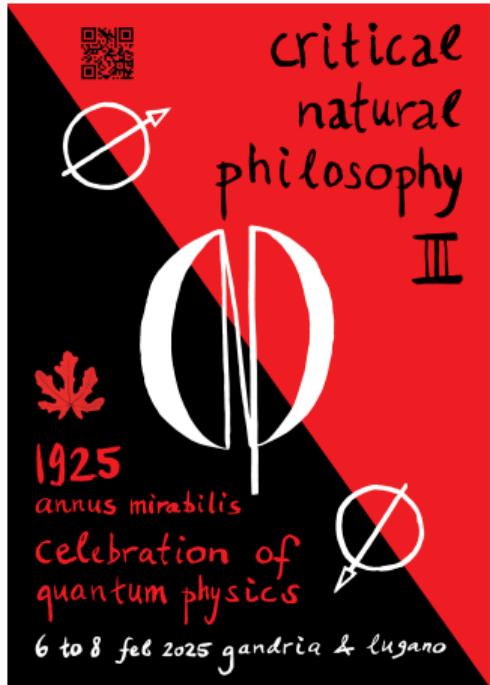


The Language of Quantum Networks

From Physical Experiment to Language

Anita Buckley

SWYSTEMS group
Università della Svizzera italiana

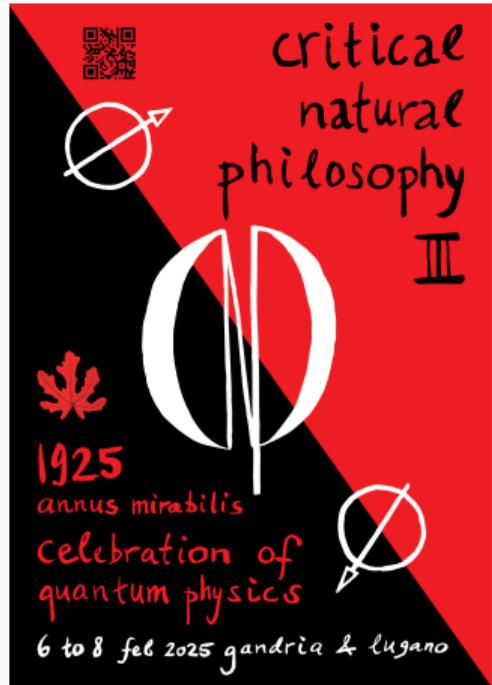


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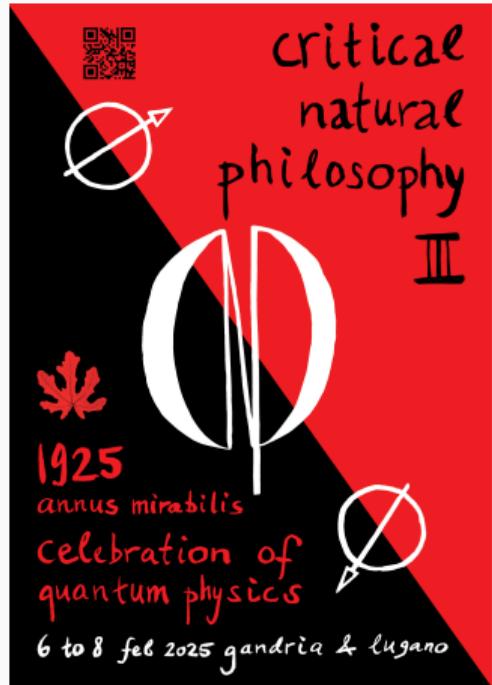


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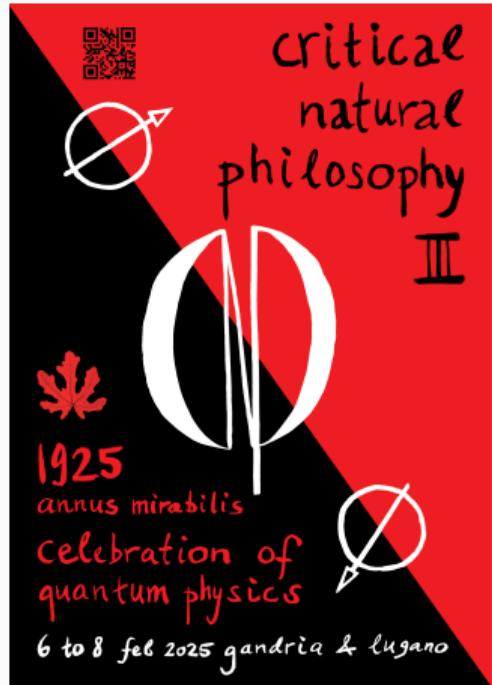


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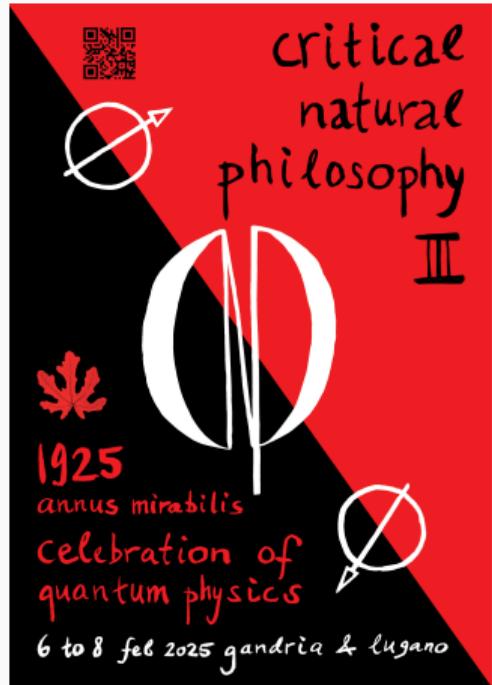
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The Language of Quantum Networks

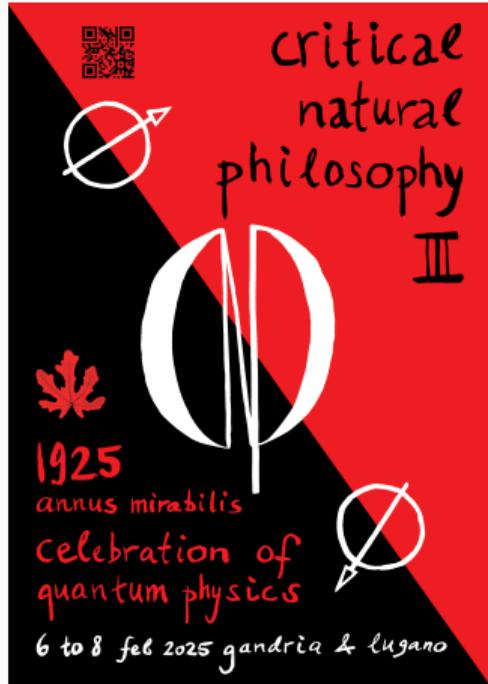
From Physical Experiment to Language

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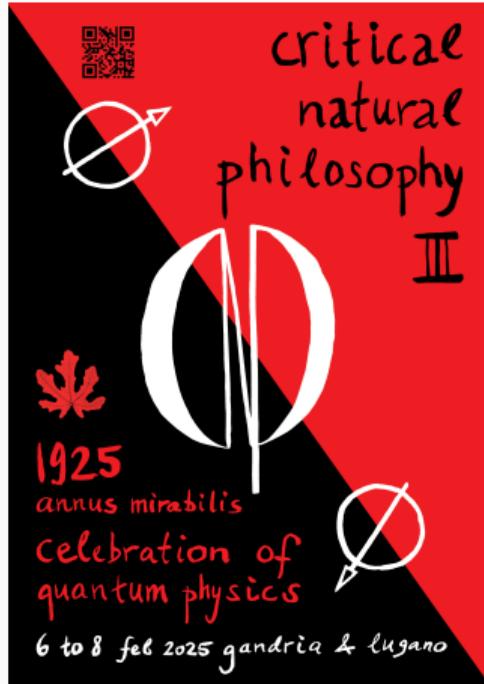
Reasoning

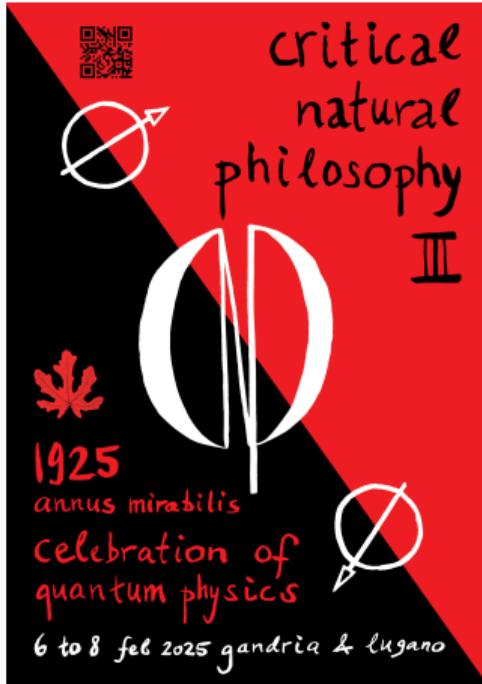


reason
understanding
knowledge
truth
limitations

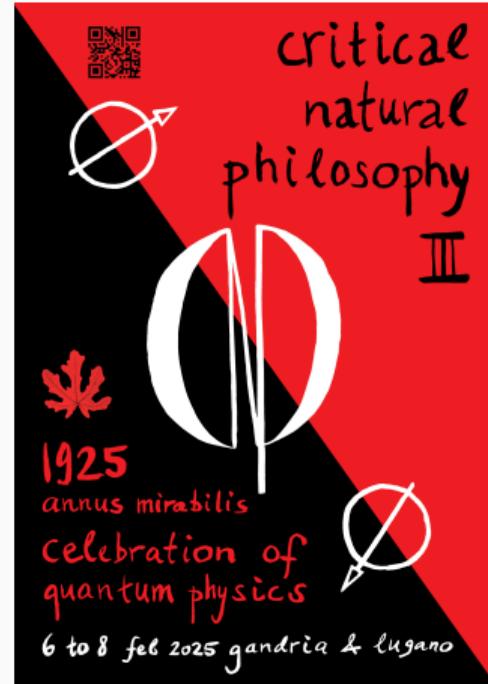
Mathematical reasoning

•••

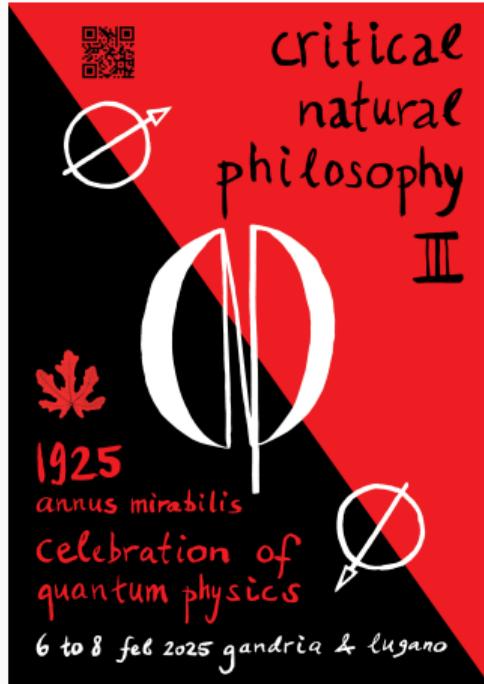




- Kant: the “thorough grounding” that mathematics finds in its definitions, axioms, and demonstrations cannot be “achieved or imitated” by philosophy or physical sciences

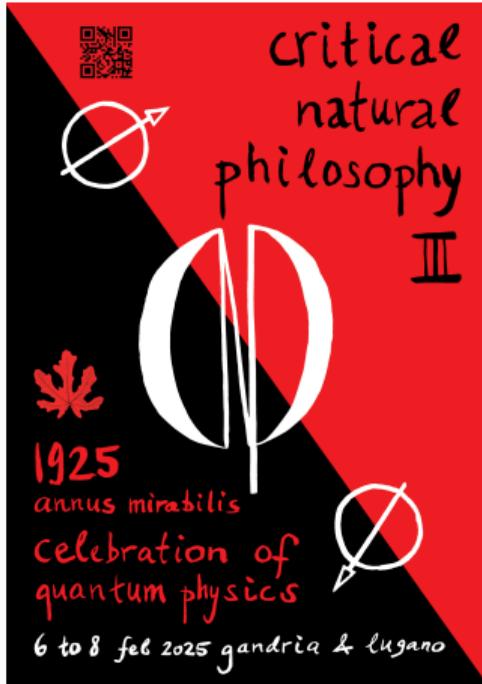


- Hegel: “reality/being \equiv thought”



- Hegel: “reality/being \equiv thought”
- Hilbert’s program: “truth \equiv proof”

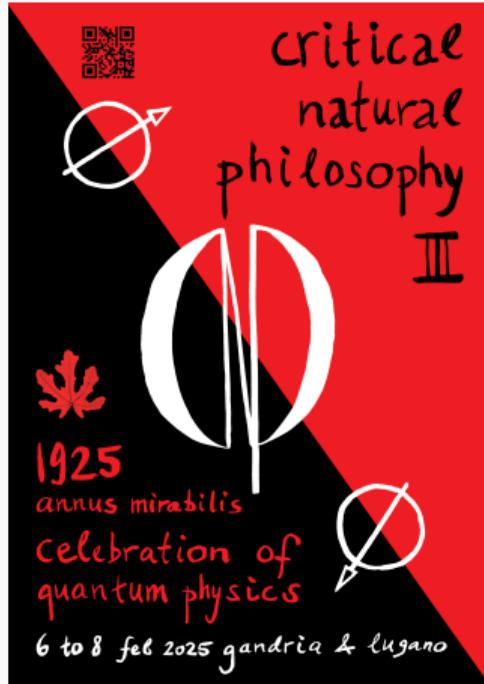
mathematical understanding = having “satisfactory” solutions that usually provide (explicitly or implicitly) mechanical procedures, which when applied to an object, determine (in finite time) whether it has the property



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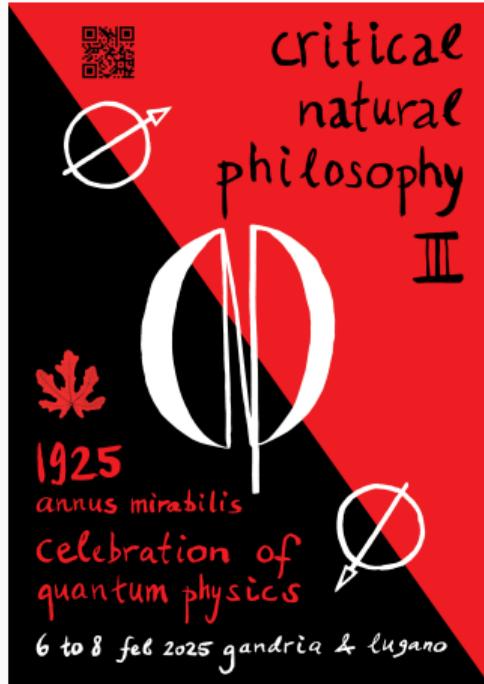
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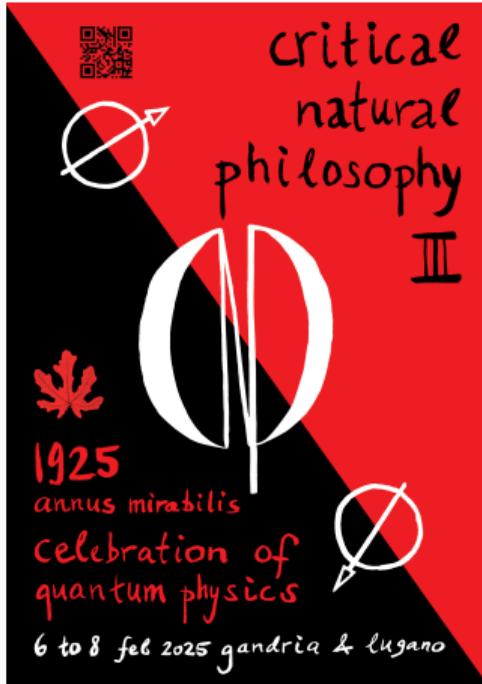
- Gödel, Church, Post, Turing: formal definitions of computation (identical in power)
- TM: limits to mathematical knowledge



hunt/gather food
run away
social activities
communication

}

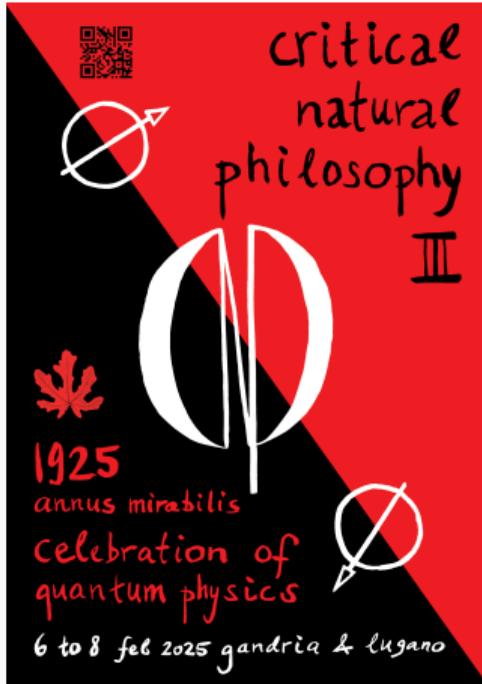
problem solving



hunt/gather food
run away
social activities
communication

problem solving

relations
decomposing
grouping
language



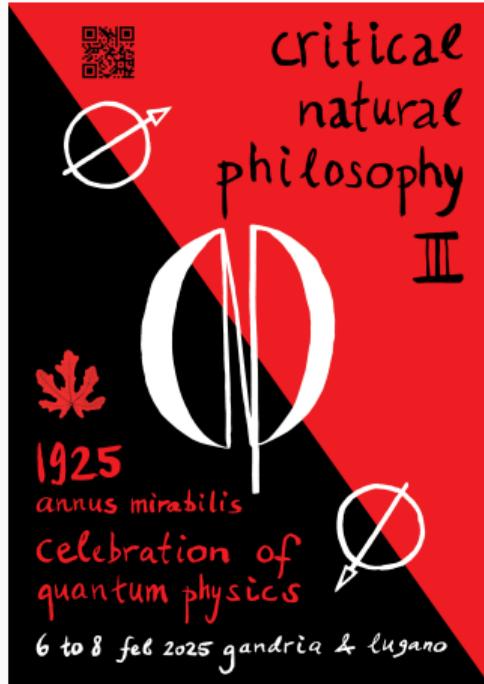
hunt/gather food
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decomposing
grouping
language

STRUCTURE

ABSTRACTION



hunt/gather food
run away
social activities
communication

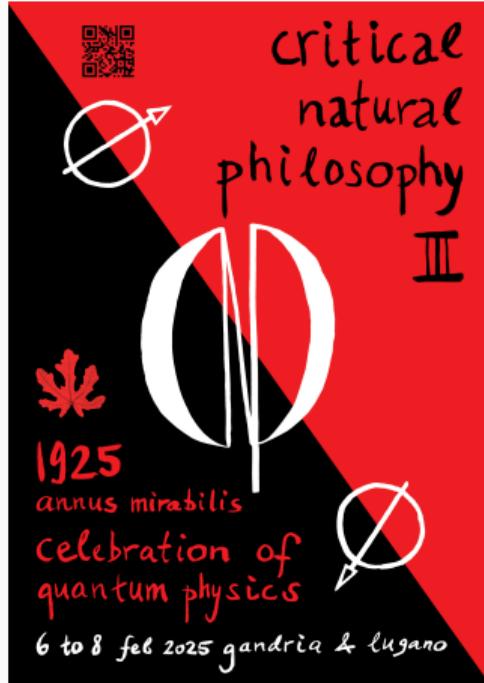
problem solving

relations
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language

STRUCTURE

IDENTITY

ABSTRACTION



hunt/gather food
run away
social activities
communication

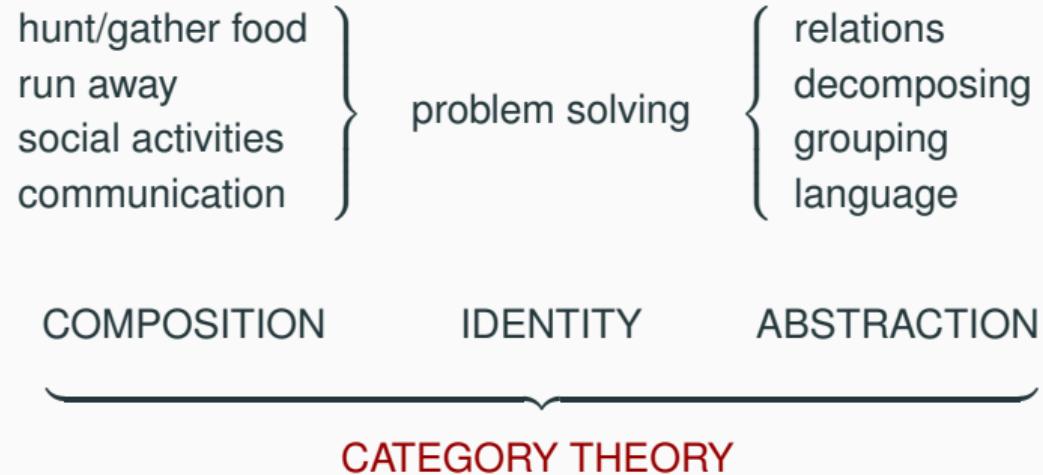
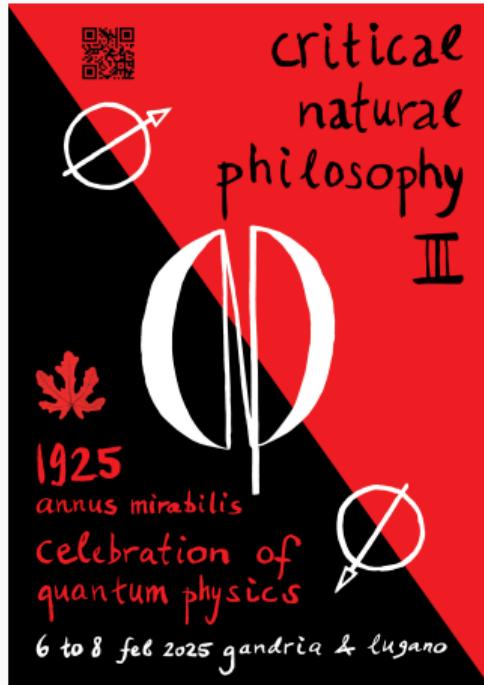
problem solving

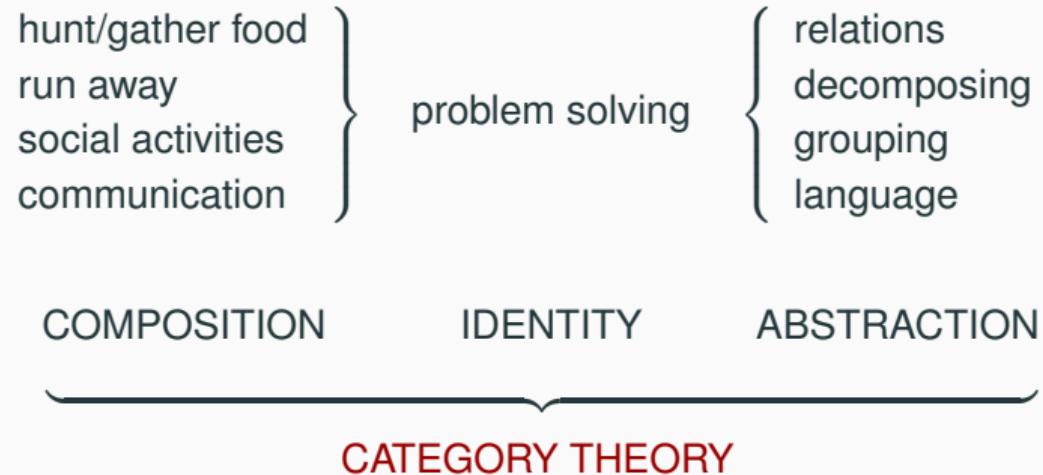
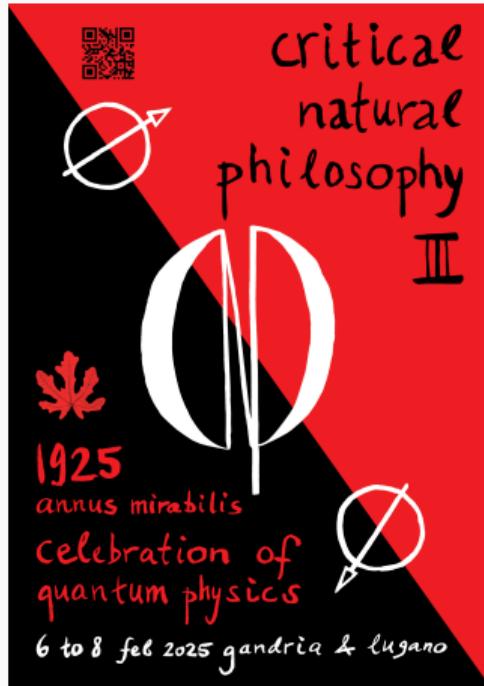
relations
decomposing
grouping
language

COMPOSITION

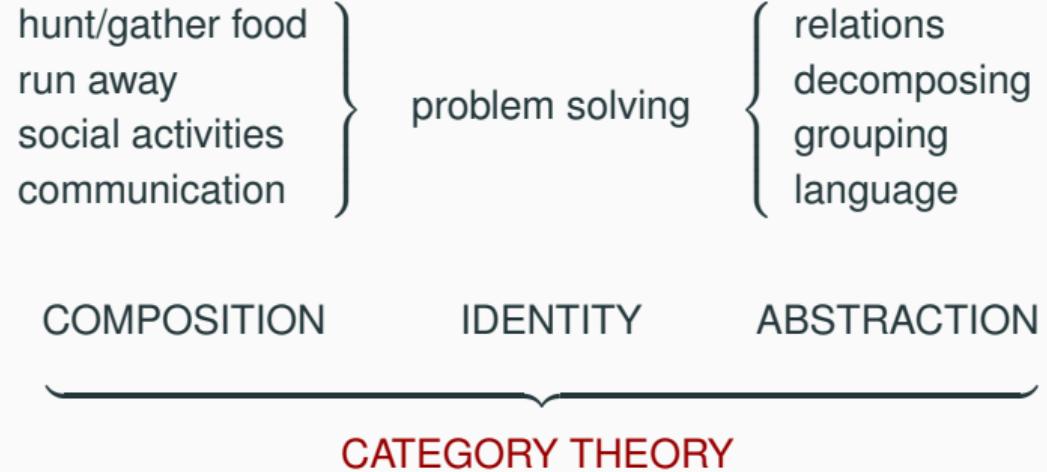
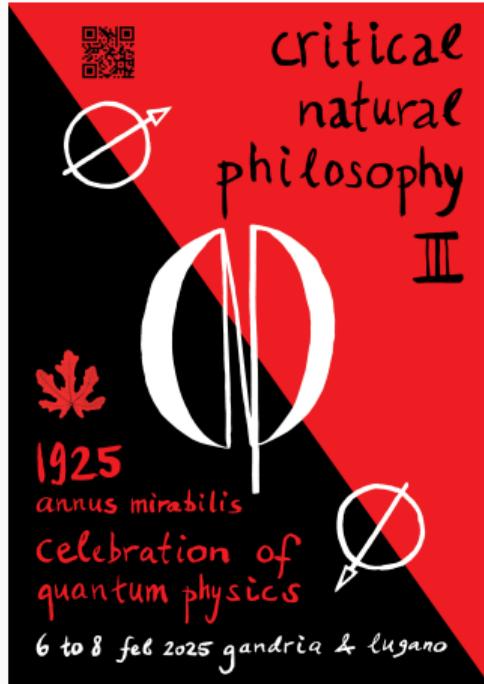
IDENTITY

ABSTRACTION





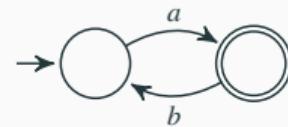
Turing: “Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two facilities, which we may call **intuition** and **ingenuity**.”



