

Probability :-

The word probability is commonly used word in our day to day conversation like probability It may rain today

probability I am not attending today's class, The probability we can another name "change".

Sample space.

Let S be the finite set all possible outcomes of the random experiment of the problem is said to be a sample space.

Ex: Tossing of a coin one time, the sample space is given by

R_x = Coin 2 time

$$S = \{H, T\}$$

Event :

A Event E is any subset of sample space

R_x :

For above example H be the one event, now
 T be the one event..

T : event
 H : event

Defination of Probability :

Let ' S ' be the finite sample space and ' E ' be the event. The probability of event E is given by the ratio of event by Sample space.

OR.

The probability of E is given by the ratio of favorable outcomes to the all possible outcomes and is denoted by.

$$P(E) = P(E) = \frac{\text{Event}}{\text{Sample space}} = \frac{O(E)}{O(S)} = \frac{|E|}{|S|} = P$$
$$= \frac{\text{Favorable outcomes}}{\text{All possible outcomes}}$$

A tossing of a coin 2 times, find the probability state

i) Atleast once head appeared in both the toss.

ii) Exactly 2 times tail appeared in the toss

A tossing of a coin 2 times. All possible outcomes (Sample space)

$$S = \{HH, HT, TH, TT\}$$

$$\text{All possible outcomes} = o(s) = |s| = 4$$

Tossing of a

$$o(s) = \text{order of } s$$

$$|s| = \text{cardinality of } s.$$

i) Let E be the event atleast once head appeared in any one of the toss and is given by

$$E = \{HH, HT, TH\}, |E| = 3$$

$$P_r(E) = \frac{|E|}{|s|} = \frac{3}{4}$$

ii) Let E_1 be the event exactly 2 tail appeared in the outcomes and event is given by

$$E_1 = \{TT\}$$

$$|E_1| = 1$$

$$P_r(E_1) = \frac{1}{4}$$

Random variable :- | Stochastic | chance variable

Let S be the sample space a real number " x " is associated with some particular rule on sample space is said to be random variable.

For example,

A tossing of a coin 2 times we can assign the random variables X and Y in the following form.

$$S = \{HH, TT, HT, TH\}$$

X : No of heads turning up.

Outcome	HH	TT	HT	TH
Random Variable X	2	0	1	1

Range of $X = \{0, 1, 2\}$
(Set of all no of x)

Y : No of tail turning up.

Outcome :	HH	TT	HT	TH
Random variable Y :	0	2	1	1

Range of $Y = \{0, 1, 2\}$.

- There are 2 types in random variables
1. Discrete random variable
 2. Continuous random variable

Dis

A random variable can take finite values or countably infinite values such a random variable is said to be discrete random variable.

Tossing of a coin observing the outcome.

Throwing the die observing, the no of faces.

In a mobile phone dialing different no is not a discrete random variable selection.

~~Continuous~~
~~Discrete random variable~~

Discrete probability distribution

Let x_i be the real number associated with a random variable such that another real no $P(x_i)$ which satisfies

i) $P(x_i) \geq 0$

ii) $\sum_{i=1}^n P(x_i) = 1$

Here $[x_i, P(x_i)]$ is said to be discrete probability distribution values on random variable X . and $P(x_i)$ is said to be discrete probability function or discrete density function.

For the set of values $[x_i, P(x_i)]$ we can find mean value

Random

2

$$\text{Mean}(\mu) = \sum_{i=1}^n x_i \cdot P(x_i)$$

$$\text{Variance}(V) = \sum_{i=1}^n (x_i - \mu)^2 P(x_i)$$

$$= \sum_{i=1}^n x_i^2 P(x_i) - \mu^2$$

$$\text{Standard deviation (SD)} : \sigma = \sqrt{V}$$

1) A coin is tossed twice a random variable X represents the no of heads turning up. Find the discrete probability distribution of X also find its mean and variance.

Sample space is given by $S = \{HH, TT, HT, TH\}$
Here X be the random variable the no of Heads

turning up like 2, 0, 1, 1 respectively

$$\begin{aligned} P(X=0) &= \text{No Head appearance} \\ &= P(TT) = \frac{1}{4} \end{aligned}$$

$$P(X=1) = \text{1 time head appearance}$$

$$= P(HT \cup TH) = P(HT) \cup P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X=2) = \text{2 times Head appearance:}$$

$$P(HH) = \frac{1}{4}$$

The discrete probability distribution table is given by

$x_i \sim x$	0	1	2
$p(x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\therefore \text{Mean } (\mu) = \sum_{i=1}^n x_i p(x_i) = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} = 1$$

$$\text{ii) Variable } (V) = \sum_{i=1}^n (x_i - \mu)^2 p(x_i) =$$

$$= (0-1)^2 \times \frac{1}{4} + (1-1)^2 \times \frac{1}{2} + (2-1)^2 \times \frac{1}{4}$$

$$= \frac{1}{4} + 0 + \frac{1}{4}$$

$$= \frac{1}{2}$$

2. A sealed box containing a dozen apples it was found that 3 apples are perished (not good), obtain the probability distribution of the number of perished apples were, 2 apples are drawn at random. also find the mean and variance of this distribution

Let X be the sample space

Given a box containing 1 dozen apples among these 3 apples are perished [not good]. Remaining 9 are good apples.

