Theorem

Derive Cauchy-Reimann (CR) eqn in cartesian form

statement:- "The necessary condition that the function $w = f(z) = u(x,y) + iv(x,y)$ may be analytic at any point $z = x+iy$ is that, there exist 4 contineous first order partial derivative $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ & satisfy the eqn

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}."$$

Proof

$$W = f(z)$$

$$u + iv = f(x+iy) \quad \text{—} (*)$$

Diff (*) Partially w.r.t x.

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = f'(x+iy) \cdot 1 \quad \text{—} (1)$$

Diff (*) P w.r.t y

$$\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y} = f'(x+iy)\, i \quad \text{—} (2).$$

use (1) in (2).

$$\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y} = \left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right) i.$$

$$\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y} = i\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}.$$

equating real & imaginary parts we get.

$$\left\langle \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \right\rangle \quad \& \quad \left\langle \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \right\rangle$$

$$u_y = -v_x \quad \& \quad u_x = v_y$$

Derive Cauchy - Reiman (CR) eqn in Polar form

Statement :- If $f(z) = f(re^{i\theta}) = u(r,\theta) + i\,v(r,\theta)$ is analytic at a point z, then $\exists$ 4 continues (there exist) I order partial derivative

$$\frac{\partial u}{\partial r}, \frac{\partial u}{\partial \theta}, \frac{\partial v}{\partial r}, \frac{\partial v}{\partial \theta}$$ & satisfy the eqn

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad \& \quad \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$$

Proof : If r, θ be the coordinates of form whose cartesian coordinates are (x, y) then $z = x+iy = re^{i\theta}$

$$w = f(z).$$

$$u + iv = f(re^{i\theta}) \quad \text{—}(*)$$

Diff $(*)$ partially w.r.t r.

$$\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r} = f'(re^{i\theta}).e^{i\theta} \quad \text{—}(1).$$

Diff $(*)$ partially w.r.t θ

$$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = f'(re^{i\theta}).re^{i\theta}i \quad \text{—}(2)$$

use (2) in (1).

$$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = \left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right) ir$$

$$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = ir\frac{\partial u}{\partial r} - r\frac{\partial v}{\partial r}$$

equating real & imaginary term

$$\frac{\partial u}{\partial \theta} = -\frac{\cancel{\;}}{r}\,\frac{\partial v}{\partial r} \quad \& \quad \frac{\partial v}{\partial \theta} = r\,\frac{\partial u}{\partial r}$$

$$\frac{\partial v}{\partial r} = -\frac{1}{r}\,\frac{\partial u}{\partial \theta} \quad \& \quad \frac{\partial u}{\partial r} = \frac{1}{r}\,\frac{\partial v}{\partial \theta}.$$

$$\left\langle v_r = -\frac{1}{r}\,u_\theta \right\rangle \quad \& \quad \left\langle u_r = \frac{1}{r}\,v_\theta \right\rangle.$$

1) ST $w = z + e^{z}$ is analytic & Hence find $\frac{dw}{dz}$.

$\Rightarrow$ $w = z + e^{z}$

put $z = x + iy$.

$w = (x+iy) + e^{x+iy}$

$f(z) = (x+iy) + e^{x} e^{iy}$

$u + iv = (x+iy) + e^{x}(\cos y + i \sin y)$

$u = x + e^{x} \cos y$ $\quad\quad$ $v = y + e^{x} \sin y$

$\langle u_x = 1 + e^{x} \cos y \rangle$ $\quad\quad$ $\underline{v_x = e^{x} \sin y}$

$\underline{u_y = -e^{x} \sin y}$. $\quad\quad$ $\langle v_y = 1 + e^{x} \cos y \rangle$.

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From CR eq^n we have

$u_x = v_y$ & $v_x = -u_y$.

$\therefore$ f(z) is analytic

Further $f'(z) = u_x + i v_x$.

$f'(z) = 1 + e^x \cos y + i e^x \sin y$

Put $x = z$ & $y = 0$.

$\langle f'(z) = 1 + e^z \rangle$.

*dAc

1) Find the analytic function $f(z) = u + iv$ given $u - v = e^x(\cos y - \sin y)$

Sol

$u - v = e^x(\cos y - \sin y)$

$u_x - v_x = e^x(\cos y - \sin y)$ — ①

$u_y - v_y = e^x(-\sin y - \cos y)$

$(u_x = v_y$
$v_x = -u_y)$

$-v_x - u_x = e^x(-\sin y - \cos y)$

$u_x + v_x = e^x(\sin y + \cos y)$ — ②

eq^n ① + ② gives

$2u_x = 2e^x \cos y$

$u_x = e^x \cos y$

① − ② gives

$-2v_x = -2e^x \sin y$

$v_x = e^x \sin y$

$f'(z) = u_x + iv_x$

$f'(z) = e^x \cos y + i e^x \sin y$

{Put $x = z$ & $y = 0$}

$f'(z) = e^z(1)$

$\boxed{f(z) = e^z + c}$

Problem

i) construct the analytic funⁿ whose real part is

$u = \log\sqrt{x^2+y^2}$.

sol

$u = \log\sqrt{x^2+y^2}$

$u = \frac{1}{2}\log(x^2+y^2)$.

