Q1. Given X be a discrete random variable with the following PMF

1. Find the range RX of the random variable X.
2. Find P(X ≤ 0.5)
3. Find P(0.25<X<0.75)
4. P(X = 0.2|X<0.6)

P\_x(x) = { 0.1 for x = 0.2, 0.2 for x = 0.4, 0.2 for x = 0.5, 0.3 for x = 0.8, 0.2 for x = 1, 0 otherwise }

1. Range of X (RX): {0.2, 0.4, 0.5, 0.8, 1}
2. P(X ≤ 0.5) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) = 0.1 + 0.2 + 0.2 = 0.5
3. P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5) = 0.2 + 0.2 = 0.4
4. P(X = 0.2 | X < 0.6) = P(X = 0.2 and X < 0.6) / P(X < 0.6) = (0.1) / (0.1 + 0.2 + 0.2) = 0.25

Q2. Two equal and fair dice are rolled, and we observed two numbers X and Y.

1. Find RX, RY, and the PMFs of X and Y.
2. Find P(X = 2,Y = 6).
3. Find P(X>3|Y = 2).
4. If Z = X + Y. Find the range and PMF of Z.
5. Find P(X = 4|Z = 8).
6. RX = {1, 2, 3, 4, 5, 6}
   1. RY = {1, 2, 3, 4, 5, 6}
   2. PMF of X: P\_X(x) = P\_Y(y) = 1/6 for x, y ∈ RX, RY
7. P(X = 2, Y = 6) = P(X = 2) \* P(Y = 6) = (1/6) \* (1/6) = 1/36
8. P(X > 3 | Y = 2) = P(X > 3) = 1/6 + 1/6 = 1/3
9. Z = X + Y, Range of Z: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

PMF of Z: P\_Z(z) = 1/36 for z ∈ Range of Z

1. P(X = 4 | Z = 8) = P(X = 4, Y = 4) / P(Z = 8) = (1/36) / (5/36) = 1/5

Q3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?

Given:

* Total questions (n) = 20
* Options per question (k) = 44
* Known correct answers (k\_correct) = 10
* Unknown questions (k\_unknown) = 10

The student is answering the unknown questions randomly, so each question has a probability of being answered correctly as 1/k (1 out of k possible options).

calculate the probability mass function (PMF) of X, which represents the probability of getting exactly x questions correct out of 20:

PMF of X: P\_X(x) = nCk\_correct \* (1/k)^k\_correct \* (1 - 1/k)^(n - k\_correct)

calculate the values:

* For x = 0 to 10 (since the student can only answer up to 10 questions correctly): P(X = x) = 20C\_x \* (1/44)^x \* (1 - 1/44)^(20 - x)
* For x = 11 to 20 (as the student can't answer more than 10 correctly, so these probabilities are 0): P(X = x) = 0

To find P(X > 15), you need to sum the probabilities for x = 16, 17, 18, 19, and 20:

P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)

PMF formula: P\_X(x) = nCk\_correct \* (1/k)^k\_correct \* (1 - 1/k)^(n - k\_correct)

For x = 16: P(X = 16) = 20C16 \* (1/44)^16 \* (1 - 1/44)^(20 - 16) ≈ 0.2363

For x = 17: P(X = 17) = 20C17 \* (1/44)^17 \* (1 - 1/44)^(20 - 17) ≈ 0.0618

For x = 18: P(X = 18) = 20C18 \* (1/44)^18 \* (1 - 1/44)^(20 - 18) ≈ 0.0103

For x = 19: P(X = 19) = 20C19 \* (1/44)^19 \* (1 - 1/44)^(20 - 19) ≈ 0.0010

For x = 20: P(X = 20) = 20C20 \* (1/44)^20 \* (1 - 1/44)^(20 - 20) ≈ 0.0001

Now, add up these probabilities to find P(X > 15):

P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20) ≈ 0.2363 + 0.0618 + 0.0103 + 0.0010 + 0.0001 ≈ 0.3095

Therefore, the probability P(X > 15) is approximately 0.3095. This means there's about a 30.95% chance that the student will score more than 15 correct answers out of the 20 questions.

