Oubrid
Anis
B 2

Devoir OOE

Exercice 23

1) calculer A :

On a $\vec{E_1} = \vec{E_2} \Rightarrow 60\, e^{j(wt)} + 20\, e^{j wt} = A$

$A = 60 + 20 = 80$

2) calculer $\vec{H_1}$ et $\vec{H_2}$:

$$rot\, \vec{E_1} = -\frac{\partial H_1}{\partial t} u_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{x_1} & 0 & 0 \end{vmatrix}$$

$$= \frac{\partial\left(60\, e^{j(wt-5z)} + 20\, e^{j(wt+5z)}\right)}{\partial z} \vec{j}$$

$$rot(E_1) = \left(100\, e^{j(wt+5z)} - 300\, e^{j(wt-5z)}\right) \vec{j}$$

$$\vec{H_1} = -\frac{1}{u_0} \int 100\, e^{j(wt+5z)} - 300\, e^{j(wt-5z)}\, dt\, \vec{j}$$

$$\boxed{\vec{H_1} = \left(\frac{300}{jwu_0} e^{j(wt+5z)} - \frac{100}{jwu_0} e^{j(wt-5z}\right)\vec{j}}$$

$$\bullet\ rot\, \vec{E_2} = -\frac{\partial H_2}{\partial t} u_0$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{x2} & 0 & 0 \end{vmatrix} = -400\, e^{j(wt\,5z)} \vec{j}$$

$$\vec{H_2} = -\frac{1}{u_0} \int -400\, e^{j(wt-5z)} \vec{j}\, dt = \frac{400}{u_0}\left[\frac{e^{j(wt-5z)}}{jw}\right]\vec{j}$$

$$H_2 = \frac{400}{j\omega\mu_0} e^{j(\omega t - \beta z)} \vec{j}$$

Exercice 3 :

1) $\vec{D} = \varepsilon \vec{E}$ et $\vec{E} = \begin{cases} E_r = -j \frac{\cos\theta}{2\pi\varepsilon} \left[\frac{1}{r^2} - j\frac{k}{r^3} \right] e^{j(\omega t - kr)} \\ E_\theta = -j \frac{\sin\theta}{4\pi\varepsilon} \left[\frac{1}{r^3} + \frac{jk}{r^2} - \frac{k}{r} \right] e^{j(\omega t - kr)} \end{cases}$

$$\Rightarrow \vec{D} = \begin{cases} D_r = -j \frac{\cos\theta}{2\pi} \left[\frac{1}{r^2} - j\frac{k}{r^3} \right] e^{j(\omega t - kr)} \\ D_\theta = j \frac{\sin\theta}{4\pi} \left[\frac{1}{r^3} + \frac{jk}{r^2} - \frac{1}{r} \right] e^{j(\omega t - kr)} \end{cases}$$

2) $\text{rot}\,\vec{E} = \begin{pmatrix} \vec{e_r} & \vec{e_\theta} & \vec{e_\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ E_r & E_\theta & 0 \end{pmatrix}$

$$= \Big(-j \frac{\sin\theta}{4\pi\varepsilon} \left(\frac{-3}{r^4} - \frac{2kj}{r^3} + \frac{1}{r^2} \right) e^{j(\omega t - kr)}$$

$$+ jk \frac{\sin\theta}{4\pi\varepsilon} \left[\frac{1}{r^3} + \frac{jk}{r^2} - \frac{1}{r} \right] e^{j(\omega t - kr)}$$

$$+ j \frac{2\sin\theta}{4\pi\varepsilon} \left(\frac{1}{r^2} - j\frac{k}{r^3} \right) e^{j(\omega t - \cancel{k}r)} \Big) \vec{e_\varphi}$$

$$= \frac{j\sin\theta}{4\pi\varepsilon} e^{j(\omega t - kr)} \Big(\frac{3}{r^4} + \frac{2kj}{r^3} + \frac{1}{r^2} + \frac{k}{r^3} +$$

$$+ \frac{2}{r^2} - \frac{jk}{r} - \frac{2jk}{r^3} \Big) \vec{e_\varphi}$$

$\cancel{\text{rot}\,\vec{E}} = \cancel{\dfrac{-1}{u_0}}$

$\text{rot}\,\vec{H} = -\dfrac{1}{u_0}\int \dfrac{j\, i\, \theta}{4\pi\varepsilon} e^{j(wt-Kt)} \cdot i\left(\dfrac{3}{r^4} - \dfrac{K}{r^3} + \dfrac{jK^2+1}{r^2} - \dfrac{jK}{r}\right) e\,\vec{p}\, \partial t$

$\vec{H} = \dfrac{-j\, i\, \theta}{u_0 4\pi\varepsilon w}\left(\dfrac{3}{r^4} + \dfrac{K}{r^3} + \dfrac{jK^2 + j}{r^2} - \dfrac{jK}{r}\right)\vec{e_\theta}$

Ex 043

- Le vecteur de poynting :

$\vec{\Pi} = \vec{E} \wedge \vec{H} = \vec{E} \wedge \dfrac{\vec{B}}{u_0}$

- on calcule $\vec{B}$:

~~on~~ ~~rot~~

$\text{rot}\,\vec{E} = -\dfrac{\partial B}{\partial t}$, $\text{rot}\,\vec{E} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{pmatrix}$

$\text{rot}\,\vec{E} = 0\vec{i} + \dfrac{\partial\left(10^{-t} e^{wt-6z}\right)}{\partial z}\vec{j} + 0\vec{k}$

$= \dfrac{-3\times 10^{-6}}{\varepsilon_0 j w} e^{wt-6z}\vec{j}$

d'autre part on a $\text{rot}\,\vec{E} = \dfrac{-\partial B}{\partial t}$

$\Rightarrow \dfrac{\partial B}{\partial t} = \dfrac{3\times 10^{-6}}{\varepsilon_0 j w} e^{wt-6z}\vec{j}$

$\Rightarrow \vec{B} = \int \dfrac{3\times 10^{-6}}{\varepsilon_0 j w} e^{wt-6z}\partial t$

$$\vec{B} = \frac{3 * 10^{-6}\, e^{\omega t - 6z}}{\varepsilon_0\, j\, \omega^2}\, \vec{j}$$

alors $\vec{\Pi} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ E_x & 0 & 0 \\ 0 & \frac{B_y}{\mu_0} & 0 \end{vmatrix} = E_x \frac{B_y}{\mu_0} \vec{k}$

$$= \frac{3 \times 10^{-12}\, e^{2(\omega t - 6z)}}{2(8{,}85 \times 10^{-12})^2 \times 4\pi \times 10^{-7}\, j^2 \omega^3}\, \vec{k}$$

$$\cancel{\vec{\Pi} = 11 \times 10^{10}\, e^{2(\omega t - 6z)}\, \vec{k}}$$

$$\boxed{\vec{\Pi} = \frac{11 \times 10^{10}\, e^{2(\omega t - 6z)}}{4\pi\, j^2 \omega^3}\, \vec{k}}$$