Hardy: “I am interested in mathematics only as a creative art.”

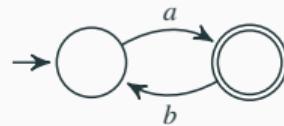
$$(ab)^*a \equiv a(ba)^*$$

$\{a, aba, ababa, \dots\}$



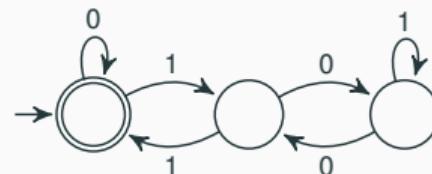
$$(ab)^*a \equiv a(ba)^*$$

{a, aba, ababa, ...}



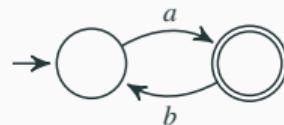
$$(0 + 1(01^*0)^*1)^*$$

multiples of 3



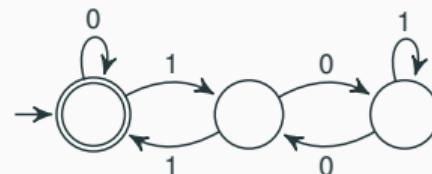
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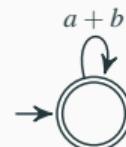
$$(0 + 1(01^*0)^*1)^*$$

multiples of 3



$$(a + b)^* \equiv a^*(ba^*)^*$$

all strings over a, b



Kleene algebra axioms

●○○

$$p + (q + r) \equiv (p + q) + r$$

$$p + q \equiv q + p$$

$$p + 0 \equiv p$$

$$p + p \equiv p$$

$$p \cdot (q \cdot r) \equiv (p \cdot q) \cdot r$$

$$1 \cdot p \equiv p$$

$$p \cdot 1 \equiv p$$

$$p \cdot (q + r) \equiv p \cdot q + p \cdot r$$

$$(p + q) \cdot r \equiv p \cdot r + q \cdot r$$

$$0 \cdot p \equiv 0$$

$$p \cdot 0 \equiv 0$$

$$1 + p \cdot p^* \equiv p^*$$

$$q + p \cdot r \leq r \Rightarrow p^* \cdot q \leq r$$

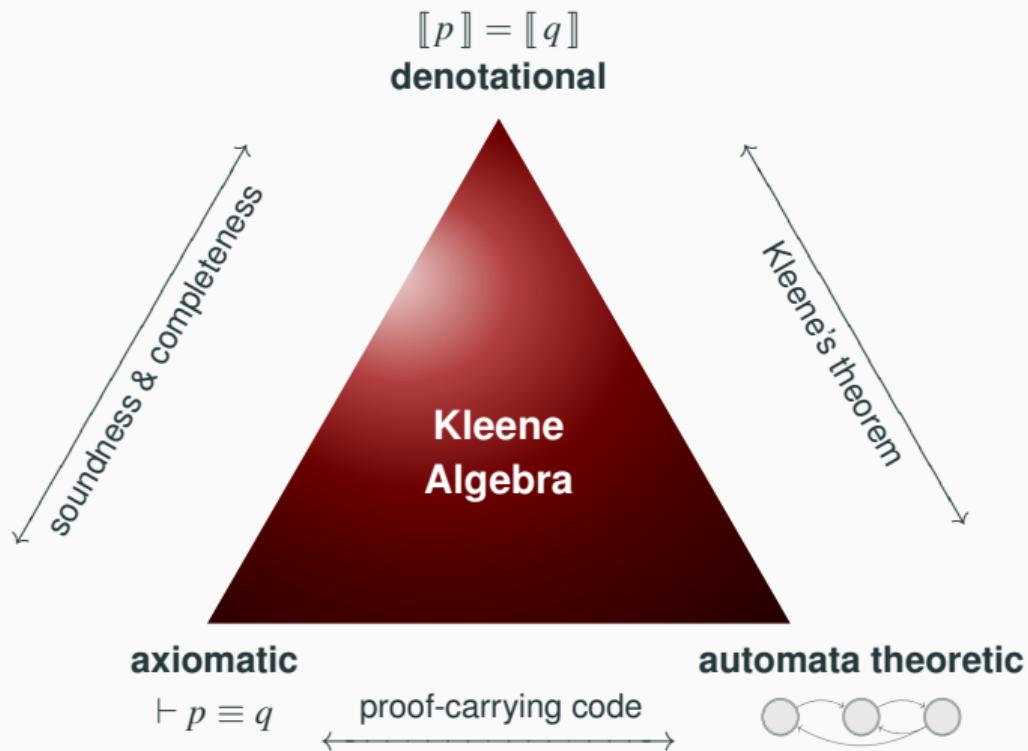
$$1 + p^* \cdot p \equiv p^*$$

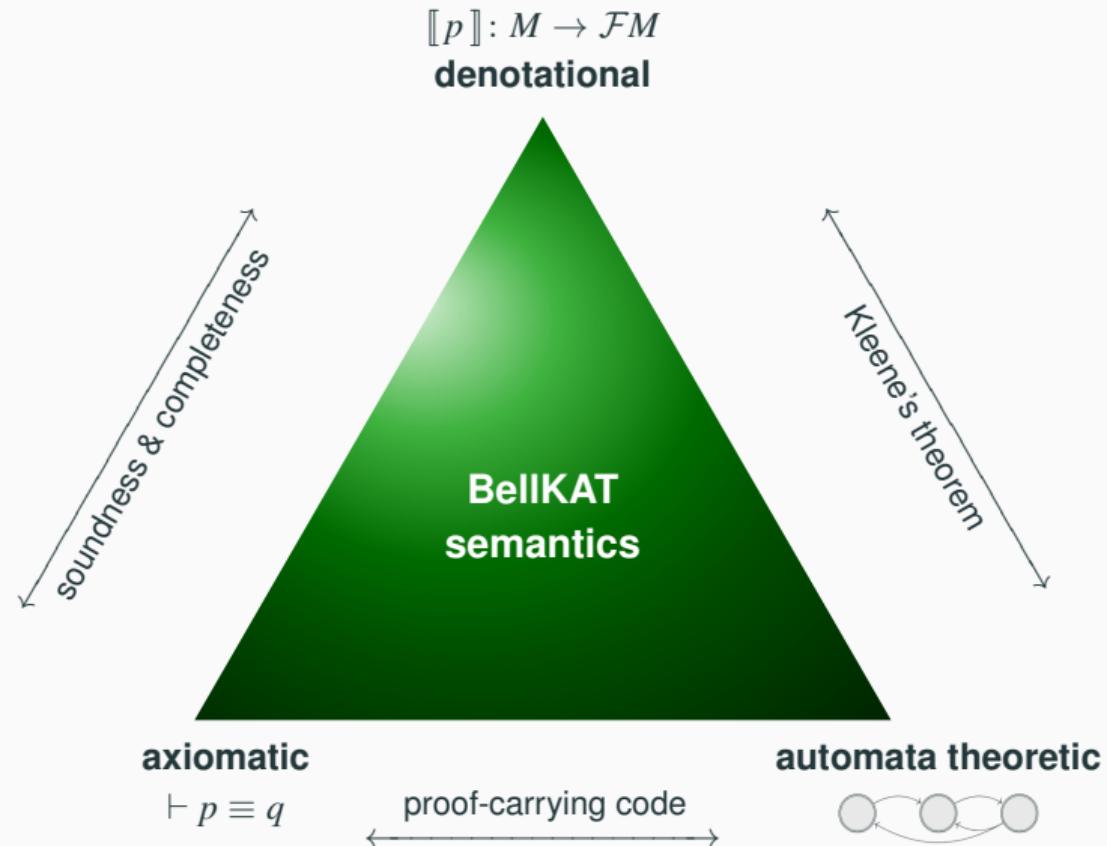
$$p + q \cdot r \leq q \Rightarrow p \cdot r^* \leq q$$

$$(ab)^* a \equiv a(ba)^* \quad \{a, aba, ababa, \dots\}$$

$$(0 + 1(01^* 0)^* 1)^* \quad \text{multiples of 3}$$

$$(a + b)^* \equiv a^*(ba^*)^* \quad \text{all strings over } a, b$$



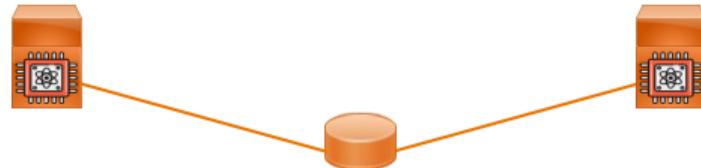


Quantum networks are networks connecting quantum capable devices

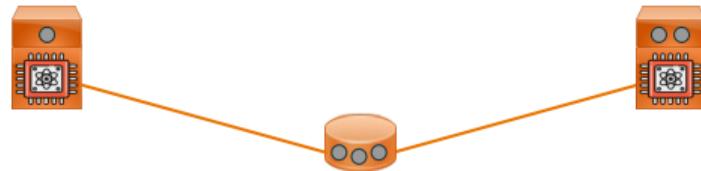
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Quantum networks are networks connecting quantum capable devices

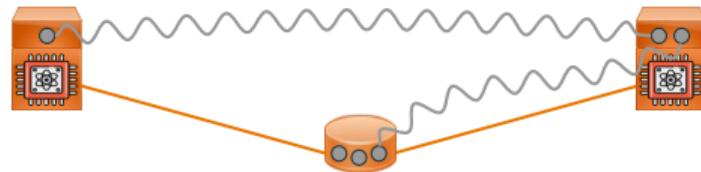


Quantum networks are networks connecting quantum capable devices



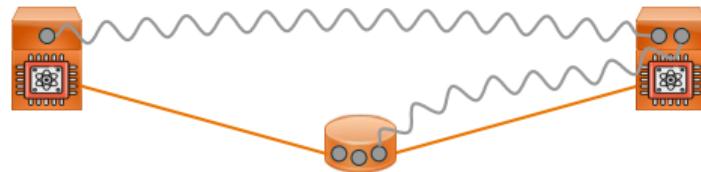
- **Communication qubits** designated to establish *connections* between devices

Quantum networks are networks connecting quantum capable devices



- **Communication qubits** designated to establish *connections* between devices
- Distributed **entanglement**: communication qubits sharing a *correlated random secret*

Quantum networks are networks connecting quantum capable devices



- **Communication qubits** designated to establish *connections* between devices
- Distributed entanglement: communication qubits sharing a *correlated random secret*

Benefits: **scaling of quantum computation** and **secure communication**



1

- teleportation
- entanglement based QKD

¹[IBM Quantum: Development Roadmap 2023]

Quantum networks are coming into reality

•••

The image shows a journal article cover from Communications of the ACM. The title is "Advances in the Quantum Internet". The authors are LASZLO GYONGYOSI AND SANDOR IMRE. The DOI is 10.1145/3524455. The abstract text reads: "A deep dive into the quantum Internet's potential to transform and disrupt."

Advances in the Quantum Internet

QUANTUM INFORMATION WILL not only reformulate our view of the nature of computation and communication but will also open up fundamentally new possibilities for realizing high-performance computer architecture and telecommunication networks. Since our data will no longer remain safe in the traditional Internet when commercial quantum computers become fully available,^{1,2,3,4,15,34} there will be a need for a fundamentally different network structure: the quantum Internet.^{22,35,32,33,45,47} While *quantum computational supremacy* refers to tasks and problems that quantum computers can solve but are beyond the capability of classical computers, the *quantum supremacy of the quantum Internet* identifies the properties and attributes that the quantum Internet offers but are unavailable in the traditional Internet.²

^a While “supremacy” is a concept used to describe the theory of computational complexity⁴⁴ and not a specific device (like a quantum computer), the supremacy of the quantum Internet in the current context refers to the collection of those advanced networking properties and attributes that are beyond the capabilities of the traditional Internet.

The quantum Internet uses the fundamental concepts of quantum mechanics for networking (see Sidebars 1–7 in the online Supplementary Information at <https://dl.acm.org/doi/10.1145/3524455>). The main attributes of the quantum Internet are advanced quantum phenomena and protocols (such as quantum superposition and quantum entanglement, quantum teleportation, and advanced quantum coding methods), unconditional security (quantum cryptography), and an entangled network structure.

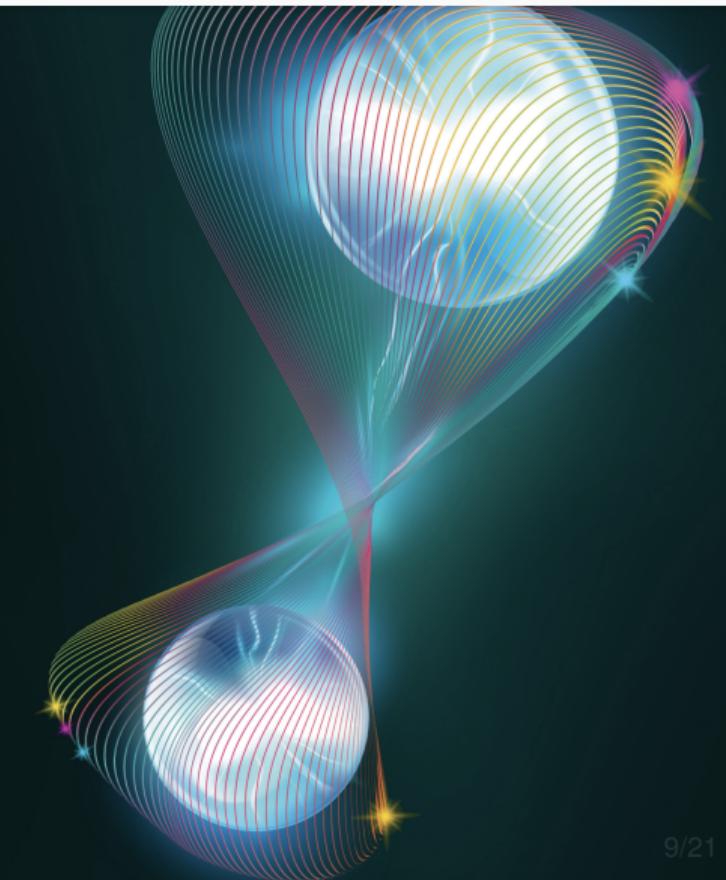
In contrast to traditional repeaters,^b quantum repeaters cannot apply the receive-copy-retransmit mechanism because of the so-called no-cloning theorem, which states that it is impossible to make a perfect copy of a quantum system (see Sidebar 4). This fundamental difference between the nature of classical and quantum information does not just lead to fundamentally different networking mechanisms; it also necessitates the definition of novel networking services in a quantum Internet scenario. Quantum memories in quantum repeater units are a fundamental part of any global-scale quantum Internet. A challenge connected to quantum memory units is the noise quantum memories add to storing quantum systems. However, while quantum repeaters can be realized without requiring quantum memories, these units are, in fact, necessary to guarantee optimal performance in any high-performance quantum-networking scenario.

In 2019, the National Quantum

^b Traditional repeaters rely on signal amplification.

» key insights

- The quantum Internet is an adequate answer to the security issues that will become relevant as commercial quantum computers hit the market.
- The quantum Internet is based on the fundamentals of quantum mechanics to provide advanced, high-security network communications.
- The quantum Internet gives users many capabilities and services not available in a traditional Internet setting.



Quantum networks are coming into reality

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Article | Published: 15 May 2024
Creation of memory–memory entanglement in a metropolitan quantum network
Jian-Liang Liu, Xi-Yu Luo, Yong Yu, Chao-Yang Wang, Bin Wan, Yi Hu, Jun Li, Ming-Yang Zi, Yao, Zi Yan, Da Teng, Jin-Wei Jiang, Xiao-Bing Liu, Xu-Ping Xie, Jun Zhang, Qing-He Mao, Qiang Zhang, Xiao-Hui Bao & Jian-Wei Pan
Nature 629, 579–585 (2024) | [Cite this article](#)
3190 Accesses | 87 Altmetric | [Metrics](#)

Abstract
The realization of a quantum Internet requires the development of quantum technologies in laboratories to demonstrate the basic principles and in field trials to demonstrate the feasibility of building a real-world quantum network. Here we report the creation of memory–memory entanglement between two metropolitan nodes connected by a 15-km-long optical fiber. The two nodes are based on silicon-vacancy (SiV) centres in nanophotonic waveguides integrated with a telecom fiber. We demonstrate heralded entanglement generation between the two nodes using time-bin entanglement storage and integrated error detection. By using quantum frequency conversion of photonic communication frequencies (1,350 nm), we demonstrate the implementation of separated nodes. Long-lived nuclear spin qubits using entanglement storage and integrated error detection.

Science Advances
DOI:10.1145/3524455
A deep dive into the quantum Internet's potential to transform and disrupt.
BY LASZLO GYONGYOSI AND SANDOR IMRE
Advances in the Quantum Internet

The quantum Internet uses the fundamental concepts of quantum mechanics for networking (see Sidebars 1–7 in the online Supplementary Information at <https://dl.acm.org/doi/10.1145/3524455>). The main attributes of the quantum Internet are advanced quantum phenomena and protocols (such as quantum superposition and quantum entanglement, quantum teleportation, and advanced quantum coding methods). *unconditional security*

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Published: 15 May 2024
Metropolitan-scale heralded entanglement of solid-state qubits
ARIAN J. STOLK, KIAN L. VAN DER ENDEN, MARIE-CHRISTINE SLATER, INGMAR TE RAA-GERCKX, PIETER BOTMA, JORIS VAN RANTWIJK, J. J. BENJAMIN BEIMOND, RONALD A. J. HAGEN, RODOLF W. HERST, I. AND RONALD HANSON, +13 authors | [Authors Info & Affiliations](#)
SCIENCE ADVANCES | 30 Oct 2024 • Vol 10, Issue 44 • DOI: 10.1126/sciadv.adb6442

Abstract
practical quantum networks for long-distance quantum just entanglement between quantum memory nodes connected el.23. Here we demonstrate a two-node quantum network isters based on silicon-vacancy (SiV) centres in nanophotonic with a telecommunication fibre network. Remote entanglement lanced interactions between the electron spin qubits of the SiVs traled spin-photon entangling gate operations with time-bin gement of separated nodes. Long-lived nuclear spin qubits g entanglement storage and integrated error detection. By al quantum frequency conversion of photonic munication frequencies (1,350 nm), we demonstra

Quantum networks are coming into reality

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Health Space Physics **Technology** Environment Mind Humans Life Mathematics Chemistry Earth Society

Technology

Quantum internet draws near thanks to entangled memory breakthroughs

Researchers aiming to create a secure quantum version of the internet called a quantum repeater, which doesn't yet exist - but now two well on the way to building one

By Alex Wilkins

15 May 2024

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Quantum networks could spread across a city
R. Zoller/Newscientist

Efforts to build a global quantum internet have received a boost from two developments that could one day make it possible to communicate securely across long distances or link up quantum computers, would use quantum bits (qubits) to transmit information. A quantum internet, which could be

The internet as it exists today involves sending strings of digital bits, or 0s at

optical signals, to transmit information. A quantum internet, which could be

communications or link up quantum computers, would use quantum bits (qubits) to transmit information. A quantum internet, which could be

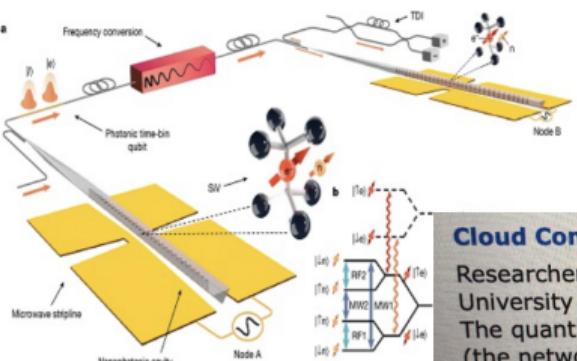
the University of Science and Technology of China entangled three nodes

AT

The quantum Internet uses the fundamental concepts of quantum mechanics for networking (see Sidebars 1–7 in the online Supplementary Information at <https://doi.org/10.1145/3524455>). The main attributes of the quantum Internet are advanced quantum phenomena and protocols

○ ACM TechNews <technews-editor@acm.org>

To: Buckley Anita



Quantum Internet Draws Nearer

Harvard University researchers assembled a quantum repeater spanning 35 kilometers across Boston, Massachusetts, with two nodes separated by a loop of optical fiber. The repeater consists of a diamond with an atom-sized hole. Meanwhile, researchers at the



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Published: 15 May 2024

of nanophotonic quantum memory com network

Y.-C. Wei, D. R. Assumpcao, P.-J. Stas, Y.-Q. Huan, B. Machieloo, E.N. Sinclair, C. De-Eknamkul, D. S. Levonian, M. K. Bhaskar, H. Park, M.

b | [Cite this article](#)

c | [Metrics](#)

Cloud Computing Under the Cover of Quantum

Researchers at the U.K.'s University of Oxford and France's Sorbonne University demonstrated blind quantum computing using trapped ions. The quantum cloud system's "server" was made from a strontium ion (the network qubit) and a calcium ion (the memory qubit). The server does not know the electronic state of the network qubit but can still process its information via a laser-based process that entangles the network and memory qubits. The system also uses one-time-padded encryption to encode information, concealing the data and operations from the server.

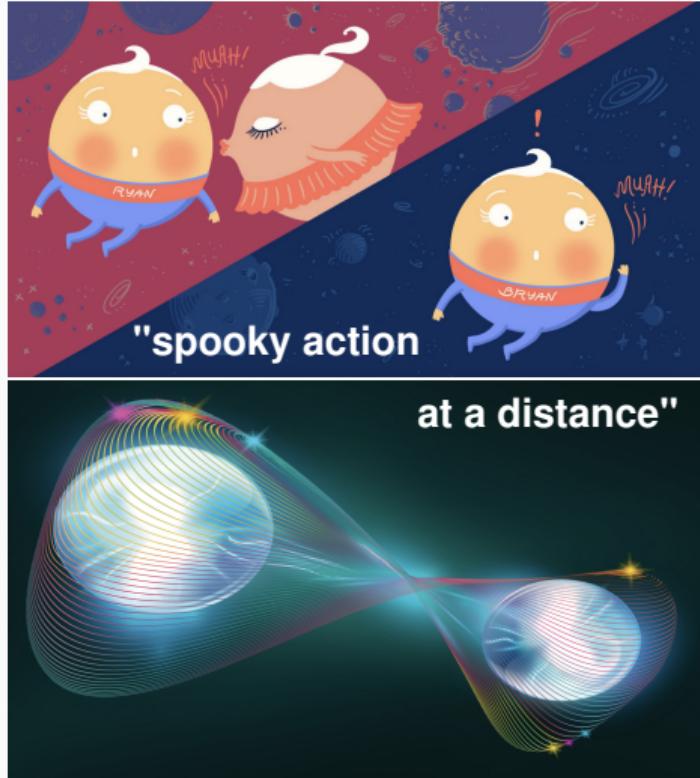
9/21

communication frequencies (1,350 nm), we demon-

Bell pair: a pair of entangled qubits



- Fundamental *resource* in quantum networks
- *Bell pair* is a pair of entangled qubits:
 $R \sim B$ distributed between nodes R and B
- No headers: control information needs to be sent via separate classical channels



Artwork by Sandbox Studio, Chicago with Ana Kova
Image by Andrij Borys Associates, using Shutterstock

Bell pair: a pair of entangled qubits

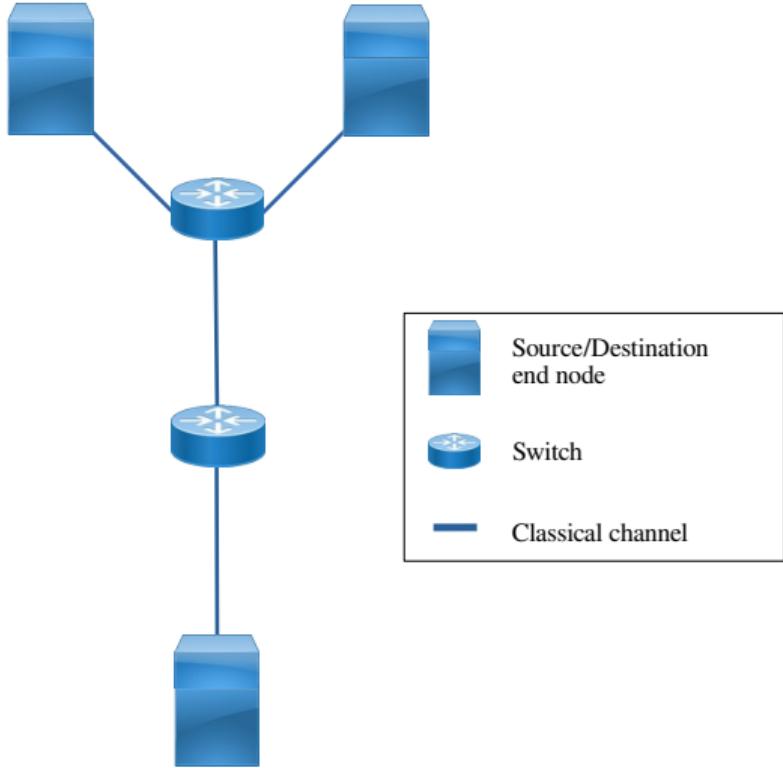


- Fundamental *resource* in quantum networks
- *Bell pair* is a pair of entangled qubits:
 $R \sim B$ distributed between nodes R and B
- No headers: control information needs to be sent via separate classical channels

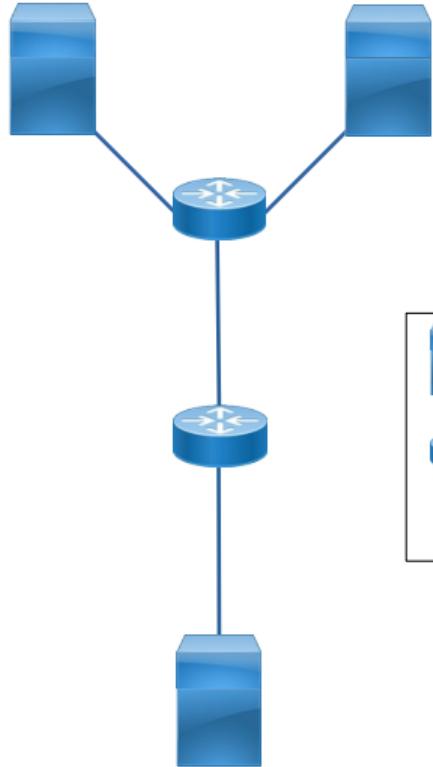


Artwork by Sandbox Studio, Chicago with Ana Kova
Image by Andrij Borys Associates, using Shutterstock