Q4. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

The Poisson distribution is often used to model the number of events that occur in a fixed interval of time or space, given a certain average rate of occurrence. In this case, we have an average arrival rate of 10 students per hour.

Let's calculate the probability P(10 < Y ≤ 15) for the number of students arriving between 10 am and 11:30 am.

Given:

- Average arrival rate (λ) = 10 students per hour

- Time interval = 1.5 hours (from 10 am to 11:30 am)

First, let's calculate the expected number of arrivals in the given time interval:

μ = λ \* time interval = 10 \* 1.5 = 15

Now, we can use the Poisson distribution formula to calculate the probability:

P(Y = y) = (e^(-μ) \* μ^y) / y!

Where y is the number of arrivals (10 < y ≤ 15).

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15)

= (e^(-15) \* 15^11) / 11! + (e^(-15) \* 15^12) / 12! + (e^(-15) \* 15^13) / 13! + (e^(-15) \* 15^14) / 14! + (e^(-15) \* 15^15) / 15!

Calculating each term:

P(10 < Y ≤ 15) ≈ 0.1705 + 0.1942 + 0.1787 + 0.1350 + 0.0899

≈ 0.7683

Therefore, the probability that the number of students arriving between 10 am and 11:30 am is between 10 and 15 (inclusive) is approximately 0.7683, or 76.83%.

Q5.Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

When two independent random variables, X and Y, follow Poisson distributions with parameters α and β respectively, their sum Z = X + Y also follows a Poisson distribution.

The Poisson distribution PMF is given by:

P(Z = z) = (e^(-λ) \* λ^z) / z!

Where λ is the mean of the distribution.

In this case, the mean of the sum Z is the sum of the means of X and Y:

λ\_z = λ\_x + λ\_y

Since X ~ Poisson(α) and Y ~ Poisson(β), their means are:

λ\_x = α

λ\_y = β

So, the mean of Z is:

λ\_z = α + β

Now, using the PMF formula for Z:

P(Z = z) = (e^(-λ\_z) \* λ\_z^z) / z!

Substitute the value of λ\_z and simplify:

P(Z = z) = (e^(-(α + β)) \* (α + β)^z) / z!

Therefore, the PMF of the random variable Z = X + Y is given by:

P(Z = z) = (e^(-(α + β)) \* (α + β)^z) / z!

Q6. There is a discrete random variable X with the pmf.

If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.

2. Find the pmf of Y.

1. Finding the range of Y: Since X can take values -2, -1, 0, 1, and 2, when we substitute these values into Y = (X + 1)^2, we get: Y(-2) = (-2 + 1)^2 = 1 Y(-1) = (-1 + 1)^2 = 0 Y(0) = (0 + 1)^2 = 1 Y(1) = (1 + 1)^2 = 4 Y(2) = (2 + 1)^2 = 9

So, the range of Y is {0, 1, 4, 9}.

1. Finding the PMF of Y: To find the PMF of Y, we need to compute the probabilities of Y taking each value in its range.

P(Y = 0) = P(X = -1) = 1/8 P(Y = 1) = P(X = -2) + P(X = 0) = 1/4 + 1/4 = 1/2 P(Y = 4) = P(X = 1) = 11/84 P(Y = 9) = P(X = 2) = 1/4

For any other value of Y not in the range {0, 1, 4, 9}, the probability is 0.

So, the PMF of Y is:

Q7.Assuming X is a continuous random variable with PDF

* + 1. Find EX and Var(X).
    2. Find *P*(*X* ≥ 1/2).

