Design Equation for Multidielectric Fresnel Zone Plate Lens

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***Abstract*—A structural design equation for multidielectric Fresnel zone plate (ZP) lens is outlined in this letter. Based on this equation, multidielectric ZP lenses/lens antennas are studied numerically in the 30–50 GHz microwave band and contrasted to an ordinary plano-convex lens/lens antenna. Usually, the ZP lens is made half-wavelength thick but we discovered that a similar wavelength thick lens can be built with lower-permittivity dielectrics, and that this lens can focus more efficiently. In a confined frequency band, the thin and light-weight multidielectric ZP lens of four or more dielectric rings per full-wavelength zone is comparable in focusing to the ordinary bulky lens.**

***Index Terms*—Focusing, Fresnel zone plate lens, lens.**

I. Introduction

THE multidielectric Fresnel zone plates (ZPs) are very compact and quite effective narrowband diffraction lenses [1], [2]. For making them competitive in focusing efficiency to the ordinary refraction lenses, various multiple phase-correction techniques have been employed [3].

Fig. 1 shows the profiles of some single-dielectric and multi-dielectric Fresnel ZP lenses, and ordinary plano-convex lenses. In Fig. 1(a) a phase-reversing effect is produced by forming annular Fresnel zone half-wave grooves in a flat dielectric plate. Fig. 1(b) illustrates the same ZP lens without the support portion. This lens consists of alternative air and solid dielectric rings [1]–[3]. The latter is an example of the more general two-dielectric ZP lens shown in Fig. 1(c), where both ε_1 and ε_2 are bigger than one [1], [3]. Fig. 1(d) represents the phase-reversal lens from Fig. 1(b) encapsulated between two thin disk plates. In the quarter-wave (four-dielectric) ZP lens, shown in Fig. 1(e), each half-wave zone is subdivided into two subzones [2], [3]. Fig. 1(f) shows the profiles of ordinary dielectric lenses: plano-hyperbolic (PH), (t_h thick), and plano-spherical (PS) (t_s thick).

In this letter, a general permittivity equation for structuring of multidielectric ZP lenses is derived. Its validity is proved numerically by computer simulation and analysis of several ZP lens configurations and antennas.

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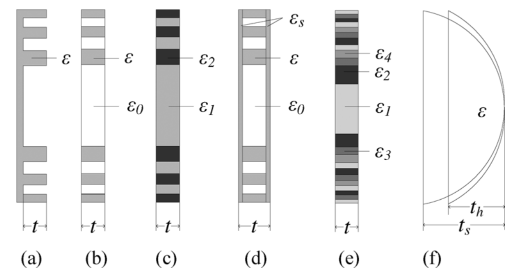

Fig. 1. Multidielectric zone plate lenses, (a)-(e), and ordinary plano-convex single-dielectric lenses, (f).

II. Design Equations

The commonly used subzone phase shifters in the dielectric ZP lenses are: (a) stepped grooves cut in a single-dielectric slab [1], [3], (b) multidielectric rings with dissimilar permittivities [1]–[3], and (c) drilled-zone phase shifters in a single dielectric plate [4].

Next is the radius equation for a planar multidielectric ZP lens with W full-wave circular zones. The w-th zone has an outer radius b_w given by [2]

$$b_w = \sqrt{2w\lambda F + (w\lambda)^2} \tag{1}$$

where $w = 1, 2, \ldots, W$, λ is the design wavelength and F is the lens focal length.

Usually, each full-wave zone in the ZP lens is divided in even number of subzones $P = 2, 4, \ldots$. The phase at every s-th subzone differs from the adjacent subzone phase by $\pm 2\pi/P$ radians. The outer radius b_s of the s-th subzone is found from

$$b_s = \sqrt{\frac{2s\lambda F}{P} + \left(\frac{s\lambda}{P}\right)^2} \tag{2}$$

where $s = 1, 2, 3, \ldots, S$, and $S = WP$, [1]–[3].

