CMSC 351 Spring 2022 Exam 3

Version TuTh

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Directions: Please do work in the spaces provided and do not spill over to other pages - the exams will be scanned into Gradescope and auto-tagged this way. Use only methods taught in this course and show work as indicated. No calculators or other devices permitted. Good luck!

1. Mark each of the following as true or false. Write the word TRUE or FALSE in the box to the right. No justification is required. [10 pts]

   (a) The graph with adjacency list $[[2],[3],[0],[4]]$ is connected. [ ]

   (b) If the adjacency matrix for a graph has a column of all 0 then the graph is disconnected. [ ]

   (c) If DFT starts at node s it will always head directly to the node furthest away. [ ]

   (d) MOM uses columns of length 5 [ ]

   (e) Karatsuba always uses fewer SDMs than Schoolbook Multiplication. [ ]

2. Draw a graph G for which the nodes are labeled with the letters A, B, C, D, E such that: [10 pts]

   - A BFT starting at node A could traverse the nodes in the order: A, E, B, C, F, D.
   - A DFT starting at node A could traverse the nodes in the order: A, B, F, D, E, C.

   You do not need to justify.

   Solution:

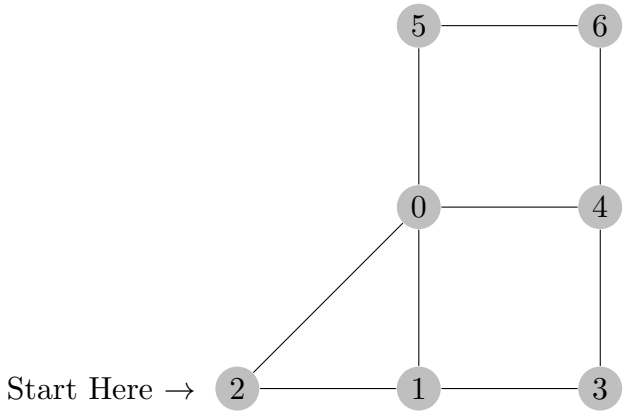
3. Just for reference, here is our pseudocode for Karatsuba: [20 pts]

```
function karatsuba(A,B)
    if either A or B is single-digit
        return(A*B)
    else
        sp = floor((minimum number of digits in A,B)/2)
        A1,A0 = split A, sp digits from the right
        B1,B0 = split B, sp digits from the right
        k1 = karatsuba(A1,B1)
        k2 = karatsuba(A1+A0,B1+B0)
        k3 = karatsuba(A0,B0)
        r = 10^(2*sp)*k1 + 10^(sp)*(k2-k3-k1) + k3
        return(r)
    end
end
```

Draw a tree diagram for Karatsuba's Algorithm applied to (1156)(9852) and use it to count the number of single-digit multiplications. You do not have to evaluate the final result of the overall multiplication, just break down the process into single-digit multiplications and count how many there are.

Solution:

4. Suppose the shortest path algorithm is applied to the following graph starting with node 2. When nodes adjoining a specific node are pushed, they will be pushed in increasing order. [20 pts]



If the queue Q starts at $Q = [2]$, list what Q will be after each pop and its associated pushes. There are 7 pops.

Solution:

After the first pop and pushes: $Q = [$

After the second pop and pushes: $Q = [$

After the third pop and pushes: $Q = [$

After the fourth pop and pushes: $Q = [$

After the fifth pop and pushes: $Q = [$

After the sixth pop and pushes: $Q = [$

After the seventh pop and pushes: $Q = [$

5. In determining the MOM of a list containing n distinct elements suppose n is an odd multiple of 5 plus 1, so $n = 5(2k + 1) + 1$. We proceed as usual, arranging the list into columns of 5 elements and one column of 1 element. [20 pts]

   Assuming that the MOM is from one of the columns of 5 elements, and is not greater than the column of 1 element, show that the MOM is greater than or equal to $\frac{3}{10}n$ elements.

   Solution:

6. Suppose T is a full binary tree with N levels and hence $n = 2^N - 1$ nodes. This means that every node has two children except the leaf nodes (level N) which have no children.

   Suppose exactly one node is EXPLOSIVE.

   We run the following process:

   (a) We run BFT starting at the root node. BFT always visits left before right.

   (b) When the EXPLOSIVE node is encountered, we suddenly run DFT on that node (without revisiting any of the nodes our BFT visited).

   (c) When DFT finishes the process exits, without worrying about any other nodes.

   Suppose visiting each node takes exactly 1 second, nothing else takes any time at all, and the EXPLOSIVE node is node i.

   (a) For any i, how much time will the BFT part of the process take, not including node i? [5 pts]
   Solution:

   (b) For any i, how much time will the DFT part of the process take? [15 pts]
   Hint: Node i is in level $1 + \lfloor \lg i \rfloor$ where the root is level 1.
   Solution: