EDU T543: RESEARCH-BASED INSTRUCTIONAL DESIGN: APPLIED COGNITIVE AND LEARNING SCIENCES

PROTOTYPE

LESSON 1: Exploring Fraction-Decimal Equivalence Through Visual Models

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LESSON -1 OVERVIEW

Prior Knowledge	Next steps
• Basic fractions from earlier grades (halves, quarters) • Recognizing parts of a whole visually • Simple decimal notation (money context: ₹0.50)	• Lesson 2: Exploring place value patterns in decimals

Learning Goals: Teacher vs. Student Perspectives

Teacher Planning Goal: What students will understand today	Student Discovery: What students will figure out today
GOAL 1: EQUIVALENCE Different fractions can represent the same quantity (equivalent fractions) **Evidence to watch for:** Students can identify that 1/2 = 2/4 = 4/8 AND explain WHY they're equal (matching length = same amount)	**Discovery 1:** "Different fractions can show the same amount!" - When I line up 1/2 and 2/4 strips, they're exactly the same length - Same length means they show the same amount. So, 1/2 = 2/4 = 4/8. - They're all equivalent! - My thinking: "I thought they were different because the numbers are different, but the strips prove they're the same amount"
GOAL 2: FRACTION-DECIMAL CONNECTION The same quantity can be represented as a fraction or a decimal **Evidence to watch for:** Students can match fractions to their decimal equivalents (e.g., 1/2 = 0.5, 1/4 = 0.25) AND explain that they're two notations for the same amount	**Discovery 2:** "Fractions and decimals are just different names for the same amounts!" - My 1/2 strip has 0.5 written on it: that's the decimal name - My 1/4 strip has 0.25 - So, 1/4 and 0.25 mean the exact same thing - It's like saying "one-half" or "point-five" - two ways to say it! My thinking: "Decimals aren't separate from fractions - they're connected!"
GOAL 3: VISUAL PROOF Physical models (fraction strips) help us see and prove mathematical relationships **Evidence to watch for:** Students use strips as evidence ("I know they're equivalent because the strips match in length")	**Discovery 3:** "I can USE strips to PROVE things in math!" - Instead of guessing if fractions are equal, I can test it with strips - If strips match in length, I have proof they're equivalent - The strips are tools for proving, not just pretty pictures - My thinking: "Now I can show WHY something is true, not just say it is"
GOAL 4: MATHEMATICAL PATTERNS Predictable patterns exist in fraction-decimal relationships **Evidence to watch for:** Students identify the pattern (0.5 → 0.25 → 0.125) AND can explain or extend it	**Discovery 4:** "There's a pattern I can predict with!" - The decimals keep getting cut in half: 0.5 → 0.25 → 0.125 - When the bottom number doubles, the decimal halves - I can use this pattern to figure out new ones - My thinking: "Math has patterns I can discover and use!"

Teacher Note:

As you teach, mentally (or on paper) track evidence of these goals: - Quick Start (Engage): Do students show prior knowledge of fractions? - Investigation Part 2(Explore & Elaborate): Are students discovering equivalence? - Explain Phase (Explain): Can students articulate the big ideas? - Exit Ticket (Evaluate): Can they apply understanding independently? This evidence will inform your reflection and planning at the end of class.		
Knowledge	**Skills**	**Practice**
Different fractions can represent the same quantity (UG1a)	Identify and explain equivalent fractions (UG1a)	Compare strips physically, justify equivalence (UG1e)
Fractions and decimals are two notations for same amounts (UG1a)	Match fractions to decimals and convert between them (UG1a)	Label strips with both notations, complete pairs (UG1a)
Physical models provide mathematical evidence (UG1e)	Use visual models to prove and explain reasoning (UG1a, UG1e)	Compare strips as proof, document with evidence (UG1e)
Predictable patterns exist in fraction-decimal relationships (UG1c)	Identify, predict, and extend patterns (UG1c)	Complete pattern table, find rules (UG1c)

