

KATHMANDU UNIVERSITY

Dhulikhel, Kavre

COMP 304

Operations Research

Assignment - 3 (Simulation + Networking)

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61

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Task 1: For the following project schedule: Construct the project network.

- i) Find the total float, free float, independent float for non-critical activities
- ii) Construct the network diagram
- iii) Find the critical and non-critical activities and critical path

Activity	Duration
0-1	2
1-2	8
1-3	10
2-4	6
2-5	3
3-4	3
3-6	7
4-7	5
5-7	2
6-7	8

→ Forward Pass:

$$\text{Set } E_0 = 0$$

$$E_1 = E_0 + t_{01} = 0 + 2 = 2$$

$$E_2 = E_1 + t_{12} = 2 + 8 = 10$$

$$E_3 = E_2 + t_{23} = 2 + 10 = 12$$

$$E_4 = \max [E_3 + t_{34}, E_3 + t_{34}]$$

$$= \max [10 + 6, 12 + 3]$$

$$= 16$$

$$E_5 = E_2 + t_{23} = 10 + 3 = 13$$

$$E_6 = E_3 + t_{36} = 12 + 7 = 19$$

$$\begin{aligned} E_7 &= \max(E_5 + t_{57}, E_4 + t_{47}, E_6 + t_{67}) \\ &= \max(13 + 2, 16 + 5, 19 + 8) \\ &= 27 \end{aligned}$$

### Backward Pass

Set  $E_7 = 27$  so,

$$L_7 = 27$$

$$L_6 = L_7 - t_{67} = 27 - 8 = 19$$

$$L_5 = L_7 - t_{57} = 27 - 2 = 25$$

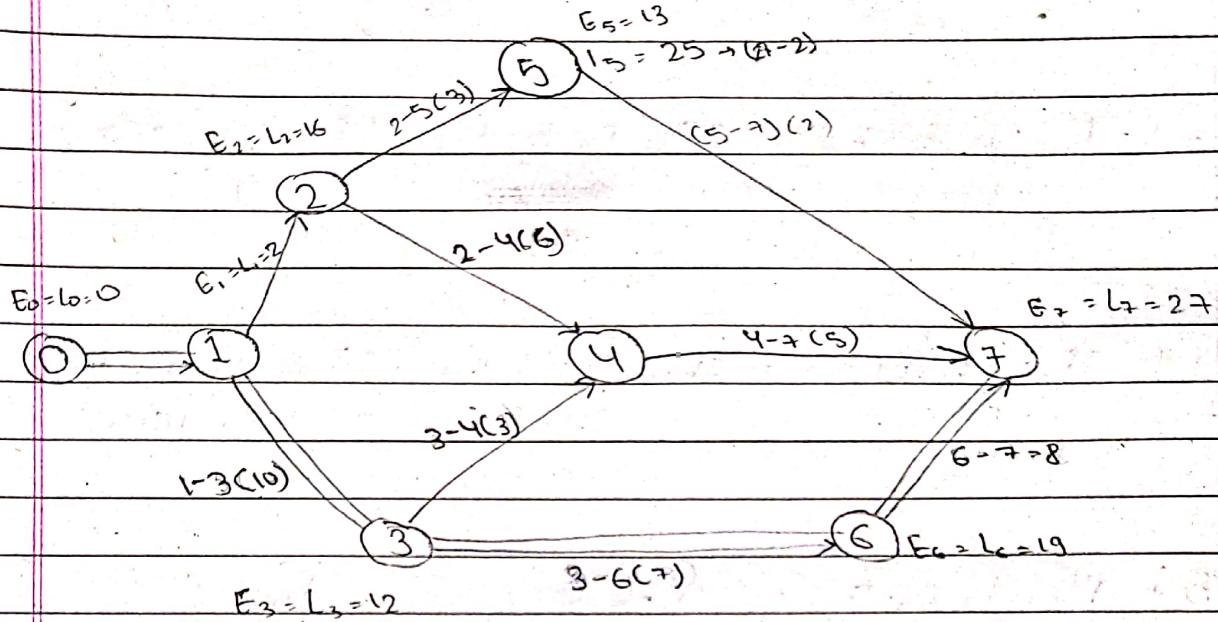
$$L_4 = L_7 - t_{47} = 27 - 5 = 22$$

$$\begin{aligned} L_3 &= \min(L_6 - t_{36}, t_4 - t_{34}) \\ &= \min(19 - 7, 22 - 3) \\ &= 12 \end{aligned}$$

