Practical No. 1

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3ax}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a}+x+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x})+\sqrt{3x})}$$

$$\frac{1}{3}\;\frac{\sqrt{3a+x}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+3\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{4}{2}\;\frac{\sqrt{a}}{3\sqrt{a}}$$

$$= \frac{2}{3}\;\frac{5}{3}$$

2. $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$= \lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$= \lim_{y\to 0} \dfrac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \lim_{y\to 0} \dfrac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}\,(2\sqrt{a})} = \dfrac{1}{2a}$

3. $\lim_{x\to \pi/6} \dfrac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By substituting $x-\pi/6 = h$

$x = h+\dfrac{\pi}{6}$

where $h\to 0$

$\lim_{h\to 0}\dfrac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\frac{\pi}{6})}{\pi-6(h+\frac{\pi}{6})}$

$\lim_{h\to 0}\dfrac{\cos h\cdot\cos\pi/6-\sin h\cdot\sin\pi/6 - \sqrt{3}\,(\sin h\cos\pi/6+\cos h\sin\frac{\pi}{6})}{\pi-6(\frac{6h+\pi}{6})}$

$\lim_{h\to 0}\dfrac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\cdot 1/2 - \sqrt{3}\,(\sin h\frac{\sqrt{3}}{2}+\cos h\cdot 1/2)}{\pi-6h-\pi}$

$\lim_{h\to 0}\dfrac{\cos\frac{\sqrt{3}}{2}h-\frac{\sin h}{2}-\sin\frac{3h}{2}-\cos\frac{\sqrt{3}h}{2}}{-6h}$

$\lim_{h\to 0} + \dfrac{\sin\frac{6h}{2}}{+6h}$

$\lim_{h\to 0}\dfrac{\sin 4h}{12h}$

$\dfrac{1}{3}\lim_{h\to 0}\dfrac{\sin h}{h} = \dfrac{1}{3}\times 1 = \dfrac{1}{3}$

4) $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator and denominator both,

$\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$

$\lim_{x\to\infty}\left[\dfrac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)}\dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$

$\lim_{x\to\infty}\dfrac{8(\sqrt{x^2+3})+\sqrt{x^2+1})}{2(\sqrt{x^2+5})+(\sqrt{x^2-3})}$

$4\lim_{x\to\infty}\dfrac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$

After Applying limit we get,

$= 4$

5) i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \quad x \le \frac{\pi}{2}$
$= \dfrac{\cos x}{x-2\pi}$ for $\frac{\pi}{2} < x < \pi$ $\}$ at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

$\therefore f$ at $x = \pi/2$ define

ii) $\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to\frac{\pi}{2}^+} \dfrac{\cos x}{x-2\pi}$

By substituting method

$x - \frac{\pi}{2} = h$

$x = h + \frac{\pi}{2}$

where $h \to 0$

$\lim_{h\to 0}\dfrac{\cos(h+\pi/2)}{x-2(h+\frac{\pi}{2})}$

$\lim_{h\to 0}\dfrac{\cos(h+\pi/2)}{x-2\left(\frac{2h+\pi}{2}\right)}$

$\lim_{h\to 0}\dfrac{\cos(h+\pi/2)}{-2h}$ using $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\lim_{h\to 0}\dfrac{\cos h\cdot\cos\frac{\pi}{2} - \sin h\cdot\sin\pi/2}{-2h}$

$\lim_{h\to 0}\dfrac{\cos h\times 0 - \sin h}{-2h}$

$\lim_{h\to 0} + \dfrac{\sin h}{2h}$

$\frac{1}{2}\lim_{h\to 0}\dfrac{\sin h}{h} = \frac{1}{2}$

3) $\lim_{x\to\pi/4} f(x) = \lim_{x\to\pi/2^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to\pi/2^-}\dfrac{2\sin x\cdot\cos x}{\sqrt{2\sin^2 x}}$ using $\sin 2x = 2\sin x\cos x$

