ECE/CS 559 - Fall 2018 - Midterm #2.

Full Name: **ID Number:**

For all questions, answers without any justification will not be given any credit.

Q1 (10 pts). Given $x, y \in \mathbb{R}^{2\times 1}$, let $K(x,y) = (x^T y)^2$. Find the feature map $\phi(\cdot)$ such that $K(x,y) = [\phi(x)]^T \phi(y)$ for every x and y.

Q2 (15 pts). Consider a two-neuron Hopfield network with weight matrix $W = \begin{bmatrix} +2 & -1 \\ -1 & +0.5 \end{bmatrix}$, zero biases, and the $\mathrm{sgn}(\cdot)$ activation function; i.e., $\mathrm{sgn}(x) = 1$ if $x \geq 0$, and $\mathrm{sgn}(x) = -1$, otherwise.

(a) **(5 pts)** Consider the initial state $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the next state using the synchronous update rule.

(b) **(5 pts)** Consider the initial state $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Does the network state converge with synchronous updates?

(c) **(5 pts)** Consider the initial state $x = \begin{bmatrix} +1 \\ -1 \end{bmatrix}$. Does the network state converge with asynchronous updates?

Q3 (10 pts). Give an example of a weight matrix and a bias vector such that the corresponding Hopfield network with the $\mathrm{sgn}(\cdot)$ activation function does not converge for any initial state regardless of whether one uses synchronous or asynchronous update rules.

Q4 (10 pts). Consider an n-input perceptron with the input-output relationship $y = (w^T x + b)^2$, where $w = [w_1 \cdots w_n]^T$ is the weight vector, b is the bias, $x = [x_1 \cdots x_n]^T$ is the input, and y is the output. The goal is to minimize the energy function $E = \frac{1}{8}(d - y)^4$. Write down the gradient descent update equations to minimize E over w and b.

Q5 (30 pts). Consider classes $\mathcal{C}^- = \{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right], \left[\begin{smallmatrix}-1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\-1\end{smallmatrix}\right],\}$, and $\mathcal{C}^+ = 9C^- = \{\left[\begin{smallmatrix}9\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\9\end{smallmatrix}\right], \left[\begin{smallmatrix}-9\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\-9\end{smallmatrix}\right],\}$.

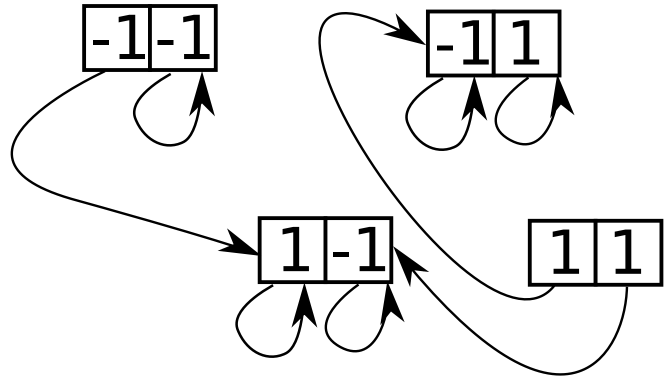
(a) **(5 pts)** Are the classes C^+ and C^- linearly separable?

(b) **(10 pts)** Find a feature mapping ϕ so that $C_0^- = \{\phi(x) : x \in C^-\}$ and $C_0^+ = \{\phi(x) : x \in C^+\}$ are linearly separable.

(c) **(10 pts)** Solve the linear SVM in the corresponding feature space for classes C_0^- and C_0^+. The result will be a discriminator function $g(x) = \mathrm{sgn}((\phi(x))^T w + \theta)$ such that $g(x) > 0$ for $x \in \mathcal{C}^+$ and $g(x) < 0$ for $x \in \mathcal{C}^-$. Indicate your discriminator $g(x)$.

(d) **(5 pts)** Indicate the support vectors in your solution in (c).

Q6 (25 pts). The state transition diagram of a two-neuron Hopfield network with the asynchronous update rule is shown below:



(a) **(5 pts)** What are the steady state(s) of the network?

(b) **(5 pts)** What are the urstate(s) of the network?

(c) **(5 pts)** True or False: The energy of state $\boxed{1}\boxed{-1}$ is less than or equal to that of state $\boxed{1}\boxed{1}$.

(d) **(5 pts)** True or False: The energy of state $\boxed{1}\boxed{-1}$ is less than or equal to that of state $\boxed{-1}\boxed{-1}$.

(e) **(5 pts)** Draw the state transition diagram for the synchronous update scenario.