PHYS 720 – FINAL PROJECT

MULTI-MODE OPTICAL CAVITY BASED QUANTUM HARMONIC OSCILLATOR

STUDENT: ANISH RAJESH MORE

CLASS INSTRUCTOR: DR. PRATIK DHOLABHAI

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1. INTRODUCTION:

1. BACKGROUND & APPLICATIONS:

In the realm of quantum optics, where light and matter interact in ways beyond classical understanding, the concept of a quantum harmonic oscillator plays a pivotal role. The harmonic oscillator, a fundamental model in physics, describes systems that exhibit oscillatory behavior around an equilibrium point. In the field of quantum physics, this oscillator represents the simplest quantum mechanical system with discrete energy levels.

Historically, the study of harmonic oscillators dates back to the early 20th century with the development of quantum mechanics. The quantum harmonic oscillator was formalized by physicists Erwin Schrödinger and Werner Heisenberg. Schrödinger's wave equation and Heisenberg's matrix mechanics offered complementary descriptions of quantum systems, demonstrating the quantization of energy levels and the probabilistic nature of quantum mechanics.

Quantum optics, as a distinct field, began to take shape in the mid-20th century with the advent of lasers and the development of quantum electrodynamics (QED). Researchers started exploring how light interacts with matter at the quantum level, leading to the realization that optical phenomena could be described using quantum mechanical principles.

One significant advancement in this field is the introduction of optical cavities. These cavities, typically composed of mirrors that confine light within a resonant structure, have enabled precise control and manipulation of light-matter interactions. This development paved the way for the realization of the multi-mode optical cavity-based quantum harmonic oscillator. In the 1980s and 1990s, groundbreaking experiments demonstrated the strong coupling regime, where individual photons interacted with atoms or quantum dots confined within optical cavities. These setups effectively created harmonic oscillator systems in which photons and atoms exchanged energy in quantized units.

The motivation behind studying such systems stems from their potential applications in quantum information processing, quantum communication, and quantum metrology. By harnessing the unique properties of multi-mode optical cavities, researchers aim to create robust platforms for quantum computation, where quantum bits (qubits) can be encoded and manipulated with high fidelity. Moreover, these systems offer insights into fundamental quantum phenomena such as entanglement, squeezing, and quantum state engineering. Understanding and mastering the dynamics of multi-mode optical cavity-based quantum harmonic oscillators not only deepen our comprehension of quantum mechanics but also hold promise for practical advancements in technology.

2. THEORY:

1. SINGLE MODE OPTICAL CAVITY:

We start by constructing a 3-dimensional Cavity. This is done by placing two perfectly reflecting parallel mirrors at a distance $'L'$ apart from each other. We assume the optical cavity to be cubical in shape. The two mirrors are placed at $z = 0$ and $z = L$ and their surface area is given by, A. For the sake of simplicity, we assume the electromagnetic field within the cavity to be characterized by a single frequency, ω. The walls of the cavity are assumed to be perfectly conducting. We also make the assumption that there are no sources of electric and magnetic fields within the cavity and the cavity is isolated from the rest of the universe.

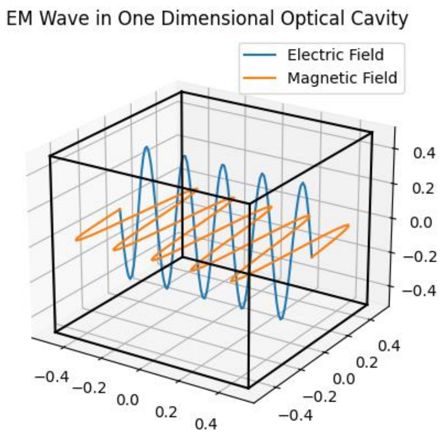


Figure 1.1: Single Mode Optical Cavity characterized by angular frequency $\omega = \frac{\pi}{5}$

The electric and the magnetic fields, owing to the strict boundary conditions vanish at the ends of the walls. The electric field is polarized in the x-direction. Thus, it can be represented in vector form as, $\boldsymbol{E}(\boldsymbol{r}, t) = E_x(z, t)\,\hat{x}$

Now, the Maxwell Equations in the absence of sources of charges and currents are given by

$$\nabla \times \boldsymbol{E} = -\frac{\partial B}{\partial t} \tag{1.1}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \tag{1.2}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{1.3}$$

$$\nabla . \mathbf{E} = 0 \tag{1.4}$$

One of the solutions for an electric field restricted to these boundary conditions is given by,

$$E_x(z,t) = \left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} q(t)\sin(kz)\,\hat{x} \qquad 1.5$$

Where ω is the angular frequency of the light wave, k is the wave number, which can be obtained from the relation $\omega = ck$, V is the volume of the cavity, and $q(t)$ is a time-dependent function analogous to position which we want to determine.

Now using the boundary conditions at $z = L$,

$$E_x(z = L, t) = 0$$

$$\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} q(t)\sin(kL) = 0$$

$$\sin(kL) = 0$$

This implies that k and thereby ω are restricted to discrete values given by,

$k = n\pi/L$ and $\omega = \frac{nc\pi}{L}$.