Classical network

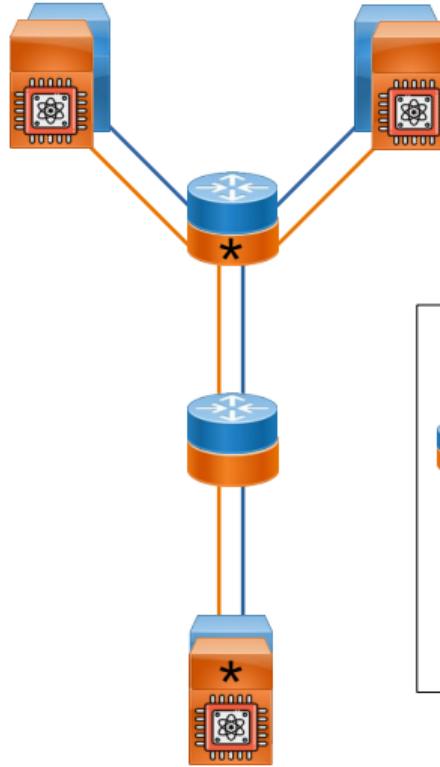


Classical network



Source/Destination end node
 Switch
— Classical channel

Quantum network ^{1,2}

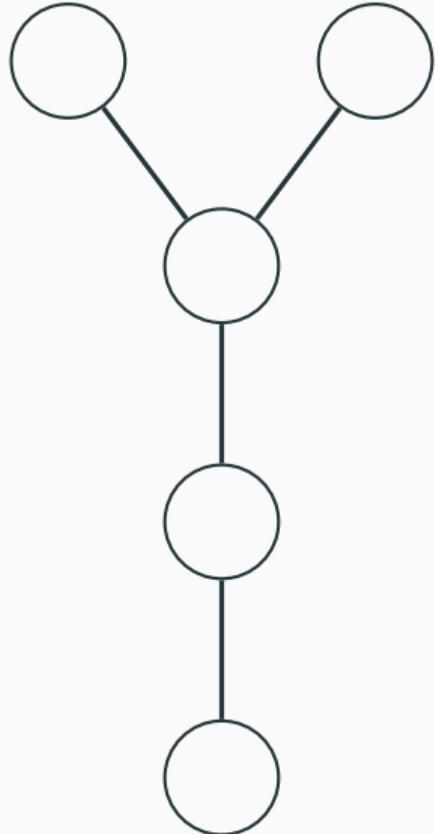


Quantum capable end node
 Repeater with classical and quantum capabilities
— Quantum channel
— Classical channel
* Quantum source

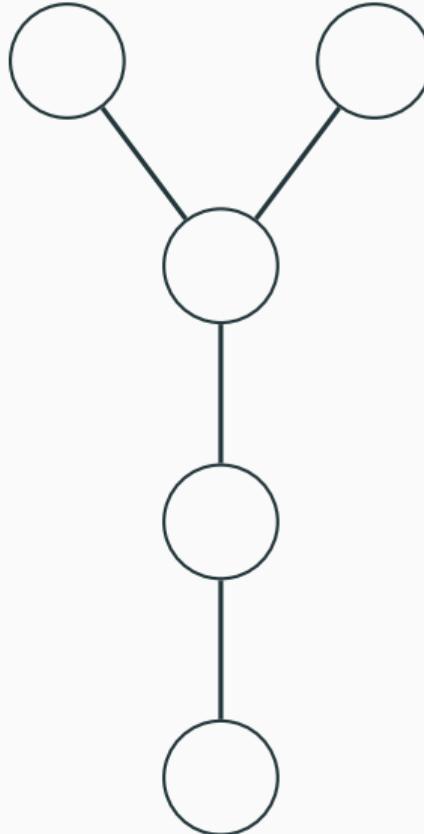
¹[Kozlowski and Wehner: NANOCOM 2019],

²[Quantum Internet Research Group: RFC 9340 2023]

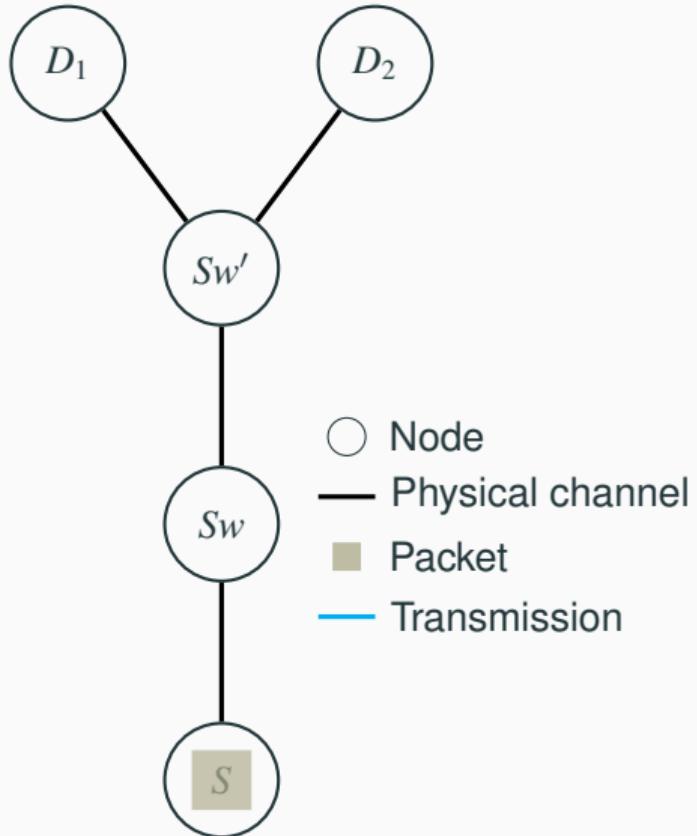
Classical network



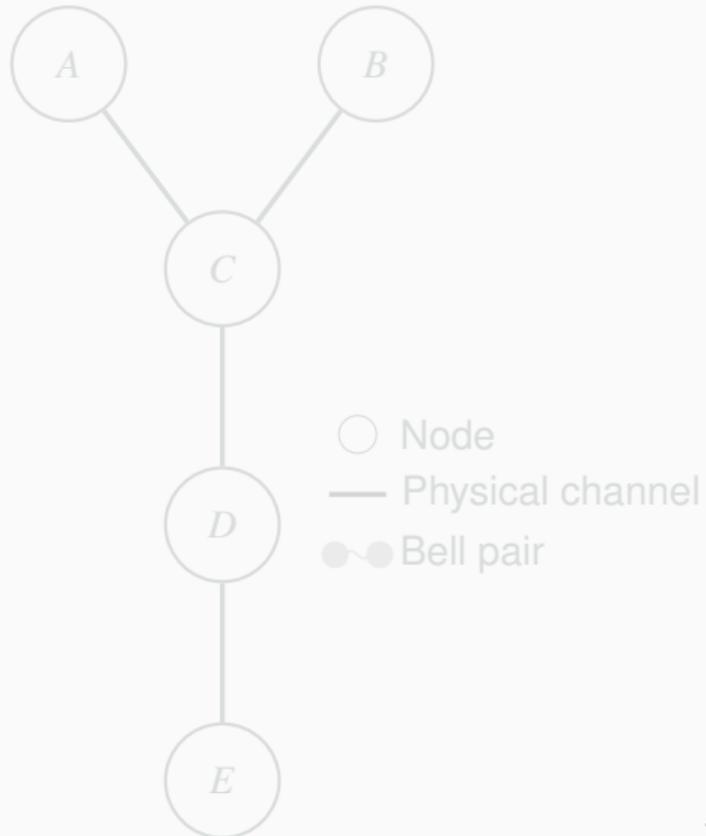
Quantum network



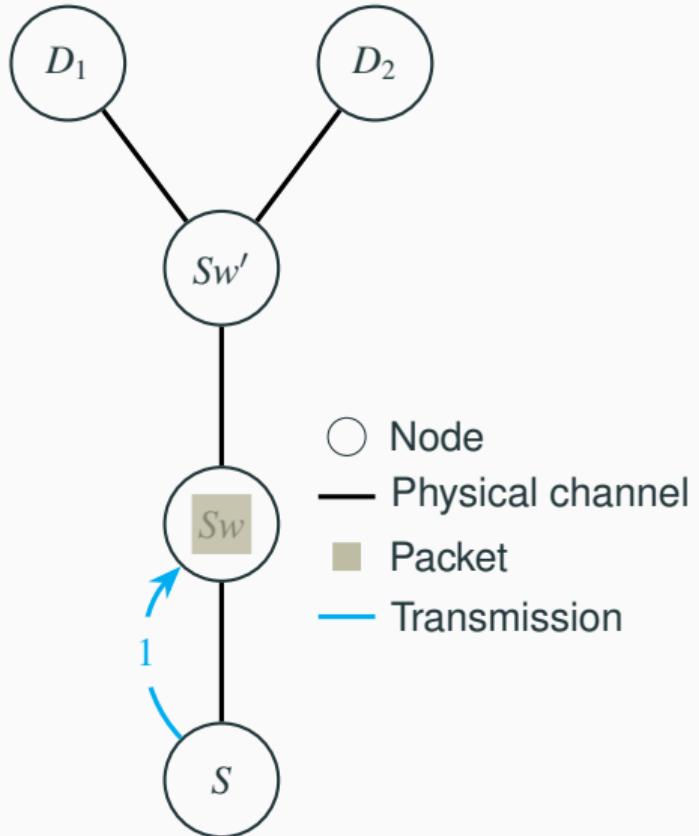
Forwarding packets



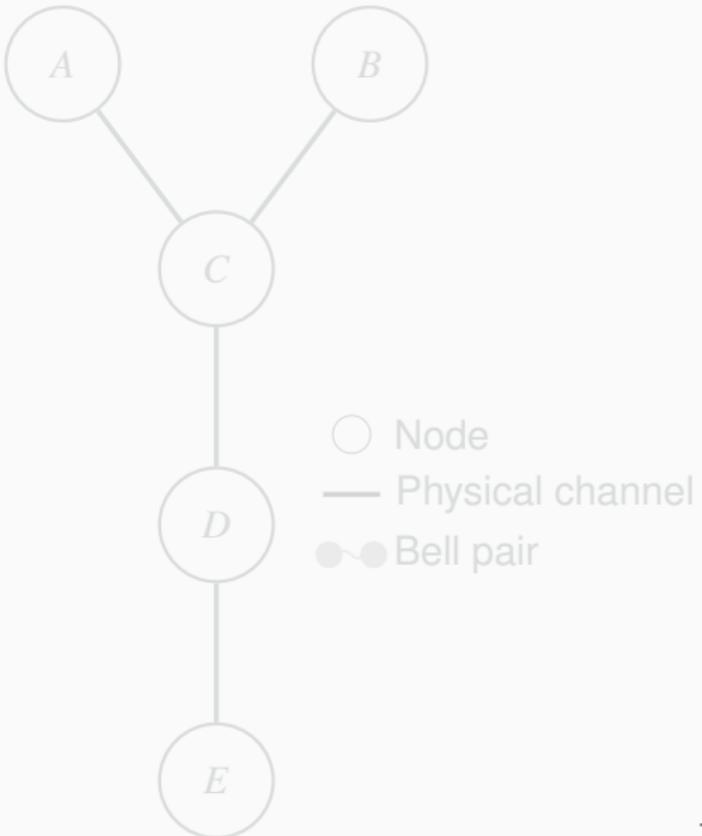
Distributing Bell pairs



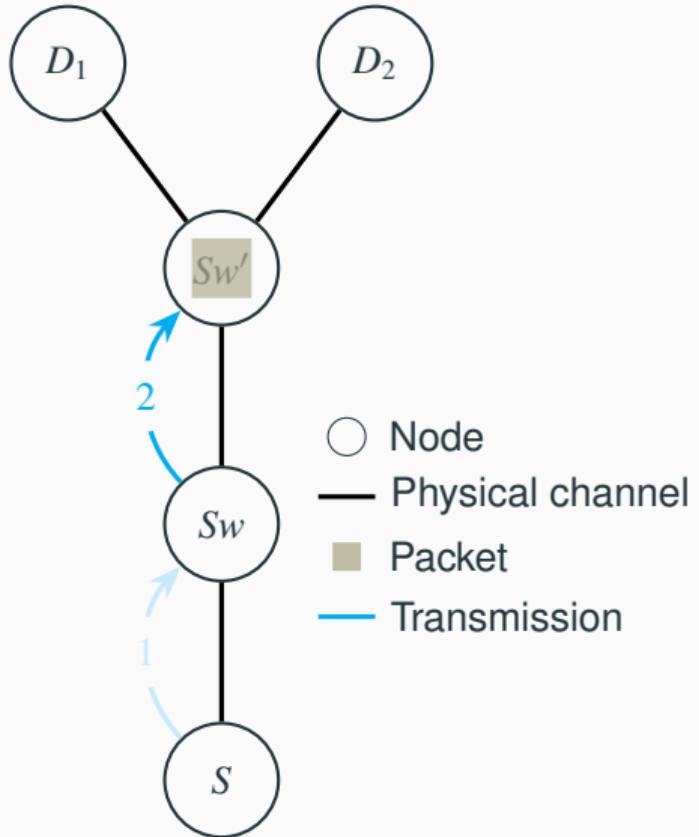
Forwarding packets



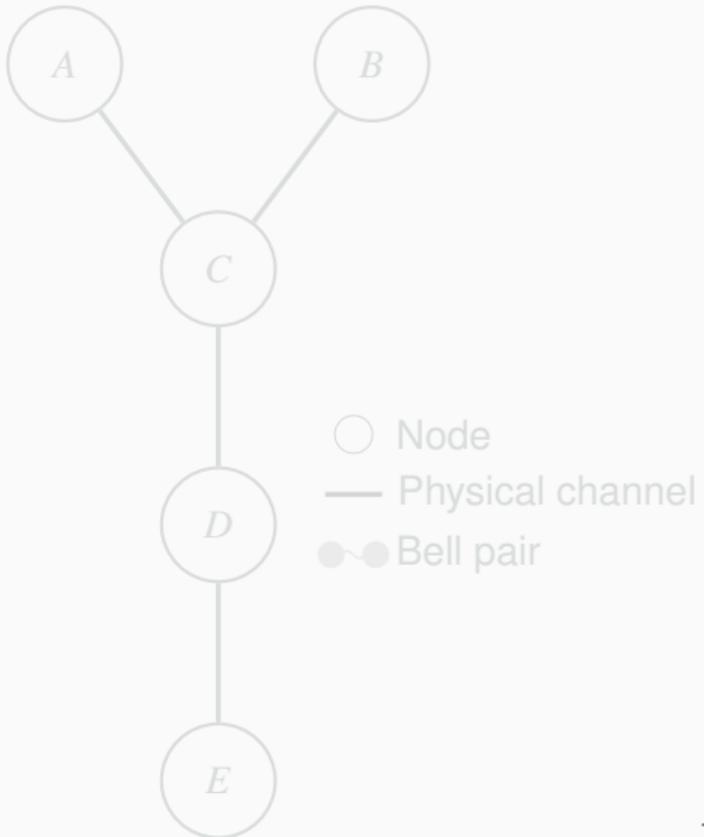
Distributing Bell pairs



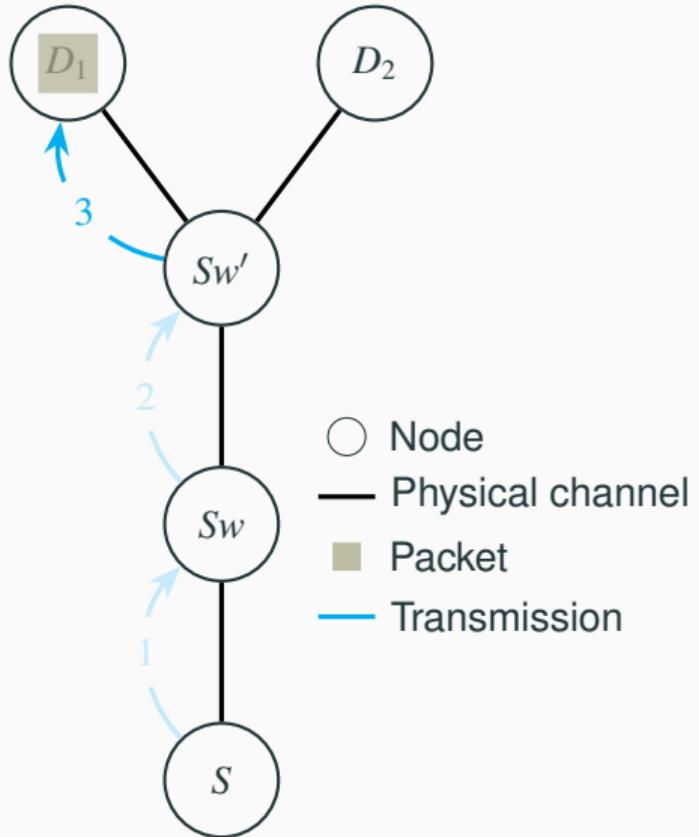
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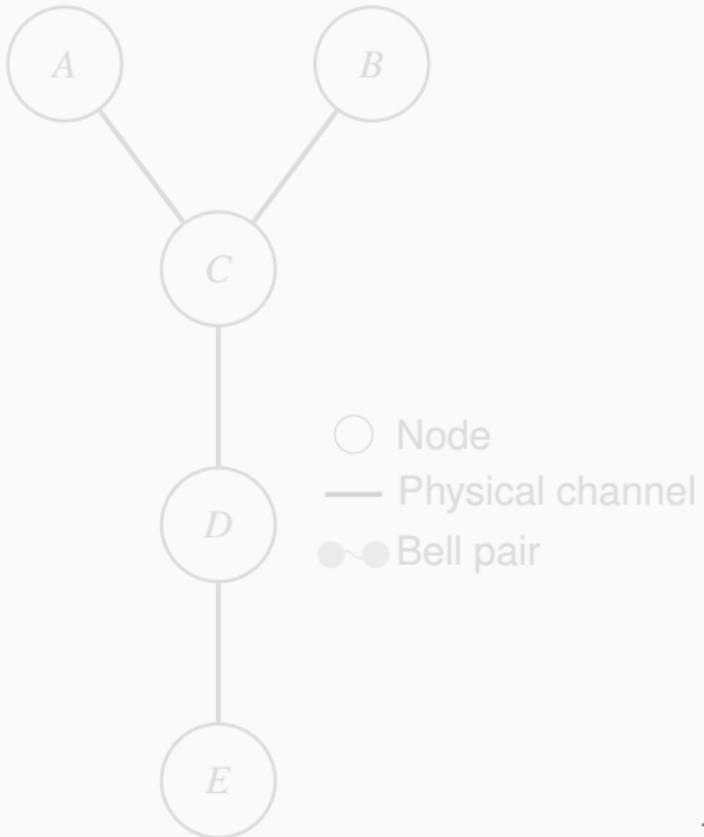
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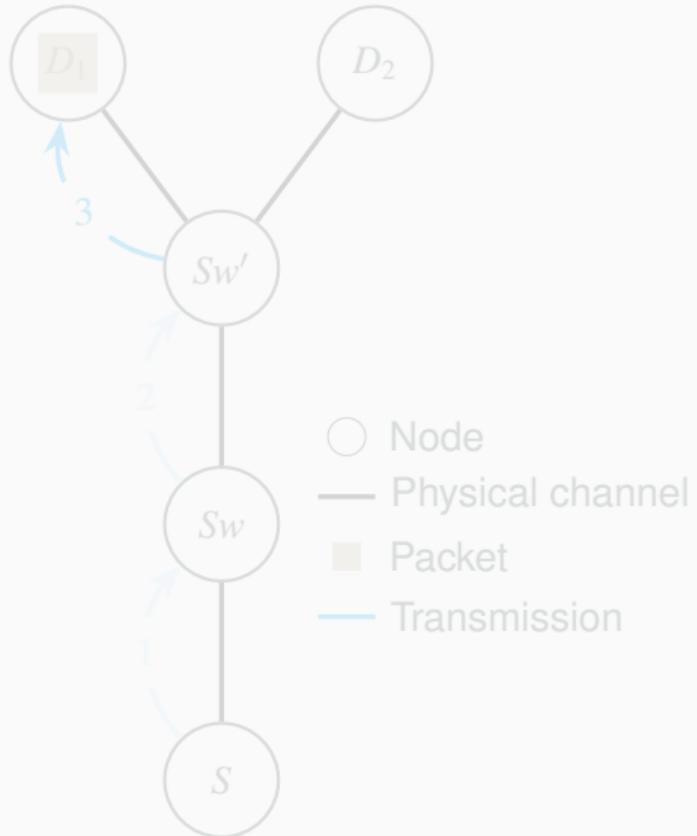
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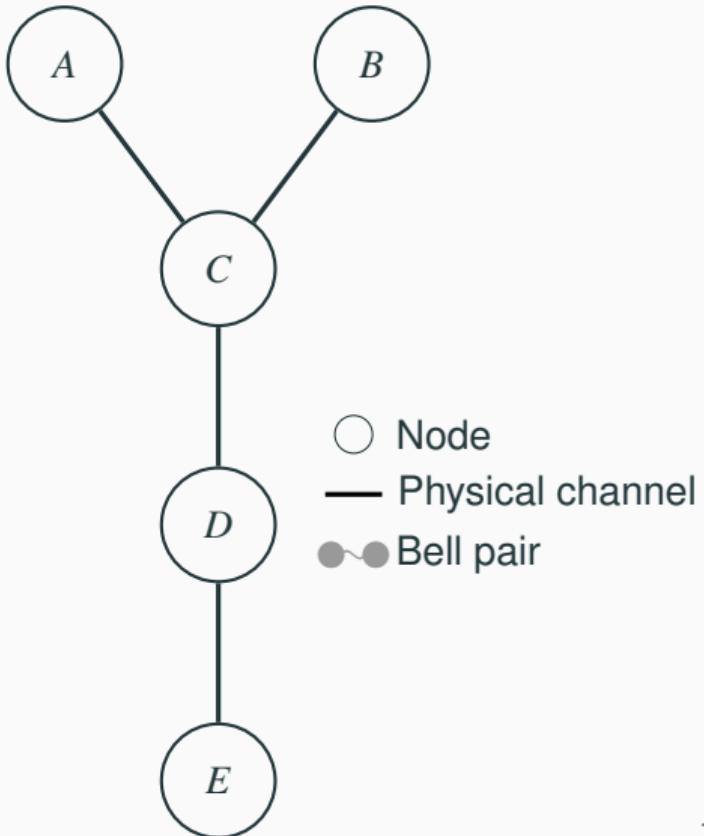
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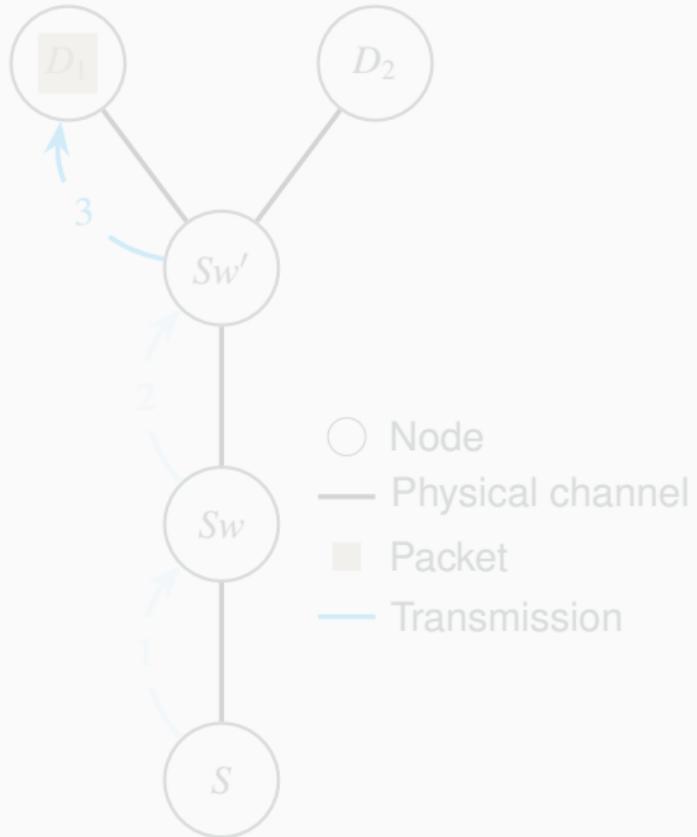
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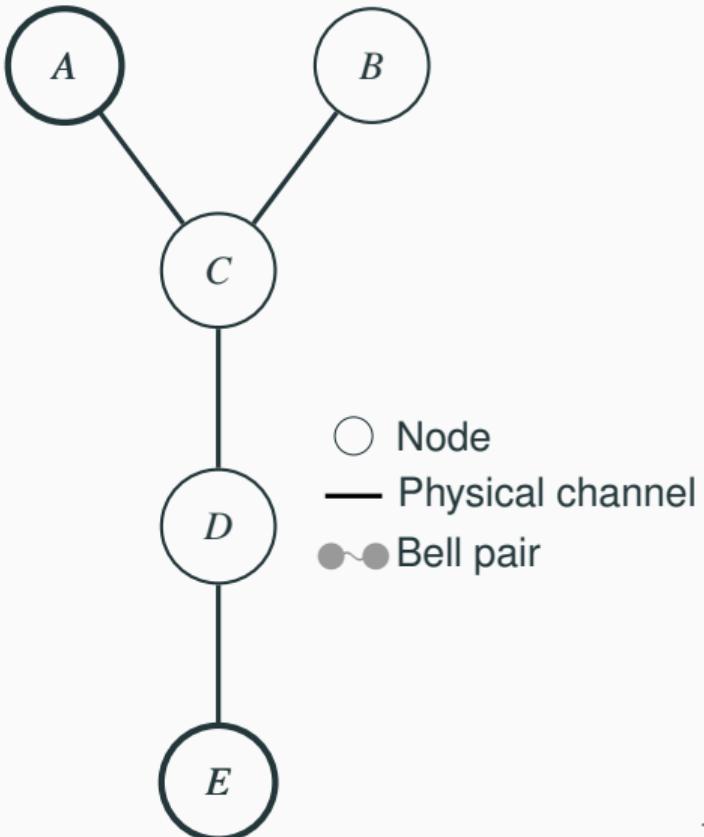
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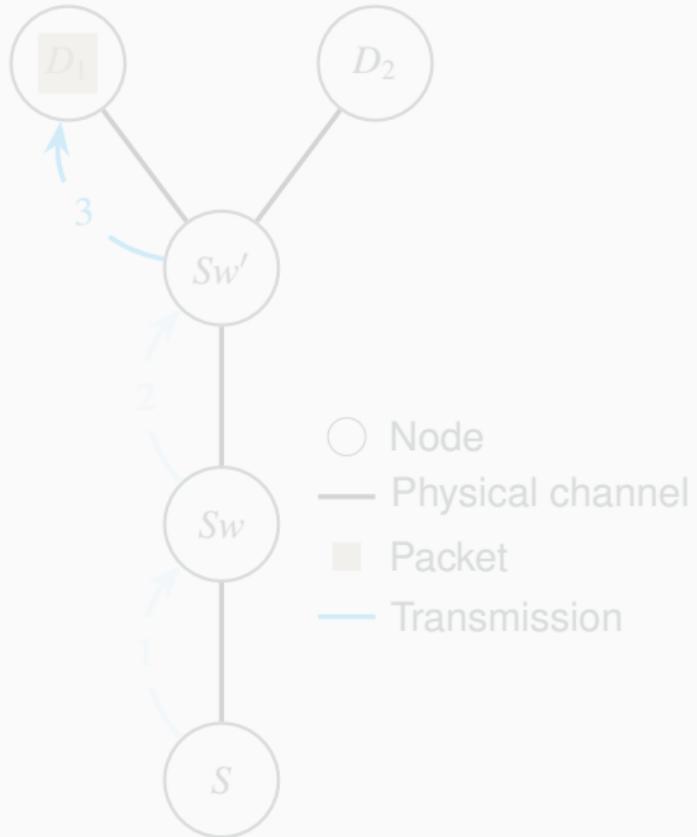
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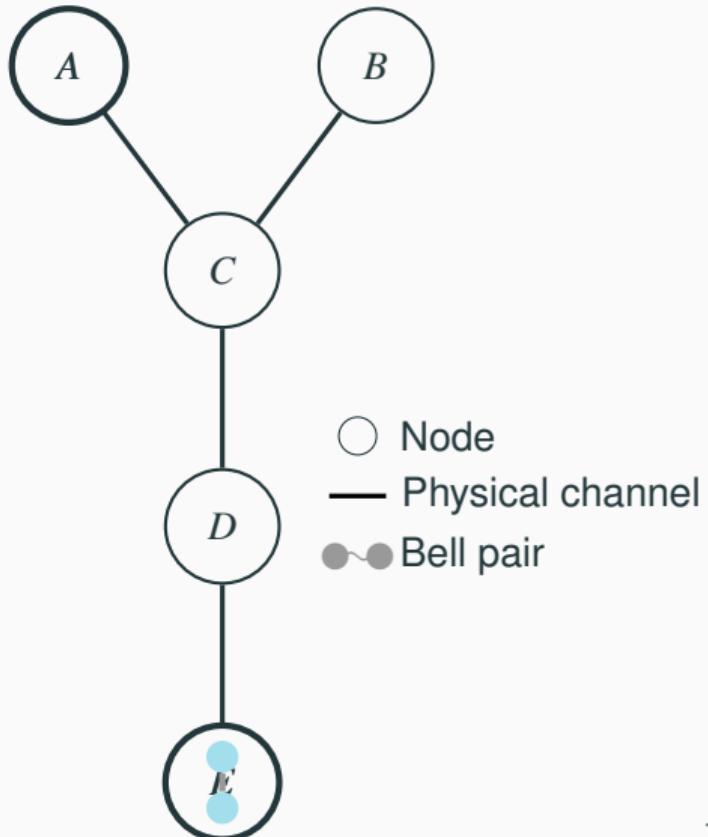
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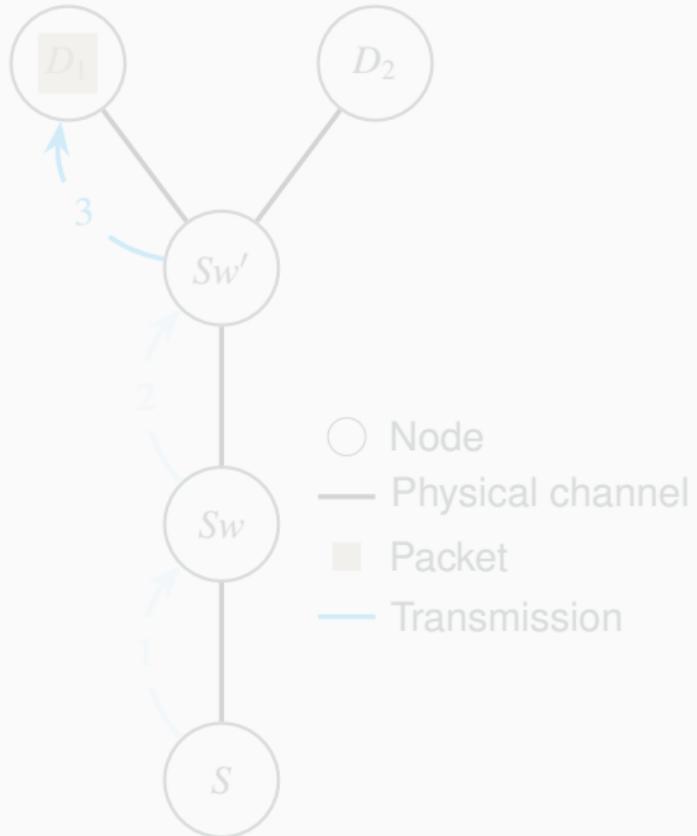
Forwarding packets