$u_x = \frac{1}{2} \times \frac{2x}{(x^2+y^2)} = \frac{x}{x^2+y^2}$

$u_y = \frac{1}{2} \times \frac{2y}{(x^2+y^2)} = \frac{y}{x^2+y^2}$.

we've

$f'(z) = u_x + i v_x$

But $v_x = -u_y$ from CReq.

$f'(z) = u_x - i u_y.$

$$f'(z) = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

Put $x = z, \ y = 0$

$$f'(z) = \frac{z}{z^2+0} - i(0) = \frac{z}{z^2} = \frac{1}{z}$$

$$f(z) = \int \frac{1}{z}\, dz + c$$

$$f(z) = \log z + c$$

1) Fitt a 2^{nd} degree parabola $y = ax^2+bx+c$ in the least square sense for the following data & hence estimate y at $x=6$.

x	1	2	3	4	5
y	10	12	13	16	19

$$y = ax^2+bx+c \quad \text{---}(*)$$

The normal eq^n are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2.$$

x	y	x^2	x^3	xy	x^4	x^2y
1	10	1	1	10	1	10
2	12	4	8	24	16	48
3	13	9	27	39	81	117
4	16	16	64	64	256	256
5	19	25	125	95	625	475
Σ 15	70	55	225	232	979	906

on substituting we get,

$$55a + 15b + 5c = 70$$

$$225a + 55b + 15c = 232$$

$$979a + 225b + 55c = 906$$

$a = 0.2857$

$b = 0.4857$

$c = 9.4$

$(*) \Rightarrow \; y = 0.2857(6)^2 + 0.4857(6) + 9.4$

$\langle y = 22.5994 \rangle$

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3) Fitt a straight line in a least square sense for the following data.

x	50	70	100	120
y	12	15	21	25

x	y	x^2	xy
50	12	2500	600
70	15	4900	1050
100	21	10000	2100
120	25	14400	3000
Σ 340	73	31800	6750

$y = ax + b$ — (*)

The normal eq^n are

$\Sigma y = a\Sigma x + nb$

$\Sigma xy = a\Sigma x^2 + b\Sigma x$

subs^ting we get.

~~340a + 340b.~~

$\begin{cases} 340a + 4b = 73 \\ 31800a + 340b = 6750 \end{cases}$

Solving we get

$\boxed{a = 0.1879}$ $\boxed{b = 2.2758}$

(*) $\Rightarrow y = 0.1879x + 2.2758.$

1) Fit a least square series for the geometric curve $y = ax^b$ for the following data.

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

$y = ax^b$ —(*)

Take log on b.s

$\log y = \log a + \log x^b$

$\log y = \log a + b \log x$

$Y = A + bX.$ where $Y = \log y$, $A = \log a$, $X = \log x$.

The normal eqn is (take ln in calci).

$\Sigma Y = nA + b\Sigma X$

$\Sigma XY = A\Sigma X + b\Sigma X^2$.

x	y	$X = \log x$	$Y = \log y$	XY	X^2
1	0.5	0	-0.6931	0	0.
2	2	0.6931	0.6931	0.4803	0.4803
3	4.5	1.0986	1.5040	1.6522	1.2069
4	8	1.3862	2.0794	2.8824	1.9215
5	12.5	1.6094	2.5254	4.0643	2.5901
		Σ. 4.7873	6.1092	9.0804	6.1993

On. sub

$5A + 4.7874 b = 6.1092$

$4.7874 A + 6.1993 b = 9.0804$

On solving

$A = -0.6931 \Rightarrow a = e^{A} = 0.5 \Rightarrow$ (*) $\Rightarrow y = 0.5x^2$

$b = 2$

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Show that if θ is the angle b/w the lines of regression then $\tan\theta = \frac{\sigma x \sigma y}{\sigma^2 x + \sigma^2 y}\left(\frac{1-r^2}{r}\right)$

Sol// WKT if θ is acute the angle b/w the lines $y = m_1 x + c_1$ & $y = m_2 x + c_2$ is given by.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 * m_2} \quad — (*)$$

