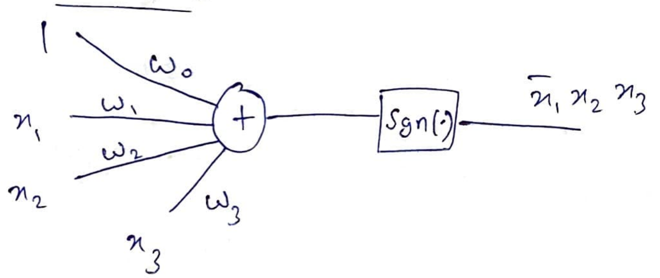
① $f(x_1, x_2, x_3) = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$

Truth Table:

x_1	x_2	x_3	$\bar{x}_1 x_2 x_3$	$x_1 \bar{x}_2$	$\bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$
-1	-1	-1	-1	-1	-1
-1	-1	1	-1	-1	-1
-1	1	-1	-1	-1	-1
-1	1	1	1	-1	1
1	-1	-1	-1	1	1
1	-1	1	-1	1	1
1	1	-1	-1	-1	-1
1	1	1	-1	-1	-1

For $\bar{x}_1 x_2 x_3$:



$$\bar{x}_1 x_2 x_3 = \text{Sgn}(\omega_0 + \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3)$$

① $x_1 = -1,\ x_2 = -1,\ x_3 = -1$

$\omega_0 - \omega_1 - \omega_2 - \omega_3 < 0$

② $x_1 = -1,\ x_2 = -1,\ x_3 = 1$

$\omega_0 - \omega_1 - \omega_2 + \omega_3 < 0$

③ $x_1 = -1,\ x_2 = 1,\ x_3 = -1$

$\omega_0 - \omega_1 + \omega_2 - \omega_3 < 0$

④ $x_1 = -1,\ x_2 = 1,\ x_3 = 1$

$\omega_0 - \omega_1 + \omega_2 + \omega_3 > 0$

⑤ $x_1 = 1,\ x_2 = -1,\ x_3 = -1$

$\omega_0 + \omega_1 - \omega_2 - \omega_3 < 0$

⑥ $x_1 = 1,\ x_2 = -1,\ x_3 = 1$

$\omega_0 + \omega_1 - \omega_2 + \omega_3 < 0$

⑦ $x_1 = 1,\ x_2 = 1,\ x_3 = -1$

$\omega_0 + \omega_1 + \omega_2 - \omega_3 < 0$

⑧ $x_1 = 1,\ x_2 = 1,\ x_3 = 1$

$\omega_0 + \omega_1 + \omega_2 + \omega_3 < 0$

From the above equations:

① + ⑧ < 0

$\Rightarrow$ $\boxed{w_0 < 0}$

① + ⑦ < 0

$\Rightarrow$ $w_0 - w_3 < 0$

$\Rightarrow$ $\boxed{w_3 > w_0}$

① + ⑥ < 0

$\Rightarrow$ $w_0 - w_2 < 0$

$\Rightarrow$ $\boxed{w_2 > w_0}$

④ − ① > 0

$\Rightarrow$ $w_2 + w_3 > 0$

$\Rightarrow$ $\boxed{w_2 > -w_3}$

④ − ② > 0

$\Rightarrow$ $\boxed{w_2 > 0}$

④ − ③ > 0

$\Rightarrow$ $\boxed{w_3 > 0}$

④ − ⑥ > 0

$\Rightarrow$ $-2w_1 + 2w_2 > 0$

$\Rightarrow$ $\boxed{w_1 < w_2}$

④ − ⑧ > 0

$\Rightarrow$ $-2w_1 > 0$

$\Rightarrow$ $\boxed{w_1 < 0}$

④ − ⑦ > 0

$\Rightarrow$ $-2w_1 + 2w_3 > 0$

$\Rightarrow$ $w_1 - w_3 < 0$

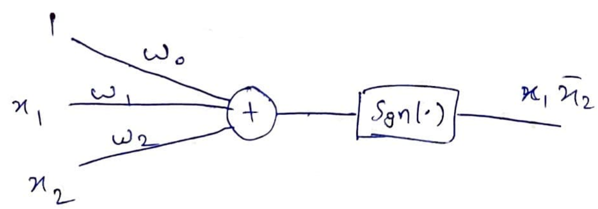
$\Rightarrow$ $\boxed{w_1 < w_3}$

From above inequalities, from trial and error

w_0	w_1	w_2	w_3
-2	-1	1	1

weights for $\bar{x}_1 x_2 x_3$ are $w_0 = -2$, $w_1 = -1$, $w_2 = 1$, $w_3 = 1$, satisfy eqns ① to ⑧