We can use 1.1, to find out the magnetic field $B_y(z,t)$,

$$\frac{1}{\mu_0\varepsilon_0}\nabla\times\boldsymbol{B} = \frac{\partial\boldsymbol{E}}{\partial t} = -\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} q(t)k\cos(kz)\ \hat{x}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_y & B_z \end{vmatrix} = \mu_0\varepsilon_0\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} \dot{q}(t)\sin(kz)\,\hat{x}$$

$$\left(\frac{\partial}{\partial y}B_z - \frac{\partial}{\partial z}\ B_y\right)\hat{x} = \mu_0\varepsilon_0\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} \dot{q}(t)\sin(kz)\,\hat{x}$$

Note that, $\frac{\partial}{\partial y}B_z = 0$

$$-\frac{\partial}{\partial z}B_y = \mu_0\varepsilon_0\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} \dot{q}(t)\sin(kz)$$

$$\boldsymbol{B}_y(z,t) = \left(\frac{\mu_0\varepsilon_0}{k}\right)\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} p(t)\cos(kz) \qquad 1.6$$

where $p(t) = \dot{q}(t)$.

A variant of the magnetic field obtained using 1.1 is,

$$\boldsymbol{B_y}(z,t) = -\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} s(t)k\cos(kz) \tag{1.7}$$

where $\dot{s}(t) = q(t)$

Similarly using 1.6 and 1.2,

$$\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\mu_0\varepsilon_0}\nabla\times\boldsymbol{B} = -\frac{1}{\mu_0\varepsilon_0}s(t)\left(\frac{2\omega^2}{V\varepsilon_0}\right)^{\frac{1}{2}} k\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_y(z,t) & 0 \end{vmatrix}$$

This relation yields,

$$\dot{q}(t) = -\frac{k^2}{\mu_0\varepsilon_0}s(t) = -k^2c^2s(t) = -\omega^2 s(t)$$

$$\ddot{q}(t) + \omega^2 q(t) = 0 \tag{1.8}$$

The general solution of this differential equation is given by,

$$q(t) = Acos(\omega t + \phi) \tag{1.9}$$

We can solve for the function using our Numerical Techniques for Second Order Ordinary Differential Equations.

The energy contained within an electromagnetic field is given by the Hamiltonian,

$$\mathcal{H} = \frac{1}{2}\int dV\left[\varepsilon_0\boldsymbol{E^2}(\boldsymbol{r},t) + \frac{1}{\mu_0}\boldsymbol{B}^2(\boldsymbol{r},t)\right]$$

After plugging in our derived expressions, this yields,

$$\mathcal{H} = \frac{p^2}{2} + \frac{1}{2}\omega^2 q^2$$

This Hamiltonian is analogous to that of a Simple Harmonic Oscillator in 1-D,

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

We can also find the energy in a Quantum Harmonic Oscillator by solving the time-independent Schrodinger's equation with $V(x) = \begin{cases} \frac{1}{2}kx^2, & |x| < L \\ \frac{1}{2}kL^2, & otherwise \end{cases}$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \qquad 1.10$$

In terms of the Eigen Value equation analogous to a Quantum Harmonic Oscillator this is given by,

$$\hat{H}|\psi\rangle = E_n|\psi\rangle$$

This leads to the introduction of the raising and lowering operators, $\hat{a}$ and $\hat{a}^\dagger$ to rewrite the Hamiltonian in the form,

$$H = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

We can also use the creation and annihilation operators to introduce the number operator which corresponds to the energy state of the Quantum System.

$$\hat{N} = \hat{a}^\dagger\hat{a}$$

$$\hat{H}|n\rangle = \hbar\omega\left(n + \frac{1}{2}\right)|n\rangle \qquad 1.11$$

We will be using both the approaches to compute the Energy eigenstates of the Hamiltonian for our given Optical Cavity.

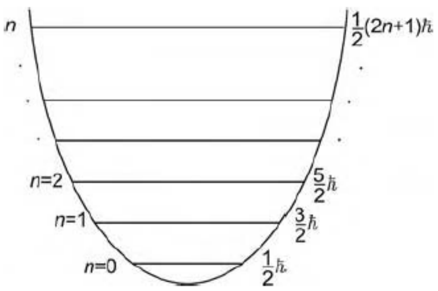


Fig. 2.2. The energy levels of a harmonic oscillator of frequency ω.

Introduction to Quantum Optics, Gerry and Knight

The raising and lowering operators act on the number states as follows:

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

This also implies that we can generate any number state,$|n\rangle$ by applying the creation operator on the vacuum state 'n' times,

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle$$

2. COHERENT STATES:

A linear combination of these number states gives rise to coherent states, which are the most classical states of light.

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{e^{-\frac{|\alpha|^2}{2}} \alpha^n}{\sqrt{n!}} |n\rangle \qquad 1.12$$

Where α can be any complex value.

We can use this to study photon statistics such as the Coherent state photon number probability distribution.

This can be achieved by projecting the coherent state onto the number states,

$$P_n(\alpha) = |\langle n|\alpha\rangle|^2$$

$$P_n(\alpha) = \left| \langle n| \sum_{m=0}^{\infty} \frac{e^{-\frac{|\alpha|^2}{2}} \alpha^m}{\sqrt{m!}} |m\rangle \right|^2$$

$$P_n(\alpha) = \left| \frac{e^{-\frac{1}{2}|\alpha|^2} \alpha^n}{\sqrt{n!}} \langle n|n\rangle \right|^2$$

$$P_n(\alpha) = \frac{e^{-|\alpha|^2} \alpha^{2n}}{n!} \qquad 1.13$$

This resembles a Poisson Distribution.

