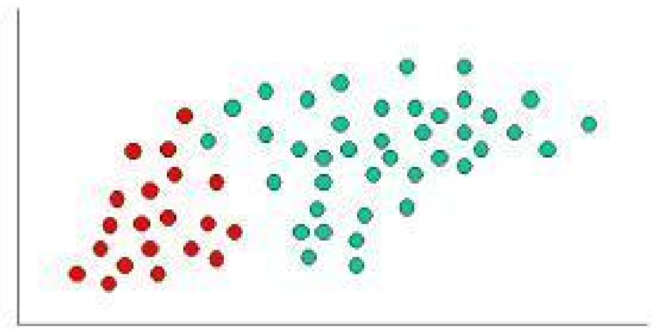
ASSIGNMENT NO. 4

AIM: Assignment on Naïve Bayes.

PREREQUISITE: Python programming

THEORY:

The Naive Bayes Classifier technique is based on the so-called Bayesian theorem and is particularly suited when the dimensionality of the inputs is high. Despite its simplicity, Naive Bayes can often outperform more sophisticated classification methods.



To demonstrate the concept of Naïve Bayes Classification, consider the example displayed in the illustration above. As indicated, the objects can be classified as either GREEN or RED. Our task is to classify new cases as they arrive, i.e., decide to which class label they belong, based on the currently exiting objects.

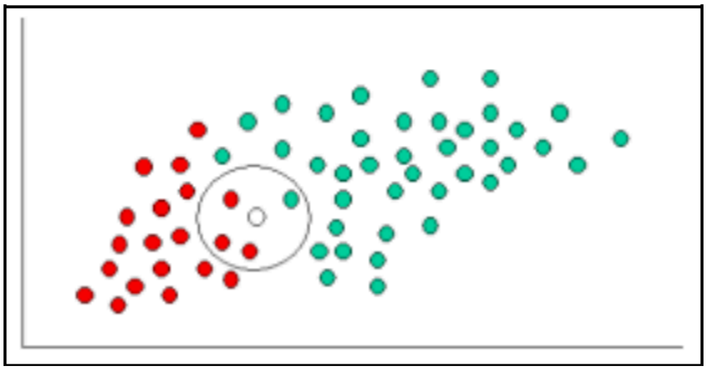
Since there are twice as many GREEN objects as RED, it is reasonable to believe that a new case (which hasn't been observed yet) is twice as likely to have membership GREEN rather than RED. In the Bayesian analysis, this belief is known as the prior probability. Prior probabilities are based on previous experience, in this case the percentage of GREEN and RED objects, and often used to predict outcomes before they actually happen.

Thus, we can write:

Since there is a total of 60 objects, 40 of which are GREEN and 20 RED, our prior probabilities for class membership are:

$$\text{Prior probability for GREEN} \propto \frac{40}{60}$$

$$\text{Prior probability for RED} \propto \frac{20}{60}$$



Having formulated our prior probability, we are now ready to classify a new object (WHITE circle). Since the objects are well clustered, it is reasonable to assume that the more GREEN (or RED) objects in the vicinity of X, the more likely that the new cases belong to that particular color. To measure this likelihood, we draw a circle around X which encompasses a number (to be chosen a priori) of points irrespective of their class labels. Then we calculate the number of points in the circle belonging to each class label. From this we calculate the likelihood:

$$\text{Likelihood of } X \text{ given GREEN} \propto \frac{\text{Number of GREEN in the vicinity of } X}{\text{Total number of GREEN cases}}$$

$$\text{Likelihood of } X \text{ given RED} \propto \frac{\text{Number of RED in the vicinity of } X}{\text{Total number of RED cases}}$$

From the illustration above, it is clear that Likelihood of X given GREEN is smaller than

Likelihood of X given RED, since the circle encompasses 1 GREEN object and 3 RED ones. Thus:

$$\text{Probability of } X \text{ given GREEN} \propto \frac{1}{40}$$

$$\text{Probability of } X \text{ given RED} \propto \frac{3}{20}$$

Although the prior probabilities indicate that X may belong to GREEN (given that there are twice as many GREEN compared to RED) the likelihood indicates otherwise; that the class membership of X is RED (given that there are more RED objects in the vicinity of X than GREEN). In the Bayesian analysis, the final classification is produced by combining both sources of information, i.e., the prior and the likelihood, to form a posterior probability using the so-called Bayes' rule (named after Rev. Thomas Bayes 1702-1761).

$$\text{Posterior probability of } X \text{ being GREEN} \propto$$
$$\text{Prior probability of GREEN} \times \text{Likelihood of } X \text{ given GREEN}$$
$$= \frac{4}{6} \times \frac{1}{40} = \frac{1}{60}$$
$$\text{Posterior probability of } X \text{ being RED} \propto$$
$$\text{Prior probability of RED} \times \text{Likelihood of } X \text{ given RED}$$
$$= \frac{2}{6} \times \frac{3}{20} = \frac{1}{20}$$

Finally, we classify X as RED since its class membership achieves the largest posterior probability.

Note. The above probabilities are not normalized. However, this does not affect the classification outcome since their normalizing constants are the same.

Technical Naïve Bayes

In the previous section, we provided an intuitive example for understanding classification using Naive Bayes. In this section are further details of the technical issues involved. Naive Bayes classifiers can handle an arbitrary number of independent variables whether continuous or categorical. Given a set of variables, X = {x1,x2,x...,xd}, we want to construct the posterior probability for the event Cj among a set of possible outcomes C = {c1,c2,c...,cd}. In a more familiar language, X is the predictors and C is the set of categorical levels present in the dependent variable. Using Bayes' rule:

$$p(X \mid C_j) \propto \prod_{k=1}^{d} p(x_k \mid C_j)$$

$$p(C_j \mid x_1, x_2, \ldots, x_d) \propto p(x_1, x_2, \ldots, x_d \mid C_j) p(C_j)$$

where p(Cj | x1,x2,x...,xd) is the posterior probability of class membership, i.e., the probability that X belongs to Cj. Since Naive Bayes assumes that the conditional probabilities of the independent variables are statistically independent we can decompose the likelihood to a product of terms:

and rewrite the posterior as:

$$p(C_j \mid X) \propto p(C_j) \prod_{k=1}^{d} p(x_k \mid C_j)$$

Using Bayes' rule above, we label a new case X with a class level Cj that achieves the highest posterior probability.

Although the assumption that the predictor (independent) variables are independent is not always accurate, it does simplify the classification task dramatically, since it allows the class conditional densities p(xk | Cj) to be calculated separately for each variable, i.e., it reduces a multidimensional task to a number of one-dimensional ones. In effect, Naive Bayes reduces a high-dimensional density estimation task to a one-dimensional kernel density estimation. Furthermore, the assumption does not seem to greatly affect the posterior probabilities, especially in regions near decision boundaries, thus, leaving the classification task unaffected.

REFERENCES:

1. Coursera Course on “What is Data Science?” offered by IBM. Available at https://www.coursera.org/learn/what-is-datascience?specialization=ibm-data-science

2. https://www.ibm.com/in-en/topics/decision-trees

CONCLUSION:

The Naïve Bayes classifier is a powerful yet simple probabilistic model based on Bayes' theorem, especially effective for high-dimensional data. Through this assignment, we understood how prior and likelihood probabilities combine to determine the posterior probability, which guides the classification of new data points. Despite its assumption of feature independence—which may not always hold true—Naïve Bayes often performs surprisingly well in real-world scenarios. It is widely used in applications like spam filtering, sentiment analysis, and document classification due to its efficiency and ease of implementation.