$\lim_{x\to\pi/2^-}\dfrac{2\sin x\cdot\cos x}{\sqrt{2}\sin x}$

$\lim_{x\to\pi/2^-}\dfrac{2\cos x}{\sqrt{2}}$

$2/\sqrt{2}\lim_{x\to\pi/2^-}\cos x)$

$LHS \ne RHS$

5)

i) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x=3$ and $x=6$

at $x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

$f(x) = \lim_{x \to 3^+} (x+3)$

~~f(3~~ $f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3}$ $\frac{(x-3)(x+3)}{(x-3)}$

$\therefore LHS = RHS$

f is continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

Practical - 2

Topic :- Derivative

Q1. Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x - a) \tan x \cdot \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$ $\quad h \to 0$

$$\cancel{as} \; \cancel{x} \; df(h) = \lim_{h \to 0} \frac{\tan(a) - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \tan(a+h) \tan a}$$

formula $= \tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A - B)(1 - \tan A \tan B)$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f'(a) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a - \sin(a+h)}$$

formula :-

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos(a+a+h) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a - \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times 2\cos\left(\frac{2a+h}{2}\right)$$

$$= -\frac{1}{2} \times 2\cos\left(\frac{2a+0}{2a}\right)$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a$$

Q.2. If $f(x) = 4x+1$, $x \le 2$

$= x^2 + 5$, ~~$x<0$~~ $x > 0$ $a + x = 2$,

whether function is differentiable or not

soln :-

LHD :-

$$Lf'(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1 - (4 \times 2 + 1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$Lf'(2^-) = 4$

RHD :-

$$Rf'(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)}$$

$= 2 + 2 = 4$

$Df(2^+) = 4$

$\therefore$ RHD = LHD

f is differentiable at $x = 2$

Q.3 If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$, then

find f is differentiable or not?

Solution :-

RHD =

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

RHD $\neq$ LHD

$\therefore$ f is not differentiable at $x=3$

Q.4 If $f(x) = 8x-5$, $x \leq 2$
$= 3x^2-4x+7$, $x > 2$ at $x=2$, then

find f is differentiable or not

solution :-

$f(2) = 8\times2-5 = 16-5 = 11$

RHD :

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3\times2+2 = 8$

$Df(2^+) = 8$

LHD :-

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$= 8$

$\therefore Df(2^-) = 8$

LHD = RHD

f is differentiable at $x = 3$

Practical No. 3

Topic : Application of Derivatives

1. Find the interval in which function is increasing or decreasing.

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f = 9f(x) = 69 - 24x - 4x^2 + 2x$

2) Find the interval in which function is concave of upwards

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

V) $y = 2x^3 + x^2 - 2x + 4$

Soln :-

Q.1.

i) $f(x) = \cancel{x^3 +} x^3 - 5x - 11$

$\therefore f(x) = 3x^2 - 5$

$\because$ f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - 5/3)(x - \sqrt{5}/3) > 0$

$+ \quad - \quad +$

$-\sqrt{5}/3 \qquad \sqrt{5}/3$

$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

and f is decreasing if $f(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 2x - 20 > 0$

$\therefore 2(3x^2 + x + 0) > 0$

$\therefore 3x^2 + x + 0 > 0$

$\therefore 3x^2 + 6x - 5x + 0 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x + 0) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$x = -2, 5/3$

$x \in (-2, 5/3)$

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(2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 + 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

$x = 1, 2$

$+ \quad - \quad +$ (1, 2)

$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18.$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty).$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + \frac{1}{6} > 0$

$\therefore x > -1/6$

$\therefore f''(x) \ (-1/6, \infty)$

$\therefore$ Thus f exist

Practical No. 4
Topic - Application of derivative and Newton's method

Q.1) Find maximum and minimum value of following function

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[2, 3]$

iv) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

Q.2 Find the root of following derivation by Newton's method (Take 4 iteration only correct) upto decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Q.1.

