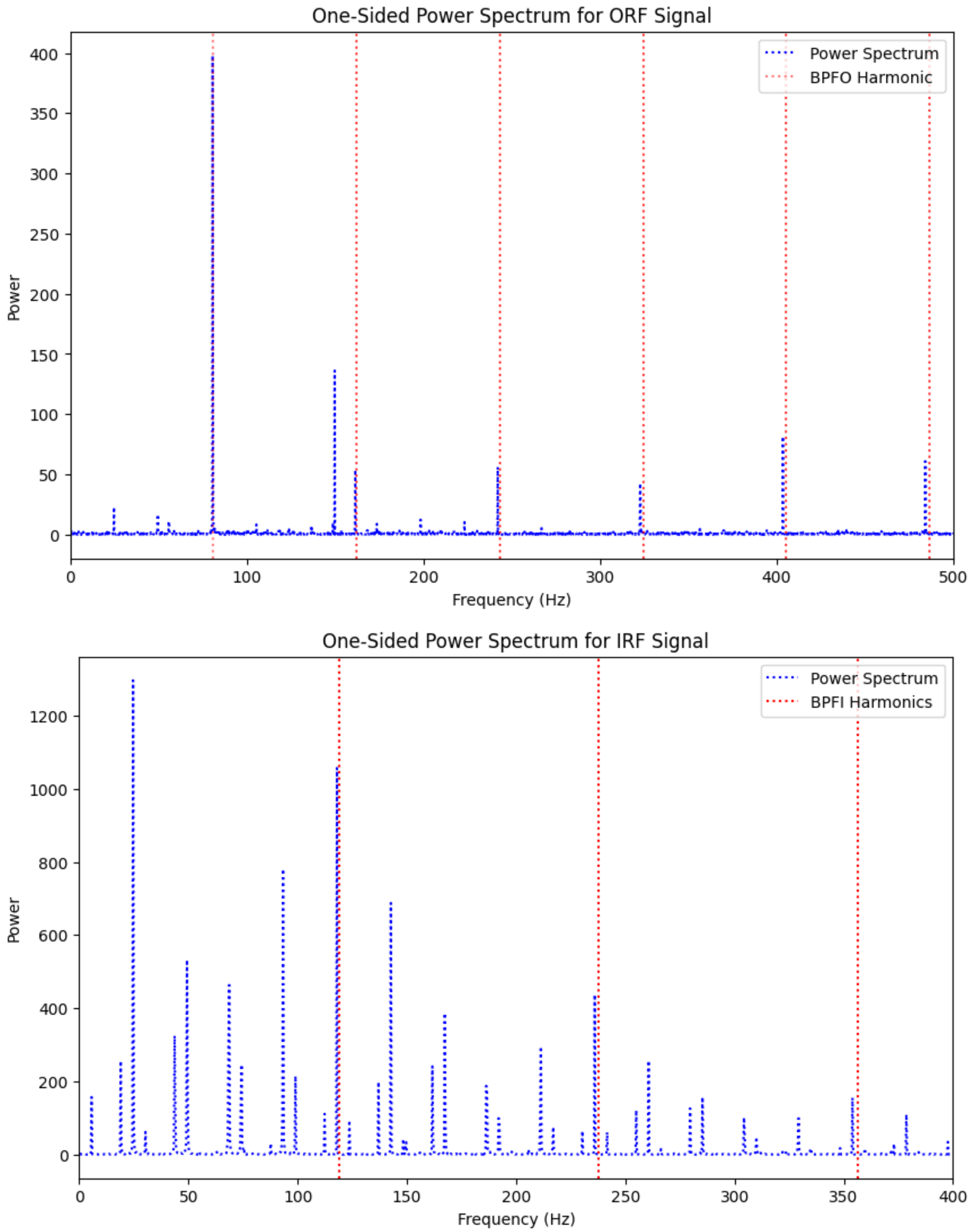

Fig. 7. One-sided Power Spectrum for ORF and IRF Signals

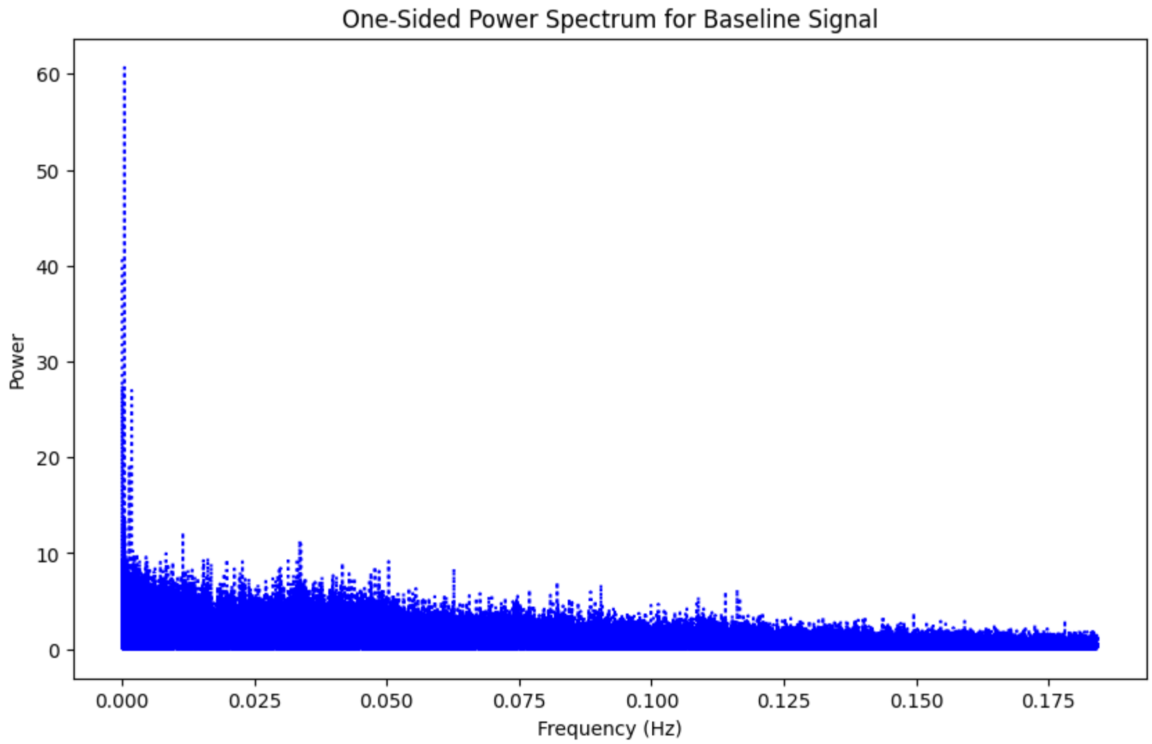

Fig. 8. One-sided Power Spectrum for Baseline Signal

6. Signal Statistics

Statistic	Inner Raceway Fault (IRF)	Outer Raceway Fault (ORF)
Envelope Mean (μ)	0.4193	0.2791
Envelope Std (σ)	0.4678	0.1794
BPFI/BPFO (Hz)	118.88	81.12
Harmonic 1 (Hz)	118.88	81.12
Harmonic 2 (Hz)	237.75	162.25
Harmonic 3 (Hz)	356.62	243.38
Harmonic 4 (Hz)	475.50	324.50
Kurtosis of the Signal	24.97	0.50

Table 1. IRF and ORF Statistics

Statistic	Envelope Mean (μ)	Envelope Std (σ)	Kurtosis of the Signal
Baseline Signal	1.2829	0.7976	0.02

Table 2. Baseline Signal Statistics

7. Observations and Conclusion

7.1. Identifying Baseline Signal—For the baseline signal, we observe the signal envelope and compute the one-sided power spectrum. The power spectrum shows the majority of power being concentrated at very low frequencies. The energy charecterisitcs of this signal is quite similar to that of a *white noise*. The charecteristics of this power spectrum confirms that the machinery corresponding to this signal is free from any kind of inner or outer raceway fault.

7.2. Identifying Outer Raceway Fault (ORF)—For the ORF signal, we can observe the signal envelope and the corresponding one-sided power spectrum. We observe that majority of the energy of the signal is present in the harmonics of the BPFO frequency i.e. 81.125 Hz. This confirms the presence of an Outer Raceway Fault in the machinery corresponding to this vibration signal.

7.3. Identifying Inner Raceway Fault (IRF)—For the IRF Signal, we observe the one-sided power spectrum of the signal envelope to identify a large portion of the energy being carried by the harmonics of the BPFI i.e. 118.88 Hz. This observation confirms the presence of an Inner Raceway Fault in this signal.