EE69205: Signal Processing System Design
Indian Institute of Technology, Kharagpur

Signal Processing for Phonocardiogram Analysis

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1. Objective

This work aims to provide a comprehensive understanding of digital signal processing for biomedical applications, with a focus on noise reduction and heart rate analysis from PCG signals. The objectives of this work are to visualize raw Phonocardiogram (PCG) signals, identify noise components, and apply digital filters for noise suppression. White Gaussian noise is introduced to simulate real-world conditions, and its removal is demonstrated using filtering techniques. Key heart rate metrics, including instantaneous heart rate, rolling average, and heart rate variability (HRV), are computed through peak detection. The results are presented via visualizations, and filter performance is evaluated by calculating the Mean Squared Error (MSE) between clean and noisy signals.

2. Phonocardiogram (PCG) Signal

A Phonocardiogram (PCG) signal records heart activity through acoustic signals produced by heartbeats. It is used to diagnose cardiovascular conditions by detecting anomalies in heart sounds (S1, S2) associated with valve closure. Due to noise contamination, digital filtering techniques are often required for effective analysis.

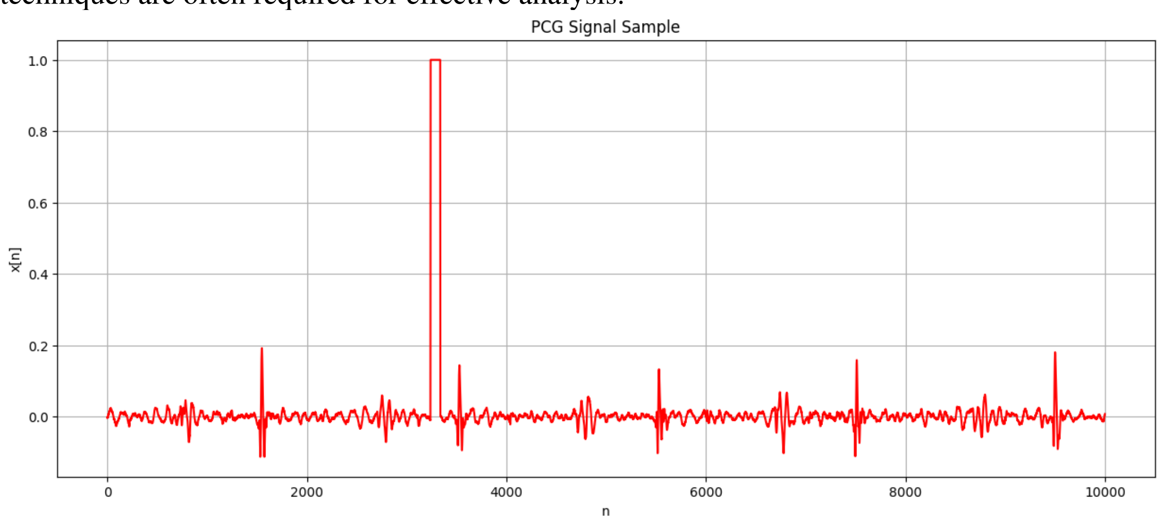

Fig. 1. Raw Phonocardiogram Signal

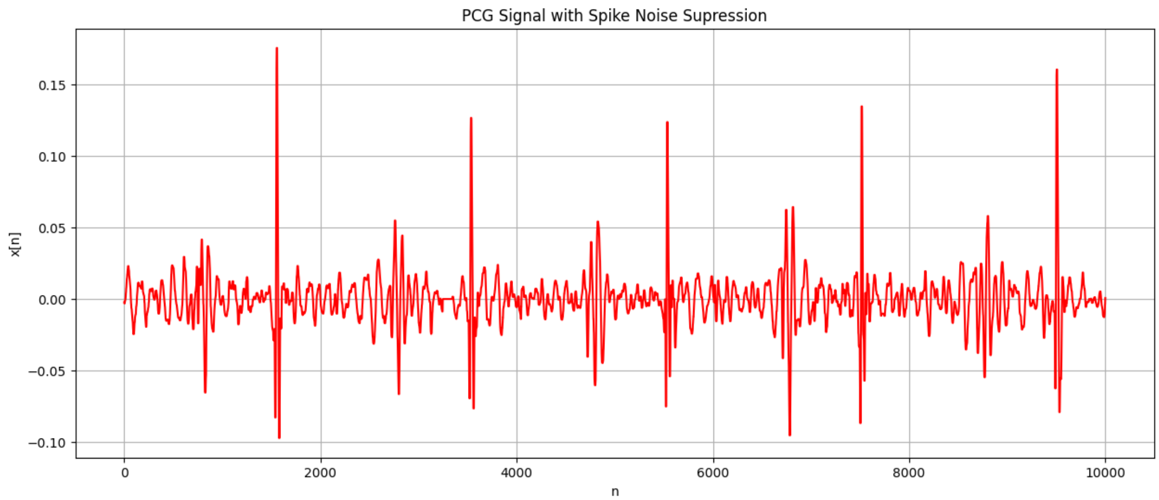

Fig. 2. PCG Signal after Spike Supression

2. Spike Noise Suppression Using Thresholding and Median Filtering

Spike noise in a signal can be suppressed using a combination of thresholding and median filtering. The process begins by applying a threshold to eliminate high-amplitude noise. Any signal values exceeding a defined threshold are set to zero:

$$x_{\text{thresholded}}[n] = \begin{cases} 0 & \text{if } |x[n]| > \text{threshold} \\ x[n] & \text{otherwise} \end{cases}$$

This step reduces the influence of extreme values in the signal.

Following this, a median filter is applied to smooth the signal and further reduce noise. The median filter computes the median of the signal over a sliding window of a specified length L, as shown:

$$y[n] = \text{median}\,(x[n-L/2],\ldots,x[n+L/2])$$

The combination of these two techniques effectively suppresses spike noise while preserving the underlying structure of the signal. In the experiment, the thresholding was applied with a threshold value of 0.2, followed by a median filter with a window length of 10.