Lesson Flow:

Phase	**Section**	**What?**	**Energy**	**Attention**	**Teaching Action**
ACTIVATION (12-15 min)	Quick Start	Diagnostic, brief individual/pair work	LOW → MEDIUM (students settling in, waking up)	Fresh, focused	Keep pace brisk, use curiosity to engage
DEMONSTRATION (12-15 min)	Introduction	Teacher models, students observe	MEDIUM (students watching, some participation)	Good if demonstration is clear and engaging	Keep demo tight, ask questions, maintain pace
HANDS-ON CREATION (15-17 min)	Investigation Part 1	Physical strip creation	MEDIUM → HIGH (active, hands-on, movement)	Sustained by doing, but can get off-task	Circulate actively, keep time visible
ATTENTIONAL BREAK NEEDED HERE (1-2 minutes) Students have been working 35-40 minutes. Before Investigation Part 2: - Brief stretch break (30 sec): **Say:** "Stand up, stretch, shake out your hands" - Reset focus: **Say:** "Great work creating! Now comes the discovery phase" - Clear transition: **Say: "**Put away scissors, keep only strips + worksheet"					
EXPLORATION & ELABORATION (17-20 min)	Investigation Part 2	Main mathematical exploration & Challenge problems	HIGH at start → gradual decline (cognitive work)	HIGH for discovery, may decline if too long	Keep energy up through questions, movement (partner talk),

					celebrating discoveries
SENSE-MAKING DISCUSSION (25-30 min)	Explain	Whole-class math talk, pattern sharing	Energy: MEDIUM (depends on discussion engagement)	Can be HIGH if discussion is interactive, or LOW if too much teacher talk	Keep students TALKING, not just listening. Use turn-and-talk frequently, call on multiple students
CLOSURE (5 min)	Close	Exit ticket, synthesis	LOW (wrapping up)	Brief burst if exit ticket is clear	Keep it tight, clear, affirming

Lesson Timeline

Section	Time	Activity
Quick Start	12 – 15 mins	Compare shaded rectangles
Introduction	12 – 15 mins	Teacher demonstrates strips
Investigation 1	15 – 17 mins	Create fraction strips
Investigation 2	20 – 25 mins	Explore & Elaboration equivalences
Explain	20 – 25 mins	Making sense
Close	5 mins	Share & exit ticket

How to use this lesson

First time teaching:	During teaching:	After teaching:
1. Read full lesson plan overview 2. Review answer key (Appendix A) 3. Prepare materials using checklist (Appendix B) 4. Skim differentiation strategies for language balance (Appendix C) 5. Print student materials (Appendix D)	1. Follow lesson flow sections 2. Watch for formative checkpoints 3. Adjust timing as needed 4. Refer appendices	1. Review exit tickets using answer key 2. Note what worked / what to adjust 3. Identify students needing support 4. Prepare for Lesson 2

This lesson plan is color coded for easy reference:

	Formative checkpoints for teacher
	Teacher actions
	Student responses and actions
	Teacher note
	Duration for each section
	Lesson flow
	Total lesson timeline
	Scaffolding strategies
	Key moments to look for discoveries

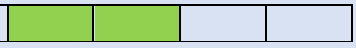
2. DETAILED LESSON PLAN

2.1 Quick Start (15 - 18 minutes)

Purpose: Activate prior knowledge about fractions and visual equivalence.
Activity: Students observe three shaded rectangles and share their initial thinking about which might show equivalent amounts.