$$\begin{aligned} L_2 &= \min(L_4 - t_{24}, L_5 - t_{23}) \\ &= \min(22 - 6, 25 - 3) \\ &= 16 \end{aligned}$$

$$\begin{aligned} L_1 &= \min(L_2 - t_{12}, L_3 - t_{13}) \\ &= \min(16 - 8, 12 - 10) \\ &= 2 \end{aligned}$$

$$\begin{aligned} L_0 &= t_1 - t_{01} \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

Network DiagramFor Non-Critical Activities $(1-2), (2-4), (2-5), (3-4), (4-7), (5-7)$ 

i)  $(1-2)$ :  $FF_{12} = E_2 - E_1 - t_{12}$ ;  $IF_{12} = F_{12} - L_1 - t_{12}$ ;  $TF_{12} = L_2 - E_1 - t_{12}$   
 $\therefore FF_{12} = 16 - 10 - 8$ ;  $\therefore IF_{12} = 16 - 10 - 8$ ;  $\therefore TF_{12} = 16 - 10 - 8$   
 $= 0$ ;  $= 0$ ;  $= 8$

ii)  $2-4$ :  $FF_{24} = E_4 - E_2 - t_{24}$ ;  $IF_{24} = F_{24} - L_2 - L_{24}$ ;  $TF_{24} = L_4 - E_2 - t_{24}$   
 $= 16 - 10 - 6$ ;  $= 16 - 16 - 6$ ;  $= 22 - 10 - 6$   
 $= 0$ ;  $= -6$ ;  $= 6$

iii) 2-5:  $FF_{25} = E_5 - E_2 - t_{23}$        $IF_{25} = E_5 - L_2 - t_{23}$        $FF_{25} = L_5 - E_2 - t_{23}$   
 $= 13 - 10 - 3$        $= 13 - 16 - 3$        $= 25 - 10 - 3$   
 $= 0$        $= -6$        $= 12$

iv) 3-4:  $FF_{34} = E_4 - E_3 - t_{34}$        $IF_{34} = E_4 - L_3 - t_{34}$        $FF_{34} = 22 - 12 - 3$   
 $= 16 - 12 - 3$        $= \cancel{16} - \cancel{12} - 3$        $= 7$   
 $= 1$        $= \cancel{1}$

v) 4-7:  $FF_{47} = E_7 - E_4 - t_{47}$        $IF_{47} = E_7 - L_4 - t_{47}$        $FF_{47} = L_7 - E_4 - t_{47}$   
 $= 27 - 16 - 5$        $= 27 - 22 - 5$        $= 27 - 16 - 5$   
 $= 6$        $= 6$        $= 6$

vi) 5-7:  $FF_{57} = E_7 - E_5 - t_{57}$        $IF_{57} = 0$        $FF_{57} = 12$   
 $= 27 - 13 - 2$   
 $= 12$

From Network diagram,

Critical activities: 0-1, 1-3, 3-6, 6-7

Non-critical activities: 2-4, 2-5, 4-7, 3-4, 5-7

Overall critical path: 0-1 → 1-3 → 3-6 → 6-7

∴ Project completion time = 27 days.