Here X be the random variable the no of selection of perished apples that means

Among 12 apples 2 apples are selected the sample space is given by

$$P(X=0: \text{ i.e no perished apples selected :}) = \frac{{}^3C_0 \times {}^9C_2}{{}^{12}C_2}$$

$$= 0.5454$$

$$P(X=1: \text{ i.e 1 perished apple selected :}) = \frac{{}^3C_1 \times {}^9C_1}{{}^{12}C_2}$$

$$= 0.4090$$

$$P(X=2: \text{ i.e 2 perished apples selected :}) = \frac{{}^3C_2 \times {}^9C_0}{{}^{12}C_2}$$

$$= 0.04545$$

The discrete probability distribution is given by

$X_i \sim X$	0	1	2
$P(X_i)$	0.5454	0.4090	0.04545

$$i) P(x_i) \geq 0$$

$$ii) \sum_{i=1}^n P(x_i) = 1$$

$$\text{Mean}(\mu) = \sum_{i=1}^n x_i P(x_i) = 0 \times 0.5454 + 1 \times 0.4090 + 2 \times 0.04545 = 0.4999$$

$$\text{Average or Mean}(\mu) = 0.4999 \approx 0.5$$

$$\text{Variance}(V) = \sum_{i=1}^n (x_i - \mu)^2 P(x_i) = (0 - 0.5)^2 \times 0.5454 + (1 - 0.5)^2 \times 0.4090 + (2 - 0.5)^2 \times 0.04545$$

$$V = 0.3408$$

$$S.D(\sigma) = \sqrt{V} = 0.583$$

The probability distribution of finite ~~random~~ variable is given by the following table

x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	K	0.2	2K	0.3	K

Find the value of K & also find mean, variance and standard deviation

Given a finite distribution table values now we can verify discrete probability distribution.

$$i) P(x_i) \geq 0$$

$$ii) \sum_{i=1}^n P(x_i) = 1$$

$$\sum_{i=1}^n P(x_i) \Rightarrow 0.1 + K + 0.2 + 2K + 0.3 + K = 1$$

$$0.6 + 4K = 1$$

$$4K = 0.4$$

$$K = 0.1$$

Table becomes

x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	0.1	0.2	0.2	0.3	0.1

$$\text{Mean } (\mu) = \sum_{i=1}^n x_i P(x_i)$$

$$\mu = (-2) \times 0.1 + (-1) \times 0.1 + 0 \times 0.2 + 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.1$$

$$\mu = 0.8$$

$$\text{Variance } (V) = \sum_{i=1}^n (x_i - \mu)^2 P(x_i) = \sum_{i=1}^n x_i^2 P(x_i) - \mu^2$$

$$V = \{(-2)^2 \times 0.1 + (-1)^2 \times 0.1 + 0^2 \times 0.2 + 1^2 \times 0.2 + 2^2 \times 0.3 + 3^2 \times 0.1\} - 0.8^2$$

$$V = 2.16$$

$$\text{Standard deviation } (\sigma) = \sqrt{V} = 1.4696$$

Q.P

A random variable x has the following probability function for the various values of x .

x	0	1	2	3	4	5	6	7
$P(x_i)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find i) k

ii) $P(x < 6)$, $P(x \geq 6)$, $P(3 < x \leq 6)$.

Given X be the a finite random variable then to verify discrete probability distribution properties.

i) $P(x_i) \geq 0$

ii) $\sum_{i=1}^n P(x_i) = 1$

$$\sum_{i=1}^{\infty} p(x_i) = 1 \quad 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$10k^2 + 10k - k + 1 - 1 = 0$$

$$10k(k+1) - (k+1) = 0$$

$$k+1 = 0$$

$$k = -1$$

$$10k - 1 = 0$$

$$k = 0.1$$

∴ k value is -1 . Then $p(x_i)$ values is negative.
Therefore neglecting $k = -1$ [∵ $p(x_i) > 0$]

x_i	0	1	2	3	4	5	6	7
$p(x_i)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

adding all
+ 1 = 1

$$P(x < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$

$$= 0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$$

$$P(x < 6) = 0.81$$

$$P(x > 6) = P(6) + P(7)$$

$$= 0.02 + 0.17$$

$$= 0.19$$

$$P(3 < x \leq 6) = P(4) + P(5) + P(6)$$

$$= 0.3 + 0.01 + 0.02$$

$$= 0.33$$