Time	Teacher Actions	Student Actions
0-2 min	**Greet students and connect to fraction representation:** **Say:** "Imagine you're sharing food with friends. You have three different chocolate bars [point to rectangles on board]. Each bar is divided differently, but you want to make sure everyone gets a FAIR amount." Display rectangles (chocolate) on board (Question 1) Compare these chocolate bars  **Display these questions:** 1. WHAT fraction is each chocolate bar? 2. WHICH bars give the same amount? 3. WHY do you think they're equal?" **Say:** "Here's the challenge: Some of these chocolate bars might give you the SAME amount of chocolate, even though they're divided differently. Which ones do you think are equal? How can you tell?" (Question 2) **Say:** "Work independently for 5 minutes. I want to see YOUR thinking: what YOU notice, what YOU predict, and WHY you think so. Write in your note book."	**Students engage with real-world framing:** • Listen to chocolate bar scenario • Look at rectangles on board • **Think:** "Which would I choose?" • **Curiosity sparked:** "Are some chocolate bars equal even though they look different?" **Student mindset:** • "This is about fairness!" • "I want to figure this out" • "I know something about fractions already" **Get ready to work:** • Open notebooks • Look at display • Read the questions: 1. WHAT fraction is each chocolate bar? 2. WHICH bars give the same amount? 3. WHY do you think they're equal?" Student: "B and C look the same." Teacher: "Tell me more - what makes you think that?" Student: "Um... they just look similar." Teacher: "Can you be more specific? What about them looks similar?" Student: "The shaded part?" Teacher: "Good - you're noticing the shaded portions. Let's think about this..."

	Connect to their competence: **Say:** "You already know about fractions from previous classes. Use what you know to figure this out!"	
2-8 min	**Circulate strategically among students (5 minutes circulation time)** **Diagnostic observation for scaffolding decisions** (helping facilitator think diagnostically): **FOR GROUP ABILITY - notice patterns across class:** • How many students can identify fractions correctly? • What's the most common prediction about equivalence? • What reasoning strategies are emerging? (visual, counting, prior knowledge) **FOR INDIVIDUAL ABILITY - note 2-3 specific students:** • Student A: Identifies fractions correctly + makes prediction with reasoning • Student B: Partially correct or interesting misconception • Student C: Uncertain or confused (needs support) **These specific questions guide observation:** **Think:** "As I circulate, I'm asking myself:" • "Can this student identify the fractions?" (4/8, 2/4, 1/4) • "What do they NOTICE?" (size, shading, divisions) • "What prediction do they make?" (which are equal) • "What REASONING do they use?" (visual comparison, fraction knowledge, guessing) **Quick diagnostic notes** (mental or jot down): ✓ Students who are ready ~ Students who are developing ✗ Students who need support	**Students work individually in notebooks (5 minutes thinking time)** **In response to these questions:** 1. "WHAT fraction is each chocolate bar?" (identifying fractions) 2. "WHICH bars give the same amount?" (predicting equivalence) 3. "WHY do you think they're equal?" (reasoning) **Student work behaviors:** • Some write fractions quickly:4/8, 2/4, 1/4 • Some count parts carefully • Most make predictions: "A and B look equal" • Some use reasoning: "They're the same size shaded" • Some uncertain: "I'm not sure how to prove it" • Some draw diagrams or visual comparisons **Range of thinking visible:** • Strong: Identifies all fractions correctly, makes reasoned prediction • Developing: Identifies most fractions, makes prediction without deep reasoning • Struggling: Unsure about fractions, relies only on visual appearance