To find the constant c, we need to ensure that the total area under the probability density function (PDF) is equal to 1. Since the PDF represents a valid probability distribution, its integral over the entire range must be 1. Therefore, we integrate the PDF over the range [-1, 1] and set it equal to 1:

∫[x=-1 to x=1] cx^2 dx = 1

Solving the integral: c \* ∫[x=-1 to x=1] x^2 dx = 1 c \* [x^3/3] from -1 to 1 = 1 c \* (1/3 - (-1/3)) = 1 2c/3 = 1 c = 3/2

Now that we have the constant c, the PDF of X becomes:

1. Expected Value (EX): The expected value of a continuous random variable X is given by the integral of x times the PDF f(x) over its entire range:

E(X) = ∫[x=-∞ to x=∞] x \* f\_x (x) dx

However, since f\_x (x) is only non-zero over the interval [-1, 1], we can restrict the integral to that interval:

E(X) = ∫[x=-1 to x=1] x \* (3/2)x^2 dx

Calculating the integral: E(X) = ∫[x=-1 to x=1] (3/2)x^3 dx E(X) = (3/2) \* [x^4/4] from -1 to 1 E(X) = (3/2) \* (1/4 - 1/4) = 0

1. Variance (Var(X)): The variance of a continuous random variable X is given by the integral of (x - E(X))^2 times the PDF f(x) over its entire range:

Var(X) = ∫[x=-∞ to x=∞] (x - E(X))^2 \* f\_x (x) dx

Again, we restrict the integral to the interval [-1, 1]:

Var(X) = ∫[x=-1 to x=1] (x - 0)^2 \* (3/2)x^2 dx

Calculating the integral: Var(X) = ∫[x=-1 to x=1] (3/2)x^4 dx Var(X) = (3/2) \* [x^5/5] from -1 to 1 Var(X) = (3/2) \* (1/5 - 1/5) = 0

1. Probability P(X ≥ 1/2):

P(X ≥ 1/2) = ∫[x=1/2 to x=∞] f\_x (x) dx

Calculating the integral: P(X ≥ 1/2) = ∫[x=1/2 to x=1] (3/2)x^2 dx P(X ≥ 1/2) = (3/2) \* [x^3/3] from 1/2 to 1 P(X ≥ 1/2) = (3/2) \* (1/3 - 1/24) = 7/16

So, the results are: Expected Value (EX) = 0 Variance (Var(X)) = 0 P(X ≥ 1/2) = 7/16

Q8.If *X* is a continuous random variable with pmf

1. If *X*~Uniform  and *Y* = sin(*X*), then find *fY*(*y*).
2. If X is a random variable with CDF
3. What kind of random variable is *X*: discrete, continuous, or mixed?

* Find the PDF of *X*, f*X*(*x*).
* Find E(eX).
* Find P(*X* = 0|X≤0.5).

**X with PMF f\_x(x) = 4x^3 (0 < x ≤ 1)**

To find: P(X ≤ 2/3 | X > 1/3)

The conditional probability can be calculated using the definition of conditional probability:

P(X ≤ 2/3 | X > 1/3) = P(X ≤ 2/3 and X > 1/3) / P(X > 1/3)

1. Calculating P(X > 1/3):

P(X > 1/3) = ∫[x=1/3 to x=1] f\_x(x) dx

P(X > 1/3) = ∫[x=1/3 to x=1] 4x^3 dx

P(X > 1/3) = [x^4] from 1/3 to 1

P(X > 1/3) = 1 - (1/3)^4

P(X > 1/3) = 80/81

1. Calculating P(X ≤ 2/3 and X > 1/3):

P(X ≤ 2/3 and X > 1/3) = ∫[x=1/3 to x=2/3] f\_x(x) dx

P(X ≤ 2/3 and X > 1/3) = ∫[x=1/3 to x=2/3] 4x^3 dx

P(X ≤ 2/3 and X > 1/3) = [x^4] from 1/3 to 2/3

P(X ≤ 2/3 and X > 1/3) = (2/3)^4 - (1/3)^4

P(X ≤ 2/3 and X > 1/3) = 7/27

Now, calculate the conditional probability:

P(X ≤ 2/3 | X > 1/3) = P(X ≤ 2/3 and X > 1/3) / P(X > 1/3)

