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**Titre :**  
Wilcoxon Test.

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## **Introduction:**

The Wilcoxon signed-rank test is a non-parametric statistical test used to compare two related samples, such as paired measurements from the same subjects under different conditions or before and after interventions. It's a robust alternative to the paired t-test, especially when the data violate the assumptions of normality or when the sample size is small.

Instead of comparing means like the t-test, the Wilcoxon test compares the medians of the paired differences. It works by ranking the absolute differences between paired observations and then summing the ranks of the positive or negative differences, depending on the direction of the hypothesis being tested. The resulting statistic is then compared to critical values from the standard normal distribution to determine statistical significance.



The Wilcoxon test is widely used in various fields, including medicine, psychology, biology, and social sciences, where it's important to assess differences between paired observations while avoiding reliance on strict distributional assumptions. It's named after its developer, Frank Wilcoxon, who introduced it in 1945 as a robust alternative to parametric tests.

### **History:**

The Wilcoxon signed-rank test is named after its creator, Frank Wilcoxon, an American statistician. He introduced the test in a paper titled "Individual Comparisons by Ranking Methods" published in 1945. The motivation behind developing this test was to provide a statistical method for comparing two related samples without making assumptions about the underlying distribution.

Wilcoxon's approach was innovative because it focused on ranking the absolute differences between paired observations rather than directly comparing means or medians. This made the test robust against deviations from normality and applicable to a wide range of data types.



Since its introduction, the Wilcoxon signed-rank test has become a fundamental tool in statistics, particularly in cases where the data violate the assumptions of parametric tests like the t-test. It's commonly used in fields such as medicine, psychology, biology, and social sciences for analyzing paired or matched data, including before-and-after studies, clinical trials, and behavioral experiments.

Over the years, variations and extensions of the Wilcoxon test have been developed to address different research questions and scenarios, further cementing its importance in statistical analysis. Today, it remains a staple in the toolkit of statisticians and researchers, offering a reliable method for comparing paired samples without relying on stringent distributional assumptions.

### **Wilcoxon test's domains of application:**

The Wilcoxon signed-rank test, along with its variations like the Wilcoxon rank-sum test (also known as the Mann-Whitney U test), finds applications across various domains where statistical comparisons between paired or independent samples are required. Some key domains of application include:

**1. Biomedical Research:** In medical and clinical studies, researchers use the Wilcoxon test to analyze data from before-and-after studies, clinical trials, and experiments involving paired measurements. It's used to assess the efficacy of treatments, compare patient outcomes, and evaluate the significance of interventions.

**2. Psychology and Behavioral Sciences:** Researchers in psychology often use the Wilcoxon test to analyze data from experiments involving human subjects, such as cognitive tests, surveys, and behavioral studies. It's useful for comparing pre- and post-intervention measurements, as well as for assessing differences between groups.

**3. Environmental Studies:** Environmental scientists use the Wilcoxon test to analyze data related to environmental impact assessments, pollution studies, and ecological monitoring. It's employed to compare measurements taken before and after interventions or to evaluate differences between treatment and control groups.

**4. Market Research and Business Analytics:** In market research and business analytics, the Wilcoxon test can be used to analyze consumer preferences, assess the effectiveness of marketing strategies, and evaluate changes in customer behavior over time.

**5. Social Sciences:** Researchers in sociology, economics, and other social sciences use the Wilcoxon test to analyze survey data, observational studies, and experiments involving paired or independent samples. It helps in comparing attitudes, behaviors, and outcomes across different groups or time points.

**6. Engineering and Quality Control:** Engineers and quality control professionals use the Wilcoxon test to analyze data from manufacturing processes, product testing, and reliability studies. It's employed to assess the impact of process improvements, compare product performance, and identify sources of variation.

Overall, the Wilcoxon test's versatility and robustness make it applicable in a wide range of research and practical contexts, particularly when the data do not meet the assumptions of parametric tests or when analyzing ordinal or non-normally distributed data.

### **Objectif of using Wilcoxon Test:**

The primary objective of using the Wilcoxon signed-rank test is to determine whether there is a statistically significant difference between paired samples or related measurements. Specifically, the test aims to:



**1. Assess Differences:** Determine whether there is a significant difference between two related groups or conditions. This could include comparing pre- and post-intervention measurements, before-and-after observations, or paired measurements from the same subjects.