	This observation informs: Who to call on in (Row 4), what support to provide in Introduction phase	
8-11 min	**Say:** "Pencils down. Turn to your partner." **Frame partner discussion with specific thinking question:** **Say:** "Share with your partner: Which chocolate bars do you think give the same amount? More importantly, WHY do you think that? What made you decide?" **Purpose:** Give students time to articulate their reasoning before whole-class share. This question asks them to explain THEIR THINKING, not just their answer. **Listen to partner conversations:** • Are students explaining their reasoning? • Are they building on each other's ideas? • Do they reference the visual (size/shading) or fraction knowledge?	**Partner share (2 minute) Students turn to partner and discuss in response to:** **Student A:** "I think A and B are the same because they look similar size." **Student B:** "Let me count... B has 2 shaded, C has 1 shaded. But C's pieces are bigger." **Student A:** "Oh! So maybe same total amount but cut differently?" **Typical partner conversations:** • Student 1: "I think A and B because they look the same size" • Student 2: "Me too! The shaded parts match" • Student 1: "But how do we KNOW for sure?" OR: • Student 1: "I think A and B are equal—that's 4/8 and 2/4" • Student 2: "Wait, how can different fractions be equal?" • Student 1: "That's what I'm wondering!" **Students practice articulating reasoning:** • Explaining what they notice • Questioning each other • Building curiosity about HOW to prove equivalence
11-18 min	**Whole-class share and diagnostic bridge (3 minutes)** **Say:** "Let's hear your thinking! I want to know what you NOTICED and what you're WONDERING." 8 parts 4 shaded **Rectangle A**  4 parts 2 shaded **Rectangle B** 	**Students share observations with whole class** **Students respond when called:** • "A and B look the same" • "I think 4/8 might equal 2/4" • "They have the same amount shaded" • "I'm not sure HOW different fractions can be equal" **Class listens and engages:**

	4 parts 1 shaded **Rectangle C**  **Call on the 2-3 students you noted during circulation (Row 2):** **Student 1 (strong reasoning):** **Say:** "[Name], what did you and your partner think?" "What made you notice that?" **Say:** "Interesting! How did you figure out they're both 'half'?" "Can you show us with your hands how they compare?" **Say:** "Does everyone agree? Who sees it differently?" - [If student says "they look the same"] "What specifically looks the same - the shaded parts or something else?" **Record on board as the student explains:** • Rectangle A = 4/8 • Rectangle B = 2/4 • Rectangle C = 1/2 • Student prediction: "A and B might be equal" **Student 2 (partial understanding):** **Say:** "[Name], what do you think?" **Say:** "That's a great question! Can different fractions show the same amount?" **Acknowledge the question everyone has:** **Say:** "Many of you noticed that A and B LOOK equal, but you're wondering: How can 4/8 equal 2/4? They're DIFFERENT fractions!" **Bridge to lesson with curiosity:** **Create cognitive tension:** **Say:** "Interesting - some of you think A and B are equal, but the NUMBERS are different (4/8 vs 2/4). How can that be? We'll find out today."	• Some nod in agreement • Some look puzzled: "Wait, can that be true?" • **Curiosity building:** "How DO we prove it?" **Student 1 Response:** "We think A and B show the same amount because they're both half-shaded" **Student 1 explains** **Student 2 Response:** "They look the same, but I'm not sure if 4/8 and 2/4 can really be equal..."] **Watch teacher record on board:** • Fractions identified • Predictions noted • Their OWN thinking is valued **Bridge moment: students are curious:** • "How can we PROVE they're equal?" • "I want to learn the tool!" • "Will we find out if I was right?" **Ready for Introduction phase:** with genuine curiosity about the method

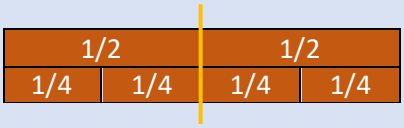
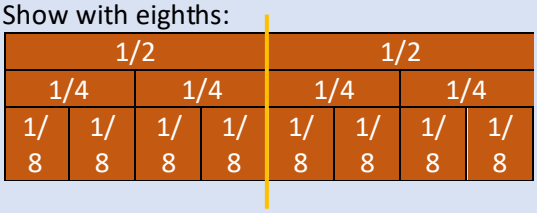
	Say: "That's EXACTLY what we're going to discover today. I'm going to show you a tool that doesn't just help you guess. It PROVES when fractions are equal. You'll be able to SEE why 4/8 equals 2/4. Ready to find out?" **Diagnostic note for teacher:** Based on this Quick Start, I now know: • General class readiness (most/some/few can identify fractions) • Common predictions and reasoning • Which students may need extra support in Introduction • What misconceptions to address	

