

# CMSC 351 (JWG) Exam 2 Spring 2021

Due by Wed Apr 14 at 12:00noon

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## Exam Logistics:

1. From the moment you download this exam you have three hours to take the exam and submit to Gradescope. This includes the entire upload and tag procedure so do not wait until the last minute to do these things.
2. Tag your problems! Please! Pretty please!
3. You may print the exam, write on it, scan and upload.
4. Or you may just write on it on a tablet and upload.
5. Or you are welcome to write the answers on separate pieces of paper if other options don't appeal to you, then scan and upload.

## Exam Rules:

1. You may ask for clarification on questions but you may not ask for help on questions!
2. You are permitted to use official class resources which means your own written notes, class Panopto recordings and the textbook. The textbook won't help much so don't worry if you don't have it.
3. You are not permitted to use other resources. Thus no friends, internet, calculators, Wolfram Alpha, etc. Question 3 is an exception - you can use technology there. Still no friends etc.!
4. By taking this exam you agree that if you are found in violation of these rules that the minimum penalty will be a grade of 0 on this exam.

## Exam Work:

1. Show all work as appropriate for and using techniques learned in this course.
2. Any pictures, work and scribbles which are legible and relevant will be considered for partial credit.
3. Arithmetic calculations do not need to be simplified unless specified.

1. (a) Suppose the partition algorithm from QuickSort is used just once on a list and the resulting list is: [5 pts]

[2, 1, 6, 9, 7, 10, 17, 16, 11, 20]

Which value(s) could have been the pivot? Explain.

**Solution:**

- (b) Draw a visual representation of MergeSort applied to the list: [5 pts]

[5, 6, 2, 8, 1, 7, 3]

See page 2 of the `mergsort.pdf` notes to see what you should be drawing here.

**Solution:**

2. Draw the tree diagram for Karatsuba's Algorithm applied to  $(4597)(358)$  and use it to count [10 pts]  
the number of single-digit multiplications.

**Solution:**

3. Suppose  $T(n) = 2T(n/8) + f(n)$ . Use the Master Theorem to solve this recurrence relation for each of the following  $f(n)$ . You do not need to check the regularity condition if that case arises.

(a) With  $f(n) = 1 + \sqrt[3]{n}$ .

[3 pts]

**Solution:**

(b) With  $f(n) = n \ln n$ .

[3 pts]

**Solution:**

(c) With  $f(n) = \sqrt[4]{n} + 256$ .

[4 pts]

**Solution:**

4. Given the following recurrence relation:

$$T(n) = 2T(n/4) + \sqrt[4]{n} \text{ with } T(1) = 3.$$

Assume for what follows that  $n$  is some power of 4. This removes any concerns about the necessity of floors and ceilings.

It's a fact that there are  $1 + \log_4 n$  layers in the associated recurrence tree. If we label them  $k = 0$  for the root node,  $k = 1$  for the next layer, and so on, up to  $k = \log_4 n$  for the leaf nodes then layer  $k$  has  $2^k$  entries. Do not prove this.

- (a) Draw the first three layers of the associated recurrence tree and fill in the expressions [5 pts]  
(involving  $n$  with all  $T$  calculated) for each node.

**Solution:**

- (b) For an arbitrary non-leaf layer  $0 \leq k \leq \log_4 n - 1$  what is the time requirement for each [5 pts]  
node and consequently what is time requirement for the entire layer?

**Solution:**

- (c) Following up from that, for the leaf-layer  $k = \log_4 n$  what is the time requirement for each node and consequently what is the time requirement for the entire layer? [5 pts]

**Solution:**

- (d) Write down a sum for the total time in the tree. This should absolutely involve  $\Sigma$  notation. [5 pts]  
You do not need to evaluate this sum.

**Solution:**

5. Suppose a list  $A$  of length  $n$  indexed at 1 represents a heap, write the pseudocode for an [10 pts]  
algorithm which returns a list of nodes which violate the maxheap property.

**Solution:**

6. A list  $A$  contains only integer powers of 2, has minimum  $min$  and maximum  $max$ . For example  $A = [2^3, 2^{-1}, 2^2]$  has  $min = \frac{1}{2}$  and  $max = 8$ .

- (a) Explain how you could modify the CountingSort pseudocode to sort this list by sorting by exponent rather by the value itself. For example if  $A = [2^3, 2^{-1}, 2^2]$  you can sort this by sorting  $[3, -1, 2]$  instead. I am not expecting/wanting pseudocode, I am just asking you to give a brief overview of how you would modify the regular pseudocode. [5 pts]

**Solution:**

- (b) What would the  $\Theta$  time complexity be in terms of  $n$ ,  $min$  and  $max$ ? [5 pts]

**Solution:**

- (c) Give two reasons why this is preferable to just using CountingSort directly. [5 pts]

**Solution:**



7. Suppose  $A$  is a list of  $n$  distinct positive integers and you wish to choose the  $k$ th order element. [10 pts]  
You have a function `where(i,k)` which will return:

- **True** if the  $k$ th order element is at index  $i$ .
- **Left** if the  $k$ th order element is to the left of index  $i$ ,
- **Right** if the  $k$ th order element is to the right of index  $i$ ,

Explain how you can find the  $k$ th order element in  $\lg(n)$  time. You don't need to write pseudocode, a decent explanation of the algorithm and why it's  $\lg(n)$  will suffice.

**Solution:**

8. Consider the following modification of Binary Search. Here A is a list containing distinct integers. Note that the only changes are what happens after the **while** loop ends.

```
function BINARYSEARCH(A,TARGET)
    L = 0
    R = len(A) - 1
    while L <= R do
        C = floor((L+R)/2)
        if TARGET = A[C] then
            return C
        end if
        if TARGET < A[C] then
            R = C - 1
        end if
        if TARGET > A[C] then
            L = C + 1
        end if
    end while
    if L <= len(A) - 1 then
        return L
    else
        return False
    end if
end function
```

- (a) What is returned for the list [1,3,5,7] with TARGET = 5? [3 pts]

**Solution:**

- (b) What is returned for the list [1,3,5,7] with TARGET = 4? [3 pts]

**Solution:**

- (c) What is returned for the list [1,3,5,7] with TARGET = 8? [4 pts]

**Solution:**

- (d) How is this algorithm different from regular Binary Search? [5 pts]

**Solution:**

9. Extra Credit: Fill in the speech bubbles with something witty.

[ $\leq 3$  pts]