P(X ≤ 2/3 | X > 1/3) = (7/27) / (80/81)

P(X ≤ 2/3 | X > 1/3) = 7/320

**X ~ Uniform((-π)/2, π), Y = sin(X)**

Given X ~ Uniform((-π)/2, π), the distribution of Y = sin(X) can be found by transforming the random variable X through the function Y = sin(X). To find the PDF fY(y), we'll use the transformation technique for continuous random variables:

fY(y) = fX(g^(-1)(y)) \* |(d/dy) g^(-1)(y)|

In this case, g^(-1)(y) is the inverse function of y = sin(x), which is arcsin(y). The derivative of arcsin(y) with respect to y is 1 / sqrt(1 - y^2).

Substitute arcsin(y) and its derivative into the formula to find fY(y).

**X with CDF F\_X(x)**

Given the cumulative distribution function (CDF) F\_X(x), we can determine the type of random variable X, find its PDF fX(x), calculate E(eX), and find P(X = 0 | X ≤ 0.5).

1. Type of Random Variable X: X is a continuous random variable since its CDF F\_X(x) is continuous.
2. PDF of X, fX(x): fX(x) = d/dx [F\_X(x)] fX(x) = d/dx [1/2 + x/2] fX(x) = 1/2
3. Expected Value E(eX): E(eX) = ∫[x=-∞ to x=∞] e^x \* fX(x) dx E(eX) = ∫[x=-∞ to x=∞] e^x \* (1/2) dx E(eX) = (1/2) \* ∫[x=-∞ to x=∞] e^x dx (integral of e^x is e^x) E(eX) = (1/2) \* ∞ (diverges)
4. Probability P(X = 0 | X ≤ 0.5): Since X is a continuous random variable, the probability of it taking exactly one value (e.g., X = 0) is zero. Therefore, P(X = 0 | X ≤ 0.5) = 0.

Q9. There are two random variables *X* and *Y* with joint PMF given in Table below

* + 1. Find *P*(*X*≤2, *Y*≤4).
    2. Find the marginal PMFs of *X* and *Y*.
    3. Find *P*(*Y* = 2|*X* = 1).
    4. Are *X* and *Y* independent?

|  |  |  |  |
| --- | --- | --- | --- |
| (L) | Y=2 | Y=4 | Y=5 |
| X=1 | 1/12 | 1/24 | 1/24 |
| X=2 | 1/6 | 1/12 | 1/8 |
| X=3 | 1/4 | 1/8 | 1/12 |

1. P(X ≤ 2, Y ≤ 4) = P(X = 1, Y ≤ 4) + P(X = 2, Y ≤ 4)

= P(X = 1, Y = 2) + P(X = 1, Y = 4) + P(X = 2, Y = 2) + P(X = 2, Y = 4)

= (1/12) + (1/24) + (1/6) + (1/12)

= 1/6

2.

* Marginal PMF of X:
  + P(X = 1) = (1/12) + (1/24) + (1/24) = 1/8
  + P(X = 2) = (1/6) + (1/12) + (1/8) = 5/12
  + P(X = 3) = (1/4) + (1/8) + (1/12) = 11/24
* Marginal PMF of Y:
  + P(Y = 2) = (1/12) + (1/6) + (1/4) = 11/24
  + P(Y = 4) = (1/24) + (1/12) + (1/8) = 11/24
  + P(Y = 5) = (1/24) + (1/8) + (1/12) = 11/24

3. P(Y = 2 | X = 1) = P(Y = 2 and X = 1) / P(X = 1)

= (1/12) / (1/8)

= 2/3

4. X and Y are independent if and only if their joint PMF is the product of their marginal PMFs for all possible values of X and Y.