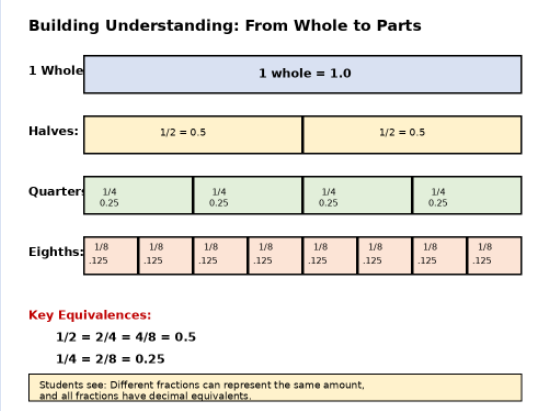
Formative Checkpoint: Before moving to Introduction, check that students can:		
Evidence of Readiness:	**Evidence of Developing Understanding:**	**Needs Additional Support:**
• Identify fractions for shaded regions (4/8, 2/4, 1/2) • Make observations about which rectangles "might be equivalent" • Articulate some reasoning (even if incorrect)	• Can identify fractions but unsure about equivalence • Makes guesses without reasoning	• Cannot identify fractions correctly • No predictions or observations about equivalence
Scaffolding Strategies: If Students Struggle, spend 2 more minutes with whole-class discussion comparing rectangles visually. Point to rectangles A and B: "Do these looks like they might show the same amount? How can you tell?"		
Transition: **Say:** "How can we PROVE if fractions are equivalent? Let me show you a tool that makes it obvious..."		
Teacher Note: At this point, students are making predictions based on visual observation. They have **not yet learned the method to PROVE equivalence** - that comes in the Introduction. This Quick Start is purely diagnostic to see what prior knowledge students bring.		

1.2 Introduction (12-15 minutes)

Purpose: Demonstrate how physical fraction strips reveal equivalence. Model the tool students will use in the investigation.
Activity: Teacher demonstrates comparing fraction strips while students observe and discuss what the visual comparison reveals.

Time	Teacher Actions	Student Actions
0-3 min	**Say:** "Remember how you predicted rectangles A and B might be equal - that 2/4 and 1/2 might show the same amount? Now I'll show you THE TOOL that proves whether you're right. These fraction strips will help us see WHY different fractions can be equivalent."	**Recall their predictions from Quick Start** **Anticipation:** "Will the tool prove I was right?"

	Show whole strip: "This is 1 whole. Like one chocolate bar."  **Show half strips:** "If I break it in half, I get 2 pieces. Each is 1/2."  **Ask:** "How many halves make one whole?" Label: "2 × 1/2 = 1 whole" **Show quarter strips:** "Divided into 4 equal pieces. Each is 1/4."  	Watch demonstration Look at whole strip Look at half strips Answer: "2 halves" See: 2 × 1/2 = 1 Watch quarter strips
3-6 min	**Ask:** "Before I compare, predict: Will two 1/4 strips equal one 1/2 strip?" "Answer using your thumb" [Wait 5 seconds - thumbs up/down] **KEY MOMENT - Physical Comparison:** Place 1/2 strip ABOVE two 1/4 strips  **Say:** "Watch carefully. Do these match?" Align precisely **Ask:** "What do you notice?" **Ask:** "What does this prove?" **Ask:** "HOW do you know they're equal?" Write on board: 1/2 = 2/4 ✓ Show with eighths:  Write: 1/2 = 4/8 ✓	**KEY OBSERVATION:** See 1/2 strip and two 1/4 strips aligned **Notice:** "They match!" "They're the same length!" **Realization:** "Oh! That's WHY 1/2 = 2/4!" **See physical proof - not just a rule** "They're equal!" "Same amount!" "2/4 = 1/2" Watch eighths demonstration See pattern: 1/2 = 2/4 = 4/8

6-10 min	**Diagnostic moment BEFORE teaching decimals:** **Ask:** "Has anyone seen numbers like 0.5 or 0.25 before? What do you think these numbers mean?" **Teacher observes:** Do they know these are decimals? Do they see any connection to fractions? **Then teacher makes the connection:** **Say:** "These numbers are called decimals. And here's something amazing - they can show the SAME AMOUNT as fractions! Watch..." **Say:** "We can also write these as decimals." **Write on board:** 1/2 = 2/4 = 4/8 = 0.5 1/4 = 2/8 = 0.25 1/8 = 0.125  **Ask:** "Why do you think 1/2 = 0.5?" **Accept responses** **Say:** "You've now seen that fractions and decimals are connected. In today's investigation, you'll discover WHY these connections work. Questions 7-10 will help you explore the patterns between fractions and decimals more deeply. We'll explore WHY these decimals work in Lesson 2."	**Share what they know about decimals** Students share (1-2 min) - revealing their current understanding • "I've seen 0.5 on a calculator" • "My mom uses decimals for money" • "I'm not sure what they mean" • "Some students don't know decimals at all" "Watch teacher label decimals, See 1/2 = 0.5" **Read:** 1/2 = 0.5 1/4 = 0.25 1/8 = 0.125 Look at image with all strips labeled Think about "Why 0.5?" **Share ideas:** • "Half of one" • "50 out of 100" • "Like money – 0.50 paise" Make connection between fractions and decimals
10-15 min	**Say:** "Now you will create your own fraction strips and discover more equivalences." **Distribute materials:** • Fraction strip templates • Scissors	Listen to instructions **Receive materials:** • Pick up templates • Get scissors

	• Investigation worksheet **Say:** "Work in groups of 3-4." "First, you'll create the strips. Then you'll use them to explore." **Check:** Do all groups have materials?	• Take worksheet Form groups of 3-4 Look at template: See strips to cut out **Anticipation:** "We get to make these!" "I want to test if mine match like the teacher's" Ready to begin creating

Formative Checkpoint: Before moving to Investigation, check that students understand:

Evidence of Readiness:	**Evidence of Developing Understanding:**	**Needs Additional Support:**
• Physical strips can be aligned to show equivalence • 1/2 = 2/4 = 4/8 are all the same amount • Fractions have decimal equivalents (1/2 = 0.5, 1/4 = 0.25) • Can explain WHY strips show equivalence (because they match in length)	• Sees that strips match but can't explain why this proves equivalence • Knows 1/2 = 0.5 but doesn't understand the connection	• Doesn't understand how strips show equivalence • Confused about decimal notation

Scaffolding Strategies: If students struggle, re-demonstrate the key comparison (1/2 = 2/4) before distributing materials.

FIRST - Diagnose what they're stuck on:
Before re-demonstrating, ask questions to understand their confusion:
Say:
- "What part is confusing you?"
- "Can you show me where you got stuck?"
- "What do you THINK equivalent means?"
- "What do you understand so far, and where does it stop making sense?"

Listen carefully to their responses. Their answers will reveal whether they're struggling with:
- The concept of equivalent (what it means)
- The physical comparison (how to line up strips)
- The connection (why matching length = same amount)
- The vocabulary/instructions

THEN - Re-demonstrate based on what you learned:

If stuck on CONCEPT: "Equivalent means 'same amount.' Watch - when these strips match in length, they show the same amount, even though one says 1/2 and the other says 2/4."

If stuck on PHYSICAL COMPARISON: Re-demonstrate the key comparison (1/2 = 2/4).
Ask: "When I line these up, what do you see? Are they the same length? What does that tell us?"

If stuck on CONNECTION: **Say:** "Why does matching length matter? Because length shows the amount. Same length = same amount = equivalent fractions." If stuck on VOCABULARY/INSTRUCTIONS: **Repeat** the instructions clearly in Tamil and English.
Teacher Note: Teacher demonstrates with physical fraction strips (or draws large strips on board). Students observe how strips physically show why fractions are equivalent. The visual alignment is key when two different fractions have the same length, they represent the same amount. This is where students learn the METHOD for proving equivalence. **Anticipated Student Thinking:** **Student Interpretations:** • "The physical strips make it obvious why they're equivalent" • "I never understood why 1/2 = 2/4 before, but now I SEE it" • "Can we make other fractions like thirds?" **Potential Misconception:** • "Bigger denominator means bigger fraction" **Teacher response:** "Let's compare the physical strips. Which piece is actually bigger, 1/2 or 1/8? Look at the strips." **Common Question:** • "How do you know what 1/8 is as a decimal?" **Teacher response:** "We can use division: 1 ÷ 8 = 0.125. Or we can think: if 1/4 = 0.25, then 1/8 is half of that = 0.125. We'll explore this more in Lesson 2."
Transition: Say: "Now it's YOUR turn to create these strips and discover equivalences for yourself..."

1.3 Investigation Part 1: Creating Strips (15-17 minutes)

Purpose: Students create physical fraction strip tools they will use for mathematical exploration in Part 2.
Activity: Students cut, fold, and color-code fraction strips (1/2, 1/4, 1/8) following step-by-step instructions, then verify accuracy before exploration.

Time	Teacher Actions	Student Actions
0-5 min	**Say:** "Now you'll create your own fraction strips. These strips will help us answer an important question: How can we PROVE that different fractions show the same amount?" **Display instructions visibly on board (write or project):** CREATE YOUR FRACTION STRIPS (Point to Activity Sheet Part 1): 1. Cut out all strips along dotted lines • Whole = one color • Halves = another color • Quarters = third color • Eighths = fourth color 3. Label each strip with BOTH: • Fraction (1/2, 1/4, etc.)	**Listen to teacher frame the purpose:** **Hear guiding question:** "How can we PROVE different fractions show the same amount?" **Understand:** "These strips are TOOLS for proving" **Anticipation:** "We're going to discover something!" **Read instructions from board:** - Look at visible display on board - See the 4 steps clearly listed - Read: Cut, Label (fractions + decimals), Organize Look at Activity

	• Decimal (0.5, 0.25, etc.) 4. Keep strips organized on your desk **Say:** "Now you all have the required materials with you for this activity" **Say:** "Follow the instructions on the board. You have 10 minutes to create your strips. Work carefully as these strips are your tools for discovery!" **Model if needed:** • Hold up template • Show cutting along lines • Show where to write labels **Say:** "Work together as a group". **Suggest roles:** **Say:** "You can divide tasks - one person cuts, one labels. Or everyone makes their own set." **Teacher's Diagnostic Focus During Creation:** **As you circulate, ask 1-2 groups:** **Ask:** "You're labeling 1/4 as 0.25. Why does one-fourth equal twenty-five hundredths?" **Listen for reasoning - accept all responses without correcting:** **Say:** "Interesting thinking. Keep that question in mind - we'll discover WHY in Part 2." **[Cognitive thinking initiated - students now curious about WHY, not just WHAT]** As students create strips, observe how they're thinking about fractions and decimals: • Do they already see connections between fractions and decimals while labeling? • Do they ask "Why does 1/2 = 0.5?" (shows curiosity about relationship) • Are they confused about which decimal goes with which fraction? • Do they treat fractions and decimals as completely separate, or see them as related?	**Sheet and template:** - See template with strips to cut - Match what's on board to what's on paper **Understand task:** - Cut carefully along dotted lines - Label BOTH fractions AND decimals (this is new awareness) - Keep strips organized **Checks the received materials:** • Get fraction strip template • Get scissors • Organize materials on desk **Decide as group (if working in groups):** • "Should we divide the work?" • "Or each make our own set?" - "Because that's what the teacher said?" - "Because 1 divided by 4 is 0.25?" - "I'm not sure - they just do?" **Some groups pause when asked WHY:** - "Hmm, I never thought about why..." - "Maybe because 25 is a quarter of 100?" - "We just know they're equal" **Quick discussion about approach Ready to begin:** • Materials ready • Instructions clear (visible on board) • Purpose understood (proving equivalence) • Motivated to start