3. Addition of White Gaussian Noise

To simulate real-world noise in the signal, white Gaussian noise is added. White Gaussian noise has a constant power spectral density across all frequencies and is commonly used to model random noise in signals. In the code, the noise is generated by sampling from a normal distribution with zero mean and a specified variance. The noisy signal $x_{\text{noisy}}[n]$ is obtained by adding the generated noise $w[n]$ to the clean signal $x[n]$ as follows:

$$x_{\text{noisy}}[n] = x[n] + w[n], \quad w[n] \sim \mathcal{N}(0, \sigma^2)$$

where σ^2 is the variance of the noise, controlling its intensity. This step models the impact of random environmental disturbances on the PCG signal. For this experiment, we are using an Additive White Gaussian Noise (AWGN) with $\sigma = 0.005$

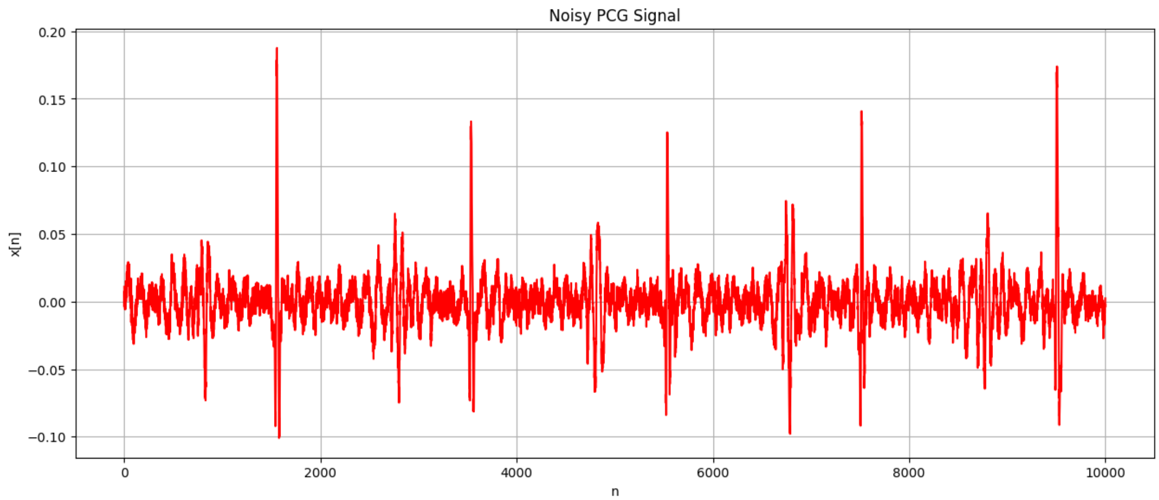

Fig. 3. PCG Signal with Additive White Gaussian Noise

4. Signal Filtering: Time Domain

4.1. Mean Filter—The mean filter replaces each sample in the signal with the average of its neighbors. For a 1D signal $x[n]$, the output is given by:

$$y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$$

where M is the filter length.

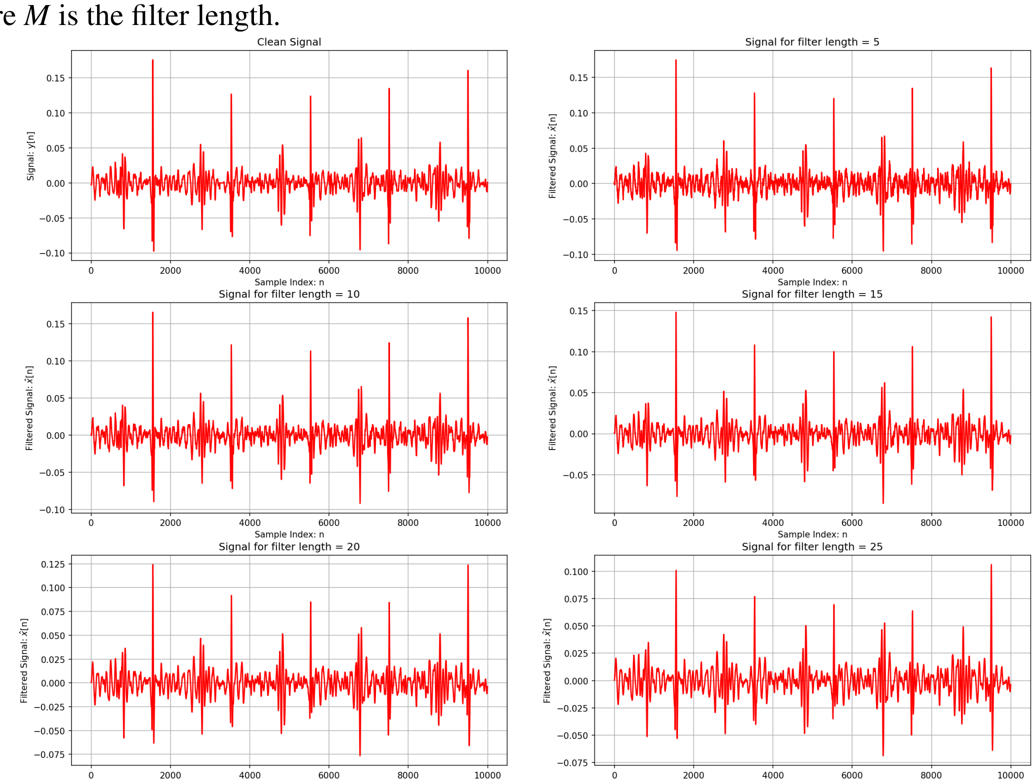

Fig. 4. Mean Filtered Signal for different Filter Lengths

Table 1. Mean Filter MSE for Different Filter Lengths

Filter Length 5	Filter Length 10	Filter Length 15	Filter Length 20	Filter Length 25
4.90×10^{-9}	4.85×10^{-9}	4.82×10^{-9}	4.74×10^{-9}	4.69×10^{-9}

4.2. Gaussian Filter—The Gaussian filter applies a weighted average based on a Gaussian function. The output for a 1D signal $x[n]$ is:

$$y[n] = x[n] * h[n]; \quad h[n] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

where σ controls the width of the Gaussian kernel.