**2. Non-Normal Data:** Handle situations where the data do not meet the assumptions of parametric tests, such as the paired t-test, due to violations of normality or other distributional assumptions. The Wilcoxon test is robust against such violations and provides reliable results even with non-normally distributed data.

**3. Ordinal Data:** Analyze ordinal or ranked data, where the observations are ranked or ordered but do not necessarily follow a normal distribution. The Wilcoxon test is suitable for analyzing ranked data without requiring interval-level measurement assumptions.



**4. Small Sample Sizes:** Perform statistical comparisons with small sample sizes, where the assumptions of parametric tests may not hold. The Wilcoxon test is less sensitive to sample size and can provide valid results with relatively few observations.

**5. Increased Power:** In cases where the data are not strictly normal but do not exhibit extreme deviations, the Wilcoxon test may offer increased statistical power compared to parametric tests. This can lead to more reliable detection of true differences between groups or conditions.

Overall, the Wilcoxon signed-rank test is a versatile and robust statistical method that allows researchers to compare paired samples or related measurements in various scenarios, without relying on stringent distributional assumptions. Its flexibility and applicability make it a valuable tool in fields ranging from biomedical research and psychology to engineering and social sciences.

**Graphic presentation with an example:**

Example:

A researcher wants to determine if a new teaching method improves student performance. The researcher selects a random sample of 15 students and measures their scores before and after the teaching method is implemented. The null hypothesis is that there is no difference in the students' scores before and after the teaching method, while the alternative hypothesis is that there is a difference.



Here are the pairs of scores (before and after) for each student:

Student:	Before:	After:

1	65	70
2	72	75
3	68	70
4	80	82
5	78	80
6	63	65
7	70	72
8	75	77
9	82	85
10	68	70
11	72	74
12	85	88
13	76	78
14	70	73
15	74	76

Step 1: Calculate the differences between paired observations:

Student:	Difference:
1	5
2	3
3	2
4	2
5	2
6	2
7	2
8	2
9	3
10	2
11	2
12	3
13	2
14	3
15	2

Step 2: Rank the absolute values of the differences:

Student:	Difference: Absolute	Difference:	Rank:
1	5	5	15
2	3	3	12.5
3	2	2	6.5
4	2	2	6.5
5	2	2	6.5
6	2	2	6.5
7	2	2	6.5
8	2	2	6.5
9	3	3	12.5
10	2	2	6.5
11	2	2	6.5
12	3	3	12.5
13	2	2	6.5
14	3	3	12.5
15	2	2	6.5



(Note: In this example, there are ties in the absolute differences, so we assign the average rank to tied observations.)

Step 3: Calculate the sum of positive ranks ( $W^+$ ) and the sum of negative ranks ( $W^-$ ):

$$W^+ = 12.5 + 12.5 + 12.5 + 12.5 = 50$$

$$W^- = 6.5 + 6.5 + 6.5 + 6.5 + 6.5 + 6.5 + 6.5 + 6.5 = 52$$

Step 4: Calculate the test statistic ( $T$ ) using the smaller of  $W^+$  and  $W^-$ :

$$T = \min(W^+, W^-) = \min(50, 52) = 50$$

Step 5: Determine the critical value for the test statistic based on the sample size and desired significance level. You can refer to statistical tables or use software.

Suppose we choose a significance level of 0.05 (5%). For this sample size ( $n = 15$ ) and significance level, the critical value for the Wilcoxon signed-rank test is 38.

Step 6: Compare the test statistic to the critical value:

Since  $T$  (50) is greater than the critical value (38), we fail to reject the null hypothesis. There is insufficient evidence to conclude that the teaching method significantly improved student performance based on this sample.

Please note that in practice, it is essential to consider the assumptions and limitations of the Wilcoxon signed-rank test and to interpret the results in the context of the specific study.

### **Conclusion :**

In conclusion, the Wilcoxon signed-rank test is a valuable statistical tool for comparing paired samples, particularly when the data do not meet the assumptions of parametric tests. In our example, we applied the Wilcoxon signed-rank test to weight measurements before and after an intervention. The analysis revealed a significant difference in weights before and after the intervention, indicating that the intervention had a measurable effect on participants' weights.

This test provides a robust method for assessing the significance of differences between paired samples without requiring stringent distributional assumptions. By considering the test statistic ( $T$ ) and p-value, researchers can make informed conclusions about the effectiveness of interventions or treatments.

Overall, the Wilcoxon signed-rank test offers a reliable approach for analyzing paired data and drawing meaningful conclusions about the underlying population from which the samples were drawn.