For $x_1 \bar{x}_2$:



$$x_1 \bar{x}_2 = \mathrm{Sgn}(w_0 + w_1 x_1 + w_2 x_2)$$

① $x_1 = -1$, $x_2 = -1$

$w_0 - w_1 - w_2 < 0$

② $x_1 = -1$, $x_2 = 1$

$w_0 - w_1 + w_2 < 0$

③ $x_1 = 1$, $x_2 = -1$

$w_0 + w_1 - w_2 > 0$

④ $x_1 = 1$, $x_2 = 1$

$w_0 + w_1 + w_2 < 0$

From the above equations:

① + ④ < 0

$\Rightarrow \boxed{w_0 < 0}$

① + ② < 0

$\Rightarrow w_0 - w_1 < 0$

$\Rightarrow \boxed{w_0 < w_1}$

② + ④ < 0

$\Rightarrow w_0 + w_2 < 0$

$\Rightarrow \boxed{w_0 < -w_2}$

③ − ① > 0

$\Rightarrow \boxed{w_1 > 0}$

③ − ② > 0

$\Rightarrow w_1 - w_2 > 0$

$\Rightarrow \boxed{w_1 > w_2}$

③ − ④ > 0

$\Rightarrow -w_2 > 0$

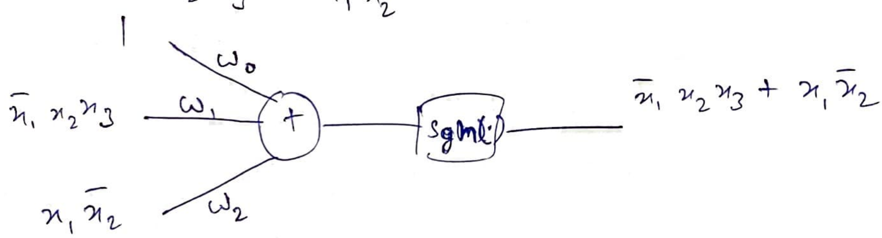
$\Rightarrow \boxed{w_2 < 0}$

From above inequalities, from trial and error:

w_0	w_1	w_2	
-2	1	-1 ×	(not satisfying eqn ③)
-2	1	-2	

weights for $x_1\bar{x}_2$ are $w_0 = -2$, $w_1 = 1$, $w_2 = -2$, satisfy eqns ① to ④