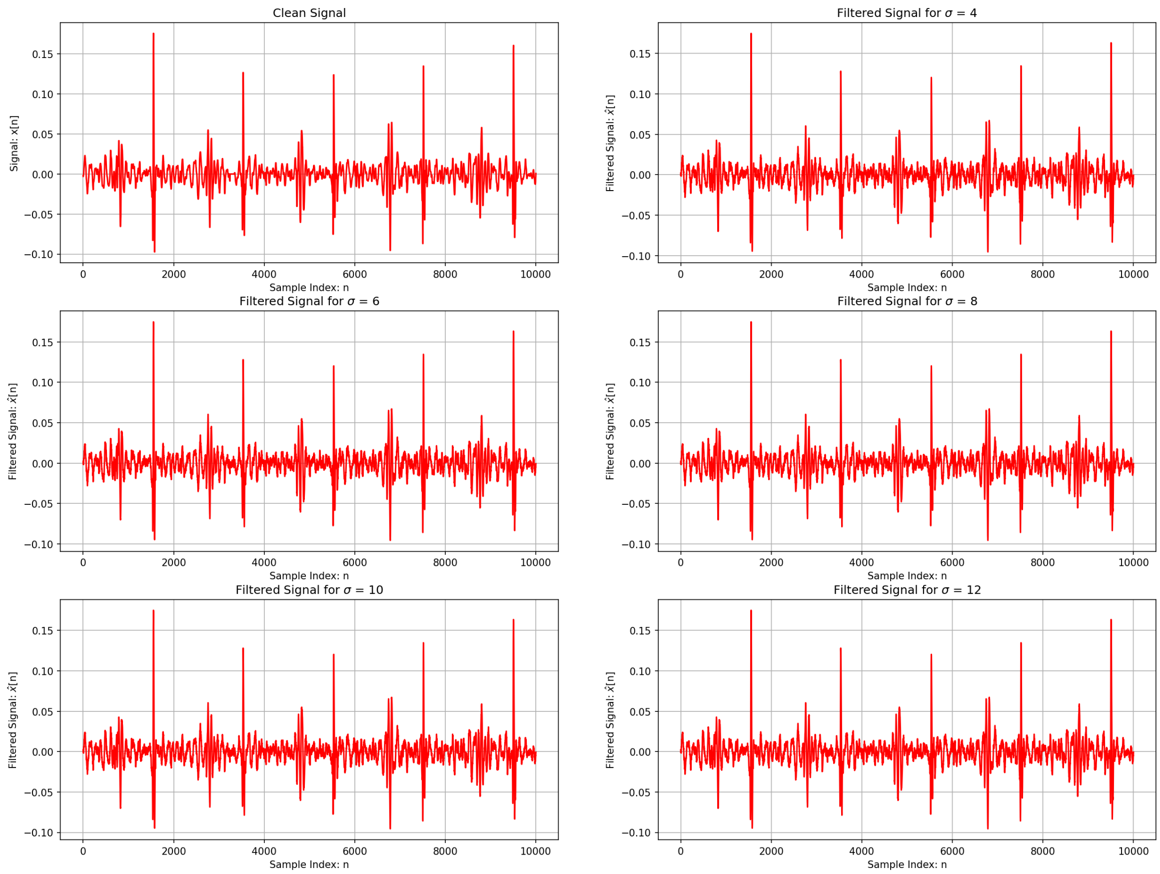

Fig. 5. Gaussian Filtered Signal for different σ

Table 2. Gaussian Filter MSE for Different σ Values

$\sigma = 4$	$\sigma = 6$	$\sigma = 8$	$\sigma = 10$	$\sigma = 12$
4.90×10^{-9}	4.90×10^{-9}	4.90×10^{-9}	4.90×10^{-9}	4.90×10^{-9}

4.3. Median Filter—The median filter replaces each sample with the median of its neighbors. For a 1D signal $x[n]$:

$$y[n] = \text{median}(x[n-k], \ldots, x[n+k])$$

where k defines the window size, effectively reducing noise while preserving edges.

Table 3. Median Filter MSE for Different Filter Lengths

Filter Length 11	Filter Length 21	Filter Length 31	Filter Length 41	Filter Length 51
2.09×10^{-9}	4.17×10^{-9}	3.36×10^{-9}	4.34×10^{-9}	7.70×10^{-8}