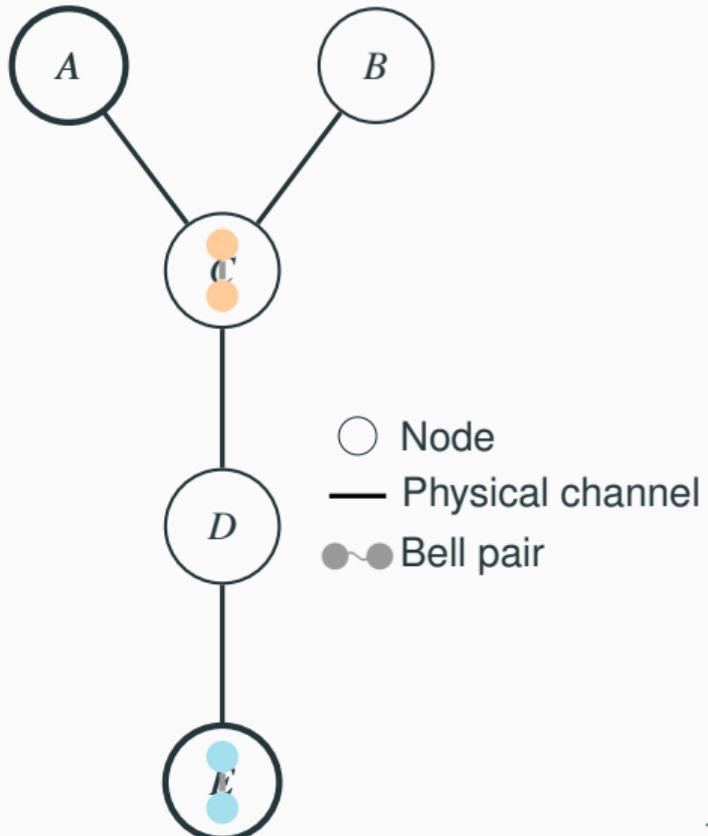
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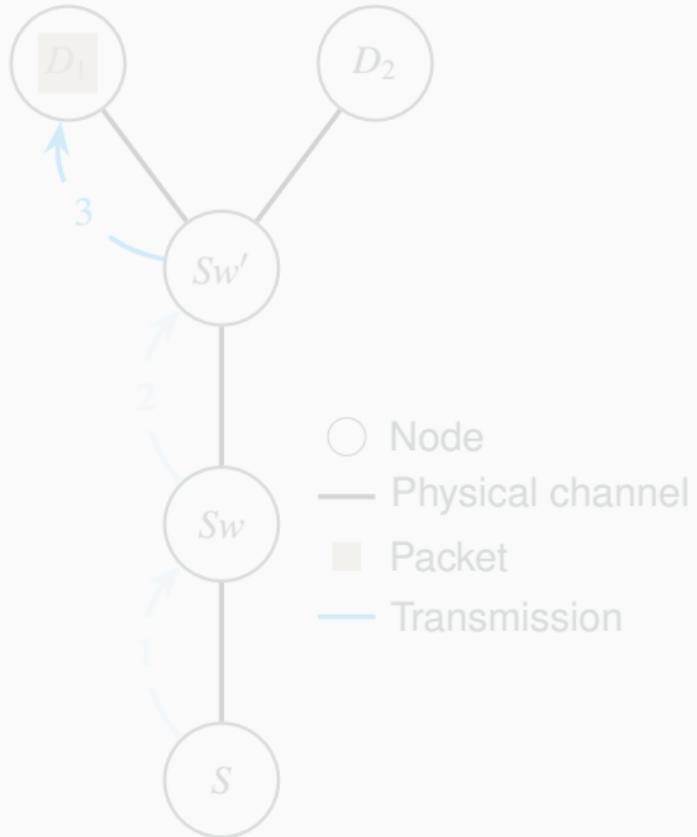
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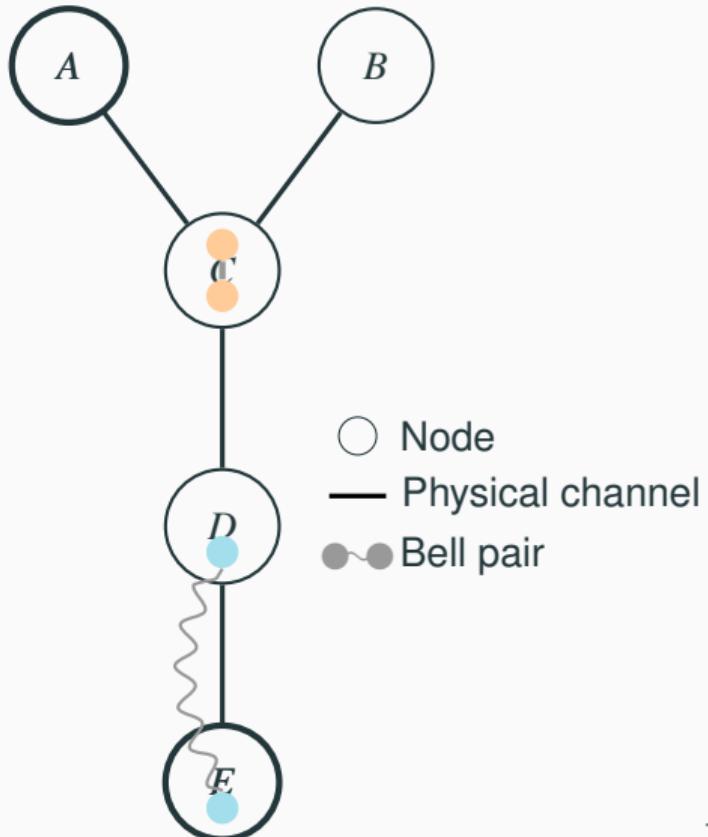
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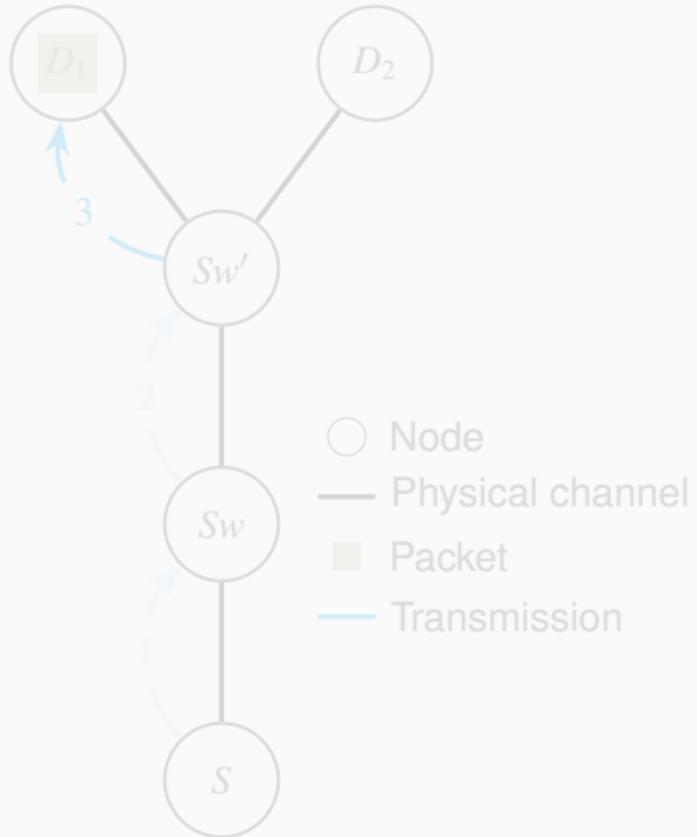
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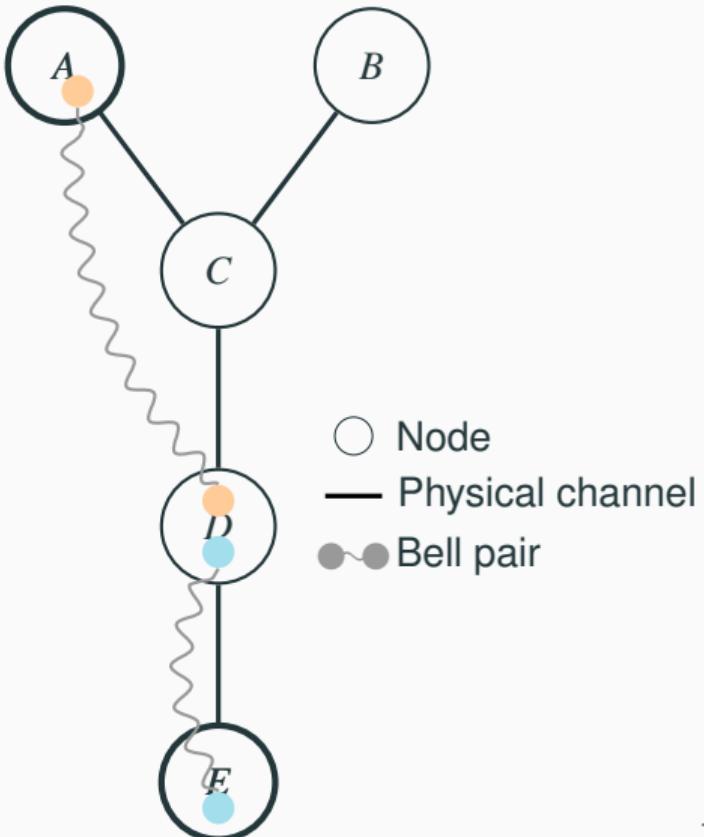
Distributing Bell pairs



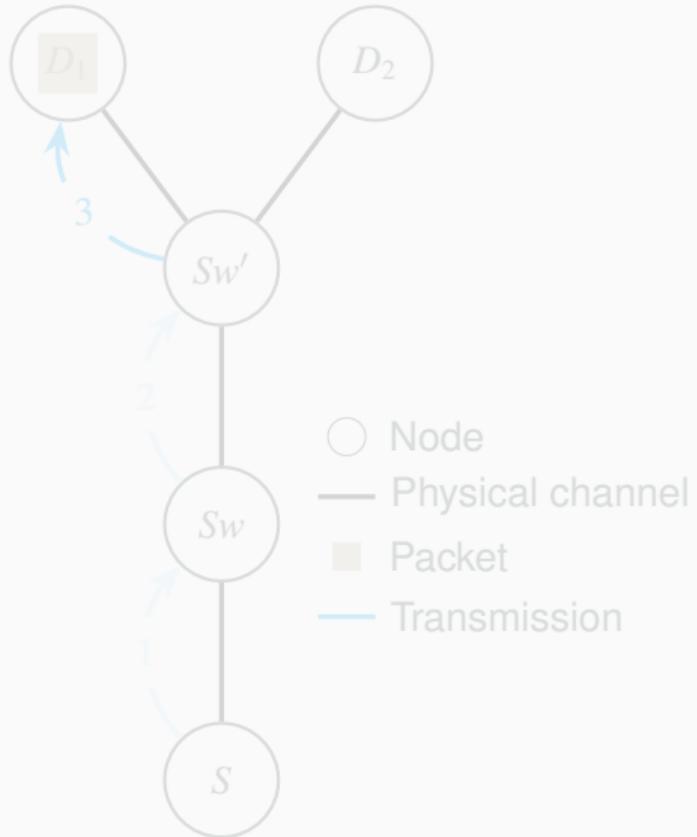
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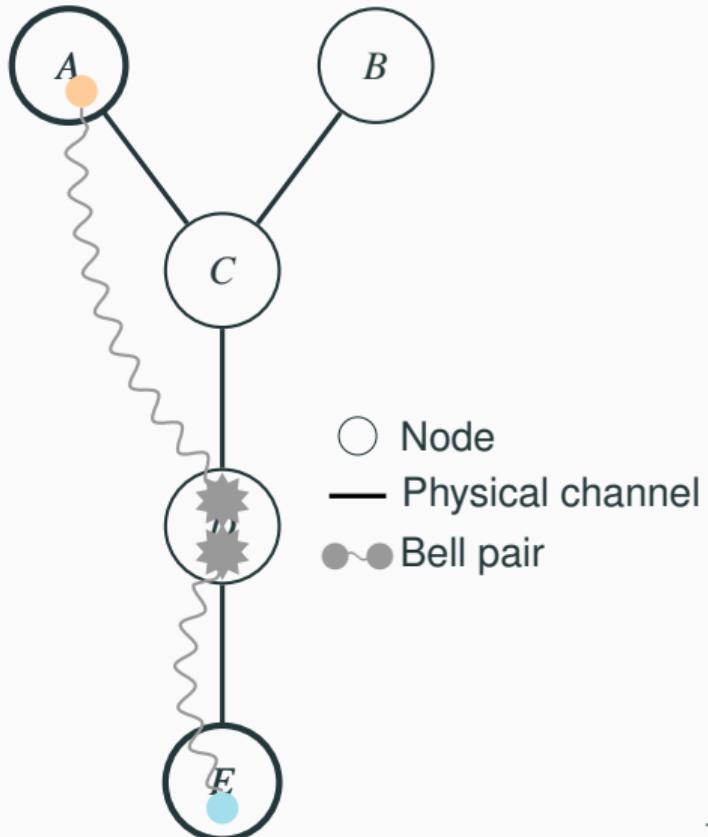
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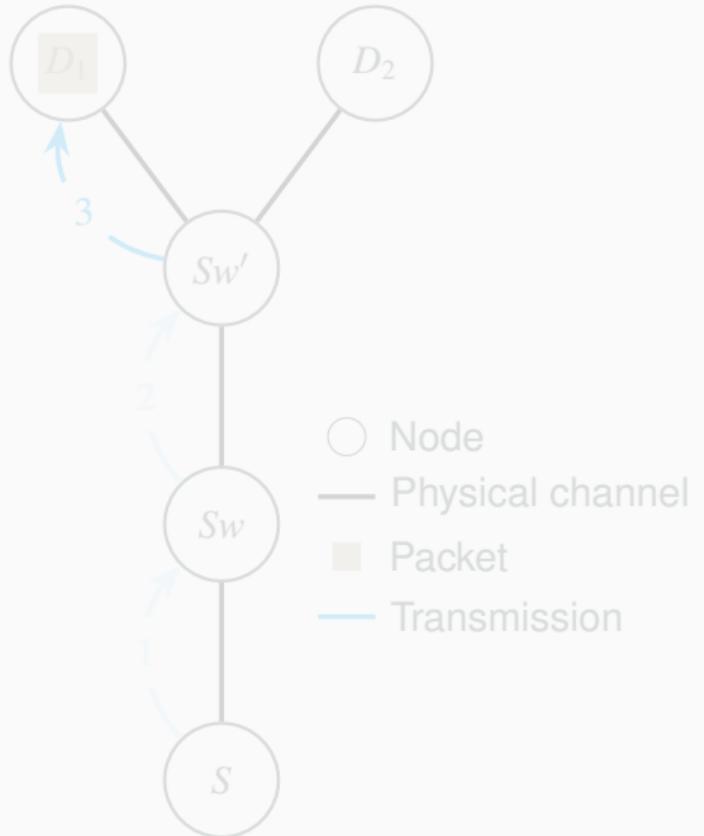
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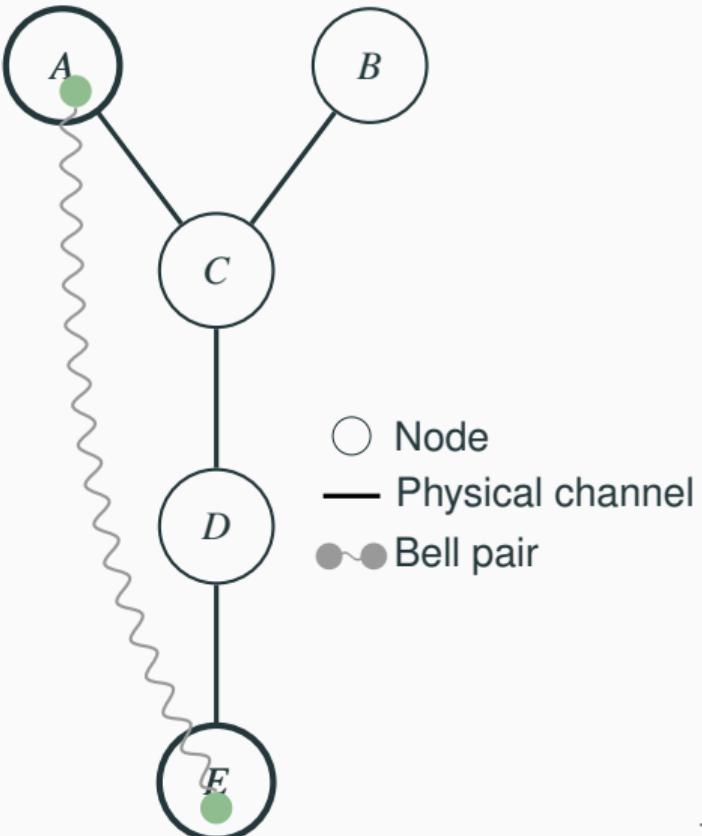
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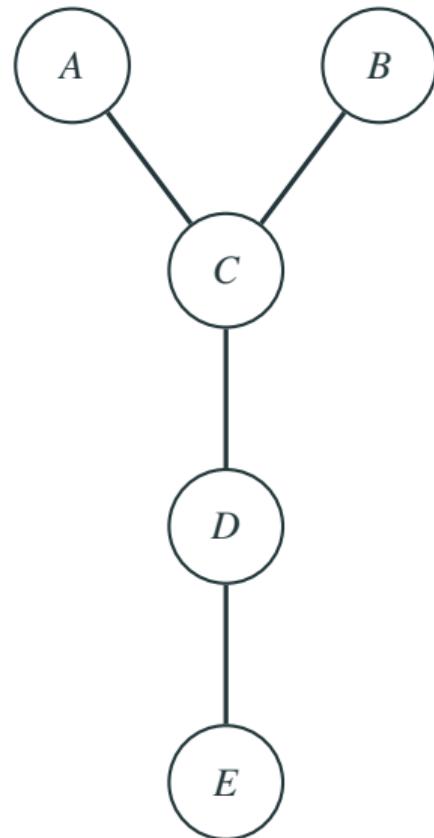


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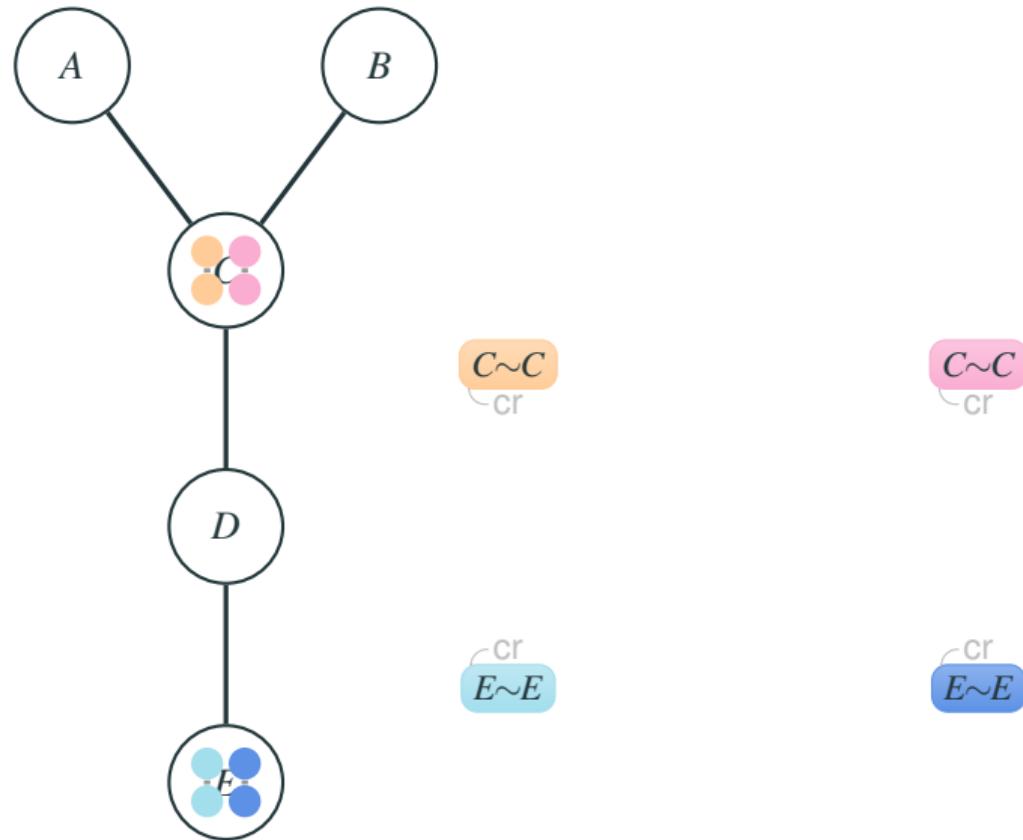
End-to-end Bell pair generation protocol

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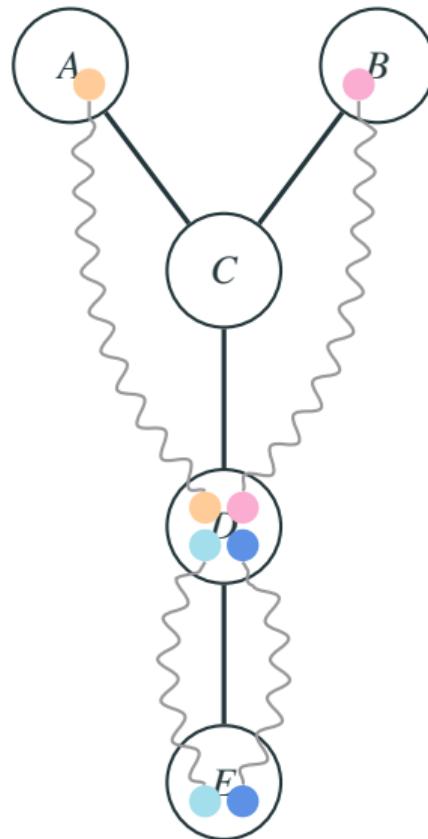
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End-to-end Bell pair generation protocol

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$\xrightarrow[\text{cr}]{\text{tr}} C \sim C \rightarrow A \sim D$

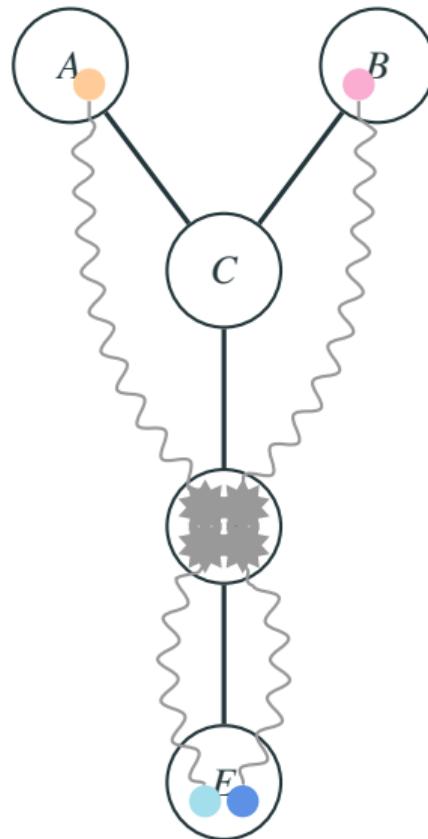
$\xrightarrow[\text{cr}]{\text{tr}} C \sim C \rightarrow B \sim D$

$\xrightarrow[\text{cr}]{\text{tr}} E \sim E \rightarrow E \sim D$

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End-to-end Bell pair generation protocol

•••



$\overset{\text{cr}}{C \sim C} \xrightarrow{\text{tr}} A \sim D$

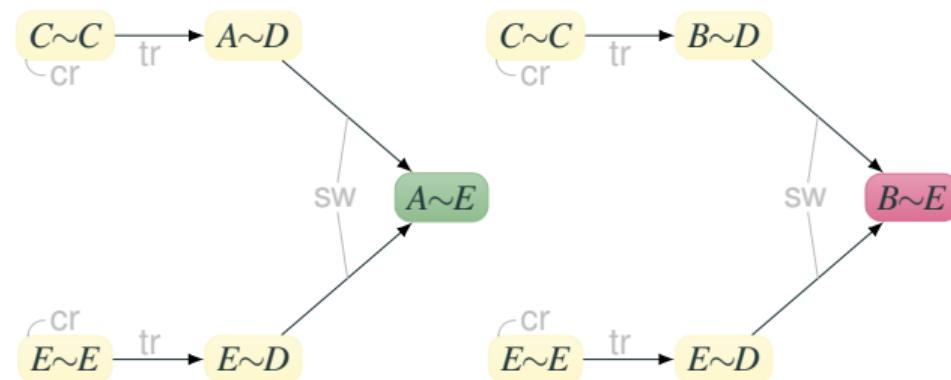
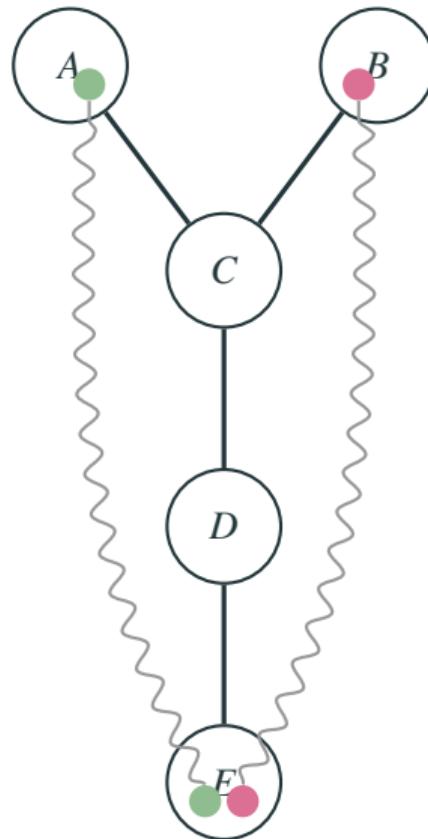
$\overset{\text{cr}}{C \sim C} \xrightarrow{\text{tr}} B \sim D$

$\overset{\text{cr}}{E \sim E} \xrightarrow{\text{tr}} E \sim D$

$\overset{\text{cr}}{E \sim E} \xrightarrow{\text{tr}} E \sim D$

End-to-end Bell pair generation protocol

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Specification language for end-to-end Bell pairs generation – BellKAT

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- Syntax and semantics
 - provide abstractions for quantum network primitives: create cr, transmit tr, swap sw, ...
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- Formal results
 - proofs of soundness and completeness of equational theory
 - decidability of semantic equivalences

$$r \triangleright o : \textcolor{brown}{a} \mapsto \begin{cases} \textcolor{green}{o} \bowtie \textcolor{blue}{a} \setminus r & \text{if } \textcolor{red}{r} \subseteq \textcolor{brown}{a} \\ \emptyset \bowtie \textcolor{blue}{a} & \text{otherwise} \end{cases}$$

required BPs

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$$r \triangleright o : a \mapsto \begin{cases} o \bowtie a \setminus r & \text{if } r \subseteq a \\ \emptyset \bowtie a & \text{otherwise} \end{cases}$$

required BPs

output BPs

The diagram illustrates the mapping of required Bell pairs (BPs) to output BPs. It features two rounded rectangular boxes: one at the top left labeled "required BPs" and one at the bottom left labeled "output BPs". A curved arrow originates from the "required BPs" box and points to the left side of the primitive definition. Another curved arrow originates from the "output BPs" box and points to the right side of the primitive definition.