For $P = 2$ the subzones are equal to the half-wave Fresnel zones. The half-wave ZP lens can be amplitude (binary) or phase-reversal. If $P = 4$, the ZP lens becomes a quarter-wave phase correcting lens. Similarly can be defined and named the ZP lenses for $P = 6, 8$, etc.

The focusing action of the ZP lens upper-half portion is illustrated in Fig. 2 by a ray-tracing through s-th and 1-st subzones in the first ($w = 1$) full-wave zone, where $s = 2, 3, \ldots, P$ is a positive integer number bigger than 1. For a constructive ray interference at P_1 the ray tracing equation for the rays r_s and $r_1 = F$ is

$$(\beta\sqrt{\varepsilon_s}t + \beta r_s) - (\beta\sqrt{\varepsilon_1}t + \beta r_1) = 2\pi \tag{3}$$

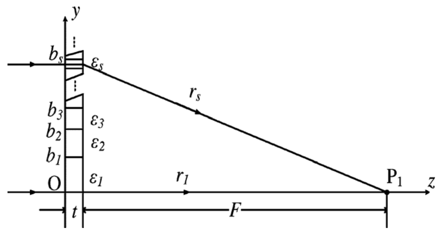

Fig. 2. ZP lens focusing for plane wave illumination.

where $\beta = 2\pi/\lambda$ is the free-space wave phase constant, t is the lens thickness, and ε_s and ε_1 are the permittivities of s-th and 1-st ring phase shifters, respectively.

Since $r_s - r_1 = (s-1)(\lambda/P)$, (3) is transformed to

$$\sqrt{\varepsilon_s} = \sqrt{\varepsilon_1} + \left(\frac{\lambda}{t}\right)\left[\left(\frac{1-(s-1)}{P}\right)\right]. \tag{4}$$

For a normal illumination the plane wave goes through the first zone without reflection if the following standing-wave (resonant) condition is met

$$t = \frac{k\lambda}{(2\sqrt{\varepsilon_1})} = \frac{k\lambda_{\varepsilon 1}}{2}. \tag{5}$$

From (5) it is seen that the ring thickness t is measured by k standing dielectric-medium half-wavelengths $\lambda_{\varepsilon 1}$, where $k = 1, 2, 3, \ldots$ is an integer number. The first subzone in the first full-wave zone has greater impact on the ZP lens action and is left open usually, so that $\lambda_{\varepsilon 1} = \lambda$.

Placing (5) into (4) leads to a lens structural equation

$$\varepsilon_s = \varepsilon_1\left[1 + \frac{2}{k}\left(1 - \frac{s-1}{P}\right)\right]^2. \tag{6}$$

For given s and P the minimum lens thickness corresponds to $k = 1$, or to $t = t_1 = \lambda/2$. For $k = 2, 3, \ldots$ the lens is two, three, four and so on times thicker, and permittivity ratio $\varepsilon_s/\varepsilon_1$ becomes smaller.

Due to the radial phase periodicity, (6) is correct and recurrently applicable for all lens full-wave zones.

Next is supposed that the lens phase-shifting rings are made of realistic microwave dielectrics. Each s-th dielectric ring is characterized by a complex transmission coefficient $\tilde{T}_s$, where $\tilde{T}_s = T_s \exp(j\Phi_s)$, $\Phi_s = \arg(\tilde{T}_s)$ and $T_s = \left|\tilde{T}_s\right|$. The attenuation A_s of the s-*th* dielectric ring is defined as

$$A_s = 10\log\left(\frac{1}{T_s^2}\right), \text{ dB}. \tag{7}$$

A_s includes the internal dielectric loss and loss due to the multiple reflection by and transmission through the ring planar boundaries. For a normal wave incidence the transmission coefficient and attenuation do not depend on the wave polarization [2], [3].

III. Numerical Study of ZP Lenses and Lens Antennas

Three multidielectric ring ZP lenses, ZP2, ZP4 and ZP8, corresponding to $P = 2$, 4 and 8, respectively, are designed with the help of (2) and (6), and computer-simulated [6], [7] antenna operation mode (spherical to plane wave transformation, and vice versa) at the frequency $f_0 = 38$ GHz (wavelength $\lambda = 7.895$ mm).