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/24$

$= 2 + 96/16$

$= 2 + 6 = 8$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4 = 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ has minimum value at

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5 = 1$

$\therefore f''(-1) = 30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore f$ has max value at $x = -1$

$\therefore f(-1) = 3 - 5(1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore f$ has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$-6 < 0$

$\therefore f$ has max value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$.

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12 = -3$

$\therefore f$ has maximum value at $x = 0$ and f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1 = -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 2(1)^3 - 3(-1)^2 + 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has max value 8 at $x = 1$

and f has min value 19 at $x = 2$

~~f(x) = (2~~

Q2.i) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.2x - 10$

$f(1) = (1)^3 - (1.8)(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation by Newtons Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x - f(x_2) / f'(x_2)$

$= 2 - 2.2 / 5.2$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.6162$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^3 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0206$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f(x_2)$

$= 1.6592 + 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(3) / f'(x_3)$

$= 1.8618 + \frac{0.0004}{7.6977}$

$= 1.6618$

Practical NO-5

Topic : Integration

Q.1 Solve the following integration

1) i) $\frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^7 \sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

ix) $\int e^{\cos^2 x} \sin 2x\,dx$

x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

i) $\int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$

= put $(x+1) = t$

$dx = 1/t \times dt$

where $t = 1$

$t = x+1$

$\int \frac{1}{\sqrt{t^2-4}}\,dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$\ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x+3}|) + C$

2) $\int (4e^{3x} + 1)\, dx$

$= \int 4e^{3x} dx + \int 1\, dx$

$= 4 \int e^{3x} dx + \int 1\, dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + 3$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$

$= \int 2x^2 dx - \int 3\sin(x)\, dx + \int 5x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\, dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\, dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}}\, dx$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \times \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, du = \frac{t^4 \times \sin(2t^2)}{8}\, du$

substituting t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)}{8}\, du$

$= \int \frac{u \times \sin(u)}{2} / 8\, du$

$= \int \frac{u \times \sin(u)}{16}\, du$

$= \frac{1}{16} \int u \times \sin(u)\, du$

6. $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1=dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dx$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int \frac{1}{3t}\,dt$

\# $\int 1/x\,dx = \ln|x|$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \ln|(x^3-3x^2+1)| + c$

7. $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

let $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3}\,dx = dt$

$I = -\frac{1}{2} \int -\frac{2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= -\frac{1}{2} \int \sin t$

$= -\frac{1}{2}(-\cos t) + c$

$= \frac{1}{2} \cos t + c$

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Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$

9. $\int e^{\cos^2 x} \cdot \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

Let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-\sin 2x \, dx = dt$

$I = \int -\sin 2x \, e^{\cos^2 x} \, dx$

$= -\int e^t \, dt$

$= e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

Practical - 6

Aim :- Application of integration and Numerical Integrations.

Q.1) Find the length of following curve

1. $x = t\sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$
2. $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$
3. $y = x^{3/2}$ in $[0, 4]$
4. $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$
5. $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2 Using simpsons rule to solve the following

1) $\int_0^2 e^{2x} e^{x^2} dx$ with $x = 4$

2) $\int_0^4 x^2 dx$ with $n = 4$

3) $\int_0^{\pi/2} \sqrt{\sin x} \, dx$ on $n = 6$

7. $x = t - \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt$$

$$= \int_0^{2\pi} 2\sin t/2\, dt$$

$$\left[\left(-4\cos(t/2)\right]_0^{2\pi} = (-4(\cos\pi) - (4\cos 0)\right)$$

$= 4 + 4$

$= 8$

c) $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$$

$$= \frac{-2x}{\sqrt{4-x^2}}$$

$$l = \int_{-2}^{2} \sqrt{1 + (dy/dx)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x}}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{2}{\sqrt{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{(2)^2 - (x)^2}}\, dx$$

$= \sin^{-1}(1) - \sin^{-1}(-1)$

$= 2\left[(\pi/2) - (-\pi/2)\right]$

$= 2\left[\pi/2 + \pi/2\right]$

$= 2$

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3) $y = x^{3/2}$ in $[0, 4]$

