KATHMANDU UNIVERSITY

Dhulikhel, Kavre

COMP 304

Operations Research

Assignment - 4 (Queue)

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Q.No. 1

Ans

i) Average number of units in the system

Here,

$$\lambda = 10 \text{ units/hr}$$

$$1/\mu = 3 \text{ min} = 3/60 \text{ hr}$$

$$\mu = 20 \text{ units/hr}$$

Now,

$$\rho = \lambda/\mu = 10/20 = 0.5$$

So, average no. of units $= \dfrac{\rho}{1-\rho} = \dfrac{0.5}{1-0.5} = 1$ unit

ii) Average waiting time for customer

$$\Rightarrow W_q = \frac{L_q}{\lambda} = \frac{1}{\lambda}\left[\frac{\lambda^2}{\mu(\mu-\lambda)}\right]$$

$$= \frac{10}{20(20-1)}$$

$$= 0.05 \text{ hr}$$

$$= 3 \text{ minute}$$

iii) Average length of queue

$$\Rightarrow L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{20(20-10)} = 0.5 \simeq 1 \text{ unit}$$

iv) Probability that a customer arriving at the pump will have to wait

$$P(n \geq 1) = (\lambda / \mu)$$
$$= \rho$$
$$= 0.5$$

$\therefore$ There is 50% chance that a customer need to wait

v) The utilization factor for the pump unit

$$\rho = 0.5 = 50\%$$

vi) Probability that the number of customers in the system is 2

$$P_n = \rho^n (1-\rho)$$
$$\Rightarrow P_2 = \rho^2 (1-\rho)$$
$$= 0.5^2 (1-0.5)$$
$$= \cancel{1.0}\ 0.125$$
$$= 12.5\%$$

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Q.No. 2

i) Find the effective arrival rate

Here,

$\lambda = 5$ per hour

$1/\mu = 10$ min $= \frac{10}{60}$ hours

$\mu = 6$ per hour

$N = 5+1 = 6$

$\rho = \frac{\lambda}{\mu} = \frac{5}{6} = 0.833$

$\therefore$ Effective arrival rate $(\lambda_{eff}) = \lambda(1-P_N)$

$$= \lambda\left(\frac{1-\rho^N(1-\rho)}{1-\rho^{N+1}}\right)$$

$$= 5\left(\frac{1-(0.833)^6(1-0.833)}{1-0.833^7}\right)$$

$= 5$ per hour

ii) What is the probability an arriving car will get service immediately upon arrival?

$\Rightarrow P_0 = \frac{1-\rho}{1-\rho^{N+1}}$

$$= \frac{1-0.833}{1-0.833^7}$$

$= 0.231$

$= 23.1\%$

iii) Find the expected number of parking spaces occupied.

$$\Rightarrow L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\,\rho^{N+1}}{1-\rho^{N+1}}$$

$$= \frac{0.833}{1-0.833} - \frac{7\cdot(0.833)^7}{1-0.833^7}$$

$$= 2.291$$

$\simeq$ 3 parking space.

Q.No.3

i) Will a queue be formed?

Here,

$$1/\lambda = 0.7 \text{ day}$$
$$\Rightarrow \lambda = 1.43/\text{day}$$

$$1/\mu = 0.5 \text{ day}$$
$$\Rightarrow \mu = 2\ /\text{day}$$

$$\Rightarrow L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{1.43^2}{2(2-1.43)} = 1.79 \simeq 2 \text{ tractors}$$

$\therefore$ There will be queue of 2 tractors.

ii) Is the queue if forms statisically stabilize?

$\Rightarrow \rho = \frac{\lambda}{\mu} = \frac{1.43}{2} = 0.715 < 1$

Here, $\rho < 1$ so it is statisically stable.

iii) What is the utilization factor of the tractors?

$\Rightarrow \rho = 0.715 = 71.5\%$

which is the req. utilization factor //

iv) What is the idle time in daily duty of 7 hours?

$\Rightarrow P_0 = 1 - \rho = 1 - 0.715 = 0.285$

$\therefore$ Ideal time $= 0.285 \times 7 = 1.995 \simeq 2$ hours //

v) What is the mean number of job orders in the system?

$\Rightarrow L_s = \frac{\rho}{1-\rho} = \frac{0.715}{1-0.715} = 2.508 \simeq 3$ orders //

vi) What is the mean waiting time for job orders in the system?

$\Rightarrow W_q = \frac{L_q}{\lambda} = \frac{1.79}{1.43} = 1.25$ days //