BellKAT primitives – basic actions

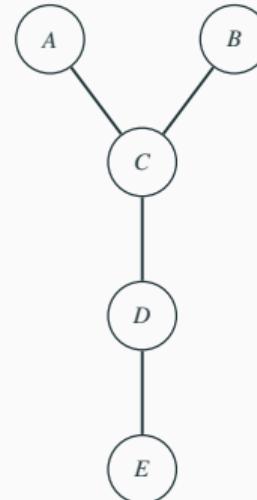
...

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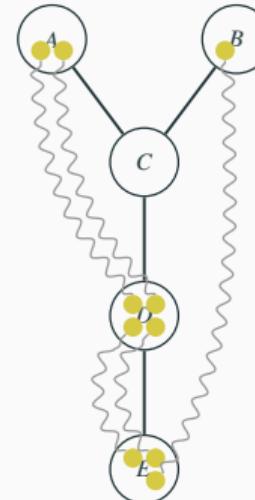
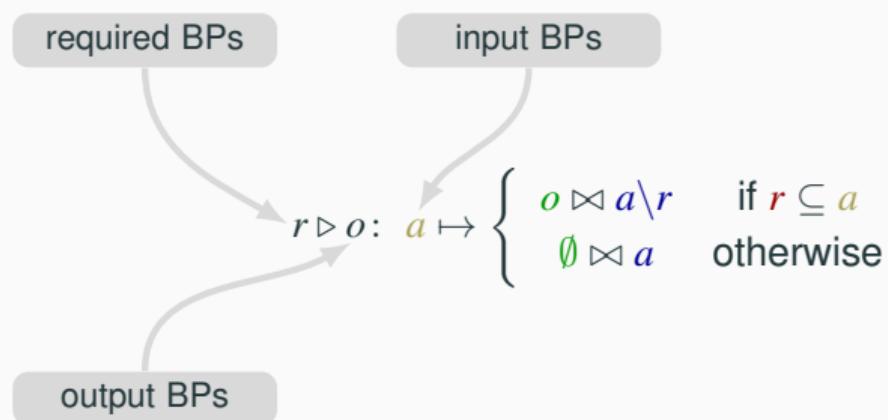
output BPs

Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$



BellKAT primitives – basic actions

...



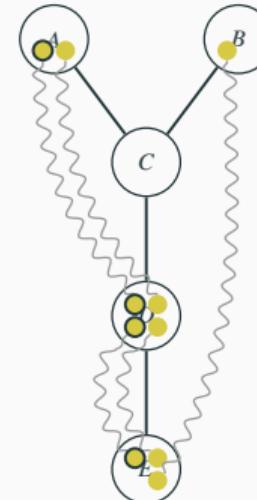
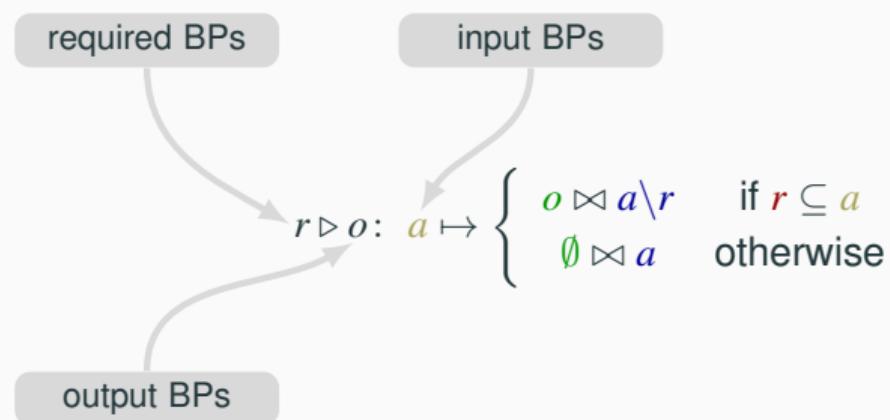
Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{A \sim D, A \sim D, D \sim E, D \sim E, B \sim E\}$

$A \sim D$ $D \sim E$ $A \sim D$ $D \sim E$ $B \sim E$

Input Bell pairs

BellKAT primitives – basic actions

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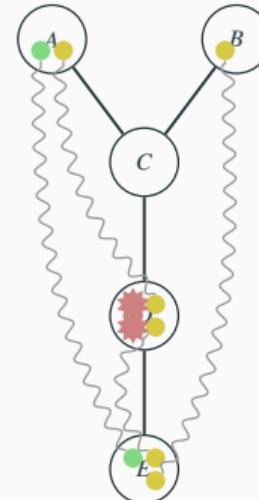
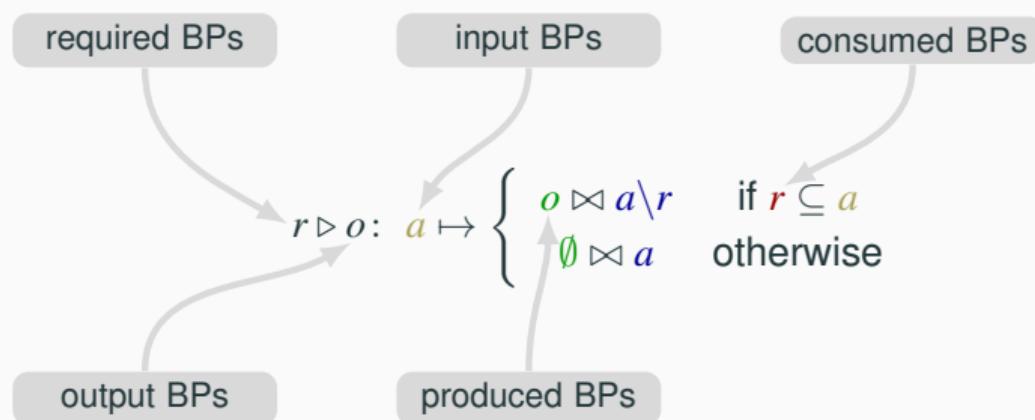


Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{\underline{A \sim D}, A \sim D, \underline{D \sim E}, D \sim E, B \sim E\}$

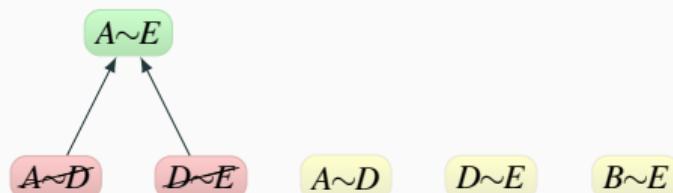
$A \sim D$ $D \sim E$ $A \sim D$ $D \sim E$ $B \sim E$

BellKAT primitives – basic actions

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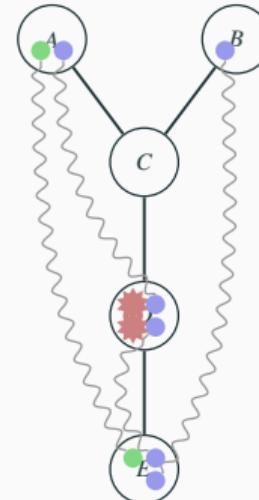
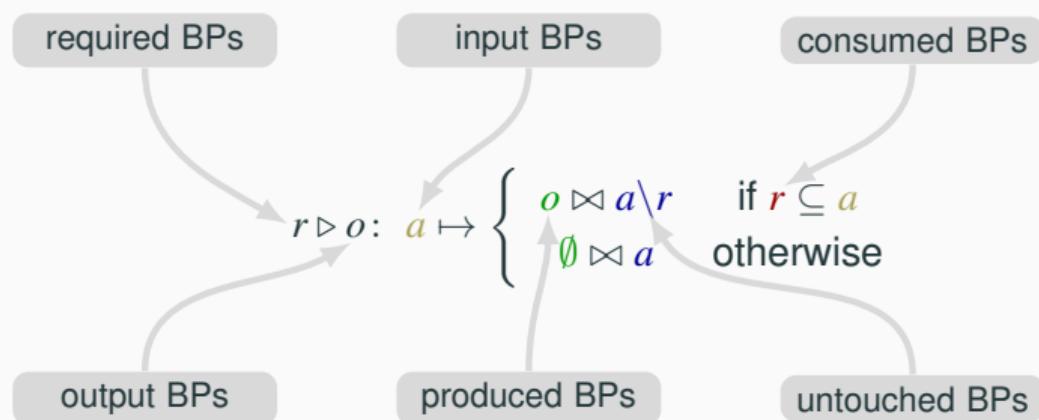
Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{A \sim D, A \sim D, D \sim E, D \sim E, B \sim E\}$



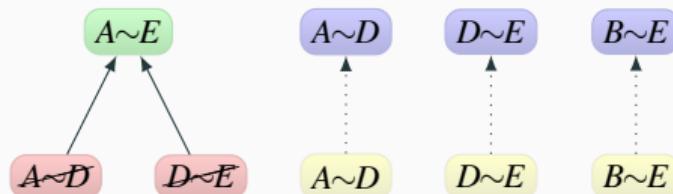
Input, consumed and produced Bell pairs

BellKAT primitives – basic actions

...



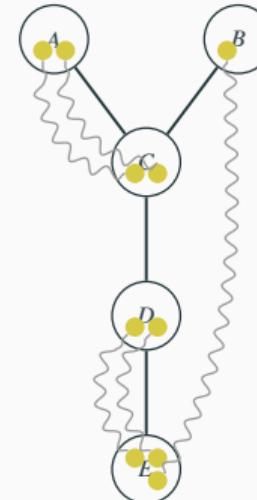
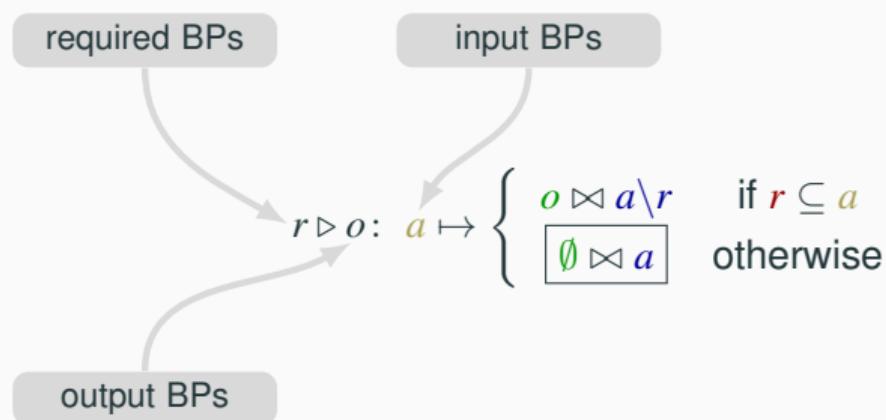
Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{A \sim D, A \sim D, D \sim E, D \sim E, B \sim E\}$



Input, consumed, produced and untouched Bell pairs

BellKAT primitives – basic actions

...

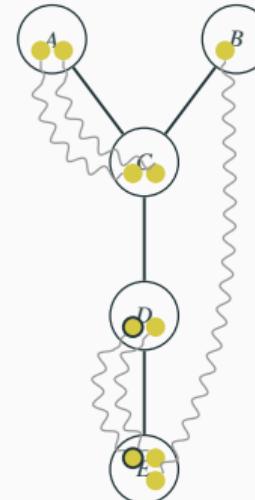
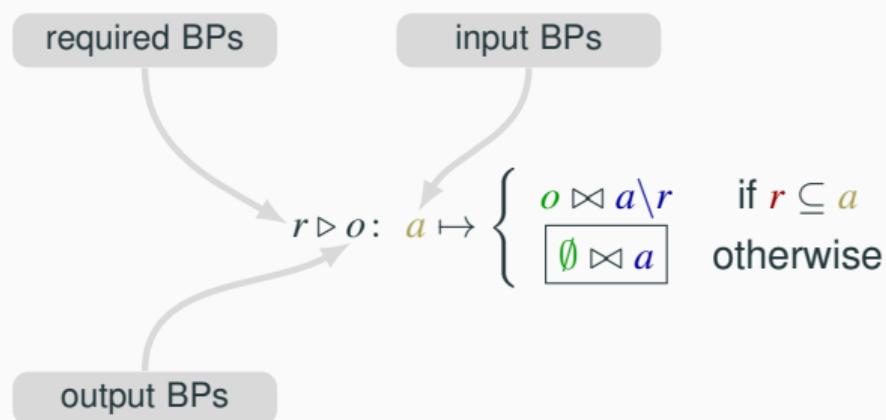


Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{A \sim C, A \sim C, D \sim E, D \sim E, B \sim E\}$

$A \sim C$ $D \sim E$ $A \sim C$ $D \sim E$ $B \sim E$

BellKAT primitives – basic actions

...

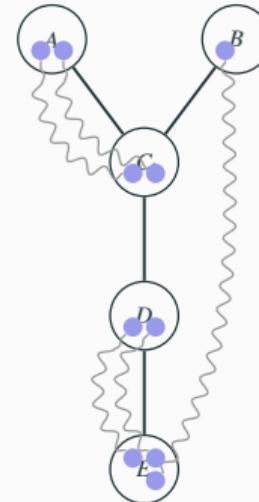
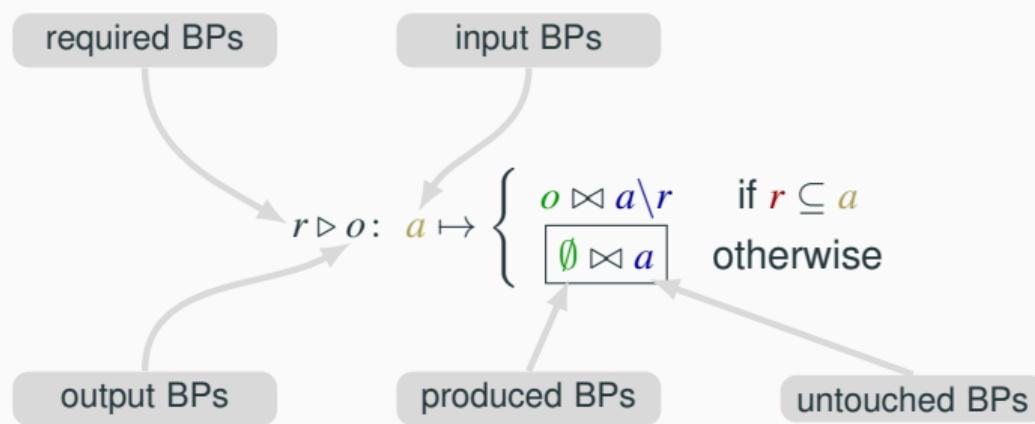


Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{A \sim C, A \sim C, \underline{D \sim E}, D \sim E, B \sim E\}$

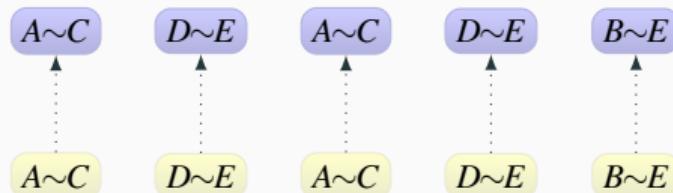
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BellKAT primitives – basic actions

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Swap $\{A \sim D, D \sim E\} \triangleright \{A \sim E\}$ acting on input $\{A \sim C, A \sim C, \underline{D \sim E}, D \sim E, B \sim E\}$



Input, consumed, produced and untouched Bell pairs

swap	$\text{sw}\langle A \sim B @ C \rangle \triangleq \{\{A \sim C, B \sim C\}\} \triangleright \{\{A \sim B\}\}$
transmit	$\text{tr}\langle A \rightarrow B \sim C \rangle \triangleq \{\{A \sim A\}\} \triangleright \{\{B \sim C\}\}$
create	$\text{cr}\langle A \rangle \triangleq \emptyset \triangleright \{\{A \sim A\}\}$
wait	$\text{wait}\langle r \rangle \triangleq r \triangleright r$
drop	$\text{drop}\langle r \rangle \triangleq r \triangleright \emptyset$

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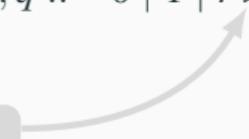
swap	$\text{sw}\langle A \sim B @ C \rangle \triangleq \{\{A \sim C, B \sim C\} \triangleright \{A \sim B\}\}$
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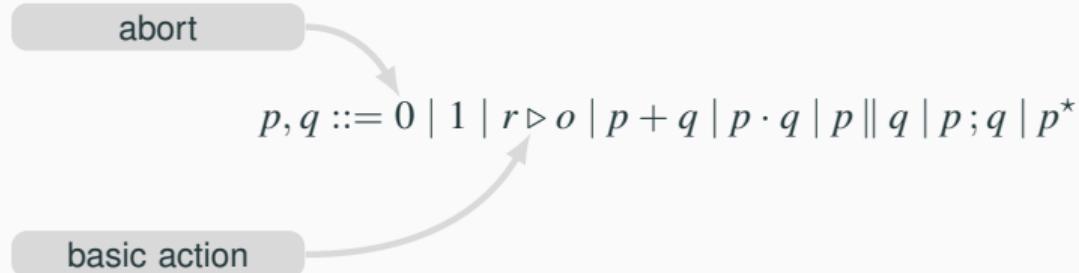
swap	$\text{sw}\langle A \sim B @ C \rangle \triangleq \{\{A \sim C, B \sim C\} \triangleright \{A \sim B\}\}$
transmit	$\text{tr}\langle A \rightarrow B \sim C \rangle \triangleq \{\{A \sim A\} \triangleright \{B \sim C\}\}$
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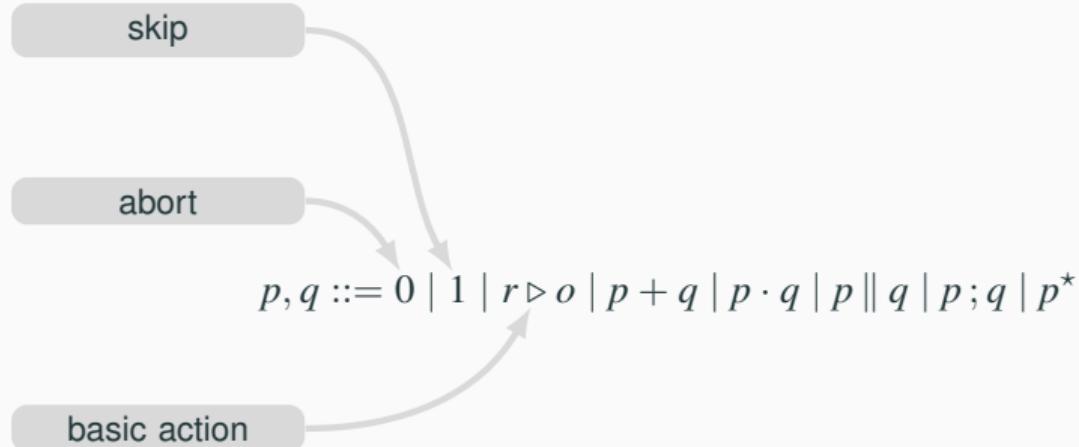
$$p, q ::= 0 \mid 1 \mid r \triangleright o \mid p + q \mid p \cdot q \mid p \parallel q \mid p ; q \mid p^*$$

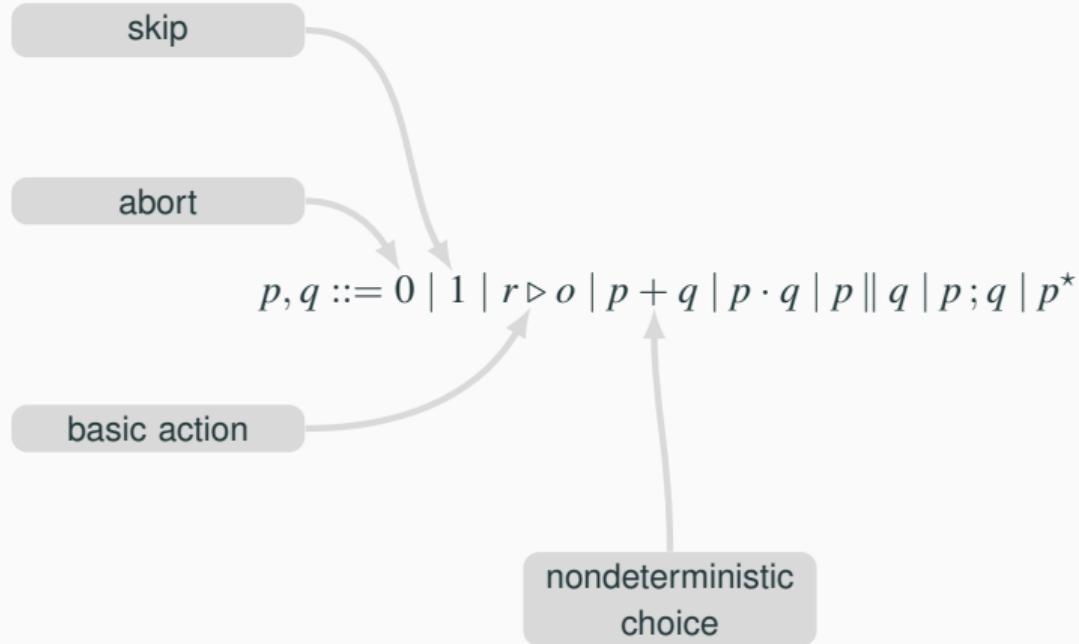
$$p, q ::= 0 \mid 1 \mid r \triangleright o \mid p + q \mid p \cdot q \mid p \parallel q \mid p ; q \mid p^*$$

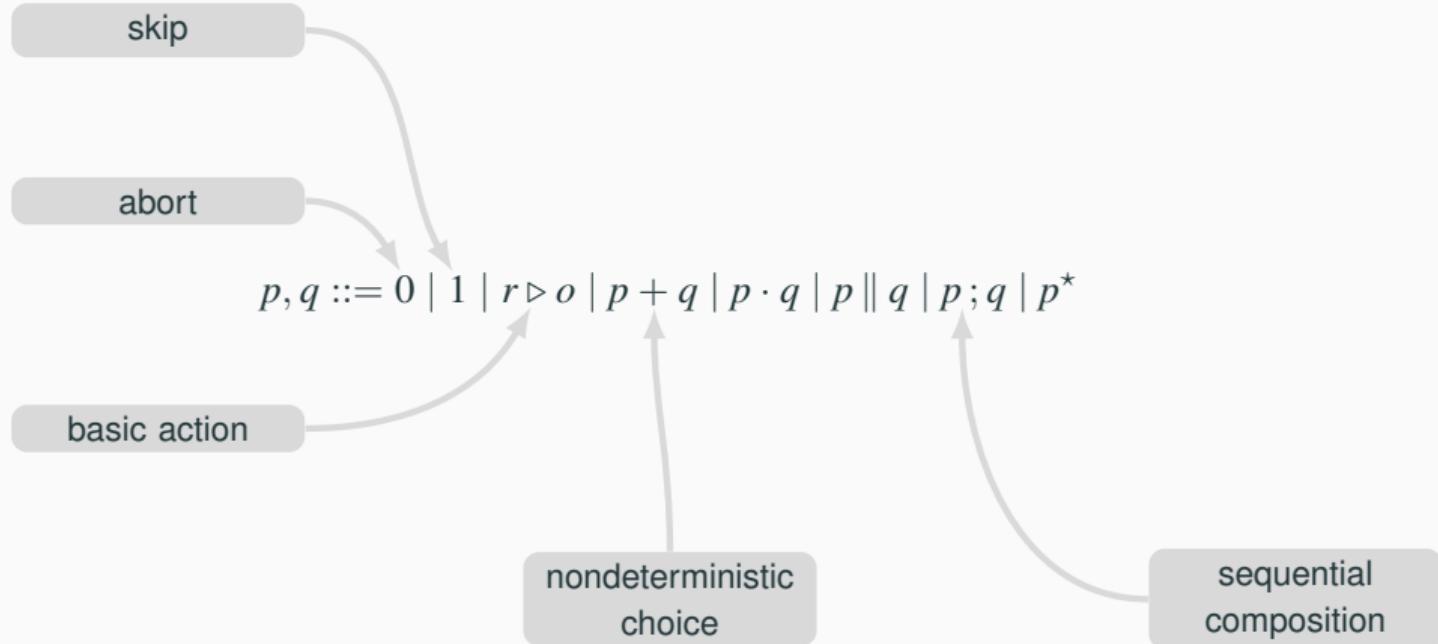
basic action

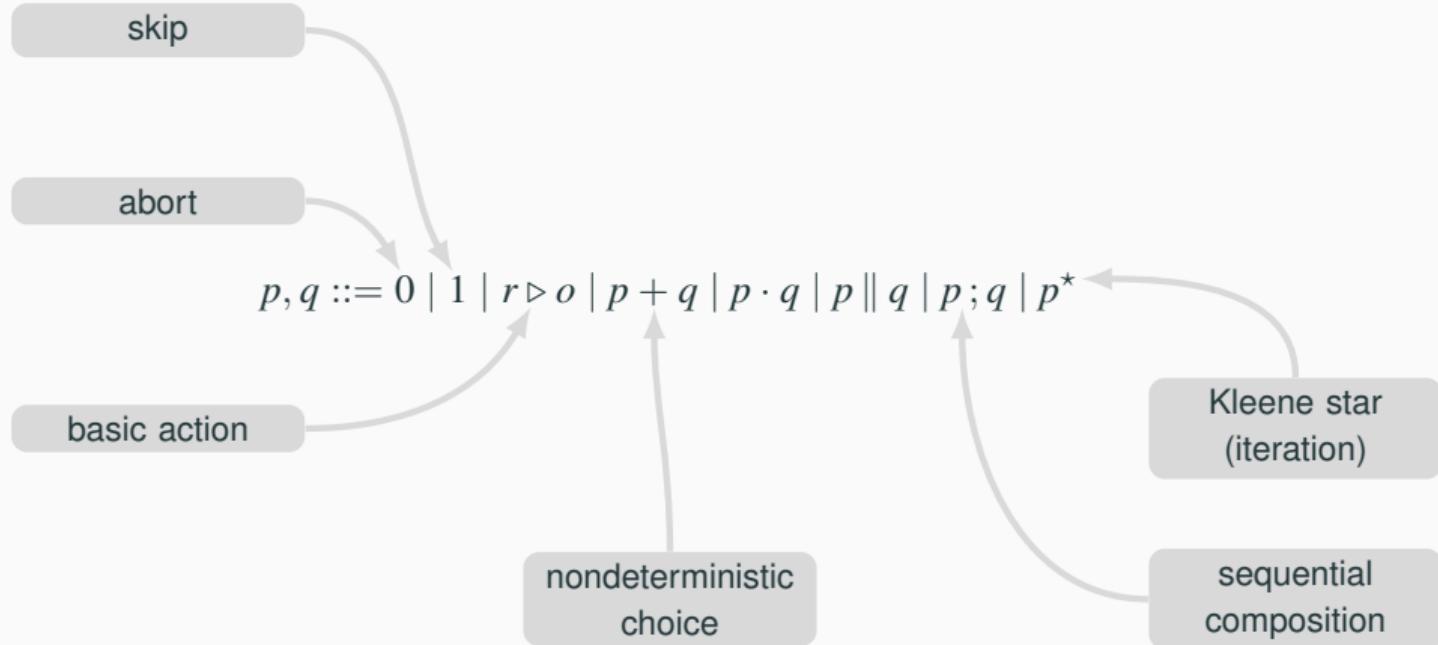


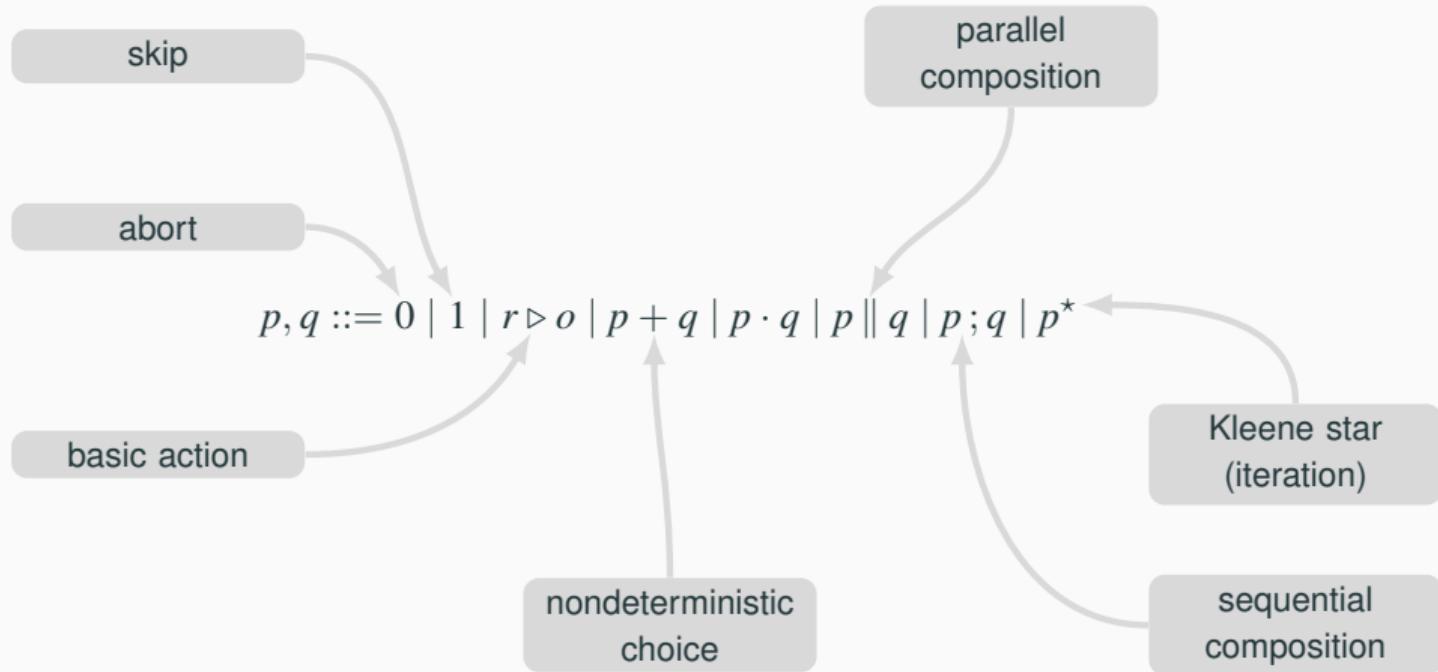


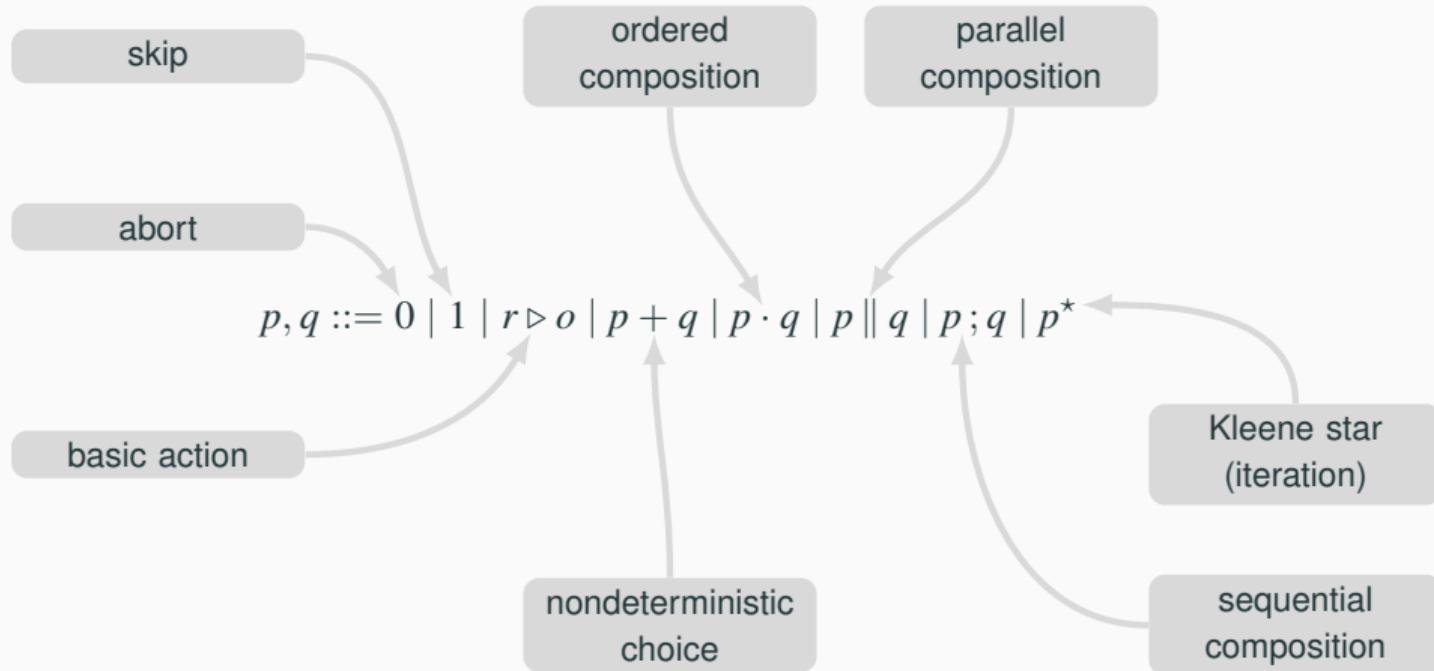






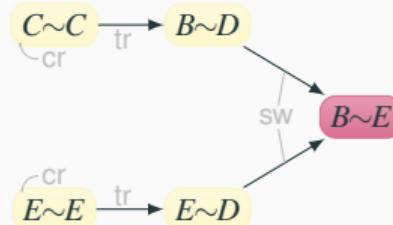
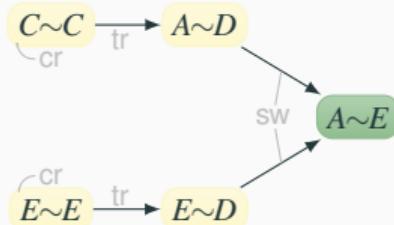






Protocol specification in BellKAT

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Protocol specification in BellKAT

•••



$(\text{cr}\langle C \rangle \parallel \text{cr}\langle C \rangle \parallel \text{cr}\langle E \rangle \parallel \text{cr}\langle E \rangle);$

$(\text{tr}\langle C \rightarrow A \sim D \rangle \parallel \text{tr}\langle C \rightarrow B \sim D \rangle \parallel \text{tr}\langle E \rightarrow E \sim D \rangle \parallel \text{tr}\langle E \rightarrow E \sim D \rangle);$

$(\text{sw}\langle A \sim E @ D \rangle \parallel \text{sw}\langle B \sim E @ D \rangle)$

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N \sim N$
Multisets	$M(BP) \ni a, b, r, o ::= \{bp_1, \dots, bp_k\}$
Tests	$T \ni t, t' ::= \begin{cases} 1 & no\ test \\ & \\ b & multiset\ absence \\ & \\ t \wedge t' & conjunction \\ & \\ t \vee t' & disjunction \\ & \\ t \uplus b & multiset\ union \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, p, q ::= \begin{cases} 0 & abort \\ & \\ 1 & skip\ or\ no-round \\ & \\ \pi & atomic\ action \\ & \\ r \triangleright o & basic\ action \\ & \\ [t]p & guarded\ policy \\ & \\ p + q & nondeterministic\ choice \\ & \\ p \cdot q & ordered\ composition \\ & \\ p \parallel q & parallel\ composition \\ & \\ p ; q & sequential\ composition \\ & \\ p^* & Kleene\ star \end{cases}$
Basic actions	$r \triangleright o ::= [1]r \triangleright o + [r]\emptyset \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$

Test semantics

$$\begin{array}{ll} \langle t \rangle \in M(BP) \rightarrow \{T, \perp\} & \\ \langle 1 \rangle a \triangleq T & \langle t \wedge b \rangle a \triangleq (\langle t \rangle a \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a \triangleq b \not\subseteq a & \langle t \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a, \text{with } \square \text{ is either } \wedge \text{ or } \vee \end{array}$$

Single round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP)) & \\ \langle \emptyset \rangle a \triangleq \emptyset & \\ \langle 1 \rangle a \triangleq \{\emptyset \bowtie a\} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{o \bowtie a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = T \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle p + q \rangle a \triangleq \langle (p) \rangle a \cup \langle (q) \rangle a & \\ \langle p \cdot q \rangle a \triangleq \langle (p) \cdot \langle (q) \rangle a & \\ \langle p \parallel q \rangle a \triangleq \langle (p) \parallel \langle (q) \rangle a & \end{cases}$$

Multi-round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP)) & \\ \langle \omega \rangle_I \in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k & \\ \langle \omega \rangle a \triangleq \bigcup_{a \in I(p)} \langle \omega \rangle_I a & \\ \langle e \rangle_I a \triangleq \{a\} & \\ \langle [t]r \triangleright o \rangle_I a \triangleq \begin{cases} \{o \bowtie a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = T \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \rangle_I a \triangleq \langle \langle \pi_1 \rangle_I \bullet \langle \pi_2 \rangle_I \ddagger \dots \ddagger \langle \pi_k \rangle_I \rangle_I a & \end{array}$$

KA axioms

$$\begin{array}{llll} (p + q) + r \equiv p + (q + r) & KA\text{-PLUS-ASSOC} & p ; 1 \equiv p & KA\text{-SEQ-ONE} \\ p + q \equiv q + p & KA\text{-PLUS-COMM} & 1 ; p \equiv p & KA\text{-ONE-SEQ} \\ p + 0 \equiv p & KA\text{-PLUS-ZERO} & 0 ; p \equiv 0 & KA\text{-ZERO-SEQ} \\ p + p \equiv p & KA\text{-PLUS-IDEM} & p ; 0 \equiv 0 & KA\text{-SEQ-ZERO} \\ (p ; q) ; r \equiv p ; (q ; r) & KA\text{-SEQ-ASSOC} & 1 + p ; p^* \equiv p^* & KA\text{-UNROLL-L} \\ p ; (q + r) \equiv p ; q + p ; r & KA\text{-SEQ-DIST-L} & p ; r \leq r \Rightarrow p^* ; r \leq r & KA\text{-LFP-L} \\ (p + q) ; r \equiv p ; r + q ; r & KA\text{-SEQ-DIST-R} & 1 + p^* ; p \equiv p^* & KA\text{-UNROLL-R} \\ & & r ; p \leq r \Rightarrow r ; p^* \leq r & KA\text{-LFP-R} \end{array}$$

SKA axioms for ||

$$\begin{array}{llll} (p \parallel q) \parallel r \equiv p \parallel (q \parallel r) & SKA\text{-PRL-ASSOC} & p \parallel q \equiv q \parallel p & SKA\text{-PRL-COMM} \\ p \parallel (q + r) \equiv p \parallel q + p \parallel r & SKA\text{-PRL-DIST} & 1 \parallel p \equiv p & SKA\text{-ONE-PRL} \\ (x : p) \parallel (y : q) \equiv (x \parallel y) : (p \parallel q) & SKA\text{-PRL-SEQ} & 0 \parallel p \equiv 0 & SKA\text{-ZERO-PRL} \end{array}$$

SKA axioms for ·

$$\begin{array}{llll} (p \cdot q) \cdot r \equiv p \cdot (q \cdot r) & SKA\text{-ORD-ASSOC} & 1 \cdot p \equiv p & SKA\text{-ONE-ORD} \\ p \cdot (q + r) \equiv p \cdot q + p \cdot r & SKA\text{-ORD-DIST-L} & p \cdot 1 \equiv p & SKA\text{-ORD-ONE} \\ (p + q) \cdot r \equiv p \cdot r + q \cdot r & SKA\text{-ORD-DIST-R} & 0 \cdot p \equiv 0 & SKA\text{-ZERO-ORD} \\ (x : p) \cdot (y : q) \equiv (x \cdot y) : (p \cdot q) & SKA\text{-ORD-SEQ} & p \cdot 0 \equiv 0 & SKA\text{-ORD-ZERO} \end{array}$$

Boolean axioms (in addition to monotone axioms)

$$\begin{array}{llll} 1 \sqcup b = 1 & BOOL\text{-ONE-U} & (t \wedge t') \sqcup b = t \uplus b \wedge t' \uplus b & BOOL\text{-CONJ-U-DIST} \\ b \wedge (b \wedge b') \equiv b & BOOL\text{-CONJ-SUBSET} & (t \vee t') \sqcup b = t \uplus b \vee t' \uplus b & BOOL\text{-DISJ-U-DIST} \\ b \vee b' = b \uplus b' & BOOL\text{-DISJ-U} & & \end{array}$$

Network axioms

$$\begin{array}{llll} [t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o} & & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & NET\text{-ORD} \\ [t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o} & & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & NET\text{-PRL} \end{array}$$

Single round axioms

$$\begin{array}{llll} \langle p \parallel p' \rangle \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q') & SR\text{-EXC} & [1]\emptyset \triangleright \emptyset \equiv 1 & SR\text{-ONE} \\ [b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o & SR\text{-CAN} & [0]r \triangleright o \equiv 0 & SR\text{-ZERO} \\ [t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o & SR\text{-PLUS} & & \end{array}$$

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$M(BP) \ni a, b, r, o ::= \{bp_1, \dots, bp_k\}$
Tests	$T \ni t, t' ::= \begin{cases} 1 & \text{no test} \\ & \\ & b & \text{multiset absence} \\ & t \wedge t' & \text{conjunction} \\ & t \vee t' & \text{disjunction} \\ & t \uplus b & \text{multiset union} \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & 1 & \text{skip or no-round} \\ & \pi & \text{atomic action} \\ & r \triangleright o & \text{basic action} \\ & [t]p & \text{guarded policy} \\ & p + q & \text{nondeterministic choice} \\ & p \cdot q & \text{ordered composition} \\ & p \parallel q & \text{parallel composition} \\ & p ; q & \text{sequential composition} \\ & p^* & \text{Kleene star} \end{cases}$
Basic actions	$r \triangleright o ::= [1]r \triangleright o + [r]0 \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$

Test semantics

$$\begin{array}{ll} \langle t \rangle \in M(BP) \rightarrow \{\top, \perp\} & \\ \langle 1 \rangle a \triangleq \top & \langle t \uplus b \rangle a \triangleq (\langle t \rangle a \setminus b \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a \triangleq b \not\subseteq a & \langle t \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ with } \square \text{ is either } \wedge \text{ or } \vee \end{array}$$

Single round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP)) & \\ \langle 0 \rangle a \triangleq \emptyset & \\ \langle 1 \rangle a \triangleq \{ \emptyset \mapsto a \} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \mapsto a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle p + q \rangle a \triangleq \langle p \rangle a \cup \langle q \rangle a & \\ \langle p \cdot q \rangle a \triangleq (\langle p \rangle \cdot \langle q \rangle) a & \\ \langle p \parallel q \rangle a \triangleq (\langle p \rangle \parallel \langle q \rangle) a & \end{array}$$

Multi-round semantics

$$\begin{array}{ll} \llbracket p \rrbracket \in M(BP) \rightarrow \mathcal{P}(M(BP)) & \\ \llbracket \omega \rrbracket_I \in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \circ \pi_2 \circ \dots \circ \pi_K & \\ \llbracket p \rrbracket a \triangleq \bigcup_{o \in I(p)} \llbracket \omega \rrbracket_I a & \\ \llbracket \epsilon \rrbracket_I a \triangleq \{ a \} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \uplus a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} & \\ \llbracket \pi_1 \circ \pi_2 \circ \dots \circ \pi_K \rrbracket_I a \triangleq (\llbracket \pi_1 \rrbracket_I \bullet \llbracket \pi_2 \rrbracket_I \circ \dots \circ \llbracket \pi_K \rrbracket_I) a & \end{array}$$

KA axioms

$$\begin{array}{llll} (p + q) + r \equiv p + (q + r) & \text{KA-PLUS-ASSOC} & p ; 1 \equiv p & \text{KA-SEQ-ONE} \\ p + q \equiv q + p & \text{KA-PLUS-COMM} & 1 ; p \equiv p & \text{KA-ONE-SEQ} \\ p + 0 \equiv p & \text{KA-PLUS-ZERO} & 0 ; p \equiv 0 & \text{KA-ZERO-SEQ} \\ p + p \equiv p & \text{KA-PLUS-IDEM} & p ; 0 \equiv 0 & \text{KA-SEQ-ZERO} \\ (p ; q) ; r \equiv p ; (q ; r) & \text{KA-SEQ-ASSOC} & 1 + p ; p^* \equiv p^* & \text{KA-UNROLL-L} \\ p ; (q + r) \equiv p ; q + p ; r & \text{KA-SEQ-DIST-L} & p ; r \leq r \Rightarrow p^* ; r \leq r & \text{KA-LFP-L} \\ (p + q) ; r \equiv p ; r + q ; r & \text{KA-SEQ-DIST-R} & 1 + p^* ; p \equiv p^* & \text{KA-UNROLL-R} \\ & & r ; p \leq r \Rightarrow r ; p^* \leq r & \text{KA-LFP-R} \end{array}$$

SKA axioms for ||

$$\begin{array}{llll} (p \parallel q) \parallel r \equiv p \parallel (q \parallel r) & \text{SKA-PRL-ASSOC} & p \parallel q \equiv q \parallel p & \text{SKA-PRL-COMM} \\ p \parallel (q + r) \equiv p \parallel q + p \parallel r & \text{SKA-PRL-DIST} & 1 \parallel p \equiv p & \text{SKA-ONE-PRL} \\ (x \parallel p) \parallel (y \parallel q) \equiv (x \parallel y) ; (p \parallel q) & \text{SKA-PRL-SEQ} & 0 \parallel p \equiv 0 & \text{SKA-ZERO-PRL} \end{array}$$

SKA axioms for ·

$$\begin{array}{llll} (p \cdot q) \cdot r \equiv p \cdot (q \cdot r) & \text{SKA-ORD-ASSOC} & 1 \cdot p \equiv p & \text{SKA-ONE-ORD} \\ p \cdot (q + r) \equiv p \cdot q + p \cdot r & \text{SKA-ORD-DIST-L} & p \cdot 1 \equiv p & \text{SKA-ORD-ONE} \\ (p + q) \cdot r \equiv p \cdot r + q \cdot r & \text{SKA-ORD-DIST-R} & 0 \cdot p \equiv 0 & \text{SKA-ZERO-ORD} \\ (x \cdot p) \cdot (y \cdot q) \equiv (x \cdot y) ; (p \cdot q) & \text{SKA-ORD-SEQ} & p \cdot 0 \equiv 0 & \text{SKA-ORD-ZERO} \end{array}$$

Boolean axioms (in addition to monotone axioms)

$$\begin{array}{llll} 1 \uplus b \equiv 1 & \text{BOOL-ONE-U} & (t \wedge t') \uplus b \equiv t \uplus b \wedge t' \uplus b & \text{BOOL-CONJ-U-DIST} \\ b \wedge (b \uplus b') \equiv b & \text{BOOL-CONJ-SUBSET} & (t \vee t') \uplus b \equiv t \uplus b \vee t' \uplus b & \text{BOOL-DISJ-U-DIST} \\ b \vee b' \equiv b \uplus b' & \text{BOOL-DISJ-U} & & \end{array}$$

Network axioms

$$\begin{array}{llll} [t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \uplus r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-ORD} \\ [t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-PRL} \end{array}$$

Single round axioms

$$\begin{array}{llll} \llbracket p \parallel p' \rrbracket \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q') & \text{SR-EXC} \\ \llbracket b \wedge t \rrbracket r \triangleright o \equiv \llbracket (r \cup b) \wedge t \rrbracket r \triangleright o & \text{SR-CAN} \\ \llbracket t \rrbracket r \triangleright o + \llbracket t' \rrbracket r \triangleright o \equiv \llbracket t \vee t' \rrbracket r \triangleright o & \text{SR-PLUS} \end{array}$$

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
BP pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$M(BP) \ni a, b, r, o ::= \{bp_1, \dots, bp_k\}$
Tests	$T \ni t, t' ::= \begin{cases} 1 & \text{no test} \\ & \\ b & \text{multiset absence} \\ & \\ t \wedge t' & \text{conjunction} \\ & \\ t \vee t' & \text{disjunction} \\ & \\ t \uplus b & \text{multiset union} \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & \\ 1 & \text{skip or no-round} \\ & \\ \pi & \text{atomic action} \\ & \\ r \triangleright o & \text{basic action} \\ & \\ [t]p & \text{guarded policy} \\ & \\ p + q & \text{nondeterministic choice} \\ & \\ p \cdot q & \text{ordered composition} \\ & \\ p \parallel q & \text{parallel composition} \\ & \\ p ; q & \text{sequential composition} \\ & \\ p^* & \text{Kleene star} \end{cases}$
Basic actions	$r \triangleright o ::= [1]r \triangleright o + [r]\emptyset \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$

Test semantics

$$\begin{array}{ll} \langle t \rangle \in M(BP) \rightarrow \{\top, \perp\} & \\ \langle 1 \rangle a \triangleq \top & \langle t \uplus b \rangle a \triangleq (\langle t \rangle a \wedge b \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a \triangleq b \not\subseteq a & \langle t \square t' \rangle a \triangleq \langle t \rangle a \Box \langle t' \rangle a, \text{ where } \Box \text{ is either } \wedge \text{ or } \vee \end{array}$$

Single round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP)) & \\ \langle \emptyset \rangle a \triangleq \emptyset & \\ \langle 1 \rangle a \triangleq \{\emptyset \triangleright a\} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{o \triangleright a \mid r\} & \text{if } r \subseteq a \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle p + q \rangle a \triangleq \langle p \rangle a \cup \langle q \rangle a & \\ \langle p \cdot q \rangle a \triangleq \langle (p) \cdot (q) \rangle a & \\ \langle p \parallel q \rangle a \triangleq \langle (p) \parallel (q) \rangle a & \end{array}$$

Multi-round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP)) & \\ \langle \omega \rangle_I \in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \circ \pi_2 \circ \dots \circ \pi_k & \\ \langle p \rangle a \triangleq \bigcup_{\omega \in I(p)} \langle \omega \rangle_I a & \\ \langle \epsilon \rangle_I a \triangleq \{a\} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{o \triangleright a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle \pi_1 \circ \pi_2 \circ \dots \circ \pi_k \rangle_I a \triangleq \langle \langle \pi_1 \rangle_I \bullet \langle \pi_2 \rangle_I \circ \dots \circ \langle \pi_k \rangle_I \rangle_I a & \end{array}$$

Basic actions

KA axioms

$$\begin{array}{llll} (p + q) + r \equiv p + (q + r) & \text{KA-PLUS-ASSOC} & p ; 1 \equiv p & \text{KA-SEQ-ONE} \\ p + q \equiv q + p & \text{KA-PLUS-COMM} & 1 ; p \equiv p & \text{KA-ONE-SEQ} \\ p + 0 \equiv p & \text{KA-PLUS-ZERO} & 0 ; p \equiv 0 & \text{KA-ZERO-SEQ} \\ p + p \equiv p & \text{KA-PLUS-IDEM} & p ; 0 \equiv 0 & \text{KA-SEQ-ZERO} \\ (p ; q) ; r \equiv p ; (q ; r) & \text{KA-SEQ-ASSOC} & 1 + p ; p^* \equiv p^* & \text{KA-UNROLL-L} \\ p ; (q + r) \equiv p ; q + p ; r & \text{KA-SEQ-DIST-L} & p ; r \leq r \Rightarrow p^* ; r \leq r & \text{KA-LFP-L} \\ (p + q) ; r \equiv p ; r + q ; r & \text{KA-SEQ-DIST-R} & 1 + p^* ; p \equiv p^* & \text{KA-UNROLL-R} \\ & & r ; p \leq r \Rightarrow r ; p^* \leq r & \text{KA-LFP-R} \end{array}$$

SKA axioms for \parallel

$$\begin{array}{llll} (p \parallel q) \parallel r \equiv p \parallel (q \parallel r) & \text{SKA-PRL-ASSOC} & p \parallel q \equiv q \parallel p & \text{SKA-PRL-COMM} \\ p \parallel (q + r) \equiv p \parallel q + p \parallel r & \text{SKA-PRL-DIST} & 1 \parallel p \equiv p & \text{SKA-ONE-PRL} \\ (x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q) & \text{SKA-PRL-SEQ} & 0 \parallel p \equiv 0 & \text{SKA-ZERO-PRL} \end{array}$$