For $\bar{x}_1 x_2 x_3 + x_1\bar{x}_2$



Let $X_1 = \bar{x}_1 x_2 x_3$, $X_2 = x_1 x_2$

$\bar{x}_1 x_2 x_3 + x_1\bar{x}_2 = \mathrm{Sgn}(w_0 + w_1 X_1 + w_2 X_2)$

① $X_1 = -1$, $X_2 = -1$

$w_0 - w_1 - w_2 < 0$

② $X_1 = 1$, $X_2 = -1$

$w_0 + w_1 - w_2 > 0$

③ $X_1 = -1$, $X_2 = 1$

$w_0 - w_1 + w_2 > 0$

From above eqns,

② + ③ > 0

$\Rightarrow \boxed{w_0 > 0}$

② − ① > 0

$\Rightarrow \boxed{w_1 > 0}$

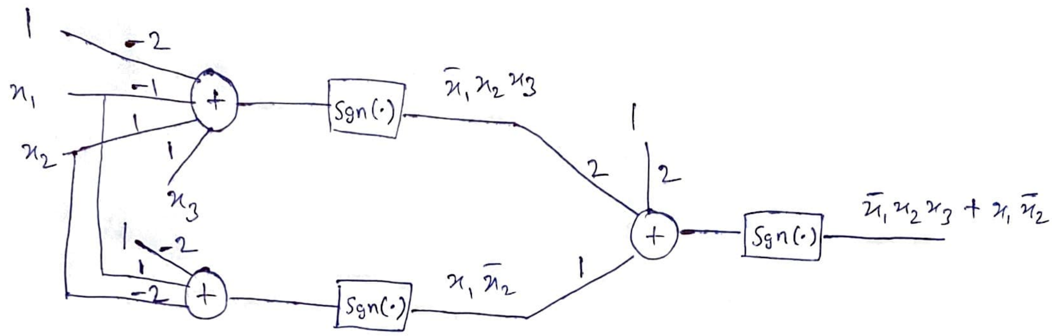
③ − ① > 0

$\Rightarrow \boxed{w_2 > 0}$

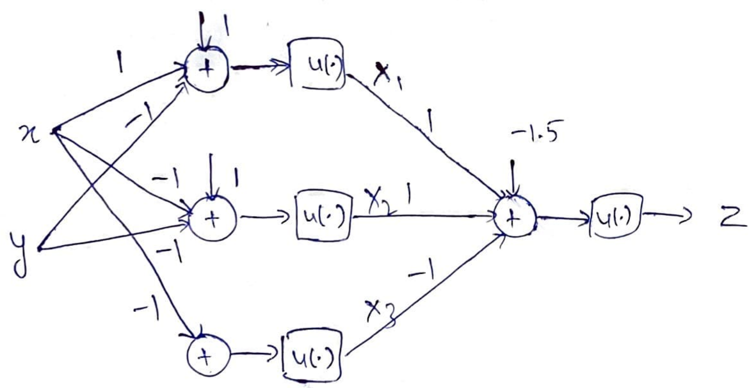
From above inequalities, from trial and error

w_0	w_1	w_2	
2	1	1	× (eqn ① not satisfied)
2	2	1	

weights for $\bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$ are $w_0 = 2,\ w_1 = 2,\ w_2 = 1$ satisfy eqns ① to ③.



②



Let outputs of first hidden layer be X_1, X_2, X_3

$$X_1 = u(1 + x - y), \quad X_2 = u(1 - x - y)$$

$$X_3 = u(-x)$$

$$Z = u(-1.5 + u(1+x-y) + u(1-x-y) - u(-x))$$

For $Z = 1$,

$$u(1+x-y) + u(1-x-y) - u(-x) \geq 1.5$$

$u(1+x-y)$	$+\ u(1-x-y)$	$-\ u(-x)$	≥ 1.5
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

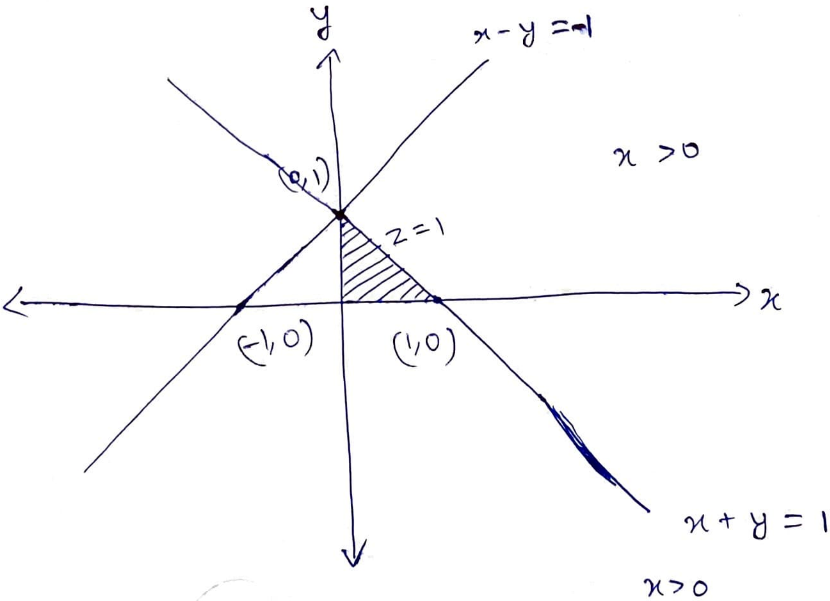
$u(1+x-y) + u(1-x-y) - u(-x) \geq 1.5$ is possible when

$u(1+x-y) = 1$, $u(1-x-y) = 1$, $u(-x) = 0$

$u(1+x-y) = 1 \Rightarrow \boxed{x - y + 1 \geq 0}$ — (1)

$u(1-x-y) = 1 \Rightarrow 1 - x - y \geq 0 \Rightarrow \boxed{x + y \leq 1}$ — (2)

$u(-x) = 0 \Rightarrow -x < 0 \Rightarrow \boxed{x > 0}$ — (3)



 is the shaded region for which $z = 1$ in x-y plane (Does not include points (x, y) lying on y-axis as $x > 0$)