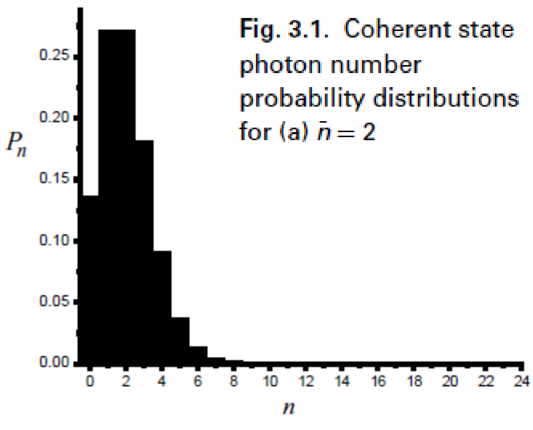


Fig. 3.1. Coherent state photon number probability distributions for (a) $\bar{n} = 2$

3. LIGHT MATTER INTERACTIONS:

Consider our earlier model for Optical Cavity. Now introduce an atom which can be in two states, a ground state and an excited state. Assume that the atom is trapped at the center of the Optical Cavity. Let the Electromagnetic field oscillate within the cavity with a frequency ω and let the frequency of oscillations generated by the two-state atom be ω_0.

Now, similar to our Optical Cavity, the Hamiltonian for the modified cavity will be given by,

$$\hat{H} = \hat{H}_{atom} + \hat{H}_{cavity} + \hat{H}_{interaction}$$

Here, $\hat{H}_{cavity} = \hbar\omega\left(n + \frac{1}{2}\right)$.

Let $|e\rangle$ and $|g\rangle$, represent the excited and ground state for the atom. Thus, the Hamiltonian for the atom will be given by,

$$\hat{H}_{atom} = \hbar\omega_0\hat{\sigma}^\dagger\hat{\sigma}$$

$\hat{\sigma}^\dagger$: Transition from $|e\rangle \rightarrow |g\rangle$

$\hat{\sigma}$: Transition from $|g\rangle \rightarrow |e\rangle$

And the interaction Hamiltonian is given by,

$$\hat{H}_{int} = \hbar g(\hat{\sigma}^\dagger\hat{a} + \hat{\sigma}\hat{a}^\dagger)$$

g: Interaction strength between the cavity and the atom

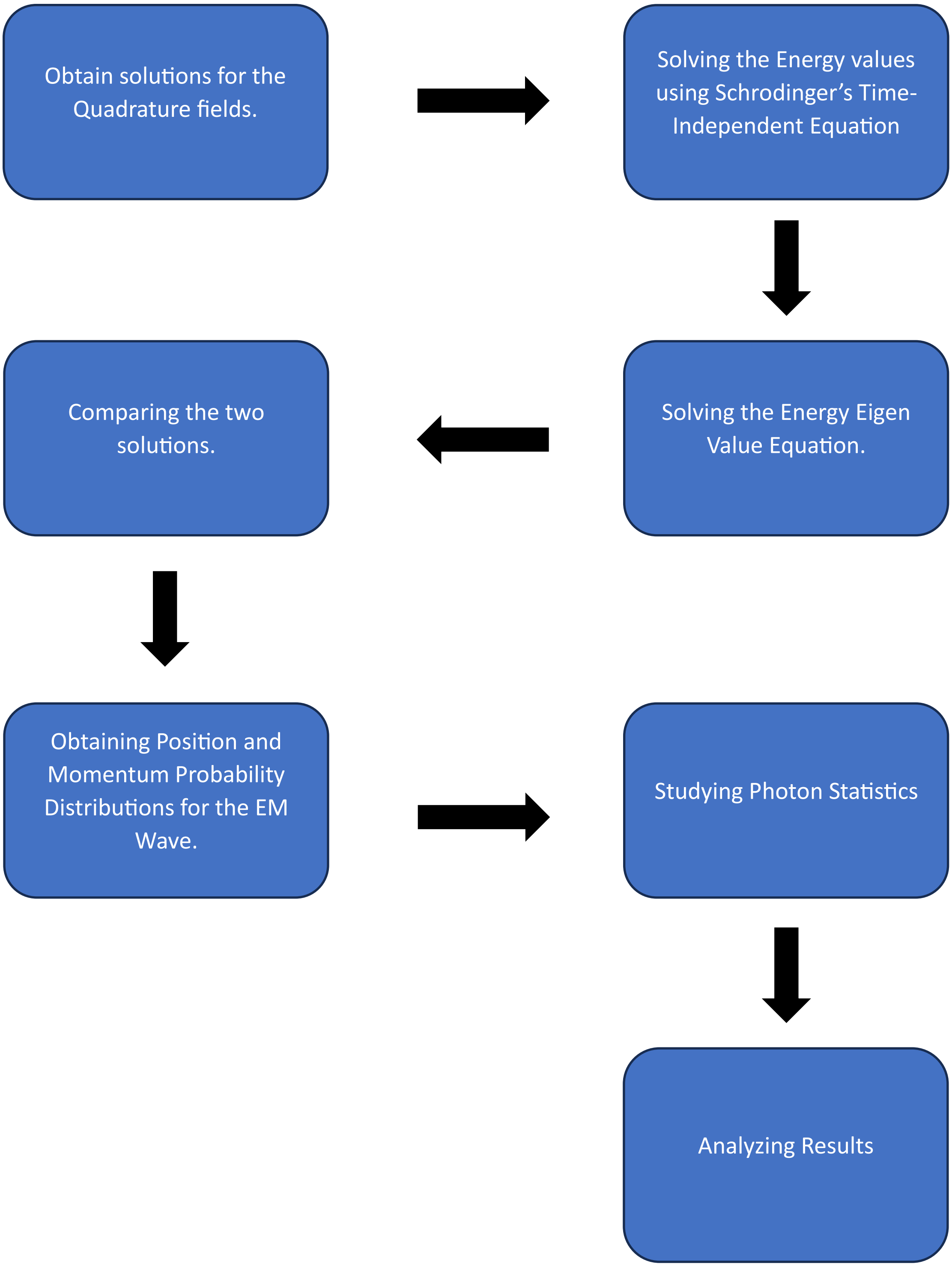
Combining these gives the simplified version known as James- Cunningham Hamiltonian, given as,