* Joint PMF P(X, Y):
  + P(X = 1, Y = 2) = 1/12
  + P(X = 1, Y = 4) = 1/24
  + P(X = 1, Y = 5) = 1/24
  + P(X = 2, Y = 2) = 1/6
  + P(X = 2, Y = 4) = 1/12
  + P(X = 2, Y = 5) = 1/8
  + P(X = 3, Y = 2) = 1/4
  + P(X = 3, Y = 4) = 1/8
  + P(X = 3, Y = 5) = 1/12
* Marginal PMFs of X and Y:
  + P(X = 1) = 1/8
  + P(X = 2) = 5/12
  + P(X = 3) = 11/24
  + P(Y = 2) = 11/24
  + P(Y = 4) = 11/24
  + P(Y = 5) = 11/24

Since the joint PMF is not equal to the product of the marginal PMFs, X and Y are not independent.

Q10.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

PMF (Probability Mass Function) of X (number of white shirts) and Y (number of black shirts) when choosing 10 shirts without replacement, we can use the hypergeometric distribution.

Given:

* n1 = 40 (number of white shirts)
* n2 = 60 (number of black shirts)
* N = 100 (total number of shirts)
* n = 10 (number of shirts drawn)

The joint PMF of X and Y can be calculated as:

P(X = x, Y = y) = (n1Cx) \* (n2Cy) / (NCn)

Where:

* n1Cx represents "n1 choose x," which is the number of ways to choose x white shirts out of n1 white shirts.
* n2Cy represents "n2 choose y," which is the number of ways to choose y black shirts out of n2 black shirts.
* NCn represents "N choose n," which is the total number of ways to choose n shirts out of N shirts.

calculate the joint PMF for various values of x and y:

* + P(X = 0, Y = 10) = (40C0) \* (60C10) / (100C10)
  + P(X = 1, Y = 9) = (40C1) \* (60C9) / (100C10)
  + P(X = 2, Y = 8) = (40C2) \* (60C8) / (100C10)
  + P(X = 3, Y = 7) = (40C3) \* (60C7) / (100C10)
  + P(X = 4, Y = 6) = (40C4) \* (60C6) / (100C10)
  + P(X = 5, Y = 5) = (40C5) \* (60C5) / (100C10)
  + P(X = 6, Y = 4) = (40C6) \* (60C4) / (100C10)
  + P(X = 7, Y = 3) = (40C7) \* (60C3) / (100C10)
  + P(X = 8, Y = 2) = (40C8) \* (60C2) / (100C10)
  + P(X = 9, Y = 1) = (40C9) \* (60C1) / (100C10)
  + P(X = 10, Y = 0) = (40C10) \* (60C0) / (100C10)

calculate each of these probabilities using the combinations formula: nCk = n! / (k!(n-k)!)

* P(X = 0, Y = 10) = (1) \* (38760) / (17310309456440) ≈ 0.0000223
* P(X = 1, Y = 9) = (40) \* (18643560) / (17310309456440) ≈ 0.0019491
* P(X = 2, Y = 8) = (780) \* (165765600) / (17310309456440) ≈ 0.0225544
* P(X = 3, Y = 7) = (9880) \* (310616185) / (17310309456440) ≈ 0.0861461
* P(X = 4, Y = 6) = (91390) \* (617614800) / (17310309456440) ≈ 0.1749642
* P(X = 5, Y = 5) = (658008) \* (829006400) / (17310309456440) ≈ 0.2459560
* P(X = 6, Y = 4) = (3894840) \* (621216192) / (17310309456440) ≈ 0.2271918
* P(X = 7, Y = 3) = (18475640) \* (186364440) / (17310309456440) ≈ 0.1320689
* P(X = 8, Y = 2) = (70590520) \* (26127360) / (17310309456440) ≈ 0.0448352
* P(X = 9, Y = 1) = (210378408) \* (2333760) / (17310309456440) ≈ 0.0073285
* P(X = 10, Y = 0) = (184756) \* (1) / (17310309456440) ≈ 0.0000086

Q11.If A and B are two jointly continuous random variables with joint PDF

* 1. Find fX(a) and fY(b).
  2. Are A and B independent of each other?
  3. Find the conditional PDF of A given B = b, fA|B(a|b).
  4. Find E[A|B = b], for 0 ≤ y ≤ 1.
  5. Find Var(A|B = b), for 0 ≤ y ≤ 1.

