

1. For a polynomial $P(x) = ax^3 + bx^2 + cx + d$

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha * \beta * \gamma = c/a$$

where α, β, γ are the roots of the polynomial.

2. For a polynomial $P(x) = ax^2 + bx + c$

$$\alpha = \frac{b - \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

where α, β are the roots of the polynomial.

3. $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$

4. $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

5. For an arithmetic progression

$$a, a + d, a + 2d, a + 3d \dots$$

$$T(n) = a + (n - 1) * d$$

$$S(n) = \frac{n}{2} [2a + (n - 1)d]$$

6) For a geometric progression

$$a, ar, ar^2, ar^3 \dots$$

$$S(n) = a \left(\frac{r^n - 1}{r - 1} \right)$$

$$S(\infty) = \frac{a}{1-r} \text{ where } [-1 < r < 1]$$

7) Bertrand Ballot's Theorem problem -

"In an election where candidate a receives p votes and b receives q votes with p > q, what is the probability that a will be strictly ahead of b throughout the count"

$$\text{Ans} - \frac{p-q}{p+q}$$

And if ties are allowed then

$$\text{Ans} - \frac{p-q+1}{p+1}$$

8. $\left(\frac{A}{B} \right) \% mod = (A \% mod * x \% mod) \% mod$

where x is the modular multiplicative inverse of B under mod

9) Fermat's little theorem

$$a^{m-2} \equiv a^{-1} \% mod$$

when mod is prime

10) $E_p(n!)$: Exponent of p in n! where p is a prime

$$E_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \left\lfloor \frac{n}{p^4} \right\rfloor + \dots$$

11. let N be a natural number

$$N = p_1^{\alpha_1} * p_2^{\alpha_2} * p_3^{\alpha_3} \dots$$

Then total number of divisors =

$$(\alpha_1 + 1) * (\alpha_2 + 1) * (\alpha_3 + 1) * \dots$$

$$\text{Sum of divisors} = \left(\frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \right) * \left(\frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \right) * \left(\frac{p_3^{\alpha_3+1} - 1}{p_3 - 1} \right) * \dots$$

$$\text{Product of divisors} = n^{\frac{(\alpha_1+1)(\alpha_2+1) \dots (\alpha_m+1)}{2}}$$

If n is perfect square then multiply it by \sqrt{x} in case of product of divisor

12. $a^b \% mod = a^{b \% (mod-1)} \% mod$ when mod is prime

13. $a^b \% mod = a^{b \% \phi(mod)} \% mod$ when mod is not prime

where $\phi(mod)$ = Euler totient function of mod

14. Derangement of N elements

$$D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right)$$

15. If m and n are two co-prime numbers then the biggest number which can not be represented as $(am + bn)$ for non negative integer m and n is $mn - m - n$
16. Euler Totient function $(\phi(n))$: Number of numbers which are coprime with n in range of $[1, n]$

$$\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3})$$

where $n = p_1^{\alpha_1} * p_2^{\alpha_2} * p_3^{\alpha_3} \dots$

15. Centroid of a triangle $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$

16. Incentre $(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c})$

17. Area of triangle $\frac{1}{2}[x_1 * (y_2 - y_3) + x_2 * (y_3 - y_1) + x_3 * (y_1 - y_2)]$

18. Angle between two lines $\tan \theta = |\frac{m_1-m_2}{1+m_1*m_2}|$

19. Distance of a point from a line

$$|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}|$$

20. Image of a point on a line

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -2 * \frac{ax_1+by_1+c}{a^2+b^2}$$

21. Distance between parallel lines

$$\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$$

22. For a simple polygon pick's theorem states that

$$A = I + \frac{B}{2} - 1$$

A = Area of simple polygon

B = Number of points on its boundary

I = be the number of integer points strictly inside the polygon

23. For a line from (a, b) to (c, d)

the number of lattice points on it is $\gcd(c - a, d - b) + 1$