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Experiment 5: Gray-scale Image Compression using Singular Value Decomposition

1 Objective of the Experiment

The objective of this experiment is to compress a gray-scale image using the Singular Value Decomposition (SVD) algorithm. Additionally, the experiment aims to evaluate how the quality of these compressed images is affected by retaining only a subset of singular value components of the original image and to analyze the compressed image quality by measuring the mean squared error (MSE), peak signal-to-noise ratio (PSNR) and structural similarity index measure (SSIM) between the original and compressed images. Also the aim is to analyze the performance when image compression is performed patch-wise.

2 Assignments to Solve and Report

Write the codes for the following tasks in a '.ipynb' file, including all visualizations, and submit the executed file. Submit also a separate PDF of your report on this experiment. This report should describe your observations and reasoning while executing these experiments. The Cameraman image can be found at: `https://drive.google.com/file/d/1nlTgGDd0dLn5Kg7ZvcW8CEHwnyzH-tFW/view?usp=sharing`.

Part 1: Gray-scale image compression using SVD algorithm.

1. Read the gray-scale Cameraman image. Show the image.
2. Crop the image to resolution 140×140.
3. Apply the inbuilt numpy.linalg.svd function to decompose the image into U, Σ, and V components. Check if the reconstructed image by taking the following sum matches with the original image for $K = 140$,

$$\sum_{k=1}^{K} U(:,k)S(k)V^T(k,:)$$

   For comparing with the original image, use mean squared error (MSE), peak signal-to-noise-ration (PSNR), and structural similarity index measure (SSIM) metrics.

4. Reconstruct the image for $K = 5, 10, 20, 40, 50, 100$. For a particular K, the image compression ratio (CR) is,

$$CR = \frac{K(1 + H + W)}{HW} \tag{1}$$

   where H and W are image height and width respectively. Compare the reconstructed images with the original image using MSE, PSNR, and SSIM metrics.

5. Plot MSE vs. CR, PSNR vs. CR, and SSIM vs. CR.

Part 2: Patch-wise gray-scale image compression using SVD algorithm.

1. Split the gray-scale image into 25 non-overlapping patches with patch size 28×28. You can use the skimage.util.shape.view_as_windows function for this purpose.

2. For each patch, perform the image compression for values of $K = 5, 10, 15, 20$. For a particular K, the image patch compression ratio (CR_p) is,

$$CR_p = \frac{K(1 + H_p + W_p)}{H_p W_p} \tag{2}$$

   where H_p and W_p are patch height and width respectively. So for each value of K, you have 25 reconstructed patches.

3. Merge the 25 reconstructed patches for each value of K to get the reconstructed images. You can use the skimage.util.shape.montage function to merge the image patches. So, you have 4 reconstructed images for values of $K = 5, 10, 15, 20$.

4. Compare the reconstructed images with the original image using MSE, PSNR, and SSIM metrics.

5. Plot MSE vs. CR_p, PSNR vs. CR_p, and SSIM vs. CR_p.

Part 3: Performance comparison.
Compare the performance when compression is performed on the whole image with that when compression is performed patchwise.

References

[1] Strang G. *Introduction to Linear Algebra*. Wellesley-Cambridge Press; 2022.