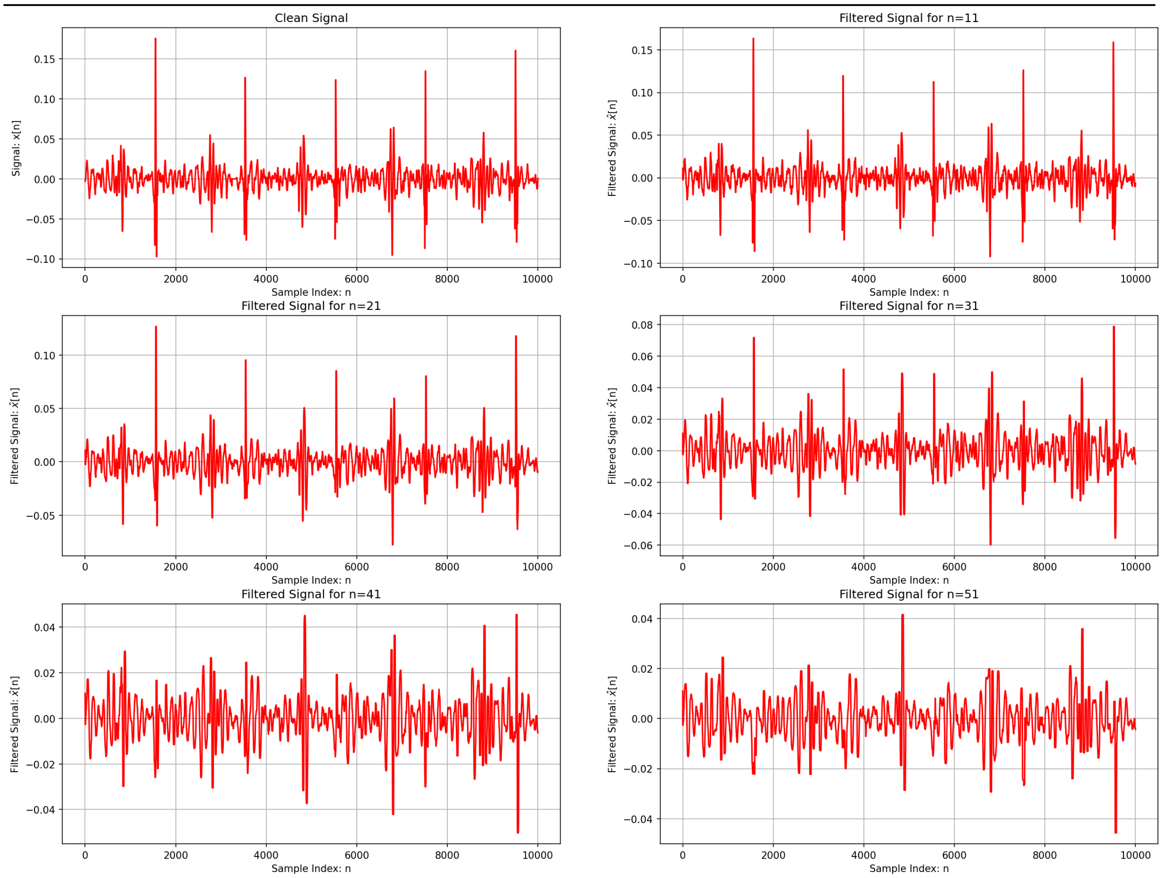

Fig. 6. Median Filtered Image with different Kernel Sizes

5. Signal Filtering: Frequency Domain

5.1. Fast Fourier Transform—The Fast Fourier Transform (FFT) is an efficient algorithm for computing the Discrete Fourier Transform (DFT) of a sequence or signal. It is widely used in various signal processing applications, including frequency analysis, filtering, and spectral estimation.

The DFT of a sequence of N complex numbers $x[n]$ is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}$$

where $X[k]$ represents the k-th frequency component of the signal.

The FFT algorithm exploits the symmetry properties of the DFT to reduce the number of computations required. It divides the input sequence into smaller sub-sequences and recursively computes their DFTs. This divide-and-conquer approach significantly reduces the computational complexity from $O(N^2)$ to $O(N \log N)$.

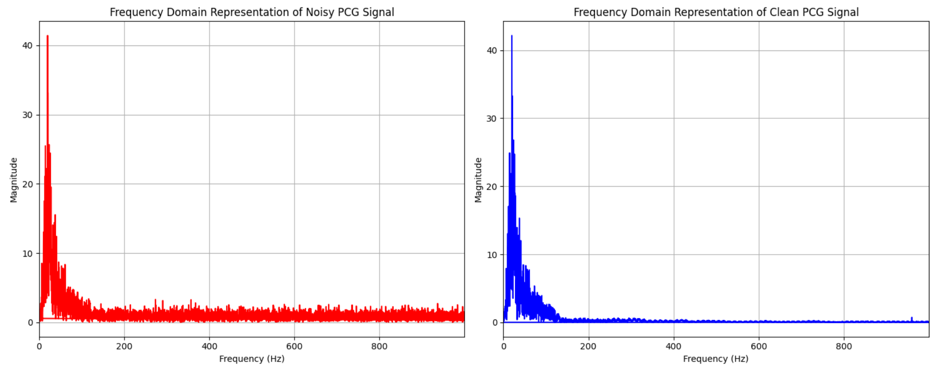

Fig. 7. Frequency Spectra of Clean and Noisy Signals

5.2. Bandpass FIR Filter—A bandpass filter is a type of filter that allows a specific range of frequencies, known as the passband, to pass through while attenuating frequencies outside the passband. FIR bandpass filters are designed to selectively filter out unwanted frequencies and retain the desired frequency range.

The general equation for an FIR filter can be represented as:

$$y[n] = \sum_{k=0}^{N-1} h[k] \cdot x[n-k]$$

where $h[k]$ is the filter coefficient at sample k. For FIR Filtering of the given signal, we'll apply bandpass FIR filter with $f_L = 20Hz$ and $f_H = 200Hz$ where f_L and f_H represent the lower and upper frequency cutoffs. We'll be implementing window method with *Hamming, Hann, Bartlet* and *Blackman* window functions and observe their performance.