$$\hat{H} = \hbar\omega_0\hat{\sigma}^\dagger\hat{\sigma} + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g(\hat{\sigma}^\dagger\hat{a} + \hat{\sigma}\hat{a}^\dagger) \qquad 1.14$$

2. PROGRAM MANUAL

The goal of this project is to study the quantized electromagnetic field within an optical cavity. The source of this EM field is considered to be an atom for the sake of simplicity. We begin by solving for the position and momentum analogues (Also called Quadratures) of Quantum Harmonic Oscillator and feeding these obtained expressions into the expressions for our Electric and Magnetic Fields.

1.ALGORITHM:



2. QUADRATURE SOLUTIONS:

```
def leapfrog(function, init_cond,a,b,n): #
  h = (b-a)/n #Spacing between 2 points
  t_points = np.arange(a,b+h,h) #Time scale
  x_points = [init_cond[0]]
  v_points = [init_cond[1]]
  half_values = init_cond + 0.5 * h * function(init_cond,t_points[0]) #introduced a new 2D array
  #to store the initial value of the function at half-step interval
  for i in range(1,len(t_points)):
    init_cond += h * function(half_values,t_points[i]+0.5*h)
    x_points.append(init_cond[0]), v_points.append(init_cond[1])
    half_values += h * function(init_cond,t_points[i]+h) #Updated the next half-integral
    #value using the previous half-integral value

  return x_points, v_points, t_points
```

Parameters:

- function: Insert the function whose differential equation is to be solved.
- init_cond: Initial conditions of the given differential equation. We assume the wave to be starting at 0 and travelling with wave velocity $c = 3 \times 10^8\ m/s$.
- a: Starting position for the solution of the differential equation.
- b: Ending position for the solution of the differential equation.
- n: Number of points to be evaluated.

Returns:

- x_points: Solution of the given differential equation. Yields the x(position values) of the solution.
- v_points: Yields the velocity component of the solution
- t_points: Yields the time dependency of the solution.

3. QUANTUM HARMONIC ENERGY SOLUTIONS:

```
def solve_schrodinger_eq(xmin, xmax, N):
    # Discretization parameters
    x = np.linspace(xmin, xmax, N)
    dx = (xmin - xmax)/N

    # Kinetic energy operator (second derivative)
    T = np.diag(-2.0*np.ones(N)) + np.diag(np.ones(N-1), k=1) +
np.diag(np.ones(N-1), k=-1)
    T /= dx**2

    # Potential energy operator
    V_mat = np.diag(V(x))

    # Hamiltonian matrix
    H = -(h_bar**2 / (2.0 * m)) * T + V_mat

    # Solve eigenvalue problem
    energies, wavefunctions = np.linalg.eigh(H)
```

```python
    # Normalize wavefunctions
    wavefunctions /= np.sqrt(dx)

    return energies, wavefunctions, x
```

Parameters:

- xmin: Input value for minimum value of x which is to be plotted.
- xmax: Input value for maximum value of x which is to be plotted.
- N: Number of datapoints between xmin to xmax

Returns:

- energies: Returns the eigenvalues for the energy using the numpy defined eigenvalue equation np.linalg.eigh(H).
- wavefunctions: Returns the data points for the Wavefunction/ eigenstate of the
- x: Returns the x values with the limits mentioned in the arguments.

```python
# Define creation and annihilation operators
def creation_operator(i):
    ket = np.zeros(N)
    ket[i] = 1
    return np.sqrt(0.5) * (np.roll(ket, -1) + (-1)**i * ket)

def annihilation_operator(i):
    ket = np.zeros(N)
    ket[i] = 1
    return np.sqrt(0.5) * (np.roll(ket, 1) + (-1)**i * ket)

# Define Hamiltonian
def Hamiltonian():
    H = np.zeros((N, N))
    for i in range(N):
        H += omega[i] * (np.outer(creation_operator(i),
annihilation_operator(i)) + 0.5 * np.eye(N))
    return H

# Calculate eigenstates and eigenvalues
eigenvalues, eigenstates = np.linalg.eigh(Hamiltonian())

# Print eigenvalues and eigenstates
print("Eigenvalues:")
print(eigenvalues)
print("\nEigenstates:")
print(eigenstates)
```

Parameters:

- creation_operator(i): Based on the matrix definition of the creation operator we define a creation operator matrix.
- annihilation_operator(i): Similar to the definition used for creation operator, it gives the matrix definition of the lowering operator.
- Hamiltonian(): This Hamiltonian equation computes the discrete energy levels using the Eigenvalue equation.