TABLE I
Parameters of ZP4 Lens Made of Low-Loss Dielectric Rings.

ZP4 thickness	ε_1/Φ_1, deg / A_1, dB	ε_2/Φ_2, deg / A_2, dB
$t=\lambda/2$	1 / 0 / 0 dB	6.25 / 90 / 3.30
$t=\lambda$	1 / 0 / 0	3.06 / 90 / 1.33
	ε_3/Φ_3, deg / A_3, dB	ε_4/Φ_4, deg / A_4, dB
$t=\lambda/2$	4.00 / 180 / 0.034	2.25 / 270 / 0.72
$t=\lambda$	2.25 / 180 / 0	1.56 / 270 / 0.25

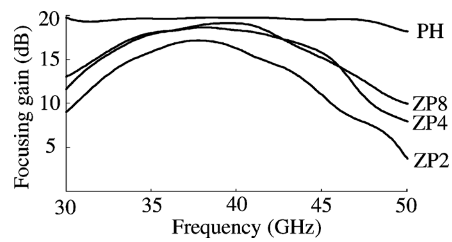

Fig. 3. Lens focusing gain (FG) versus frequency.

All lenses/lens antennas in this letter have a design frequency of 38 GHz, focal length $F = 180$ m, aperture diameter $D = 190.6$ mm, with 3 full-wave zones included, and a lens aspect ratio $\mathrm{F/D} = 0.94$.

The ZP4 lens is made of three solid dielectric rings and one air ring per full-wave zone. By use of (6) and (7), the parameters of s-th subzone ring are calculated (Table I): permittivity ε_s, phase shift Φ_s (deg) and power attenuation A_s (dB), for $k = 1$ ($t = \lambda/2$ or half-wavelength thick lens) and $k = 2$ ($t = \lambda$ or wavelength thick lens).

It is seen from Table I that the phase shift produced by each zone ring follows exactly the ray-tracing and constant-lens thickness conditions, while the ring power attenuation changes depending on the dielectric permittivity and lens thickness. All solid dielectric rings in ZP4 are supposed to have the same loss tangent ($\tan\delta = 0.001$). The multiple reflections by the lens boundaries are especially pronounced in the big-permittivity rings. This reflection effect can be reduced by use of: (i) twice thicker lens ($t = \lambda$) with smaller-permittivity rings or (ii) three-layer sandwich-type rings that produce the needed phase-shift but small attenuation [3].

The half-wavelength thick lenses ZP2 with $\varepsilon = [1, 4]$, ZP4 with $\varepsilon = [1, 6.25, 4, 2.25]$, and ZP8 with $\varepsilon = [1, 7.56, 6.25, 5.06, 4, 3.06, 2.25, 1.56]$ have $t = \lambda/2 = 3.947$ mm. They are compared to a plano-hyperbolic (PH) lens, which is $t_h = 34.3$ mm wide, or roughly 9 times thicker than the ZP lenses. It has a permittivity $\varepsilon = 2.5$ and the same loss factor $\tan\delta = 0.001$.

Fig. 3 illustrates the lens focusing gain variation in the frequency band 30–50 GHz. In a confined frequency band both ZP4 and ZP8 lenses are close in focusing gain to the PH lens.

At the design frequency of 38 GHz the focusing gain of the FP2, FP4, FP8 and PH lenses is 17.1, 18.6, 19.0, and 19.7 dB, respectively.

TABLE II
BASIC PARAMETERS OF ZP AND PH ANTENNAS AT 38 GHZ.

