

- 1 For a polynomial $P(x) = ax^3 + bx^2 + cx + d$
 $\alpha + \beta + \gamma = -b/a$
 $\alpha * \beta * \gamma = c/a$
 where α, β, γ are the roots of the polynomial.

- 2 For a polynomial $P(x) = ax^2 + bx + c$

$$\alpha = \frac{-b + \sqrt{b^2 - 4 * a * c}}{2 * a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4 * a * c}}{2 * a}$$
 where α, β are the roots of the polynomial.

3 $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n*(n+1)}{2}\right]^2$

4 $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n*(n+1)*(2n+1)}{6}$

- 5 For an arithmetic progression
 $a, a + d, a + 2d, a + 3d, \dots$
 $T(n) = a + (n - 1) * d$
 $S(n) = \frac{n}{2} [2a + (n - 1)d]$

6) For a geometric progression

a, ar, ar^2, ar^3, \dots

$$S(n) = a \left(\frac{r^n - 1}{r - 1} \right)$$

$$S(\infty) = \frac{a}{1-r} \text{ where } [-1 < r < 1]$$

7) Bertrand Ballot's Theorem problem -

"In an election where candidate a receives p votes and b receives q votes with $p > q$, what is the probability that a will be strictly ahead of b throughout the count"

Ans - $\frac{p-q}{p+q}$

And if ties are allowed then

Ans - $\frac{p-q+1}{p+1}$

8) $\left(\frac{A}{B}\right) \% mod = (A \% mod * x \% mod) \% mod$

where x is the modular multiplicative inverse of B under mod

9) Fermat's little theorem

$$a^{m-2} \equiv a^{-1} \% mod$$

when mod is prime

10) $E_p(n!)$: Exponent of p in n! where p is a prime

$$E_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \left\lfloor \frac{n}{p^4} \right\rfloor + \dots$$

- 11 let N be a natural number

$$N = p_1^{\alpha_1} * p_2^{\alpha_2} * p_3^{\alpha_3} \dots$$

Then total number of divisors =
 $(\alpha_1 + 1) * (\alpha_2 + 1) * (\alpha_3 + 1) * \dots$

Sum of divisors = $\left(\frac{p_1^{\alpha_1+1}-1}{p_1-1}\right) * \left(\frac{p_2^{\alpha_2+1}-1}{p_2-1}\right) * \left(\frac{p_3^{\alpha_3+1}-1}{p_3-1}\right) \dots$

Product of divisors = $n^{\frac{(\alpha_1+1)(\alpha_2+1)\dots(\alpha_m+1)}{2}}$

If n is perfect square then multiply it by \sqrt{x} in case of product of divisor

12 $a^b \% \text{mod} = a^{b \% (\text{mod}-1)} \% \text{mod}$ when mod is prime

13 $a^b \% \text{mod} = a^{b \% \phi(\text{mod})} \% \text{mod}$ when mod is not prime
 where $\phi(\text{mod})$ = Euler totient function of mod

14 Derangement of N elements

$$D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}\right)$$

15 If m and n are two co-prime numbers then the biggest number which can not be represented as $(am + bn)$ for non negative integer m and n is $mn - m - n$

16 Euler Totient function($\phi(n)$) :Number of numbers which are coprime with n in range of $[1, n]$

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \left(1 - \frac{1}{p_3}\right) \dots$$

where $n = p_1^{\alpha_1} * p_2^{\alpha_2} * p_3^{\alpha_3} \dots$

15 Centroid of a triangle $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

16 Incentre $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

17 Area of triangle $\frac{1}{2} [x_1 * (y_2 - y_3) + x_2 * (y_3 - y_1) + x_3 * (y_1 - y_2)]$

18 Angle between two lines $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 * m_2} \right|$

19 Distance of a point from a line

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

20 Image of a point on a line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2 * \frac{ax_1 + by_1 + c}{a^2 + b^2}$$

21 Distance between parallel lines

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

22 For a simple polygon pick's theorem states that

$$A = I + \frac{B}{2} - 1$$

A = Area of simple polygon

B = Number of points on its boundary

I = be the number of integer points stricly inside the polygon

23 For a line from (a, b) to (c, d)

the number of lattice points on it is $\gcd(c - a, d - b) + 1$

24 A circle with radius r and a central angle α in degrees has the corresponding chord with length

$$\text{sqrt}(2 \times r^2 \times (1 - \cos(\alpha)))$$

25 A triangle with 3 sides: a, b, c and semi-perimeter s has an area

$$A = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}.$$

26 A triangle with 3 sides: a, b, c and area A has an circumscribed circle (circumcircle) with radius

$$R = a \times b \times c / (4 \times A).$$