Assignment 1

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CH-2, EX-2, Q.4(III)

1. Show that the following triad of points form an equilateral triangle

$$\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}4\\\frac{\pi}{3}\end{pmatrix}, \begin{pmatrix}4\\\frac{2\pi}{3}\end{pmatrix} \tag{1}$$

Solution:

A triangle is said to be equilateral triangle if all the sides are equal. If d_1, d_2, d_3 are three sides of the triangle. Then, the triangle is equilateral only if d_1 = d_2 = d_3.

The given polar points are:

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}4\\\frac{\pi}{3}\end{pmatrix}, \mathbf{C} = \begin{pmatrix}4\\\frac{2\pi}{3}\end{pmatrix} \tag{2}$$

First we need to convert polar to rectangular coordinates using x = r cos(θ) and y = r sin(θ).

So the rectangular coordinates of the polar coordinate of $\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix}$ is:

$$x = rcos(\theta) = 0cos(0) = 0 \tag{3}$$

$$y = rsin(\theta) = 0sin(0) = 0 \tag{4}$$

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix} \tag{5}$$

The rectangular coordinates of the polar coordinate of $\mathbf{B} = \begin{pmatrix}4\\\frac{\pi}{3}\end{pmatrix}$ is:

$$x = rcos(\theta) = 4cos(\frac{\pi}{3}) = 2 \tag{6}$$

$$y = rsin(\theta) = 4sin(\frac{\pi}{3}) = 2 \tag{7}$$

$$\mathbf{B} = \begin{pmatrix}2\\2\end{pmatrix} \tag{8}$$

The rectangular coordinates of the polar coordinate of $\mathbf{C} = \begin{pmatrix}4\\\frac{2\pi}{3}\end{pmatrix}$ is:

$$x = rcos(\theta) = 4cos(\frac{2\pi}{3}) = -2 \tag{9}$$

$$y = rsin(\theta) = 4sin(\frac{2\pi}{3}) = 3.5 \tag{10}$$

$$\mathbf{C} = \begin{pmatrix}-2\\3.5\end{pmatrix} \tag{11}$$

The rectangular coordinates of given polar points from equation (5), (8), (11) are,

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}2\\2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-2\\3.5\end{pmatrix} \tag{12}$$

Let V be a inner product space, and $\|\|$ be its associated norm. The distance between u,v $\in$ V is given by dist(u, v) = $\|u, v\|$

$$\begin{aligned}\|\mathbf{A}-\mathbf{B}\|^2 &= (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B})\\ \mathbf{A}-\mathbf{B} &= \begin{pmatrix}0-2\\0-2\end{pmatrix}\\ &= \begin{pmatrix}-2\\-2\end{pmatrix}\\ \|\mathbf{A}-\mathbf{B}\| &= \sqrt{(-2)^2+(-2)^2} = 3 \end{aligned} \tag{13}$$

$$\begin{aligned}\|\mathbf{B}-\mathbf{C}\|^2 &= (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})\\ \mathbf{B}-\mathbf{C} &= \begin{pmatrix}2-(-2)\\2-3.5\end{pmatrix}\\ &= \begin{pmatrix}4\\-1.5\end{pmatrix}\\ \|\mathbf{B}-\mathbf{C}\| &= \sqrt{(4)^2+(-1.5)^2} = 4 \end{aligned} \tag{14}$$

$$\begin{aligned}\|\mathbf{A}-\mathbf{C}\|^2 &= (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{C})\\ \mathbf{A}-\mathbf{C} &= \begin{pmatrix}0-(-2)\\0-3.5\end{pmatrix}\\ &= \begin{pmatrix}2\\-3.5\end{pmatrix}\end{aligned}$$

$$\|\mathbf{A}-\mathbf{C}\| = \sqrt{(2)^2+(-3.5)^2} = 4 \tag{15}$$

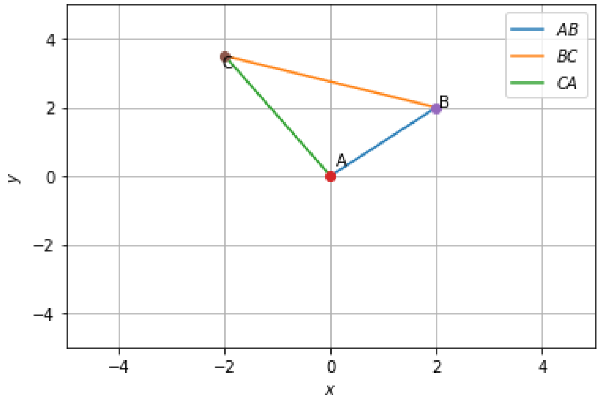

Fig. 1. The given points form a triangle

From the Fig.1 it is clear that sides BC and AC has same side length, which is different from AB. So the given triangle is not an equilateral triangle.

Here from equations (13),(14) and (15), only two sides of triangle are equal ($\|\mathbf{A}-\mathbf{B}\| \neq \|\mathbf{B}-\mathbf{C}\| = \|\mathbf{A}-\mathbf{C}\|$). **So the given triad of points does not form an equilateral triangle.**

Download python code at

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https://github.com/AnishAntony11
/Assignment1
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