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Assignment 10:

1) Given KB:

-> Isaac is Roman: $Roman(Isaac)$
-> Isaac does not hate Caesar: $\neg hate(Isaac, Caesar)$
-> Paulus hates Mark: $hate(Paulus, Mark)$
-> Isaac does not think Paulus is crazy: $\neg thinkcrazy(Isaac, Paulus)$

Given implication:
$\forall x[Roman(x) \wedge know(x, Mark)] \implies [hate(x, Caesar) \vee (\forall y(\exists z hate(y, z)) \implies thinkcrazy(x, y))]$

Simplifying the implication we get:

- Using implication elimination $(a \implies b \equiv \neg a \vee b)$ on the second implies in the implication we get:
  $\forall x[Roman(x) \wedge know(x, Mark)] \implies [hate(x, Caesar) \vee (\forall y(\neg \exists z hate(y, z)) \vee thinkcrazy(x, y))]$

- Using De Morgan rules for quantifiers we can write $\neg \exists z hate(y, z) \equiv \forall z \neg hate(y, z)$
  $\forall x[Roman(x) \wedge know(x, Mark)] \implies [hate(x, Caesar) \vee (\forall y(\forall z \neg hate(y, z)) \vee thinkcrazy(x, y))]$

- Nesting both quantifiers $\forall y$ and $\forall z$ together:
  $\forall x[Roman(x) \wedge know(x, Mark)] \implies [hate(x, Caesar) \vee \forall y, z((\neg hate(y, z)) \vee thinkcrazy(x, y))]$

- Using implication elimination $(a \implies b \equiv \neg a \vee b)$ on the first implies we get:
  $\forall x \neg [Roman(x) \wedge know(x, Mark)] \vee [hate(x, Caesar) \vee (\forall y, z(\neg hate(y, z)) \vee thinkcrazy(x, y))]$

- Using De Morgan rules we can write
  $\forall x[\neg Roman(x) \vee \neg know(x, Mark)] \vee [hate(x, Caesar) \vee (\forall y, z(\neg hate(y, z)) \vee thinkcrazy(x, y))]$

Bringing out the universal quantifiers we get:
$\forall x, y, z \neg Roman(x) \vee \neg know(x, Mark) \vee hate(x, Caesar) \vee \neg hate(y, z) \vee thinkcrazy(x, y)$

Since the above implication is true for all x, y, z we can make the following substitution for x, y, z and the implication should still hold true. Thus by Universal elimination for the substitution $\{x/Isaac, y/Paulus, z/Mark\}$ we get the implication:

$\neg Roman(Isaac) \vee \neg know(Isaac, Mark) \vee hate(Isaac, Caesar) \vee \neg hate(Paulus, Mark) \vee thinkcrazy(Isaac, Paulus)$

We can infer the following from KB:

- From the KB $Ask(Roman(Isaac))$ returns True.
  $False \vee \neg know(Isaac, Mark) \vee hate(Isaac, Caesar) \vee \neg hate(Paulus, Mark) \vee thinkcrazy(Isaac, Paulus)$
- From the KB $Ask(hate(Isaac, Caesar)$ returns False.
  $\neg know(Isaac, Mark) \vee False \vee \neg hate(Paulus, Mark) \vee thinkcrazy(Isaac, Paulus)$
- From the KB $Ask(hate(Paulus, Mark)$ returns True.
  $\neg know(Isaac, Mark) \ \vee \ False \ \vee \ thinkcrazy(Isaac, Paulus)$
- From the KB $Ask(thinkcrazy(Isaac, Paulus)$ returns False.
  $\neg know(Isaac, Mark) \ \vee \ False$

The implication left is $\neg know(Isaac, Mark)$ which must be True for the overall implication to be held True.

Therefore, $\neg know(Isaac, Mark) \ = \ True$ which infers that Isaac does not know Mark.