Returns:

- H: Returns the Hamiltonian based on 1.11. The Hamiltonian is normalized.

4. PROBABILITY DISTRIBUTION ANALYSIS:

```
# Function to calculate the wavefunction for position. This basically gives the discrete wavefunction solutions
def wavefunction_position(n, x):
    prefactor = 1.0 / np.sqrt(2**n * math.factorial(n)) * (m * omega / (np.pi * hbar)) ** 0.25
    return prefactor * np.exp(-m * omega * x**2 / (2 * hbar)) * np.polynomial.hermite.hermval(np.sqrt(m * omega / hbar) * x, np.eye(n + 1)[-1])
#hermite.hermval returns the solutions of the wavefunctions. More efficient than LU Decomposition
```

Parameters:

- n: Energy state of the system. Takes integer values
- x: Position input. Can be any n-dimensional array.

Returns:

- prefactor*np.exp(-m*omega*x**2/(2*hbar))* np.polynomial.hermite.hermval(np.sqrt(m*omega/hbar)*x,np.eye*(n+1)[-1]): This returns the solved wavefunction similar to the method employed above. Makes use of the hermeval function from numpy to evaluate the solutions of the wavefunction which assume the form of the Hermite Polynomials.

```
# Function to calculate the probability distribution for position
def probability_distribution_position(n, x_values):
    psi = wavefunction_position(n, x_values)
    return np.abs(psi)**2

# Function to calculate the probability distribution for momentum
def probability_distribution_momentum(n, x_values):
    psi_p = np.fft.fftshift(np.fft.fft(wavefunction_position(n, x_values)))
    return np.abs(psi_p)**2
```

Parameters:

- n: Energy state of the system.
- x_values: Position input. Can be any n-dimensional array.

Returns:

- np.abs(psi)**2: Returns the probability density, which is the square of the absolute value of the wavefunction as an array.
- np.abs(psi_p)**2: Returns the probability density for the momentum, which is obtained by using Fourier Transform on the solved wavefunction.

5. COHERENT STATE ANALYSIS:

```
# Define photon number probability distribution P_n(alpha)
def photon_prob_distribution(alpha, n_mean):
    alpha_abs_sq = np.abs(alpha) ** 2
    prob_distribution = np.zeros(n_mean+1)
    for n in range(n_mean+1):
        prob_distribution[n] = np.abs((np.exp(-0.5 * alpha_abs_sq) * alpha ** n) / np.sqrt(factorial(n))) ** 2
    return prob_distribution
```

Parameters:

- alpha: A complex variable alpha, which represents the coherent state of the system.
- n_mean: Represents the mean number of photons in the system.

Returns:

- prob_distribution: Returns the probability distribution for the number of photons in the system.

6. TWO STATE ATOM IN OPTICAL CAVITY:

```
times = np.linspace(0.0, 10.0, 200)
psi0 = tensor(fock(2,0), fock(10, 5))
a = tensor(qeye(2), destroy(10)) #Lowering operator for the optical cavity
sm = tensor(destroy(2), qeye(10)) #Sigma + vector for the atom (from ground to excited state)
#Hamiltonian Equation
H = 2 * np.pi * a.dag() * a + 2 * np.pi * sm.dag() * sm + 2 * np.pi * 0.25 * (sm * a.dag() + sm.dag() * a)
#Solves the Hamiltonian equation to give discretized energy states functions
result = mesolve(H, psi0, times, [np.sqrt(0.1)*a], e_ops=[a.dag()*a, sm.dag()*sm])

#Plotting
plt.figure()
plt.plot(times, result.expect[0])
plt.plot(times, result.expect[1])
plt.xlabel('Time')
plt.ylabel('Expectation values')
plt.legend(("cavity photon number", "atom excitation probability"))
plt.show()
```

Installation:

If you aleady have your conda environment set up, and have the `conda-forge` channel available, then you can install QuTiP using:

```
conda install qutip
```

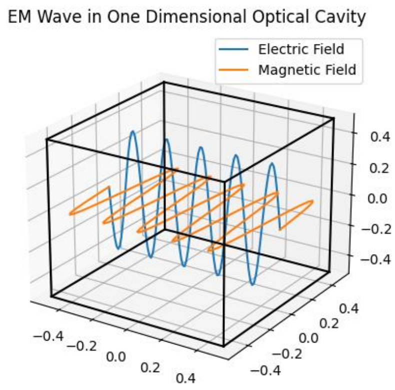
For other types of installation, it is often easiest to use the Python package manager pip.

```
pip install qutip
```

Outcome:

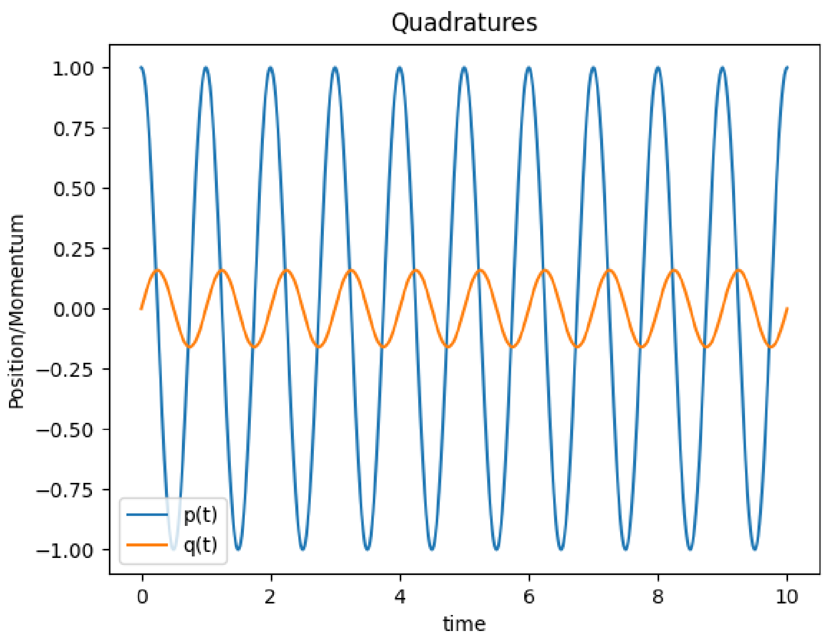
Solves the Hamiltonian by using Von-Neumann Equation using qutip.mesolve() to return the energy eigenstates and energy eigenvectors of the given Hamiltonian equation.

3. RESULTS AND ANALYSIS:



1. QUADRATURE SOLUTIONS:

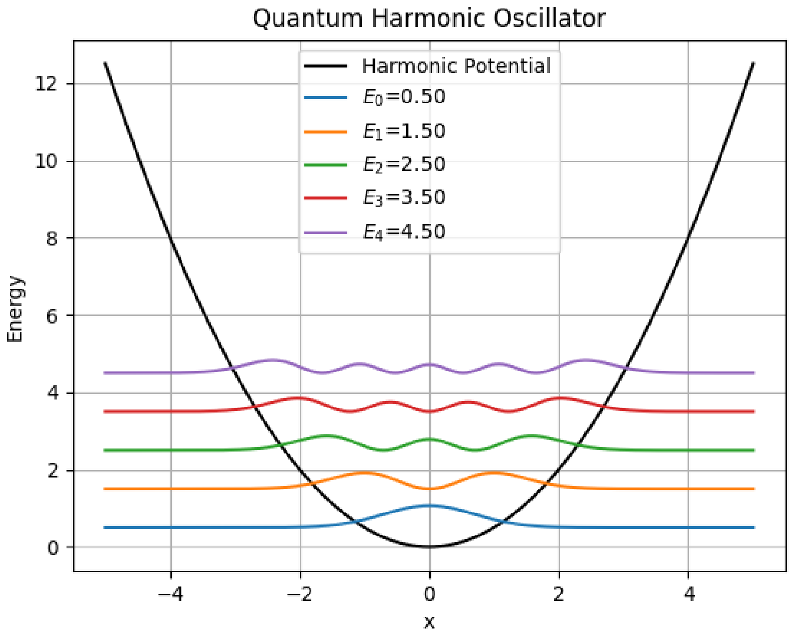
Using 1.8, we were able to find the solutions for the Quadrature, which are the position and momentum functions, $p(t)$ and $q(t)$ which appear in the expressions for Electric and Magnetic Field. We used normalized $\hbar$ to compute the solution for the position operator and discrete central difference method to find the solution of the momentum operator. These solutions are exactly $\frac{\pi}{2}$ out of phase with each other which is to be predicted for position and momentum functions.



2. QUANTUM HARMONIC ENERGY SOLUTIONS:

We use an analogue of 1.10 to compute the Hamiltonian for our given Optical Cavity system. We use matrix operations instead of a differential equation approach to find the discrete solutions. We generate a Kinetic and Potential Energy Operator and define the Hamiltonian Matrix. We use the predefined numpy.linalg.eigh() function to compute the discrete solutions to the Hamiltonian Matrix. We then normalize the wavefunction which is to be plotted.

The result corresponds well with the Quantum Energy Ladder which is well established in Quantum Optics. The difference between two consecutive states is constant is given by $\frac{\hbar}{2}$.



We compare these obtained Energy values with the Eigen value equation for the Hamiltonian given in 1.11. We generate the matrix representations of the creation and annihilation operator which are just diagonal matrices offset either vertically or horizontally. Next, we define the Eigenvalue equation for the Hamiltonian by making use of the numpy defined outer product function.

Results:

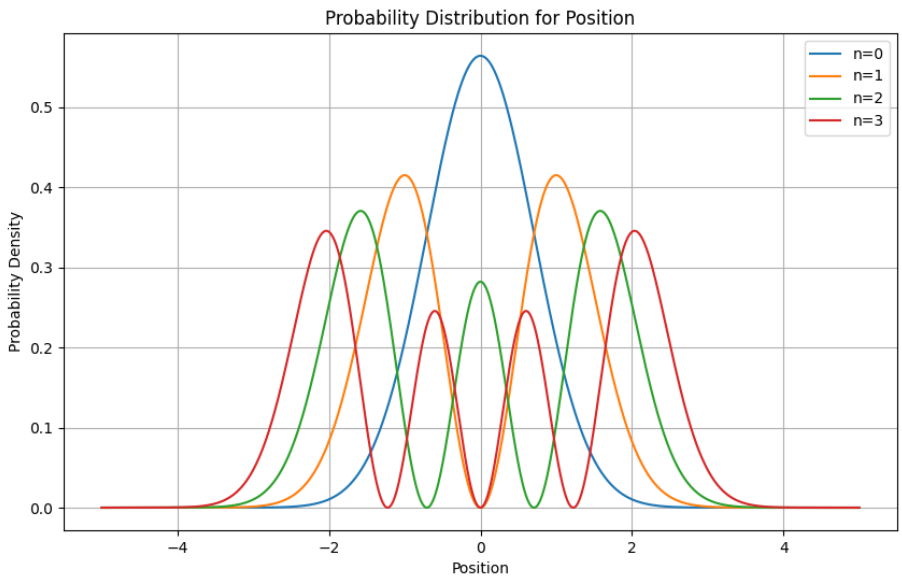
```
Eigenvalues:
[1.33430472 2.61564    5.05005528]

Eigenstates:
[[-0.78594152  0.04450141 -0.6166973 ]
 [ 0.31151967 -0.83306118 -0.45712643]
 [ 0.53408935  0.55138798 -0.6408743 ]]
```