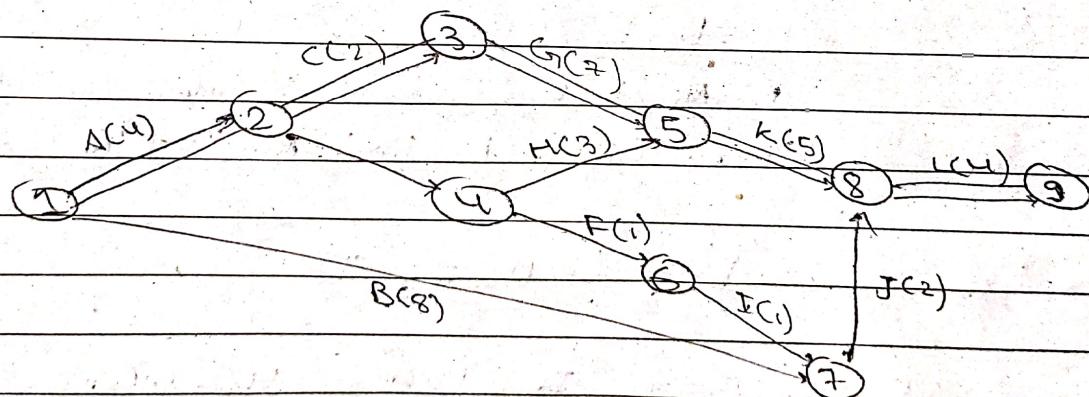
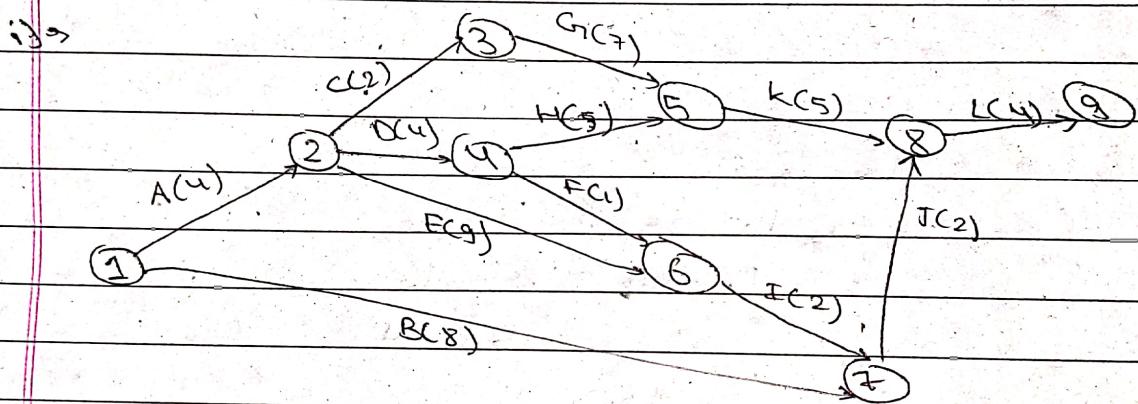
Task 2: A project with the following precedence relationships and durations:

Activity: A B C D F F G H I J K L

Predecessor: - - A A A D C D F, F B, I G, H J, K

Duration, (week) 4 8 2 4 9 1 7 3 2 2 5 4

- Draw the network and find the critical path
- List the total float, free float and independent float for all the activities.



Forward Pass:

$$E_1 = 0$$

$$E_2 = E_1 + t_{12} = 0 + 4 = 4$$

$$E_3 = E_2 + t_{23} = 4 + 2 = 6$$

$$E_4 = E_3 + t_{34} = 6 + 4 = 10$$

$$E_5 = \max(E_2 + t_{25}, E_4 + t_{45}) = 13$$

$$E_6 = \max(E_4 + t_{46}, E_5 + t_{56}) = \max(10 + 3, 13 + 3) = 13$$

$$E_7 = \max(E_5 + t_{57}, E_6 + t_{67}) = \max(13 + 2, 13 + 2) = 15$$

$$E_8 = \max(E_7 + t_{78}, E_8 + t_{88}) = \max(15 + 2, 15 + 2) = 18$$

$$E_9 = E_8 + t_{89} = 18 + 4 = 22$$

Backward Pass:

$$\text{Set } L_9 - E_9 = 22$$

$$L_8 = L_9 - t_{89} = 22 - 4 = 18$$

$$L_7 = L_8 - t_{78} = 18 - 2 = 16$$

$$L_6 = L_7 - t_{76} = 16 - 2 = 14$$

$$L_5 = L_8 - t_{58} = 18 - 5 = 13$$

$$L_4 = \min(L_5 - t_{45}, L_6 - t_{46}) = \min(13 - 3, 14 - 1) = 10$$

$$L_3 = L_5 - t_{35} = 13 - 2 = 11$$

$$L_2 = \min(L_3 - t_{23}, L_4 - t_{24}, L_5 - t_{52}) = \min(11 - 3, 10 - 4, 13 - 9) = 4$$

$$L_1 = \min(L_1 - t_{12}, L_2 - t_{12}) = \min(4 - 4, 11 - 3) = 6$$

For Non-Critical Path (B, D, E, F, H, T)

$$BCI-7) \quad FF_{12} = E_2 - E_1 - t_{12} \quad IF_{12} = 15 - 0 - 8 \quad IF_{11} = 16 - 0 - 8$$

$$= 10 - 0 - 8 \quad = 7 \quad = 8$$

$$= 7$$

D(2-4):  $FF_{24} = 8-4-4 = 0$     $IF_{24} = 8-4-4 = 0$     $TF_{24} = 14-4-4 = 6$

F:  $FF_{46} = 13-8-1 = 4$     $IF_{46} = 13-10-1 = 2$     $TF_{46} = 14-8-3 = 3$

E:  $FF_{26} = 13-4-9 = 0$     $IF_{26} = 13-4-9 = 0$     $TF_{26} = 14-4-9 = 1$

H:  $FF_{93} = 13-8-3 = 2$     $IF_{93} = 13-10-3 = 0$     $TF_{93} = 13-8-5 = 0$

I:  $FF_{67} = 15-13-2 = 0$     $IF_{67} = 15-14-2 = -1$     $TF_{67} = 16-13-2 = 1$

J:  $FF_{28} = 18-15-2 = 1$     $IF_{28} = 18-16-2 = 0$     $TF_{28} = 18-15-2 = 1$

For Critical:

A:  $FF_{12} = 4-0-4 = 0$     $IF_{12} = 4-0-4 = 0$     $TF_{12} = 4-0-4 = 0$

C:  $FF_{23} = 6-4-2 = 0$     $IF_{23} = 6-4-2 = -2$     $TF_{23} = 6-4-4 = -2$

G:  $FF_{35} = 13-6-7 = 0$     $IF_{35} = 13-6-4 = 3$     $TF_{35} = 13-4-4 = 5$

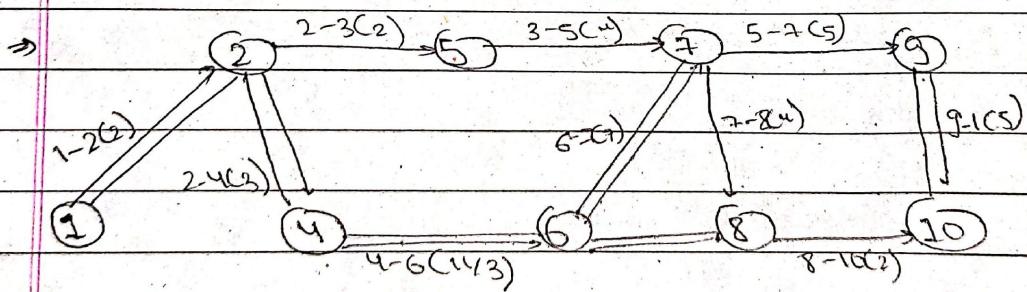
K:  $FF_{58} = 18-13-8 = -3$     $IF_{58} = 18-13-4 = 1$     $TF_{58} = 18-13-4 = 1$

L:  $FF_{84} = 22-18-4 = 0$     $IF_{84} = 22-18-4 = 0$     $TF_{84} = 22-18-4 = 0$

Task 3: Project schedule has the following characteristics:

Activity	to	$t_m$	$t_p$
1-2	1	2	3
2-3	1	2	3
2-4	1	3	5
3-5	3	4	5
4-5	2	3	4
4-6	3	3	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	4	6	8
8-10	1	2	3
9-10	3	5	7

- i) Draw the project network and identify all the paths through it.



