(1) $f(x,y) = -\log(1-x-y) - \log x - \log y$

(a) gradient of $f(x,y)$

$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial}{\partial x} f(x,y) \\ \frac{\partial}{\partial y} f(x,y) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial x}(-\log(1-x-y) - \log x - \log y \\ \frac{\partial}{\partial y}(-\log(1-x-y) - \log x - \log y \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{(1-x-y)} \cdot (0-1-0) - \frac{1}{x} - 0 \\ \frac{-1}{(1-x-y)} \cdot (0-0-1) - 0 - \frac{1}{y} \end{bmatrix}$$

$$\nabla f(x,y) = \begin{bmatrix} \frac{1}{(1-x-y)} - \frac{1}{x} \\ \frac{1}{(1-x-y)} - \frac{1}{y} \end{bmatrix}$$

Hessian of $f(x,y)$

$$\nabla^2 f(x,y) = \begin{bmatrix} \frac{\partial^2}{\partial x^2} f(x,y) & \frac{\partial^2}{\partial x \partial y} f(x,y) \\ \frac{\partial^2}{\partial y \partial x} f(x,y) & \frac{\partial^2}{\partial y^2} f(x,y) \end{bmatrix}$$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial^2}{\partial x^2}\left(-\log(1-x-y) - \log x - \log y\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{-1}{(1-x-y)} \cdot (0-1-0) - \frac{1}{x} - 0\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{1}{(1-x-y)} - \frac{1}{x}\right)$$

$$= \frac{-1}{(1-x-y)^2} \cdot (0-1-0) + \frac{1}{x^2}$$

$$= \frac{1}{(1-x-y)^2} + \frac{1}{x^2}$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial^2}{\partial x \partial y}\left(-\log(1-x-y) - \log x - \log y\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{-1}{(1-x-y)} \cdot (0-0-1) - 0 - \frac{1}{y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{1}{(1-x-y)} - \frac{1}{y}\right)$$

$$= \frac{-1}{(1-x-y)^2} \cdot (0-1-0) - 0$$

$$= \frac{1}{(1-x-y)^2} = \frac{\partial^2}{\partial y \partial x} f(x,y)$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial^2}{\partial y^2}\left(-\log(1-x-y) - \log x - \log y\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{-1}{(1-x-y)} \cdot (0-0-1) - 0 - \frac{1}{y}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{1}{(1-x-y)} - \frac{1}{y}\right)$$

$$= \frac{-1}{(1-x-y)^2} \cdot (0-0-1) + \frac{1}{y^2}$$

$$= \frac{1}{(1-x-y)^2} + \frac{1}{y^2}$$

$$H = \nabla^2 f(x,y) = \begin{bmatrix} \frac{1}{(1-x-y)^2} + \frac{1}{x^2} & \frac{1}{(1-x-y)^2} \\ \frac{1}{(1-x-y)^2} & \frac{1}{(1-x-y)^2} + \frac{1}{y^2} \end{bmatrix}$$