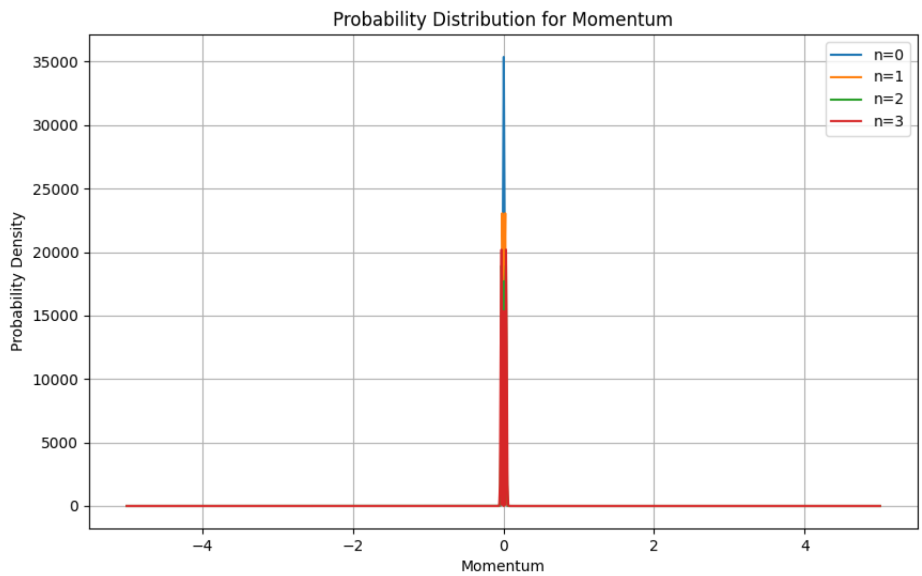
I wasn’t able to figure out why the outer product wasn’t giving me the right eigenvalues for the energy despite working out the equation analytically.

4. PROBABILITY DISTRIBUTION ANALYSIS:

We make use of the Hermite polynomial solver to find the wavefunction solutions found above. The Hermite polynomial solutions for a Quantum Harmonic Oscillator are well-defined and we know that the solution of the Wavefunction solution for a Quantum Harmonic Oscillator yields Hermite Polynomials. So instead of working through the entire function used earlier I used a more concise version to find the wavefunction solutions. The probability density is then given by the square of the absolute value of the wavefunction for discrete intervals.

Next, we perform a fourier transform on the obtained wavefunction to ger the wavefunction in momentum space.





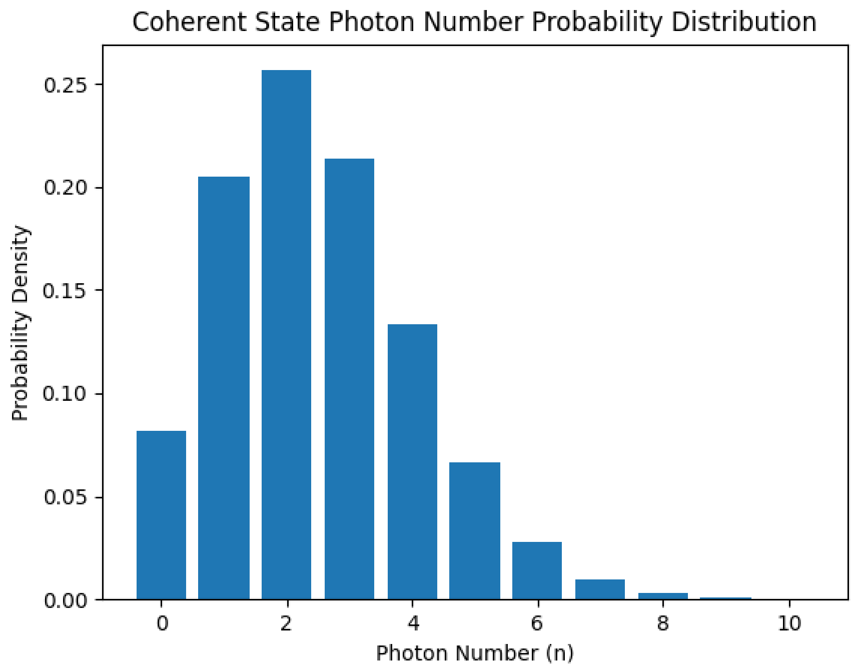
This result matches perfectly with our classical predictions for classical waves. The position probability distribution varies according to the amplitude of the wavefunction and the momentum of a given EM wave is fixed since it travels at a constant wave velocity, c. This also explains why we have no fluctuations in the probability for the momentum since it is very well defined. This also agrees with the commutation relation for our momentum and position operators as well as with the Heisenberg's uncertainty Principle.