Paths from node 1 to 10 : Duration

1-2 → 2-3 → 3-5 → 5-7 → 7-9 → 9-10 24

1-2 → 2-3 → 3-5 → 5-7 → 7-8 → 8-10 19

1-2 → 2-4 → 4-5 → 5-7 → 7-9 → 9-10 24

1-2 → 2-4 → 4-5 → 5-7 → 7-8 → 8-10 19

1-2 → 2-4 → 4-6 → 6-7 → 7-8 → 8-10 26.67

1-2 → 2-4 → 4-6 → 6-7 → 7-9 → 9-10 26.67

Since max. value  $\rightarrow 26.67$ , so, critical path is:

1-2 → 2-4 → 4-6 → 6-7 → 7-9 → 9-10

ii) Determine expected project length

$\therefore$  Expected project length  $\therefore 26.67$  weeks

iii) Probability that the project will be completed at least

4 weeks earlier than expected time is:  $S_e^2$

Since critical path is  $1-2 \rightarrow 2-4 \rightarrow 4-6 \rightarrow 6-7 \rightarrow 7-9 \rightarrow 9-10$

$$S_e^2 = (1-2) \rightarrow 1/9$$

$$(2-4) \rightarrow 4/9$$

$$(4-6) \rightarrow 4/9$$

$$(6-7) \rightarrow 1/9$$

$$(7-9) \rightarrow 1/9$$

$$(9-10) \rightarrow 4/9$$

$$\therefore S_e^2 = 1/9 + 4/9 + 4/9 + 1/9 + 4/9 + 4/9 = 2$$

$$\therefore S_e = \sqrt{2} = 1.41,$$

At least 4 weeks early means time deviation is ?  
 $22.67 - 50, P(Z \leq \frac{T_g - T_e}{\sigma_e})$

$$\begin{aligned} &= P\left(Z \leq \frac{22.67 - 26.67}{52}\right) \\ &= P(Z \leq -0.8) \\ &\approx 0.1924 \end{aligned}$$

$\therefore$  Prob. at least 4 week = 0.1924.

iv.) 4 weeks later is  $26.67 + 4 = 30.67$  week So

$$\begin{aligned} &P\left(Z \leq \frac{T_g - T_e}{\sigma_e}\right) \\ &= P\left(Z \leq \frac{30.67 - 26.67}{52}\right) \\ &= P(Z \leq 0.828) \\ &\approx 0.9976 \end{aligned}$$

$\therefore$  Prob. no more than 4 week later = 99.76%

v) Given:

$$T_s = 19$$

$$\sigma_e = \sqrt{2}$$

$$t_e = 26.6^2$$

$$\therefore P\left(\frac{z \leq 19 - 26.6^2}{\sqrt{2}}\right) = P(z \leq -5.423) = 0.0001$$

Thus, probability of not meeting 19 weeks is  $1 - 0.0001$   
 $= 0.9999$   
 $\approx 99.99\%$

vi) Given:

$$T_s = 20$$

$$\sigma_e = \sqrt{2}$$

$$t_e = 26.6^2$$

$$\therefore P\left(\frac{z \leq 20 - 26.6^2}{\sqrt{2}}\right) = P(z \leq -4.71) = 0.0001$$

 $\approx 0.01\%$ 

vii) Given:

$$Z_{0.9} = \frac{T_s - t_e}{\sigma_e} = \frac{19 - 26.6^2}{\sqrt{2}}$$

$$\Rightarrow 1.28155 \times \sqrt{2} + 26.6^2 = T_s$$

$$\therefore T_s = 28.48 \text{ weeks}$$

Thus, scheduled completion time for probability of completion is 90% is 28.48 weeks.