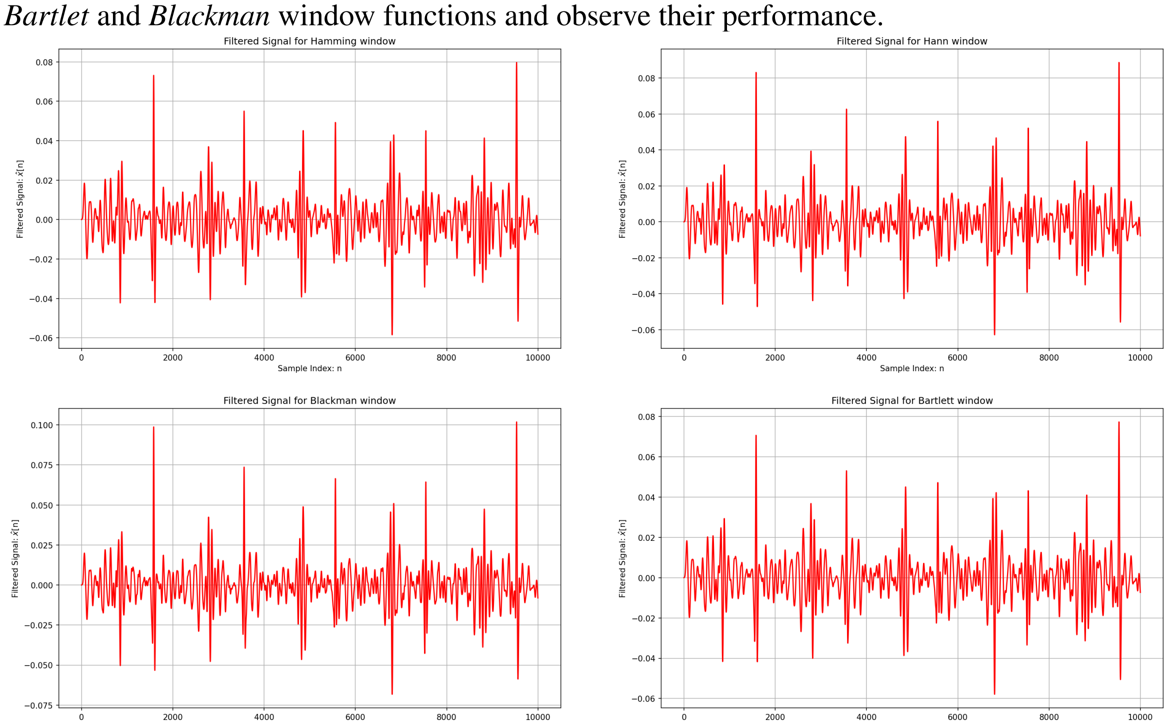

Fig. 8. FIR Bandpass Filtering for Different Window Functions

Table 4. Mean Squared Error (MSE) for Different Window Types

Hamming	Hann	Blackman	Bartlett
2.43×10^{-9}	2.39×10^{-9}	2.36×10^{-9}	2.43×10^{-9}

5.3. Bandpass Butterworth Filtering—Butterworth filters are widely used in signal processing for their maximally flat frequency response in the passband, which means they do not have ripples. A Bandpass Butterworth filter allows frequencies within a certain range (between ω_1 and ω_2) to pass while attenuating frequencies outside this range. The frequency response of an nth order bandpass butterworth filter with cutoffs ω_1 and ω_2 is defined as:

$$|H_{BP}(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega^2-\omega_1\omega_2}{\omega(\omega_2-\omega_1)}\right)^{2n}}}$$

Table 5. Mean Squared Error (MSE) for Different Filter Orders

Order 2	Order 4	Order 6	Order 8
1.68×10^{-9}	2.23×10^{-9}	2.88×10^{-9}	8.02×10^{-9}