1. To find the marginal PDF of A (fX(a)), integrate the joint PDF over the range of y: fX(a) = ∫(from 0 to √a) 6xy dy = 6a∫(from 0 to √a) y dy = 6a[y^2/2] (from 0 to √a) = 3a√a, for 0 ≤ a ≤ 1.
2. To find the marginal PDF of B (fY(b)), integrate the joint PDF over the range of x: fY(b) = ∫(from 0 to 1) 6xy dx = 3b∫(from 0 to 1) 2x^2 dx = 2b[x^3/3] (from 0 to 1) = 2b/3, for 0 ≤ b ≤ √1.

A and B are independent if and only if their joint PDF is the product of their marginal PDFs. In this case, if f\_XY(x, y) = fX(a) \* fY(b), then A and B are independent.

1. To find the conditional PDF of A given B = b (fA|B(a|b)), we use the conditional probability formula: fA|B(a|b) = f\_XY(a, b) / fY(b), for 0 ≤ a ≤ 1 and 0 ≤ b ≤ √a.
2. To find E[A|B = b] (expected value of A given B = b), we integrate the conditional PDF: E[A|B = b] = ∫(from 0 to 1) afA|B(a|b) da.
3. To find Var(A|B = b) (variance of A given B = b), we use the formula for conditional variance: Var(A|B = b) = E[A^2|B = b] - (E[A|B = b])^2.

Q12.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

We have 100 men with individual weights Xi, where Xi is independently and identically distributed with mean μ = 170 and standard deviation σ = 30.

The total weight of the 100 men can be represented as the sum of their individual weights:

Total weight = X1 + X2 + ... + X100

The mean of the total weight is the sum of the means of the individual weights:

E[Total weight] = E[X1 + X2 + ... + X100] = E[X1] + E[X2] + ... + E[X100] = 100 \* μ = 100 \* 170 = 17000.

The variance of the total weight is the sum of the variances of the individual weights:

Var[Total weight] = Var[X1 + X2 + ... + X100] = Var[X1] + Var[X2] + ... + Var[X100] = 100 \* σ^2 = 100 \* 30^2 = 90000.

The standard deviation of the total weight is the square root of the variance:

σ[Total weight] = √(Var[Total weight]) = √(90000) = 300.

Now, we want to find the probability that the men's total weight on the ship exceeds 18,000. In terms of standard deviations from the mean, this is:

P(Total weight > 18000) = P((Total weight - E[Total weight]) / σ[Total weight] > (18000 - 17000) / 300)

P(Z > 0.3333)

Using a standard normal distribution table or calculator, we can find that the probability P(Z > 0.3333) is approximately 0.3707.

So, the probability that the men's total weight on the ship exceeds 18,000 is approximately 0.3707.

Q13.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF

If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.

E[Xi] = 1/3 and Var[Xi] = 4/9 (as provided in the PMF), we have: Mean of Y (μ\_Y) = 25 \* (1/3) = 25/3 Variance of Y (σ\_Y^2) = 25 \* (4/9) = 100/9

We want to estimate P(4 ≤ Y ≤ 6). Using the normal distribution approximation, we can standardize the values and then use the standard normal distribution table or calculator.

Standardized values: Z\_4 = (4 - μ\_Y) / σ\_Y = (4 - 25/3) / √(100/9) ≈ -4.04 Z\_6 = (6 - μ\_Y) / σ\_Y = (6 - 25/3) / √(100/9) ≈ -3.64

Now, we can calculate the probability using the standard normal distribution: P(4 ≤ Y ≤ 6) ≈ P(-4.04 ≤ Z ≤ -3.64)

Using a standard normal distribution table or calculator, you can find the corresponding probabilities for -4.04 and -3.64 and then subtract the smaller probability from the larger probability to get the estimated value of P(4 ≤ Y ≤ 6).