$\frac{dy}{dx} = 3/2 \cdot x^{3/2 - 1}$

$L = \int_0^4 \sqrt{1 + (dy/dx)^2}\, dx$

$= \int_0^4 \sqrt{1 + \left(\frac{3\sqrt{x}}{2}\right)^2} \cdot dx$

$= \int_0^4 \sqrt{\left(1 + \frac{9x}{4}\right)}\, dx$

$= \int_0^4 \sqrt{\left(\frac{4+9x}{4}\right)}\, dx$

$= 1/2 \int_0^4 \sqrt{4+9x}\, dx$

$= 1/2 \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]^4 \cdot dx$

$= 1/2 \left\{\frac{(4+9x)^{3/2}}{3/2} \times 1/9\right]^4$

$= 1/27 \left[(9x+4)^{3/2}\right]^4$

$= 1/27 \left[(4)^{3/2} - 8\right]$

4) $x = 3\sin t$, $y = 3\cos t$, $t \in (0, 2\pi)$

$\frac{dx}{dt} = 3\cos t$ $\quad \frac{dy}{dt} = -3\,\cancel{\cos}\, \sin t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t} \cdot dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos t} \cdot dt$

$= \int_0^{2\pi} 3 \cdot \sqrt{x}\, dt$

$= 3 \int_0^{2\pi} dt$

$= 3\,[x]_0^{2\pi}$

$= 3(2\pi - 0)$

$L = 6\pi$ units.

2) $\int_0^4 x^2 \, dx$: $n = 4$

$x = \frac{4-0}{4} = 1$

$\int_0^n f(x)\,dx = \frac{x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$

$= \frac{1}{3}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right]$

$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$

$= \frac{64}{3} \approx 21.333$

3. $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ $n = 6$

$x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$\int_0^{\pi/3} \sqrt{\sin x}\, dx \approx \frac{dx}{3}\left(y_0 + 4(y_1 + y_2 + y_3) + 2(y_2 + y_4)\right)$

$= \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.8715) +\right.$

$\left.2(0.584 + 0.801) + 0.930\right)$

≈ 0.681

Practical No. 7

Topic :- Differential Equation

Q.1.

1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int P(x)dx}$

$= e^{\int 1/x\, dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int \frac{e^x}{x} x \cdot dx + c$

$= \int e^x dx + 1$

$x \cdot y = e^x + c$

2) $e^{x}\frac{dy}{dx} + 2e^{x}y = 1$

$(\div \text{ by } e^{x})$

$\frac{dy}{dx} + \frac{2e^{x}}{e^{x}}y = 1/e^{x}$

$\frac{dy}{dx} + 2y = 1/e^{x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c$

$y \cdot e^{2x} = \int e^{-x} \cdot e^{2x}\,dx + c$

$= \int e^{x}\,dx + c$

$y \cdot e^{2x} = e^{x} + c$

vi) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$

$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^{c}$

vii) $\frac{dy}{dx} = \sin^2(x - y + 1)$

put $x - y + 1 = V$

differentiating on both sides

$x - y + 1 = V$

$1 - \frac{dy}{dx} = \frac{dV}{dx}$

$1 - \frac{dV}{dx} = \frac{dy}{dx}$

$1 - \frac{dV}{dx} = \sin^2 V$

$\frac{dv}{dx} = 1-\sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x+c$

viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3\,dx$

$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = 3x$

$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = 3x$

$= v + \log|x| = 3x+C$

$= 2x + 3y + \log|2x+3y+11| = 3x+C$

$= 3y = x + \log 2x + 3y + 11 + C$

Practical No. 8.

1) $\frac{dy}{dx} = y + e^x - 2 \quad y(0) = 2 \quad h = 0.5$ find $y(2)$

Soln :-

$f(x) = y + e^x - 2$

$y(0) = 2$

$y(x_0) = y_0$

$\therefore x_0 = 0 \quad y_0 = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2) $\frac{dy}{dx} = 1 + y^2 \quad y(0) = 0 \quad h = 0.2$ find $y(1)$

Soln :-

$x_0 = 0$

$y(0) = 0$

$y(x_0) = y_0$

$x_0 = 0 \quad y_0 = 0 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.412	0.9234
4	0.8	0.9234	1.8526	1.2939
5	0.1	1.2939		

$\therefore y(1) = 1.2439$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$.