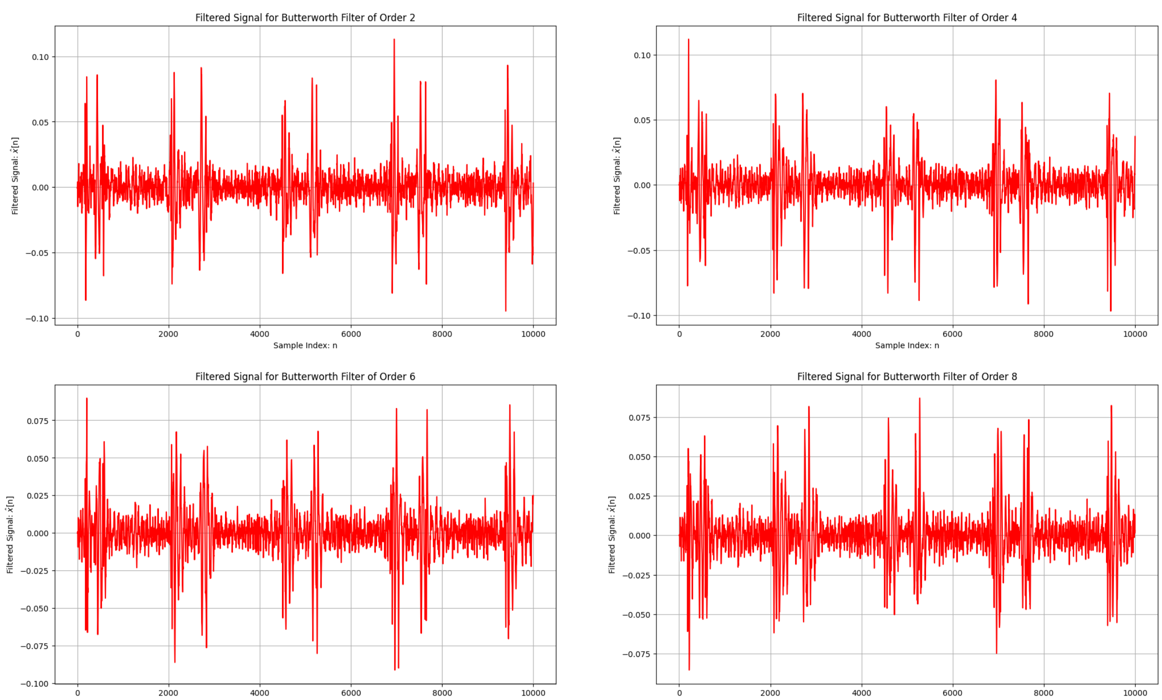

Fig. 9. Buuterworth Filtered Signal for Different Filter Orders

6. Filter Comparision based on Reconstruction Signal to Noise Ratio (RSNR)

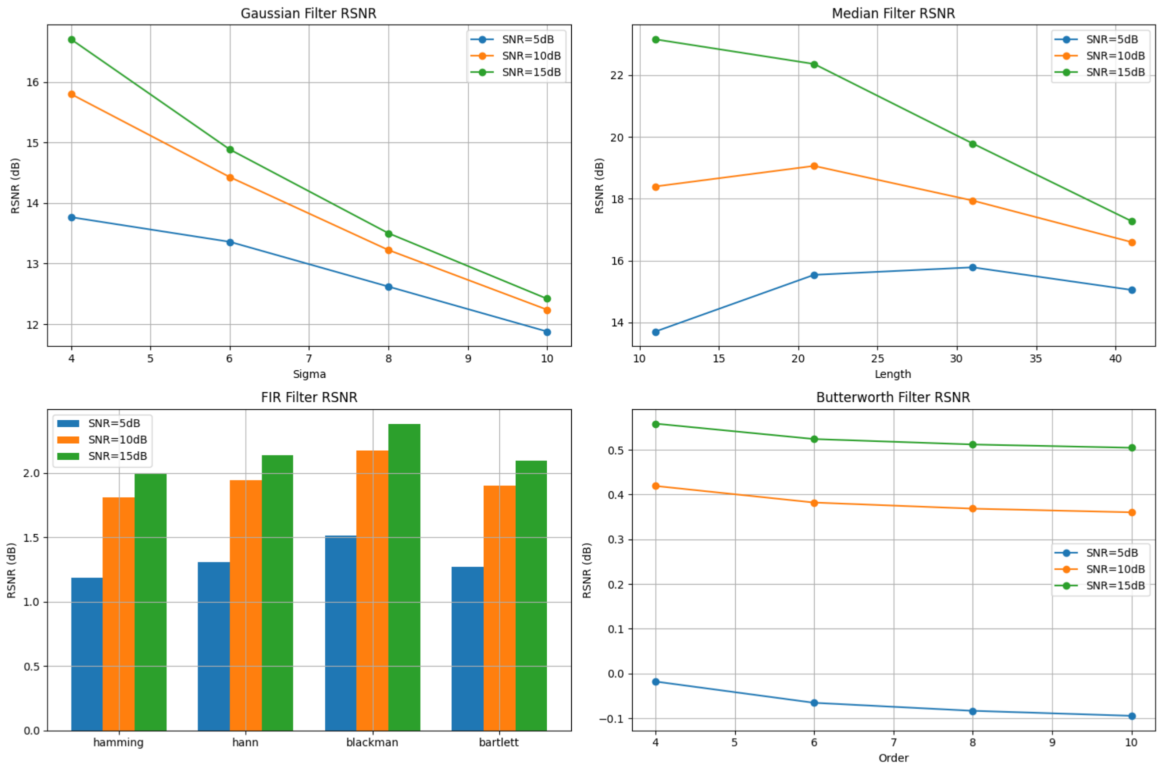

Fig. 10. Filter Performance Comparsion based on PSNR Values

Table 6. RSNR for Gaussian Filter

σ | SNR	5 dB	10 dB	15 dB
4	13.77	15.80	16.70
6	13.36	14.43	14.89
8	12.62	13.22	13.50
10	11.88	12.24	12.42

Table 7. RSNR for Median Filter

Length | SNR	5 dB	10 dB	15 dB
11	13.71	18.40	23.16
21	15.54	19.06	22.36
31	15.78	17.94	19.79
41	15.05	16.60	17.28

Table 8. RSNR for FIR Filter

Window | SNR	5 dB	10 dB	15 dB
Hamming	1.19	1.81	2.00
Hann	1.31	1.94	2.14
Blackman	1.51	2.17	2.38
Bartlett	1.27	1.90	2.09

Table 9. RSNR for Butterworth Filter

Order | SNR	5 dB	10 dB	15 dB
4	-0.02	0.42	0.56
6	-0.07	0.38	0.52
8	-0.08	0.37	0.51
10	-0.09	0.36	0.50

7. Envelope Formation: Hilbert Transform

7.1. Hilbert Transform—The Hilbert transform of a real-valued signal $x(t)$ is a fundamental tool in signal processing, particularly in the analysis of analytic signals. The Hilbert transform

$\mathscr{H}\{x(t)\}$ of $x(t)$ is defined as:

$$\mathscr{H}\{x(t)\} = \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau \tag{1}$$

The Hilbert transform shifts the phase of each frequency component of the signal by $-90°$, resulting in a new signal $\hat{x}(t)$, which is the imaginary part of the corresponding analytic signal.

7.2. Analytic Signal—An analytic signal $z(t)$ is a complex-valued signal composed of the original signal $x(t)$ and its Hilbert transform $\hat{x}(t)$. It is given by:

$$z(t) = x(t) + j\hat{x}(t) \tag{2}$$

where $j = \sqrt{-1}$. The analytic signal removes the negative frequency components of $x(t)$ and provides a representation suitable for envelope detection.

7.3. Signal Envelope Formation—The envelope of a signal represents the smooth curve outlining the extremes of the oscillations of the signal. For an analytic signal $z(t) = x(t) + j\hat{x}(t)$, the envelope $E(t)$ is defined as the magnitude of the analytic signal:

$$E(t) = |z(t)| = \sqrt{x(t)^2 + \hat{x}(t)^2} \tag{3}$$

The envelope $E(t)$ gives a measure of the instantaneous amplitude of the signal $x(t)$.