SKA axioms for \cdot

$$\begin{array}{llll} (p \cdot q) \cdot r \equiv p \cdot (q \cdot r) & \text{SKA-ORD-ASSOC} & 1 \cdot p \equiv p & \text{SKA-ONE-ORD} \\ p \cdot (q + r) \equiv p \cdot q + p \cdot r & \text{SKA-ORD-DIST-L} & p \cdot 1 \equiv p & \text{SKA-ORD-ONE} \\ (p + q) \cdot r \equiv p \cdot r + q \cdot r & \text{SKA-ORD-DIST-R} & 0 \cdot p \equiv 0 & \text{SKA-ZERO-ORD} \\ (x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q) & \text{SKA-ORD-SEQ} & p \cdot 0 \equiv 0 & \text{SKA-ORD-ZERO} \end{array}$$

Boolean axioms (in addition to monotone axioms)

$$1 \uplus b \equiv 1 \quad \text{BOOL-ONE-U} \quad (x \wedge x') \uplus b = x \uplus b, x \cdot x' \uplus b = \text{BOOL-COMM-U-DIST}$$

$$\begin{array}{lll} [t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \wedge r)]\hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-ORD} \\ [t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-PRL} \end{array}$$

Single round axioms

$$\begin{array}{lll} [1]0 \triangleright \emptyset \equiv 1 & \text{SR-ONE} & (p \parallel p') \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q') \\ [\emptyset]r \triangleright o \equiv 0 & \text{SR-ZERO} & [b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o \\ [t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o & & [t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o \end{array}$$

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$M(BP) \ni a, b, r, o ::= \{bp_1, \dots, bp_k\}$
Tests	$T \ni t, t' ::= \begin{cases} 1 & \text{no test} \\ b & \text{multiset absence} \\ t \wedge t' & \text{conjunction} \\ t \vee t' & \text{disjunction} \\ t \uplus b & \text{multiset union} \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \blacktriangleright o$
Policies	$p \ni p, q ::= \begin{cases} 0 & \text{abort} \\ 1 & \text{skip or no-round} \\ \pi & \text{atomic action} \\ r \blacktriangleright o & \text{basic action} \\ [t]p & \text{guarded policy} \\ p + q & \text{nondeterministic choice} \\ p \cdot q & \text{ordered composition} \\ p \parallel q & \text{parallel composition} \\ p ; q & \text{sequential composition} \\ p^* & \text{Kleene star} \end{cases}$
Basic actions	$r \blacktriangleright o ::= [\mathbb{1}]r \blacktriangleright o + [r]\emptyset \blacktriangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \blacktriangleright \emptyset \cdot p$

Test semantics

$$\begin{aligned} \langle t \rangle &\in M(BP) \rightarrow \{\top, \perp\} \\ \langle \mathbb{1} \rangle a &\triangleq \top & \langle t \uplus b \rangle a &\triangleq (\langle t \rangle a \wedge b \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a &\triangleq b \not\subseteq a & \langle t \square t' \rangle a &\triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ where } \square \text{ is either } \wedge \text{ or } \vee \end{aligned}$$

Single round semantics

$$\begin{aligned} \langle p \rangle &\in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP)) \\ \langle \emptyset \rangle a &\triangleq \emptyset \\ \langle 1 \rangle a &\triangleq \{\emptyset \blacktriangleright a\} \\ \langle [t]r \blacktriangleright o \rangle a &\triangleq \begin{cases} \{o \blacktriangleright a \mid r\} & \text{if } r \subseteq a \\ \emptyset & \text{otherwise} \end{cases} \\ \langle p + q \rangle a &\triangleq \langle p \rangle a \cup \langle q \rangle a \\ \langle p \cdot q \rangle a &\triangleq (\langle p \rangle \cdot \langle q \rangle) a \\ \langle p \parallel q \rangle a &\triangleq (\langle p \rangle \parallel \langle q \rangle) a \end{aligned}$$

Multi-round semantics

$$\begin{aligned} \llbracket p \rrbracket &\in M(BP) \rightarrow \mathcal{P}(M(BP)) \\ \llbracket \omega \rrbracket_I &\in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \ddot{\wedge} \pi_2 \ddot{\wedge} \dots \ddot{\wedge} \pi_k \\ \llbracket p \rrbracket a &\triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a \\ \llbracket \epsilon \rrbracket a &\triangleq \{a\} \\ \llbracket [t]r \blacktriangleright o \rrbracket a &\triangleq \begin{cases} \{o \blacktriangleright a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \\ \llbracket \pi_1 \ddot{\wedge} \pi_2 \ddot{\wedge} \dots \ddot{\wedge} \pi_k \rrbracket_I a &\triangleq (\llbracket \pi_1 \rrbracket_I \bullet \llbracket \pi_2 \rrbracket_I \ddot{\wedge} \dots \ddot{\wedge} \llbracket \pi_k \rrbracket_I) a \end{aligned}$$

KA axioms

$(p + q) + r \equiv p + (q + r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$	KA-SEQ-ONE
$p + q \equiv q + p$	KA-PLUS-COMM	$1 ; p \equiv p$	KA-ONE-SEQ
$p + 0 \equiv p$	KA-PLUS-ZERO	$0 ; p \equiv 0$	KA-ZERO-SEQ
$p + p \equiv p$	KA-PLUS-IDEM	$p ; 0 \equiv 0$	KA-SEQ-ZERO
$(p ; q) ; r \equiv p ; (q ; r)$	KA-SEQ-ASSOC	$1 + p ; p^* \equiv p^*$	KA-UNROLL-L
$p ; (q + r) \equiv p ; q + p ; r$	KA-SEQ-DIST-L	$p ; r \leq r \Rightarrow p^* ; r \leq r$	KA-LFP-L
$(p + q) ; r \equiv p ; r + q ; r$	KA-SEQ-DIST-R	$1 + p^* ; p \equiv p^*$	KA-UNROLL-R
$r ; p \leq r \Rightarrow r ; p^* \leq r$		$r ; p \leq r \Rightarrow r ; p^* \leq r$	KA-LFP-R

Atomic actions

$$\Pi \ni \pi, x, y ::= [t]r \blacktriangleright o$$

$$(x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q) \quad \text{SKA-PRL-SEQ} \quad 0 \parallel p \equiv 0 \quad \text{SKA-ZERO-PRL}$$

SKA axioms for \cdot

$(p \cdot q) \cdot r \equiv p \cdot (q \cdot r)$	SKA-ORD-ASSOC	$1 \cdot p \equiv p$	SKA-ONE-ORD
$p \cdot (q + r) \equiv p \cdot q + p \cdot r$	SKA-ORD-DIST-L	$p \cdot 1 \equiv p$	SKA-ORD-ONE
$(p + q) \cdot r \equiv p \cdot r + q \cdot r$	SKA-ORD-DIST-R	$0 \cdot p \equiv 0$	SKA-ZERO-ORD
$(x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q)$	SKA-ORD-SEQ	$p \cdot 0 \equiv 0$	SKA-ORD-ZERO

Boolean axioms (in addition to monotone axioms)

$$1 \uplus b \equiv 1 \quad \text{BOOL-ONE-U} \quad (x \wedge x') \text{ if } b = x \wedge b \wedge x' \text{ if } b, \text{ BOOL-COME-U-Drop}$$

Basic actions

$$r \blacktriangleright o ::= [\mathbb{1}]r \blacktriangleright o + [r]\emptyset \blacktriangleright \emptyset$$

IN/OUT AXIOMS

$$\begin{aligned} [t]r \blacktriangleright o \cdot [t']r' \blacktriangleright o' &\equiv [t \wedge (t' \wedge r)]\hat{r} \blacktriangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-ORD} \\ [t]r \blacktriangleright o \parallel [t']r' \blacktriangleright o' &\equiv [(t \uplus r') \wedge (t' \uplus r)]\hat{r} \blacktriangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-PRL} \end{aligned}$$

Single round axioms

$[1]r \blacktriangleright \emptyset \parallel \emptyset \equiv 1$	SR-ONE	$(p \parallel p') \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q')$	SR-EXC
$[\emptyset]r \blacktriangleright o \equiv 0$	SR-ZERO	$[b \wedge t]r \blacktriangleright o \equiv [(r \cup b) \wedge t]r \blacktriangleright o$	SR-CAN
$[t]r \blacktriangleright o + [t']r \blacktriangleright o \equiv [t \vee t']r \blacktriangleright o$		$[t]r \blacktriangleright o + [t']r \blacktriangleright o \equiv [t \vee t']r \blacktriangleright o$	SR-PLUS

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$M(BP) \ni a, b, r, o ::= \{\!\{bp_1, \dots, bp_k\}\!\}$
Tests	$T \ni t, t' ::= \begin{cases} 1 & \text{no test} \\ b & \text{multiset absence} \\ t \wedge t' & \text{conjunction} \\ t \vee t' & \text{disjunction} \\ t \uplus b & \text{multiset union} \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \blacktriangleright o$
Policies	$p \ni p, q ::= \begin{cases} 0 & \text{abort} \\ 1 & \text{skip or no-round} \\ \pi & \text{atomic action} \\ r \blacktriangleright o & \text{basic action} \\ [t]p & \text{guarded policy} \\ p + q & \text{nondeterministic choice} \\ p \cdot q & \text{ordered composition} \\ p \parallel q & \text{parallel composition} \\ p ; q & \text{sequential composition} \\ p^* & \text{Kleene star} \end{cases}$
Basic actions	$r \blacktriangleright o ::= [1]r \blacktriangleright o + [r]\emptyset \blacktriangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \blacktriangleright \emptyset \cdot p$

Test semantics

$$\begin{aligned} \langle t \rangle &\in M(BP) \rightarrow \{\top, \perp\} \\ \langle \emptyset \rangle a &\triangleq \top & \langle t \uplus b \rangle a &\triangleq (\langle t \rangle a \wedge b \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a &\triangleq b \not\subseteq a & \langle t \square t' \rangle a &\triangleq \langle t \rangle a \Box \langle t' \rangle a, \text{ where } \Box \text{ is either } \wedge \text{ or } \vee \end{aligned}$$

Single round semantics

$$\begin{aligned} \langle p \rangle &\in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP)) \\ \langle \emptyset \rangle a &\triangleq \emptyset \\ \langle 1 \rangle a &\triangleq \{\emptyset \blacktriangleright a\} \\ \langle [t]r \blacktriangleright o \rangle a &\triangleq \begin{cases} \{o \blacktriangleright a \mid r\} & \text{if } r \subseteq a \\ \emptyset & \text{otherwise} \end{cases} \\ \langle p + q \rangle a &\triangleq \langle p \rangle a \uplus \langle q \rangle a \\ \langle p \cdot q \rangle a &\triangleq \langle (p) \cdot (q) \rangle a \\ \langle p \parallel q \rangle a &\triangleq \langle (p) \parallel (q) \rangle a \end{aligned}$$

Multi-round semantics

$$\begin{aligned} \llbracket p \rrbracket &\in M(BP) \rightarrow \mathcal{P}(M(BP)) \\ \llbracket \omega \rrbracket_I &\in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \circ \pi_2 \circ \dots \circ \pi_k \\ \llbracket p \rrbracket a &\triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a \\ \llbracket \epsilon \rrbracket a &\triangleq \{a\} \\ \llbracket [t]r \blacktriangleright o \rrbracket a &\triangleq \begin{cases} \{o \uplus a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \\ \llbracket \pi_1 \circ \pi_2 \circ \dots \circ \pi_k \rrbracket_I a &\triangleq (\llbracket \pi_1 \rrbracket_I \bullet \llbracket \pi_2 \rrbracket_I \circ \dots \circ \llbracket \pi_k \rrbracket_I) a \end{aligned}$$

KA axioms

$(p + q) + r \equiv p + (q + r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$	KA-SEQ-ONE
$p + q \equiv q + p$	KA-PLUS-COMM	$1 ; p \equiv p$	KA-ONE-SEQ
$p + 0 \equiv p$	KA-PLUS-ZERO	$0 ; p \equiv 0$	KA-ZERO-SEQ
$p + p \equiv p$	KA-PLUS-IDEM	$p ; 0 \equiv 0$	KA-SEQ-ZERO
$(p + q) \cdot r \equiv p ; (q ; r)$	KA-SEQ-ASSOC	$1 + p ; p^* \equiv p^*$	KA-UNROLL-L
$p ;$	Tests	$T \ni t, t'$	A-LFP-L
$(p ;$		$r ; p \Rightarrow r \Rightarrow r ; p^* \leq r$	INROLL-R
			KA-LFP-R

Atomic actions

$$\Pi \ni \pi, x, y ::= [t]r \blacktriangleright o$$

$$(x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q) \quad \text{SKA-PRL-SEQ} \quad 0 \parallel p \equiv 0 \quad \text{SKA-ZERO-PRL}$$

SKA axioms for \cdot

$(p \cdot q) \cdot r \equiv p \cdot (q \cdot r)$	SKA-ORD-ASSOC	$1 \cdot p \equiv p$	SKA-ONE-ORD
$p \cdot (q + r) \equiv p \cdot q + p \cdot r$	SKA-ORD-DIST-L	$p \cdot 1 \equiv p$	SKA-ORD-ONE
$(p + q) \cdot r \equiv p \cdot r + q \cdot r$	SKA-ORD-DIST-R	$0 \cdot p \equiv 0$	SKA-ZERO-ORD
$(x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q)$	SKA-ORD-SEQ	$p \cdot 0 \equiv 0$	SKA-ORD-ZERO

Boolean axioms (in addition to monotone axioms)

$$1 \uplus b \equiv 1 \quad \text{BOOL-ONE-U}$$

$(t \wedge t') \uplus b = t \uplus b \wedge t' \uplus b$, BOOL-CONST-U-DIST

Basic actions

$$r \blacktriangleright o ::= [1]r \blacktriangleright o + [r]\emptyset \blacktriangleright \emptyset$$

INCL/WORK AXIOMS

$$\begin{aligned} [t]r \blacktriangleright o \cdot [t']r' \blacktriangleright o' &\equiv [t \wedge (t' \wedge r)]\hat{r} \blacktriangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-ORD} \\ [t]r \blacktriangleright o \parallel [t']r' \blacktriangleright o' &\equiv [(t \uplus r') \wedge (t' \uplus r)]\hat{r} \blacktriangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-PRL} \end{aligned}$$

Single round axioms

$[1]r \blacktriangleright \emptyset \equiv 1$	SR-ONE	$(p \parallel p') \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q')$	SR-EXC
$[\emptyset]r \blacktriangleright o \equiv 0$	SR-ZERO	$[b \wedge t]r \blacktriangleright o \equiv [(r \cup b) \wedge t]r \blacktriangleright o$	SR-CAN
$[t]r \blacktriangleright o + [t']r \blacktriangleright o \equiv [t \vee t']r \blacktriangleright o$		$[t \vee t']r \blacktriangleright o + [t' \vee t]r \blacktriangleright o \equiv [t \vee t' \vee t' \vee t]r \blacktriangleright o$	SR-PLUS

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
BP pairs	$BP \ni bp ::= N \sim N$
Multisets	$M(BP) \ni a, b, r, o ::= \{bp_1, \dots, bp_k\}$
Tests	$T \ni t, t' ::= \begin{cases} \perp & \text{no test} \\ & \text{multiset absence} \\ & t \wedge t' \\ & t \vee t' \\ & t \uplus b \\ & t \uplus b \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$p \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & 1 \\ & \pi \\ & r \triangleright o \\ & [t]p \\ & p + q \\ & p \cdot q \\ & p \parallel q \\ & p : q \\ & p^* \end{cases}$
Basic actions	$r \triangleright o ::= [\perp]r \triangleright o + [r]\emptyset \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$

Test semantics

$$\begin{array}{ll} \langle t \rangle \in M(BP) \rightarrow \{\top, \perp\} & \\ \langle \perp \rangle a \triangleq \top & \langle t \uplus b \rangle a \triangleq (\langle t \rangle a \setminus b \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a \triangleq b \not\subseteq a & \langle t \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ with } \square \text{ is either } \wedge \text{ or } \vee \end{array}$$

Single round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP)) & \\ \langle \perp \rangle a \triangleq \emptyset & \\ \langle 1 \rangle a \triangleq \{ \theta \mapsto a \} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \mapsto a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle p + q \rangle a \triangleq \langle (p) \rangle a \cup \langle (q) \rangle a & \\ \langle p \cdot q \rangle a \triangleq \langle (p) \cdot \langle (q) \rangle \rangle a & \\ \langle p \parallel q \rangle a \triangleq \langle (p) \parallel \langle (q) \rangle \rangle a & \end{array}$$

Multi-round semantics

$$\begin{array}{ll} \langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP)) & \\ \langle \omega \rangle_I \in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k & \\ \langle p \rangle a \triangleq \bigcup_{\omega \in I(p)} \langle \omega \rangle_I a & \\ \langle e \rangle a \triangleq \{ a \} & \\ \langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \uplus a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} & \\ \langle \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \rangle a \triangleq \langle \langle \pi_1 \rangle \bullet \langle \pi_2 \rangle \bullet \dots \bullet \langle \pi_k \rangle \rangle a & \end{array}$$

KA axioms

$$\begin{array}{llll} (p + q) + r \equiv p + (q + r) & \text{KA-PLUS-ASSOC} & p ; 1 \equiv p & \text{KA-SEQ-ONE} \\ p + q \equiv q + p & \text{KA-PLUS-COMM} & 1 ; p \equiv p & \text{KA-ONE-SEQ} \\ p + 0 \equiv p & \text{KA-PLUS-ZERO} & 0 ; p \equiv 0 & \text{KA-ZERO-SEQ} \\ p + p \equiv p & \text{KA-PLUS-IDEM} & p ; 0 \equiv 0 & \text{KA-SEQ-ZERO} \\ (p ; q) ; r \equiv p ; (q ; r) & \text{KA-SEQ-ASSOC} & 1 + p ; p^* \equiv p^* & \text{KA-UNROLL-L} \\ p ; (q + r) \equiv p ; q + p ; r & \text{KA-SEQ-DIST-L} & p ; r \leq r \Rightarrow p^* ; r \leq r & \text{KA-LFP-L} \\ (p + q) ; r \equiv p ; r + q ; r & \text{KA-SEQ-DIST-R} & 1 + p^* ; p \equiv p^* & \text{KA-UNROLL-R} \\ & & r ; p \leq r \Rightarrow r ; p^* \leq r & \text{KA-LFP-R} \end{array}$$

SKA axioms for ||

$$\begin{array}{llll} (p \parallel q) \parallel r \equiv p \parallel (q \parallel r) & \text{SKA-PRL-ASSOC} & p \parallel q \equiv q \parallel p & \text{SKA-PRL-COMM} \\ p \parallel (q + r) \equiv p \parallel q + p \parallel r & \text{SKA-PRL-DIST} & 1 \parallel p \equiv p & \text{SKA-ONE-PRL} \\ (x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q) & \text{SKA-PRL-SEQ} & 0 \parallel p \equiv 0 & \text{SKA-ZERO-PRL} \end{array}$$

SKA axioms for ·

$$\begin{array}{llll} (p \cdot q) \cdot r \equiv p \cdot (q \cdot r) & \text{SKA-ORD-ASSOC} & 1 \cdot p \equiv p & \text{SKA-ONE-ORD} \\ p \cdot (q + r) \equiv p \cdot q + p \cdot r & \text{SKA-ORD-DIST-L} & p \cdot 1 \equiv p & \text{SKA-ORD-ONE} \\ (p + q) \cdot r \equiv p \cdot r + q \cdot r & \text{SKA-ORD-DIST-R} & 0 \cdot p \equiv 0 & \text{SKA-ZERO-ORD} \\ (x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q) & \text{SKA-ORD-SEQ} & p \cdot 0 \equiv 0 & \text{SKA-ORD-ZERO} \end{array}$$

Boolean axioms (in addition to monotone axioms)

$$\begin{array}{llll} 1 \wedge b \equiv 1 & \text{BOOL-ONE-U} & (t \wedge t') \wedge b \equiv t \wedge b \wedge t' \wedge b & \text{BOOL-CONJ-U-DIST} \\ b \wedge (b \wedge b') \equiv b & \text{BOOL-CONJ-SUBSET} & (t \vee t') \wedge b \equiv t \vee b \vee t' \vee b & \text{BOOL-DISJ-U-DIST} \\ b \vee b' \equiv b \cup b' & \text{BOOL-DISJ-U} & & \end{array}$$

Network axioms

$$\begin{array}{llll} [t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \wedge r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-ORD} \\ [t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' & \text{NET-PRL} \end{array}$$

Single round axioms

$$\begin{array}{llll} \langle p \parallel p' \rangle \cdot \langle q \parallel q' \rangle \leq (p \cdot q) \parallel (p' \cdot q') & \text{SR-EXC} \\ \langle \perp \rangle r \triangleright \emptyset \equiv 1 & \text{SR-ONE} & [b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o & \text{SR-CAN} \\ \langle \emptyset \rangle r \triangleright o \equiv 0 & \text{SR-ZERO} & [t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o & \text{SR-PLUS} \end{array}$$

Syntax

Nodes	$N ::= A, B, C, \dots$			
Bell pairs	$BP \ni bp ::= N\text{-}N$			
Multisets	$\mathcal{M}(BP) \ni a, b, r, o ::= \{\!\{bp_1, \dots, bp_k\}\!\}$			
Tests	$T \ni t, t' ::= \begin{cases} 1 & \text{no test} \\ b & \text{multiset absence} \\ t \wedge t' & \text{conjunction} \end{cases}$			
		KA axioms		
		$(p + q) + r \equiv p + (q + r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$
		$p + q \equiv q + p$	KA-PLUS-COMM	$1 ; p \equiv p$
				KA-SEQ-ONE
				KA-ONE-SEQ

Multi-round semantics

$$\begin{aligned} \llbracket p \rrbracket &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP)) \\ \llbracket \omega \rrbracket_I &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP)), \text{ where } \omega = \pi_1 ; \pi_2 ; \dots ; \pi_k \\ \llbracket p \rrbracket_I a &\triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a \\ \llbracket \epsilon \rrbracket_I a &\triangleq \{a\} \\ \llbracket [t]r \triangleright o \rrbracket_I a &\triangleq \begin{cases} \{o \uplus a \setminus r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{array}{ll} \langle t \rangle a = \top & \langle t \rangle a \wedge b \subseteq a \\ \langle b \rangle a \triangleq b \not\subseteq a & \langle t \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ with } \square \text{ is either } \wedge \text{ or } \vee \end{array}$$

Single round semantics

$$\begin{aligned} \langle p \rangle &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP) \times \mathcal{M}(BP)) \\ \langle \emptyset \rangle a &\triangleq \emptyset \\ \langle 1 \rangle a &\triangleq \{\emptyset \bowtie a\} \\ \langle [t]r \triangleright o \rangle a &\triangleq \begin{cases} \{o \bowtie a \setminus r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \\ \langle p + q \rangle a &\triangleq \langle p \rangle a \cup \langle q \rangle a \\ \langle p \cdot q \rangle a &\triangleq (\langle p \rangle \cdot \langle q \rangle) a \\ \langle p \parallel q \rangle a &\triangleq (\langle p \rangle \parallel \langle q \rangle) a \end{aligned}$$

Multi-round semantics

$$\begin{aligned} \llbracket p \rrbracket &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP)) \\ \llbracket \omega \rrbracket_I &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP)), \text{ where } \omega = \pi_1 ; \pi_2 ; \dots ; \pi_k \\ \llbracket p \rrbracket_I a &\triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a \\ \llbracket \epsilon \rrbracket_I a &\triangleq \{a\} \\ \llbracket [t]r \triangleright o \rrbracket_I a &\triangleq \begin{cases} \{o \uplus a \setminus r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \\ \llbracket \pi_1 ; \pi_2 ; \dots ; \pi_k \rrbracket_I a &\triangleq (\llbracket \pi_1 \rrbracket_I \bullet \llbracket \pi_2 \rrbracket_I ; \dots ; \llbracket \pi_k \rrbracket_I) a \end{aligned}$$

$$(x : p) \cdot (y : q) \equiv (x \cdot y) ; (p \cdot q) \quad \text{SKA-ORD-SEQ} \quad p \cdot 0 \equiv 0 \quad \text{SKA-ORD-ZERO}$$

Boolean axioms (in addition to monotone axioms)

$$\begin{array}{ll} 1 \uplus b \equiv 1 & \text{BOOL-ONE-U} \\ b \wedge (b \uplus b') \equiv b & \text{BOOL-CONJ-SUBSET} \quad (t \wedge t') \uplus b \equiv t \uplus b \wedge t' \uplus b \quad \text{BOOL-CONJ-U-DIST} \\ b \vee b' \equiv b \cup b' & \text{BOOL-DISJ-U} \quad (t \vee t') \uplus b \equiv t \uplus b \vee t' \uplus b \quad \text{BOOL-DISJ-U-DIST} \end{array}$$

Network axioms

$$\begin{array}{ll} [t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \uplus r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' \quad \text{NET-ORD} \\ [t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \uplus r' \text{ and } \hat{o} = o \uplus o' \quad \text{NET-PRL} \end{array}$$

Single round axioms

$$\begin{array}{ll} [\mathbb{1}] \parallel p' \triangleright \emptyset \equiv 1 & \text{SR-ONE} \quad (p \parallel p') \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q') \quad \text{SR-EXC} \\ [\emptyset]r \triangleright o \equiv 0 & \text{SR-ZERO} \quad [b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o \quad \text{SR-CAN} \\ [t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o & \text{SR-PLUS} \end{array}$$

BellKAT at a glance



Syntax

Nodes

 $N ::= A, B, C, \dots$

BP pairs

 $BP \ni bp ::= N \sim N$

Multisets

 $M(BP) \ni a, b \in \omega \text{ s.t. } \llbracket a \rrbracket_B = \llbracket b \rrbracket_B$

Tests

 $M(BP) \ni a, b \in \omega \text{ s.t. } \llbracket a \rrbracket_B = \llbracket b \rrbracket_B$

Atomic action
Policies

Single round semantics

Basic actions
Guarded policy

 $r \triangleright o ::= \llbracket 1 \rrbracket r \triangleright o + [r] \emptyset \triangleright \emptyset$
 $[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$

$$\begin{aligned} \langle p \rangle &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP) \times \mathcal{M}(BP)) \\ \langle \emptyset \rangle a &\triangleq \emptyset \\ \langle 1 \rangle a &\triangleq \{\emptyset \bowtie a\} \\ \langle [t]r \triangleright o \rangle a &\triangleq \begin{cases} \{o \bowtie a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \end{aligned}$$

KA axioms

Test semantics

$$\begin{aligned} \langle t \rangle &\in \mathcal{M}(BP) \rightarrow \{\top, \perp\} \\ \langle 1 \rangle a &\triangleq \top & \langle t \bowtie b \rangle a &\triangleq (\langle t \rangle a \setminus b \wedge b \subseteq a) \vee \langle b \rangle a \\ \langle b \rangle a &\triangleq b \not\subseteq a & \langle t \square t' \rangle a &\triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ with } \square \text{ is either } \wedge \text{ or } \vee \end{aligned}$$

Single round semantics

$$\begin{aligned} \langle p \rangle &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP) \times \mathcal{M}(BP)) \\ \langle \emptyset \rangle a &\triangleq \emptyset \\ \langle 1 \rangle a &\triangleq \{\emptyset \bowtie a\} \\ \langle [t]r \triangleright o \rangle a &\triangleq \begin{cases} \{o \bowtie a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \\ \langle p + q \rangle a &\triangleq \langle p \rangle a \cup \langle q \rangle a \\ \langle p \cdot q \rangle a &\triangleq (\langle p \rangle \cdot \langle q \rangle) a \\ \langle p \parallel q \rangle a &\triangleq (\langle p \rangle \parallel \langle q \rangle) a \end{aligned}$$

Multi-round semantics

$$\begin{aligned} \llbracket p \rrbracket &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP)) \\ \llbracket p \rrbracket_I &\in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP)), \text{ where } \omega = \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \\ \llbracket p \rrbracket a &\triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a \\ \llbracket e \rrbracket a &\triangleq \{a\} \\ \llbracket [t]r \triangleright o \rrbracket a &\triangleq \begin{cases} \{o \bowtie a \mid r\} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases} \\ \llbracket \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \rrbracket a &\triangleq (\llbracket \pi_1 \rrbracket \bullet (\llbracket \pi_2 \rrbracket \bullet (\dots \bullet \llbracket \pi_k \rrbracket))) a \end{aligned}$$