Soln :-

$y(0) = 1$

$y(x_0) = y_0$

$\therefore x_0 = 0$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7699	1.5051
5	1	1.5051		

$y(1) = 1.5051$

4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$

for $h = 0.5$ & $h = 0.25$

Soln :-

$y(1) = 2$

$x_0 = 1$ & $y_0 = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	1.875
2	2	7.875		

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6589	19.3310
3	1.75	19.3360	1122.6426	299.9966
4	2	299.996		

$y(2) = 299.9966$

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5. $\frac{dy}{dx} = \sqrt{xy} + 2$

$y(1) = 1$ find 0.2

Sol :-

$y(1) = 1$

$y(x_0) = y_0$

$\therefore x_0 = 1 \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
	1.2	3.6		

Practical No. 9

Topic = limits and partial order derivative

1. Evaluate the following limits

(i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$

2) Find f_x, f_y for each of the following f

i) $f(x,y) = xy\, e^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

iii) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

3) Using definition find values of f_x, f_y at $(0,0)$ for

$f(x,y) = \frac{2x}{1+y^2}$

Soln :-

(1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$\Rightarrow \lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$ denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(-4)^3-3(-4)+(-4)^2-1}{-4(-1)+5}$

$= -6\ 1/9$

(2) $\lim_{(x,y)\to(-2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{(x+3y)}$

$\Rightarrow \lim_{(x,y)\to(-2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\therefore$ At $(-2,0)$ denominator $\neq 0$

By applying limit

$\dfrac{(0+1)(4+0-8)}{2}$

$= -4/2$

$= -2$

(3) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2}$

$\Rightarrow$ At $(1,1,1)$ Denominator $= 0$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

On applying limit

$= \dfrac{1+1(1)}{1^2}$

$= 1$

Q-2

(i) $f(x,y) = xy\,e^{x^2+y^2}$

$\therefore f_x = \dfrac{df}{dx}(x,y)$

$= \dfrac{d(xy\,e^{x^2+y^2})}{dx}$

$= y\,e^{x^2+y^2}(2x)$

$\therefore f_x = 2xy\,e^{x^2+y^2}$

$f_y = \dfrac{df(x,y)}{dy}$

$= \dfrac{d(xy\,e^{x^2+y^2})}{dx}$

$= x e^{x^2+y^2} (2y) \cdot$

$\therefore f_y = 2y \cdot xe^{x^2+y^2}$

② $f(x,y) = e^x \cos y$

$f_x = \frac{d}{dx} f(x,y)$

$= \frac{d}{dx} (e^x \cos y)$

$f_x = e^x \cos y$

$f_y = \frac{d f(x,y)}{dy}$

$= \frac{d (e^x \cos y)}{dy}$

$f_y = e^x \sin y$

3) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

$f_x = \frac{df}{dy} (x, y)$

$= \frac{d}{dx} f(x^3y^2 - 3x^2 y + y^3 + 1)$

$= 3x^2 y^2 - 6xy$

$f_y = \frac{df}{dy} (x, y)$

$= \frac{d}{dy} (x^3 y^2 - 3x^2 y + y^3 + 1)$

$= 2x^3 y - 3x^2 + 3y^2$

Q7.

① $f(x,y) = \frac{2x}{1+y^2}$

$= f_x = \frac{d}{dx} f(x,y)$

$= \frac{df}{dx} (2x/1+y^2)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2}{(1+y^2)^2}$

$= 2/1+y^2$

At $f(0,0)$

$= \frac{2}{1+0} = 2$

$f_y = \frac{df(2x/1+y^2)}{dy}$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$

$= \frac{-4(0)}{(1+0)^2} = 0$

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