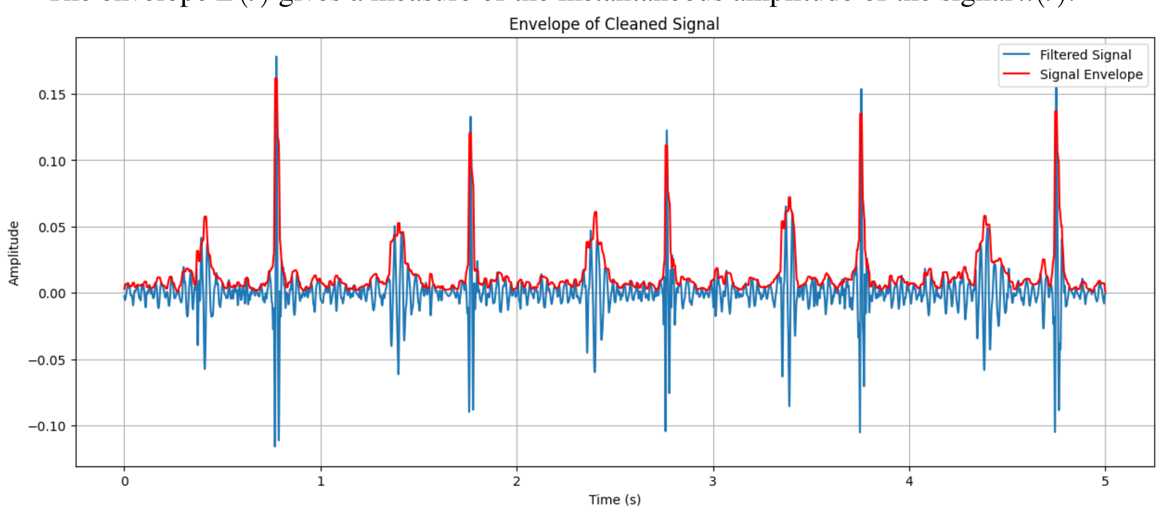

Fig. 11. PCG Signal with its Envelope

8. Systole and Diastole Detection: Interval and Ratio Analysis

In phonocardiogram (PCG) signals, systole and diastole detection is crucial for cardiac function assessment. *Systole* corresponds to the contraction phase, marked by the first heart sound, S_1, while *Diastole* is the relaxation phase, indicated by the second heart sound, S_2. The detection method extracts the envelope of the PCG signal using the Hilbert transform, followed by peak detection for S_1 and S_2. Intervals between the peaks are computed as:

$$\text{Intervals} = \frac{\Delta\text{Peaks}}{f_s}$$

where f_s is the sampling frequency. Accurate classification aids in calculating heart rate and assessing heart rate variability (HRV).

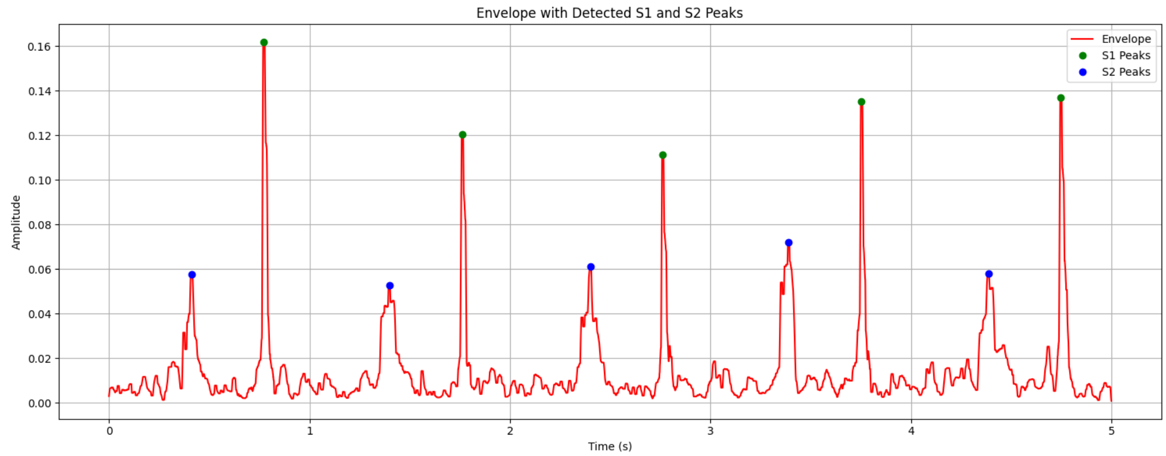

Fig. 12. PCG Envelope with Lablled S_1 and S_2 Phases

The following table presents the number of detected S_1 and S_2 peaks, along with the time intervals between consecutive peaks in seconds. We detect 5 peaks each for S_1 and S_2

The intervals between the detected S_1 and S_2 peaks represent the time between systole and diastole phases of the cardiac cycle. These intervals are crucial for calculating heart rate and assessing heart rate variability (HRV).

8.1. Ratio Analysis—The following table presents the ratios between consecutive intervals:

Interval Pairs	**Ratios**
Interval 1 to Interval 2	0.58
Interval 2 to Interval 3	1.72
Interval 3 to Interval 4	0.57
Interval 4 to Interval 5	1.77
Interval 5 to Interval 6	0.58
Interval 6 to Interval 7	1.72
Interval 7 to Interval 8	0.58
Interval 8 to Interval 9	1.75

Table 10. Ratios between Consecutive Intervals

The ratios are calculated by comparing consecutive intervals of the cardiac cycle:

$$\text{Ratio}_n = \frac{\text{Interval}_n}{\text{Interval}_{n+1}} \tag{4}$$

This formula compares the duration of one interval to the next, providing insights into the relative timing of systole and diastole phases.

8.2. Importance of the Ratios—

(1) *Assessment of Cardiac Rhythm*: The ratios help evaluate the regularity of the heart's rhythm. In a healthy heart, the intervals between heart sounds (S_1 and S_2) follow expected patterns (e.g., diastole lasting longer than systole). This can indicate efficient cardiac function.

(2) *Detection of Cardiac Abnormalities*: Significant deviations from expected ratios (like

those close to 2:1 or 1:2) can suggest potential cardiac issues. For example, an imbalance may indicate conditions such as arrhythmias, heart failure, or other structural abnormalities.

(3) *Analysis of Heart Rate Variability (HRV)*: The variability in these ratios provides insights into heart rate variability (HRV), a measure of autonomic nervous system function and cardiovascular health. A higher HRV is generally associated with better health and adaptability.

