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<u>Assignment 8:</u>

1) From the given sentences in the knowledge base, let the atomic sentences be:
P: Sam plays baseball
Q: Paul plays baseball
R: Ryan plays baseball

Then the sentences in the knowledge base can be represented as:

- Sam plays baseball or Paul plays baseball: $P \vee Q$
- Sam plays baseball or Ryan doesn't play baseball: $P \vee \neg R$

The Knowledge base can then be represented in using propositional symbols as:

- $(P \vee Q) \wedge (P \vee \neg R)$

1.A) Sam and Ryan both play baseball: $P \wedge R$
We need to check if $KB \models (P \wedge R)$

Truth Table:

P	**Q**	**R**	$(P \vee Q)$	$(P \vee \neg R)$	$(P \vee Q) \wedge (P \vee \neg R)$	$(P \wedge R)$
F	F	F	F	T	F	F
F	F	T	F	F	F	F
<u>F</u>	**<u>T</u>**	**<u>F</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>F</u>**
F	T	T	T	F	F	F
<u>T</u>	**<u>F</u>**	**<u>F</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>F</u>**
<u>T</u>	**<u>F</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**
<u>T</u>	**<u>T</u>**	**<u>F</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>F</u>**
<u>T</u>	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**	**<u>T</u>**

All the underlined rows in the above truth table show the models for which the knowledge base is True, but we see that sentence $P \wedge R$ is not true for all models in which KB is true.
Some counterexamples for the models are for which KB does not entail the sentence are:
(P, Q, R) - (F, T, F), (T, F, F), (T, T, F)

1.B) At Least one among Sam, Ryan and Paul play baseball: $(P \lor Q \lor R)$
We need to check if $KB \models (P \lor Q \lor R)$

Truth Table:

P	**Q**	**R**	$(P \lor Q)$	$(P \lor \neg R)$	$(P \lor Q) \land (P \lor \neg R)$	$(P \lor Q \lor R)$
F	F	F	F	T	F	F
F	F	T	F	F	F	T
F	**T**	**F**	**T**	**T**	**T**	**T**
F	T	T	T	F	F	T
T	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**	**T**	**T**
T	**T**	**T**	**T**	**T**	**T**	**T**

All the underlined rows in the above truth table show the models for which the knowledge base is True, and we see that for all models in which KB is true, the sentence $(P \lor Q \lor R)$ is also true in these models. Therefore, $KB \models (P \lor Q \lor R)$.

2) Let the atomic sentences be represented as:
A - Ana eats; B - Bret eats; C - Charles eats; D - Derek eats; E - Earl eats; F - Fred eats; G - Gary eats

a. If Ana eats, Bret eats: $A \implies B$
b. Charles eats and Derek doesn't eat: $C \land \neg D$
c. Bret doesn't eat: $\neg B$
d. If Derek doesn't eat at least one among Ana, Earl and Fred eats: $\neg D \implies (A \lor E \lor F)$
e. If at least one of Charles and Gary eats, Earl doesn't eat: $(C \lor G) \implies \neg E$

3) Taking the sentences from the above question as premises, we need to prove F - True.
-> **From sentence c** we have $\neg B = True$ which implies B - False
-> **From sentence a, using contraposition** $(A \implies B) \equiv (\neg B \implies \neg A)$ and since we have $\neg B = True$ from above, we can imply $\neg A = True$. So A - False
-> **From sentence b** we have $(C \land \neg D) = True$ which implies C - True and D - False.
-> **From sentence e** we have $(C \lor G) \implies \neg E$ and since C - True we can imply $(C \lor G) - True$ and from this we can imply $\neg E - True$. So E - False
-> **From sentence d, using implication elimination**, we have $\neg D \implies (A \lor E \lor F) \equiv D \lor (A \lor E \lor F)$.
Since we have A - False, D - False, E - False, we must have F - True for the above sentence to be true.

Therefore statement F which Fred eats is True.