We've Regression lines

$$y - \bar{y} = \frac{r\sigma y}{\sigma x}(x - \bar{x}) \quad — (1)$$

& $$x - \bar{x} = \frac{r\sigma x}{\sigma y}(y - \bar{y})$$

$$y - \bar{y} = \frac{\sigma y}{r\sigma x}(x - \bar{x}) \quad — (2)$$

From (1) & (2) we've

$$m_1 = \frac{r\sigma y}{\sigma x} \quad \& \quad m_2 = \frac{\sigma y}{r\sigma x}.$$

Substitute in (*), then

$$\tan\theta = \frac{\frac{\sigma y}{r\sigma x} - \frac{r\sigma y}{\sigma x}}{1 + \frac{\sigma y}{\cancel{r}\sigma x}\frac{\cancel{r}\sigma y}{\sigma x}}$$

$$\tan\theta = \frac{\frac{\sigma y}{\sigma x}\left(\frac{1}{r} - r\right)}{1 + \frac{\sigma^2 y}{\sigma x^2}}$$

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$$\tan\theta = \frac{\frac{\sigma_y}{\sigma_x}\left(\frac{1-r^2}{r}\right)}{\frac{\sigma_x^2+\sigma_y^2}{\sigma_x^2}}$$

$$= \frac{\sigma_y}{\cancel{\sigma_x}} \times \frac{\cancel{\sigma_x^2}}{(\sigma_x^2+\sigma_y^2)}\left(\frac{1-r^2}{r}\right)$$

$$\left\langle \tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right) \right\rangle$$

Note:

* If $r = \pm 1$, $\tan\theta = 0$

$\therefore \theta = 0$

Which implies that the 2 regression lines coinside & hence the variables are perfectly correlated.

* If $r = 0$, $\tan\theta = \infty$

$\therefore \theta = \frac{\pi}{2}$

Which implies that the lines are perpendicular & hence the variables are uncorrelated.

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(10).

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Production (x)	Export y	z $(x-y)$	x^2	y^2	z^2
55	35	20	3025	1225	400
56	38	18	3136	1444	324
58	38	20	3364	1444	400
59	39	20	3481	1521	400
60	44	16	3600	1936	256
60	43	17	3600	1849	289
60	45	15	3600	2025	225
Σ 408	282	126	23806	11444	2294

$$r = \frac{\sigma^2 x + \sigma^2 y - \sigma^2 z}{2\sigma x \sigma y}$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{408}{7} = 58.285$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{282}{7} = 40.285$$

$$\bar{z} = \frac{\Sigma z}{n} = \frac{126}{7} = 18.$$

$$\sigma^2 x = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{23806}{7} - (58.285)^2 = 3.7159$$

$$\sigma^2 y = \frac{\Sigma y^2}{n} - (\bar{y})^2 = \frac{11444}{7} - (40.285)^2 = 11.9759$$

$$\sigma^2 z = \frac{\Sigma z^2}{n} - (\bar{z})^2 = \frac{2294}{7} - (18)^2 = 3.7142$$

$$r = \frac{3.7159 + 11.9759 - 3.7142}{2\sqrt{3.7159}\sqrt{11.9759}}$$

$$\boxed{r = 0.8977}$$

a) compute $\bar{x}$, $\bar{y}$ & r from the following regression of line $2x+3y+1=0$; $x+6y-4=0$.

WKT the regression line passes through $\bar{x}$, $\bar{y}$.

$2\bar{x} + 3\bar{y} = -1$

$\bar{x} + 6\bar{y} = 4$

$\boxed{\bar{x} = -2}$, $\boxed{\bar{y} = 1}$

now rewrite the eq^n of regression line to find the correlation coefficient

$2x + 3y + 1 = 0$

$x = \frac{-3y - 1}{2}$

$x = -1.5y - 0.5$

$x + 6y - 4 = 0$

~~$y = 0.16y + 0.666$~~

$y = -0.166x + 0.666$

$r = \sqrt{(-1.5) \times (-0.166)}$

$r = -0.498$

12).

judje judje B.

A	(4)
(2 1	
1	6
6	4
5	9.
3	8.
10	1
2	2
4	3
9	10
7	5
8	7.

$$\rho = 1 - \frac{6 \Sigma d_i^2}{n^3 - n}.$$

$n \rightarrow 10$

now $d_i = x_i - y_i$

$d_i = -5, 2, -4, -5, 9, 0, 1, -1, 2, 1$

$\Sigma d_i^2 = 25 + 4 + 16 + 25 + 81 + 1 + 1 + 1 + 4 + 1.$

$\Sigma d_i^2 = 158.$

$$\rho = 1 - \frac{6(158)}{10^3 - 10}$$

$\boxed{\rho = 0.0424}$ //.