The ratio analysis yields important insights into the relative timing of systole and diastole. Below is a summary of the key findings:

(1) **Number of Ratios close to 2:1**: 4, These ratios indicate that diastole lasted approximately twice as long as systole, which is typical in a healthy cardiac cycle.
(2) **Number of Ratios Close to 1:2**: 4, These ratios suggest that systole lasted about half the time of diastole. This balance is consistent with a regular heart rhythm.
(3) **Total Number of Ratios**: A total of 8 ratios were computed from the 9 intervals, showing the alternating pattern of systole and diastole phases.

9. Analysis of Heart Rate Variability Metrics

Heart Rate Variability (HRV) analysis involves assessing the time intervals between successive heartbeats, which is essential for understanding autonomic nervous system function and cardiovascular health. In the context of the analysis, the following metrics are calculated:

(1) **RR Intervals**: The intervals between consecutive S1 peaks (indicative of heartbeats) are calculated to form the RR intervals. These intervals are essential for deriving instantaneous heart rates.

$$\text{RR Interval}_n = \frac{\text{S1 Peak}_{n+1} - \text{S1 Peak}_n}{f_s}$$

(2) **Instantaneous Heart Rate**: The instantaneous heart rate (HR) is computed using the RR intervals. It is defined as:

$$\text{Instantaneous HR} = \frac{60}{\text{RR Interval}} \quad \text{(in BPM)}$$

(3) **Average Heart Rate**: The average heart rate is derived by taking the mean of the instantaneous heart rates.
(4) **HRV Metrics**:
    - **SDNN** (Standard Deviation of NN intervals): It quantifies the variability of RR intervals, reflecting overall heart rate variability.
    - **RMSSD** (Root Mean Square of Successive Differences): This metric focuses on short-term variations between successive RR intervals, indicating the parasympathetic regulation of heart rate.
    - **Minimum and Maximum HR**: These values provide insights into the heart rate range during the analysis.
    - **HR Standard Deviation**: It reflects the variability of instantaneous heart rates.
(5) **Confidence Intervals**: The 95% confidence interval for the mean heart rate is calculated, providing a statistical measure of reliability for the mean value:

$$\mathrm{CI}_{HR} = \mathrm{SEM} \times 1.96$$

where SEM is the standard error of the mean calculated from the instantaneous heart rates.

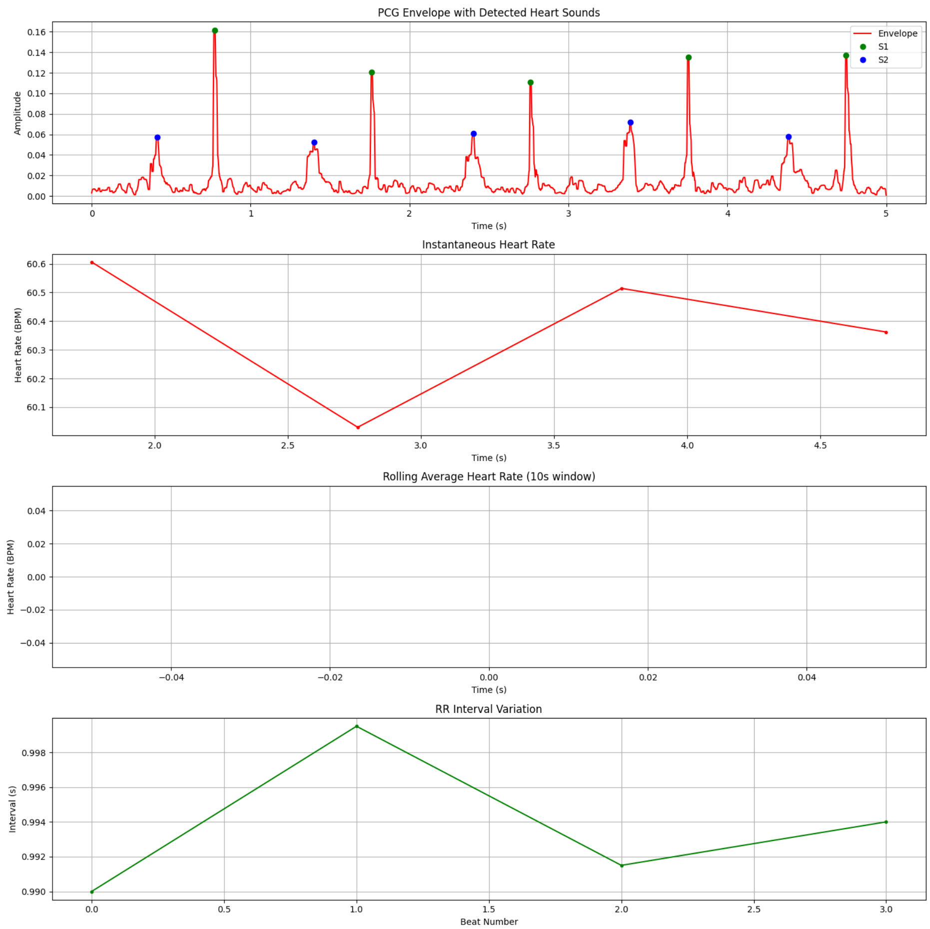

Fig. 13. Heart Rate Variability Metrics over time

The following results summarize the heart rate and HRV analysis:

Heart Rate and HRV Analysis:

- Mean Heart Rate: 60.4 BPM
- Minimum Heart Rate: 60.0 BPM
- Maximum Heart Rate: 60.6 BPM
- Heart Rate Standard Deviation: 0.2 BPM

Heart Rate Variability Metrics:

- SDNN (Standard Deviation of RR intervals): 3.6 ms
- RMSSD (Root Mean Square of Successive Differences): 7.3 ms

Heart Rate 95% Confidence Interval: 60.4 ± 0.2 BPM

By analyzing these HRV metrics, clinicians and researchers can gain insights into cardiac autonomic control and potential cardiovascular issues, thus aiding in diagnostic processes and monitoring health status.