5. COHERENT STATE ANALYSIS:

We use the expression for Coherent state photon probability distribution derived using 1.12 and Fock states (Number States).

$$P_n(\alpha) = \frac{e^{-|\alpha|^2}\alpha^{2n}}{n!}$$

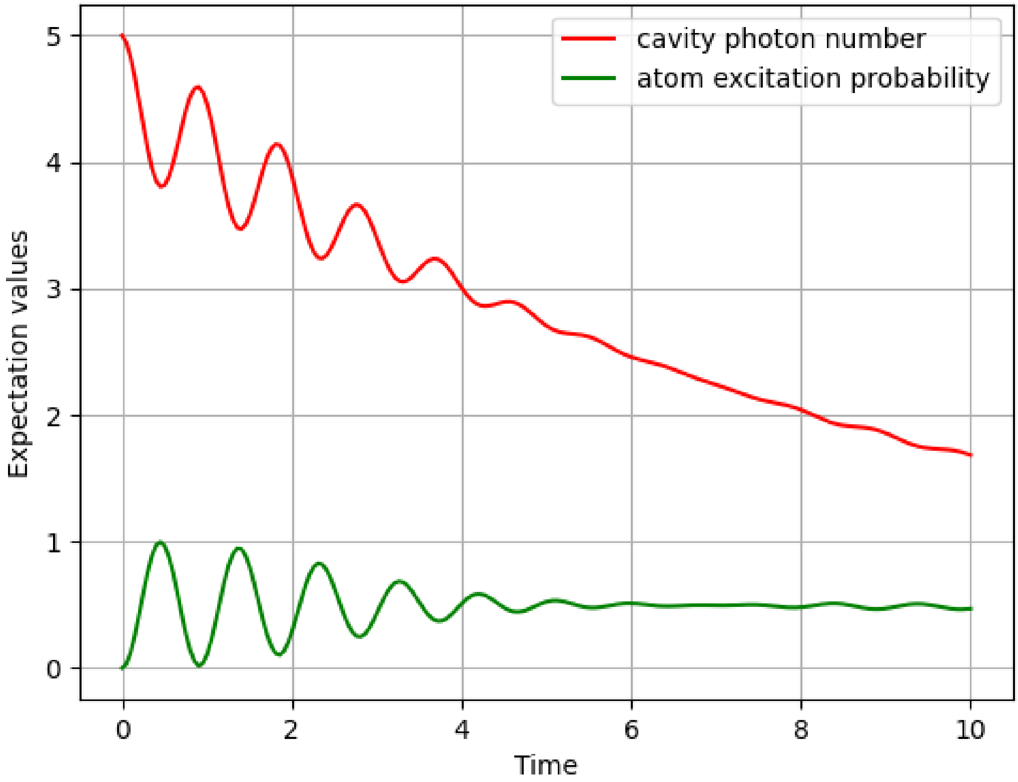
The initial goal was to project the number state onto the coherent state using Linear Algebraic methods. This basically involves taking the inner product of the number state and coherent state. Next, we take the square of that inner product which gives the probability distribution for that particular coherent state. However, I wasn't able to make the program work for some reason so I went ahead and explicitly plotted the probability we derived.



As we expected, we do have a Poisson distribution which peaks at 2 photons. The coherent state we are using is $\alpha = 1.5 + 0.5i$ with mean number of photons $\bar{n} = 10$.

6. LIGHT MATTER INTERACTIONS:

We defined destruction and σ operators in their matrix version. We set the atom to be initially in the ground state and the cavity to be initially in state characterized by, $|\psi\rangle = |5\rangle$ i.e. The state having 5 photons. We use the library QuTip, which is an open-source software for simulating the dynamics of open quantum systems. We use the function, qutip.mesolve(), which uses the Lindblad Master Equation to solve the time evolution of the Schrodinger equation and returns the solutions. We then plot the expectation values of the cavity photon number and the Atom Excitation, by assigning a variable $result$ to the output of the mesolve function, and taking the expectation of the first and second arguments of $result$ using $result.expect()$. The Hamiltonian used is of the form of 1.14.



This outcome is in sync with our classical predictions. Over time, the light within the cavity is absorbed by the atom, which excites it to the higher energy state. The atom at the same time can undergo spontaneous emission to emit the photon within the cavity once again. The peak in the atom excitation probability corresponds to the atom absorbing a cavity photon and getting excited, which is why we get a dip in the number of cavity photons at the same time. This process eventually reaches an equilibrium if allowed to settle over time.

7. CONCLUSION:

Based on our results, we were able to study how optical cavities can give rise to non-classical states of light. We were also able to make quantifiable and classically verifiable predictions about photon statistics such as the photon distribution for coherent states. We were able to verify the Quantize the electromagnetic field within the cavity. We were also able to introduce further complexity within our model by introducing a two-state atom in the system and then examining the characteristics of the system.

Our model, was able to sufficiently predict the Quantum Optical predictions of a 1D Quantum Optical Cavity.

Further work on this model can include inducing additional modes to our Optical Cavity, including all directions of propagation for the EM waves, introducing traveling waves instead of standing waves and so on.

4. REFERENCES:

5.

Christopher Gerry, P. K. (2004). *Introduction to Quantum Optics.* Cambridge University Press.