SKA axioms for \cdot

$$\begin{array}{lll} (p \cdot q) \cdot r \equiv p \cdot (q \cdot r) & \text{SKA-ORD-ASSOC} & 1 \cdot p \equiv p & \text{SKA-ONE-ORD} \\ p \cdot (q + r) \equiv p \cdot q + p \cdot r & \text{SKA-ORD-DIST-L} & p \cdot 1 \equiv p & \text{SKA-ORD-ONE} \\ (p + q) \cdot r \equiv p \cdot r + q \cdot r & \text{SKA-ORD-DIST-R} & 0 \cdot p \equiv 0 & \text{SKA-ZERO-ORD} \\ (x \cdot p) \cdot (y \cdot q) \equiv (x \cdot y) \cdot (p \cdot q) & \text{SKA-ORD-SIQ} & p \cdot 0 \equiv 0 & \text{SKA-ORD-ZERO} \end{array}$$

Boolean axioms (in addition to monotone axioms)

$$\begin{array}{lll} 1 \wedge b \equiv 1 & \text{BOOL-ONE-U} & (t \wedge t') \vee b \equiv t \vee b \wedge t' \vee b & \text{BOOL-CONJ-U-DIST} \\ b \wedge (b \wedge b') \equiv b & \text{BOOL-CONJ-SUBSET} & (t \vee t') \wedge b \equiv t \wedge b \vee t' \wedge b & \text{BOOL-DISJ-U-DIST} \\ b \vee b' \equiv b \cup b' & \text{BOOL-DISJ-U} & \end{array}$$

Network axioms

$$\begin{array}{lll} [t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \vee r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \wedge r' \text{ and } \hat{o} = o \vee o' & \text{NET-ORD} \\ [t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \vee r') \wedge (t' \vee r)] \hat{r} \triangleright \hat{o} & \text{if } \hat{r} = r \vee r' \text{ and } \hat{o} = o \vee o' & \text{NET-PRL} \end{array}$$

Single round axioms

$$\begin{array}{lll} \llbracket p \parallel p' \rrbracket \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q') & \text{SR-EXC} & \\ \llbracket 1 \rrbracket \emptyset \triangleright \emptyset \equiv 1 & \text{SR-ONE} & \llbracket b \wedge t \rrbracket r \triangleright o \equiv \llbracket (r \cup b) \wedge t \rrbracket r \triangleright o & \text{SR-CAN} \\ \llbracket \emptyset \rrbracket r \triangleright o \equiv 0 & \text{SR-ZERO} & \llbracket t \rrbracket r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o & \text{SR-PLUS} \end{array}$$

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$M(BP) \ni a, b, r, o ::= \{\{bp_1, \dots, bp_k\}\}$
Tests	$T \ni t, t' ::= \begin{cases} \perp & \text{no test} \\ & b \\ & t \wedge t' \\ & t \vee t' \\ & t \uplus b \end{cases} \begin{array}{l} \text{multiset absence} \\ \text{conjunction} \\ \text{disjunction} \\ \text{multiset union} \end{array}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & 1 \\ & \pi \\ & r \triangleright o \\ & [t]p \\ & p + q \\ & p \cdot q \\ & p \parallel q \\ & p ; q \\ & p^* \end{cases} \begin{array}{l} \text{skip or no-round} \\ \text{atomic action} \\ \text{basic action} \\ \text{guarded policy} \\ \text{nondeterministic choice} \\ \text{ordered composition} \\ \text{parallel composition} \\ \text{sequential composition} \\ \text{Kleene star} \end{array}$
Basic actions	$r \triangleright o ::= [\perp]r \triangleright o + [r]\emptyset \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$
Test semantics	$\langle t \rangle \in M(BP) \rightarrow \{\top, \perp\}$
	$\langle \perp \rangle a \triangleq \top$
	$\langle t \wedge b \rangle a \triangleq (\langle t \rangle a \wedge b \subseteq a) \vee \langle b \rangle a$
	$\langle \perp \wedge b \rangle a \triangleq b \not\subseteq a$
	$\langle t \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ where } \square \text{ is either } \wedge \text{ or } \vee$
Single round semantics	$\langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP))$
	$\langle \emptyset \rangle a \triangleq \emptyset$
	$\langle 1 \rangle a \triangleq \{ \emptyset \Rightarrow a \}$
	$\langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \Rightarrow a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases}$
	$\langle p + q \rangle a \triangleq \langle p \rangle a \cup \langle q \rangle a$
	$\langle p \cdot q \rangle a \triangleq (\langle p \rangle \cdot \langle q \rangle) a$
	$\langle p \parallel q \rangle a \triangleq (\langle p \rangle \parallel \langle q \rangle) a$
Multi-round semantics	$\llbracket p \rrbracket \in M(BP) \rightarrow \mathcal{P}(M(BP))$
	$\llbracket \omega \rrbracket_I \in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k$
	$\llbracket p \rrbracket a \triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a$
	$\llbracket e \rrbracket a \triangleq \{ a \}$
	$\llbracket [t]r \triangleright o \rrbracket a \triangleq \begin{cases} \{ o \triangleright a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases}$
	$\llbracket \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \rrbracket a \triangleq (\llbracket \pi_1 \rrbracket a \bullet \llbracket \pi_2 \rrbracket a \bullet \dots \bullet \llbracket \pi_k \rrbracket a)$

KA axioms

$(p + q) + r \equiv p + (q + r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$	KA-SEQ-ONE
$p + q \equiv q + p$	KA-PLUS-COMM	$1 ; p \equiv p$	KA-ONE-SEQ
$p + 0 \equiv p$	KA-PLUS-ZERO	$0 ; p \equiv 0$	KA-ZERO-SEQ
$p + p \equiv p$	KA-PLUS-IDEM	$p ; 0 \equiv 0$	KA-SEQ-ZERO
$(p ; q) ; r \equiv p ; (q ; r)$	KA-SEQ-ASSOC	$1 + p ; p^* \equiv p^*$	KA-UNROLL-L
$p ; (q + r) \equiv p ; q + p ; r$	KA-SEQ-DIST-L	$p ; r \leq r \Rightarrow p^* ; r \leq r$	KA-LFP-L
$(p + q) ; r \equiv p ; r + q ; r$	KA-SEQ-DIST-R	$1 + p^* ; p \equiv p^*$	KA-UNROLL-R
		$r ; p \leq r \Rightarrow r ; p^* \leq r$	KA-LFP-R

SKA axioms for \parallel

$(p \parallel q) \parallel r \equiv p \parallel (q \parallel r)$	SKA-PRL-ASSOC	$p \parallel q \equiv q \parallel p$	SKA-PRL-COMM
$p \parallel (q + r) \equiv p \parallel q + p \parallel r$	SKA-PRL-DIST	$1 \parallel p \equiv p$	SKA-ONE-PRL
$(x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q)$	SKA-PRL-SEQ	$0 \parallel p \equiv 0$	SKA-ZERO-PRL

SKA axioms for \cdot

$(p \cdot q) \cdot r \equiv p \cdot (q \cdot r)$	SKA-ORD-ASSOC	$1 \cdot p \equiv p$	SKA-ONE-ORD
$p \cdot (q + r) \equiv p \cdot q + p \cdot r$	SKA-ORD-DIST-L	$p \cdot 1 \equiv p$	SKA-ORD-ONE
$(p + q) \cdot r \equiv p \cdot r + q \cdot r$	SKA-ORD-DIST-R	$0 \cdot p \equiv 0$	SKA-ZERO-ORD
$(x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q)$	SKA-ORD-SEQ	$p \cdot 0 \equiv 0$	SKA-ORD-ZERO

Boolean axioms (in addition to monotone axioms)

$1 \sqcup b \equiv 1$	BOOL-ONE-U	$(t \wedge t') \sqcup b \equiv t \sqcup b \wedge t' \sqcup b$	BOOL-CONJ-U-DIST
$b \wedge (b \sqcup b') \equiv b$	BOOL-CONJ-SUBSET	$(t \vee t') \sqcup b \equiv t \vee b \wedge t' \vee b$	BOOL-DISJ-U-DIST
$b \vee b' \equiv b \cup b'$	BOOL-DISJ-U		

Network axioms

$[t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \sqcup r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \sqcup r'$ and $\hat{o} = o \sqcup o'$	NET-ORD
$[t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \sqcup r') \wedge (t' \sqcup r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \sqcup r'$ and $\hat{o} = o \sqcup o'$	NET-PRL

Single round axioms

$(p \parallel p') \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q')$		SR-EXC	
$[1] \emptyset \triangleright \emptyset \equiv 1$	SR-ONE	$[b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o$	SR-CAN
$[0]r \triangleright o \equiv 0$	SR-ZERO	$[t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o$	SR-PLUS

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$M(BP) \ni a, b, r, o ::= \{\!\{bp_1, \dots, bp_k\}\!\}$
Tests	$T \ni t, t' ::= \begin{cases} \perp & \text{no test} \\ & \text{multiset absence} \\ & t \wedge t' \\ & t \vee t' \\ & t \uplus b \\ & t \uplus b \end{cases}$ <i>conjunction</i> <i>disjunction</i> <i>multiset union</i>
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & 1 \\ & \pi \\ & r \triangleright o \\ & [t]p \\ & p + q \\ & p \cdot q \\ & p \parallel q \\ & p ; q \\ & p^* \end{cases}$ <i>skip or no-round</i> <i>atomic action</i> <i>basic action</i> <i>guarded policy</i> <i>nondeterministic choice</i> <i>ordered composition</i> <i>parallel composition</i> <i>sequential composition</i> <i>Kleene star</i>
Basic actions	$r \triangleright o ::= [\perp]r \triangleright o + [r]\emptyset \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$
Test semantics	$\langle t \rangle \in M(BP) \rightarrow \{\top, \perp\}$
	$\langle \perp \rangle a \triangleq \top$
	$\langle t \uplus b \rangle a \triangleq (\langle t \rangle a \setminus b \wedge b \subseteq a) \vee \langle b \rangle a$
	$\langle \perp \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a$, where \square is either \wedge or \vee
Single round semantics	$\langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP))$
	$\langle \emptyset \rangle a \triangleq \emptyset$
	$\langle 1 \rangle a \triangleq \{ \emptyset \Rightarrow a \}$
	$\langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \Rightarrow a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases}$
	$\langle p + q \rangle a \triangleq \langle p \rangle a \cup \langle q \rangle a$
	$\langle p \cdot q \rangle a \triangleq (\langle p \rangle \cdot \langle q \rangle) a$
	$\langle p \parallel q \rangle a \triangleq (\langle p \rangle \parallel \langle q \rangle) a$
Multi-round semantics	$\llbracket p \rrbracket \in M(BP) \rightarrow \mathcal{P}(M(BP))$
	$\llbracket \omega \rrbracket_I \in M(BP) \rightarrow \mathcal{P}(M(BP))$, where $\omega = \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k$
	$\llbracket p \rrbracket a \triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a$
	$\llbracket e \rrbracket a \triangleq \{ a \}$
	$\llbracket [t]r \triangleright o \rrbracket a \triangleq \begin{cases} \{ o \uplus a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases}$
	$\llbracket \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \rrbracket a \triangleq (\llbracket \pi_1 \rrbracket a \bullet \llbracket \pi_2 \rrbracket a \bullet \dots \bullet \llbracket \pi_k \rrbracket a)$

KA axioms

$(p + q) + r \equiv p + (q + r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$	KA-SEQ-ONE
$p + q \equiv q + p$	KA-PLUS-COMM	$1 ; p \equiv p$	KA-ONE-SEQ
$p + 0 \equiv p$	KA-PLUS-ZERO	$0 ; p \equiv 0$	KA-ZERO-SEQ
$p + p \equiv p$	KA-PLUS-IDEM	$p ; 0 \equiv 0$	KA-SEQ-ZERO
$(p ; q) ; r \equiv p ; (q ; r)$	KA-SEQ-ASSOC	$1 + p ; p^* \equiv p^*$	KA-UNROLL-L
$p ; (q + r) \equiv p ; q + p ; r$	KA-SEQ-DIST-L	$p ; r \leq r \Rightarrow p^* ; r \leq r$	KA-LFP-L
$(p + q) ; r \equiv p ; r + q ; r$	KA-SEQ-DIST-R	$1 + p^* ; p \equiv p^*$	KA-UNROLL-R
		$r ; p \leq r \Rightarrow r ; p^* \leq r$	KA-LFP-R

SKA axioms for \parallel

$(p \parallel q) \parallel r \equiv p \parallel (q \parallel r)$	SKA-PRL-ASSOC	$p \parallel q \equiv q \parallel p$	SKA-PRL-COMM
$p \parallel (q + r) \equiv p \parallel q + p \parallel r$	SKA-PRL-DIST	$1 \parallel p \equiv p$	SKA-ONE-PRL
$(x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q)$	SKA-PRL-SEQ	$0 \parallel p \equiv 0$	SKA-ZERO-PRL

SKA axioms for \cdot

$(p \cdot q) \cdot r \equiv p \cdot (q \cdot r)$	SKA-ORD-ASSOC	$1 \cdot p \equiv p$	SKA-ONE-ORD
$p \cdot (q + r) \equiv p \cdot q + p \cdot r$	SKA-ORD-DIST-L	$p \cdot 1 \equiv p$	SKA-ORD-ONE
$(p + q) \cdot r \equiv p \cdot r + q \cdot r$	SKA-ORD-DIST-R	$0 \cdot p \equiv 0$	SKA-ZERO-ORD
$(x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q)$	SKA-ORD-SEQ	$p \cdot 0 \equiv 0$	SKA-ORD-ZERO

Boolean axioms (in addition to monotone axioms)

$1 \wedge b \equiv 1$	BOOL-ONE-U	$(t \wedge t') \uplus b \equiv t \uplus b \wedge t' \uplus b$	BOOL-CONJ-U-DIST
$b \wedge (b \uplus b') \equiv b$	BOOL-CONJ-SUBSET	$(t \vee t') \uplus b \equiv t \uplus b \vee t' \uplus b$	BOOL-DISJ-U-DIST
$b \vee b' \equiv b \uplus b'$	BOOL-DISJ-U		

Network axioms

$[t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \uplus r'$ and $\hat{o} = o \uplus o'$	NET-ORD
$[t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \uplus r'$ and $\hat{o} = o \uplus o'$	NET-PRL

Single round axioms

$\langle p \parallel p' \rangle \cdot \langle q \parallel q' \rangle \leq (p \cdot q) \parallel (p' \cdot q')$		SR-EXC	
$[1] \emptyset \triangleright \emptyset \equiv 1$	SR-ONE	$[b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o$	SR-CAN
$[0]r \triangleright o \equiv 0$	SR-ZERO	$[t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o$	SR-PLUS

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N \multimap N$
Multisets	$M(BP) \ni a, b, r, o ::= \{\{bp_1, \dots, bp_k\}\}$
Tests	$T \ni t, t' ::= \begin{cases} \perp & \text{no test} \\ & \text{multiset absence} \\ & t \wedge t' \\ & t \vee t' \\ & t \uplus b \\ & t \uplus b \end{cases}$ <i>conjunction</i> <i>disjunction</i> <i>multiset union</i>
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & 1 \\ & \pi \\ & r \triangleright o \\ & [t]p \\ & p + q \\ & p \cdot q \\ & p \parallel q \\ & p ; q \\ & p^* \end{cases}$ <i>skip or no-round</i> <i>atomic action</i> <i>basic action</i> <i>guarded policy</i> <i>nondeterministic choice</i> <i>ordered composition</i> <i>parallel composition</i> <i>sequential composition</i> <i>Kleene star</i>
Basic actions	$r \triangleright o ::= [\perp]r \triangleright o + [r]\emptyset \triangleright \emptyset$
Guarded policy	$[t]p ::= [t]\emptyset \triangleright \emptyset \cdot p$
Test semantics	$\langle t \rangle \in M(BP) \rightarrow \{\top, \perp\}$
	$\langle \perp \rangle a \triangleq \top$
	$\langle t \uplus b \rangle a \triangleq (\langle t \rangle a \setminus b \wedge b \subseteq a) \vee \langle b \rangle a$
	$\langle \emptyset \rangle a \triangleq b \not\subseteq a$
	$\langle t \square t' \rangle a \triangleq \langle t \rangle a \square \langle t' \rangle a, \text{ where } \square \text{ is either } \wedge \text{ or } \vee$
Single round semantics	$\langle p \rangle \in M(BP) \rightarrow \mathcal{P}(M(BP) \times M(BP))$
	$\langle \emptyset \rangle a \triangleq \emptyset$
	$\langle 1 \rangle a \triangleq \{ \emptyset \multimap a \}$
	$\langle [t]r \triangleright o \rangle a \triangleq \begin{cases} \{ o \multimap a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases}$
	$\langle p + q \rangle a \triangleq \langle p \rangle a \cup \langle q \rangle a$
	$\langle p \cdot q \rangle a \triangleq (\langle p \rangle \cdot \langle q \rangle) a$
	$\langle p \parallel q \rangle a \triangleq (\langle p \rangle \parallel \langle q \rangle) a$
Multi-round semantics	$\llbracket p \rrbracket \in M(BP) \rightarrow \mathcal{P}(M(BP))$
	$\llbracket \omega \rrbracket_I \in M(BP) \rightarrow \mathcal{P}(M(BP)), \text{ where } \omega = \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k$
	$\llbracket p \rrbracket a \triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a$
	$\llbracket e \rrbracket a \triangleq \{ a \}$
	$\llbracket [t]r \triangleright o \rrbracket a \triangleq \begin{cases} \{ o \uplus a \mid r \} & \text{if } r \subseteq a \text{ and } \langle t \rangle a = \top \\ \emptyset & \text{otherwise} \end{cases}$
	$\llbracket \pi_1 \ddagger \pi_2 \ddagger \dots \ddagger \pi_k \rrbracket a \triangleq (\llbracket \pi_1 \rrbracket a \bullet \llbracket \pi_2 \rrbracket a \bullet \dots \bullet \llbracket \pi_k \rrbracket a)$

KA axioms

$(p + q) + r \equiv p + (q + r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$	KA-SEQ-ONE
$p + q \equiv q + p$	KA-PLUS-COMM	$1 ; p \equiv p$	KA-ONE-SEQ
$p + 0 \equiv p$	KA-PLUS-ZERO	$0 ; p \equiv 0$	KA-ZERO-SEQ
$p + p \equiv p$	KA-PLUS-IDEM	$p ; 0 \equiv 0$	KA-SEQ-ZERO
$(p ; q) ; r \equiv p ; (q ; r)$	KA-SEQ-ASSOC	$1 + p ; p^* \equiv p^*$	KA-UNROLL-L
$p ; (q + r) \equiv p ; q + p ; r$	KA-SEQ-DIST-L	$p ; r \leq r \Rightarrow p^* ; r \leq r$	KA-LFP-L
$(p + q) ; r \equiv p ; r + q ; r$	KA-SEQ-DIST-R	$1 + p^* ; p \equiv p^*$	KA-UNROLL-R
		$r ; p \leq r \Rightarrow r ; p^* \leq r$	KA-LFP-R

SKA axioms for \parallel

$(p \parallel q) \parallel r \equiv p \parallel (q \parallel r)$	SKA-PRL-ASSOC	$p \parallel q \equiv q \parallel p$	SKA-PRL-COMM
$p \parallel (q + r) \equiv p \parallel q + p \parallel r$	SKA-PRL-DIST	$1 \parallel p \equiv p$	SKA-ONE-PRL
$(x ; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q)$	SKA-PRL-SEQ	$0 \parallel p \equiv 0$	SKA-ZERO-PRL

SKA axioms for \cdot

$(p \cdot q) \cdot r \equiv p \cdot (q \cdot r)$	SKA-ORD-ASSOC	$1 \cdot p \equiv p$	SKA-ONE-ORD
$p \cdot (q + r) \equiv p \cdot q + p \cdot r$	SKA-ORD-DIST-L	$p \cdot 1 \equiv p$	SKA-ORD-ONE
$(p + q) \cdot r \equiv p \cdot r + q \cdot r$	SKA-ORD-DIST-R	$0 \cdot p \equiv 0$	SKA-ZERO-ORD
$(x ; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q)$	SKA-ORD-SEQ	$p \cdot 0 \equiv 0$	SKA-ORD-ZERO

Boolean axioms (in addition to monotone axioms)

$1 \wedge b \equiv 1$	BOOL-ONE-U	$(t \wedge t') \uplus b \equiv t \uplus b \wedge t' \uplus b$	BOOL-CONJ-U-DIST
$b \wedge (b \uplus b') \equiv b$	BOOL-CONJ-SUBSET	$(t \vee t') \uplus b \equiv t \uplus b \vee t' \uplus b$	BOOL-DISJ-U-DIST
$b \vee b' \equiv b \uplus b'$	BOOL-DISJ-U		

Network axioms

$[t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \uplus r'$ and $\hat{o} = o \uplus o'$	NET-ORD
$[t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus t') \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \uplus r'$ and $\hat{o} = o \uplus o'$	NET-PRL

Single round axioms

$\langle p \parallel p' \rangle \cdot \langle q \parallel q' \rangle \leq (p \cdot q) \parallel (p' \cdot q')$		SR-EXC	
$[1] \emptyset \triangleright \emptyset \equiv 1$	SR-ONE	$[b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o$	SR-CAN
$[0]r \triangleright o \equiv 0$	SR-ZERO	$[t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o$	SR-PLUS

BellKAT at a glance



Syntax

Nodes	$N ::= A, B, C, \dots$
Bell pairs	$BP \ni bp ::= N\text{-}N$
Multisets	$\mathcal{M}(BP) \ni a, b, r, o ::= \{\!\{bp_1, \dots, bp_k\}\!}$
Tests	$T \ni t, t' ::= \begin{cases} \mathbb{1} & \text{no test} \\ & \\ & b & \text{multiset absence} \\ & t \wedge t' & \text{conjunction} \\ & t \vee t' & \text{disjunction} \\ & t \uplus b & \text{multiset union} \end{cases}$
Atomic actions	$\Pi \ni \pi, x, y ::= [t]r \triangleright o$
Policies	$P \ni p, q ::= \begin{cases} 0 & \text{abort} \\ & 1 & \text{skip or no-round} \\ & \pi & \text{atomic action} \\ & r \triangleright o & \text{basic action} \\ & [t]p & \text{guarded policy} \\ & p + q & \text{nondeterministic choice} \\ & p \cdot a & \text{ordered composition} \end{cases}$

KA axioms

$(p+q)+r \equiv p+(q+r)$	KA-PLUS-ASSOC	$p ; 1 \equiv p$	KA-SEQ-ONE
$p+q \equiv q+p$	KA-PLUS-COMM	$1 ; p \equiv p$	KA-ONE-SEQ
$p+0 \equiv p$	KA-PLUS-ZERO	$0 ; p \equiv 0$	KA-ZERO-SEQ
$p+p \equiv p$	KA-PLUS-IDEM	$p ; 0 \equiv 0$	KA-SEQ-ZERO
$(p ; q) ; r \equiv p ; (q ; r)$	KA-SEQ-ASSOC	$1 + p ; p^* \equiv p^*$	KA-UNROLL-L
$p ; (q+r) \equiv p ; q + p ; r$	KA-SEQ-DIST-L	$p ; r \leq r \Rightarrow p^* ; r \leq r$	KA-LFP-L
$(p+q) ; r \equiv p ; r + q ; r$	KA-SEQ-DIST-R	$1 + p^* ; p \equiv p^*$	KA-UNROLL-R
		$r ; p \leq r \Rightarrow r ; p^* \leq r$	KA-LFP-R

SKA axioms for \parallel

$(p \parallel q) \parallel r \equiv p \parallel (q \parallel r)$	SKA-PRL-ASSOC	$p \parallel q \equiv q \parallel p$	SKA-PRL-COMM
$p \parallel (q+r) \equiv p \parallel q + p \parallel r$	SKA-PRL-DIST	$1 \parallel p \equiv p$	SKA-ONE-PRL
$; p) \parallel (y ; q) \equiv (x \parallel y) ; (p \parallel q)$	SKA-PRL-SEQ	$0 \parallel p \equiv 0$	SKA-ZERO-PRL
axioms for \cdot			

$(p \cdot q) \cdot r \equiv p \cdot (q \cdot r)$	SKA-ORD-ASSOC	$1 \cdot p \equiv p$	SKA-ONE-ORD
$p \cdot (q+r) = p \cdot q + p \cdot r$	SKA-ORD-DIST-L	$p \cdot 1 \equiv p$	SKA-ORD-ONE
$(p+q) \cdot r \equiv p \cdot r + q \cdot r$	SKA-ORD-DIST-R	$0 \cdot p \equiv 0$	SKA-ZERO-ORD
$; p) \cdot (y ; q) \equiv (x \cdot y) ; (p \cdot q)$	SKA-ORD-SEQ	$p \cdot 0 \equiv 0$	SKA-ORD-ZERO

can axioms (in addition to monotone axioms)

$\mathbb{1} \uplus t' \equiv \mathbb{1}$	BOOL-ONE-U	$(t \wedge t') \uplus b \equiv t \uplus b \wedge t' \uplus b$	BOOL-CONJ-U-DIST
$b \wedge (b \uplus b') \equiv b$	BOOL-CONJ-SUBSET	$(t \vee t') \uplus b \equiv t \uplus b \vee t' \uplus b$	BOOL-DISJ-U-DIST
$b \vee b' \equiv b \cup b'$	BOOL-DISJ-U		

Network axioms

$[t]r \triangleright o \cdot [t']r' \triangleright o' \equiv [t \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \uplus r'$ and $\hat{o} = o \uplus o'$	NET-ORD
$[t]r \triangleright o \parallel [t']r' \triangleright o' \equiv [(t \uplus r') \wedge (t' \uplus r)]\hat{r} \triangleright \hat{o}$		if $\hat{r} = r \uplus r'$ and $\hat{o} = o \uplus o'$	NET-PRL

Single round axioms

$[\mathbb{1}] \emptyset \triangleright o \equiv 1$	SR-ONE	$(p \parallel p') \cdot (q \parallel q') \leq (p \cdot q) \parallel (p' \cdot q')$	SR-EXC
$[\emptyset]r \triangleright o \equiv 0$	SR-ZERO	$[b \wedge t]r \triangleright o \equiv [(r \cup b) \wedge t]r \triangleright o$	SR-CAN
		$[t]r \triangleright o + [t']r \triangleright o \equiv [t \vee t']r \triangleright o$	SR-PLUS

Network axioms

$\llbracket p \rrbracket \in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP))$	
$\llbracket \omega \rrbracket_I \in \mathcal{M}(BP) \rightarrow \mathcal{P}(\mathcal{M}(BP))$, where $\omega = \pi_1 \ddot{\wedge} \pi_2 \ddot{\wedge} \dots \ddot{\wedge} \pi_k$	
$\llbracket p \rrbracket a \triangleq \bigcup_{\omega \in I(p)} \llbracket \omega \rrbracket_I a$	
$\llbracket e \rrbracket a \triangleq \{a\}$	
$\llbracket [t]r \triangleright o \rrbracket_I a \triangleq \begin{cases} \{o \uplus a \setminus r\} & \text{if } r \subseteq a \text{ and } \{t\}a = \top \\ \emptyset & \text{otherwise} \end{cases}$	
$\llbracket \pi_1 \ddot{\wedge} \pi_2 \ddot{\wedge} \dots \ddot{\wedge} \pi_k \rrbracket_I a \triangleq (\llbracket \pi_1 \rrbracket_I \star \llbracket \pi_2 \rrbracket_I \ddot{\wedge} \dots \ddot{\wedge} \llbracket \pi_k \rrbracket_I) a$	