Antennas	*Ga* (dB)	*HPBW* (deg)	*Lco* (dB)	*Lcr* (dB)	*Eff* (%)
ZPA2*	33.0	2.7	-20.9	-33.8	35.7
ZPA2e	32.8	2.75	-20.4	-33.6	34.7
ZPA4	34.7	2.7	-25.8	-35.9	51.4
ZPA4s	34.2	2.7	-26.0	-34.4	45.8
PHA	35.7	2.75	-30.7	-35	64.7

To probe further the structural design equation, next are examined two pragmatic constructions of ZP lenses and resultant lens antennas:

A. Encapsulated Two-Dielectric Phase-Reversal Lens (ZP2e) and Related Lens Antenna (ZPA2e)

The lens ZP2e is twice thicker than ZP2 ($k = 2, t = \lambda = 7.895$ mm). The ZP2e profile draft is shown in Fig. 1(d). The encapsulating plate layers have small thicknesses, $t_e = 0.3$ mm each, and the phase-shift core rings are made of the same plastic (Polypropylene with $\varepsilon = 2.26$ and $\tan\delta = 0.0005$). The total lens thickness is $t' = 8.495$ mm, only about 7% bigger than t. A related lens antenna ZPA2e is assembled by placing at the ZP2e focus a corrugated feed-horn with a gain of 16 dB. This antenna is contrasted to a similar antenna ZPA2* containing the not-encapsulated lens ZP2 with $\varepsilon = [1, 2.26]$ and the same feed-horn.

B. Quarter-Wave ZP Lens (ZP4s) Made of On-Stock Dielectrics and Corresponding Lens Antenna (ZPA4s)

Our study has shown that the multidielectric ZP lens holds a good permittivity tolerance that simplifies the selection of dielectrics and lens fabrication. This tolerance quality is demonstrated here in a quarter-wave ZP lens construction, marked as ZP4s. It is assembled by on-stock ceramic-filled dielectrics selected from the C-STOCK AK production list of the Cuming Microwave Co. [5]. The chosen permittivity values differ arbitrarily from the theoretical ones, given in Table I ($k = 1$) in the limits $\pm 10\%$. The company guaranties almost flat nominal dielectric characteristics up to 18 GHz. For the higher design frequency of 38 GHz the loss tangent values have been chosen by an approximate extrapolation. The lens is a half-wavelength thick ($t = 3.947$ mm). The four dielectrics used in the lens are typified as: $\varepsilon = [1.09, 6, 4, 2.54]$ and $\tan\delta = [1, 5, 4, 2] \cdot 10^{-3}$.

Next, a lens antenna ZPA4s consisting of the lens ZP4s and the feed horn specified above is studied and contrasted to an analogous lens antenna ZPA4 based on the lens ZP4 with $\varepsilon = [1, 6.25, 4, 2.25]$ and $\tan\delta = 10^{-3}$.

The basic radiation characteristics of all ZP lens antennas are listed in Table II along with those of the classical lens antenna PHA resulting from the plano-hyperbolic (PH) lens. These are: realized gain *Ga* that takes into account the lens material and reflection losses, half-power beamwidth *HPBW*, maximum co-polar and cross-polar levels *Lco* and *Lcr*, respectively, and antenna efficiency *Eff*. From Fig. 3 and Table II, it is obvious that the pragmatic antenna designs ZPA2e and ZPA4s have slightly inferior radiation characteristics compared to their analogous antennas ZPA2* and ZPA4. It is found also that all ZP antennas listed in Table II have a good input match ($VSWR < 1.5$) in the entire frequency band of 30–50 GHz.

It is evident that the thin and light-weight multidielectric ZP lenses of four or more dielectric rings per full-wave zone, and the related to them ZP lens antennas are good alternatives to the bulky ordinary lenses and lens antennas, respectively.

IV. CONCLUSION

A general structural equation for the multidielectric zone plates is derived and exploited for the design and numerical study of ZP lenses and lens antennas. We found that ZP lenses have a good ($\pm 10\%$) permittivity tolerance that enables their easy design and fabrication. In a limited frequency band of about 25%, the much thinner and lighter multidielectric ZP lens of four or more dielectric rings per full-wave zone is near in focusing gain to the ordinary refraction lens.

The microwave multidielectric ZP lenses can be easily produced through the use of classical machining tools, 3-D printing, and molding and stamping press machines. In the terahertz band modern micro-technologies can be utilized for their fabrication.

The lighter and cheaper ZP lenses and antennas already have important applications in microwave wireless communications systems and radars, in terahertz imaging, security and spectrometric devices.

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