	This creation phase reveals current understanding of fraction-decimal relationships. Note student comments and questions for your investigation Part 2.	
5-12 min	**Circulate among groups** **Monitor progress and provide scaffolding:** **Check for accurate work:** • Are strips cut carefully along lines? • Are labels correct? (Both fraction AND decimal) **Provide instructional scaffolding based on what you observe:** **IF struggling with cutting:** **Say:** "Try cutting right on the line so the strips will be accurate when we compare them. Accuracy matters for seeing patterns." **IF labels are incorrect (e.g., wrong decimal):** **Don't just correct:** **Say:** "Let's think about 1/4. That means 1 divided by 4. What's 1 ÷ 4?" So, what should you write?" **Write:** 1/4 = 0.25 **IF confused about decimals:** **Say:** "Have you seen numbers like 0.5 before? Where?" [Build on their experiences – reiterating the quick start questions]. These are called decimals. Today you'll discover how they connect to fractions."	**Work in groups** **Students work on creating strips (7 minutes of work time) Physical creation:** • Cutting along dotted lines • Labeling with fractions AND decimals • Organizing strips on desk Labeling: • Write on each strip: - 1 whole = 1.0 - 1/2 = 0.5 - 1/4 = 0.25 - 1/8 = 0.125 **Different pacing observed:** • Some students work quickly and efficiently • Some work slowly but carefully • Some struggle with cutting or labelling • Some finish early **Thinking while creating:** • Some students notice: "Hey, 1/2 and 0.5 are the same!" • Some ask: "Why is 1/4 equal to 0.25?" • Some wonder: "Are these strips going to PROVE equivalence?" • Some are curious: "Will we discover patterns?" **Student response:** 0.25 **Engagement with scaffolding:** **Students who struggle:** - Ask teacher for help - Work with partner to figure it out - Refer to board instructions - Use teacher's questions to guide thinking

	IF one student doing all work: **Say:** "Everyone needs their own understanding. How can you divide the tasks so everyone participates AND learns?" **IF rushing through without thinking:** **Say:** "Slow down: I want you to THINK as you create. What do you notice about the sizes? The numbers?" **Teacher has diagnostic data about student understanding** **Additional Scaffolds (beyond materials):** **FOR STUDENTS STRUGGLING CONCEPTUALLY:** - Pair them with a peer who can explain (not just do it for them) - Ask them to explain what "1/4" means before labelling - Have them compare sizes and these questions: "Which is bigger: 1/2 or 1/4? How do you know?" **FOR STUDENTS FINISHING EARLY (problematizing/extending):** **Say:** "You finished? Great!" - "Now make a prediction: Without using your strips yet, which fractions do you think will be equivalent? Write down your predictions." - "Can you find THREE different fractions that equal the same amount? Write them as a challenge question." - "Look at your decimals. Do you notice any patterns?" **Time check:** - At 5 minutes: "5 more minute left! Keep working."- At 2 minutes: "2 minutes - finish up the strip you're working on."	**Students who finish early:** - Make predictions about equivalence - Compare strips informally - Notice patterns in decimals - Create challenge questions for peers Most students have completed or nearly completed strip sets All students have engaged with fractions AND decimals Some finish early: Look at Part 2 questions
12-17 min	**Get attention:** "Strips down. Eyes here." **Say:** "You just made strips showing fractions AND decimals. Now comes the	**Stop working and transition Put materials down:** - Strips finished (or mostly finished) - Put scissors and pencils aside

	investigation: You're going to use these strips to answer this question" "Let's talk about what you just created" **Say:** "Hold up your 1/2 strip." [Check most are correct size and labeled] **Say:** "Hold up your 1/4 strips." [Check labels: should say "1/4 = 0.25"] If most are correct: "Excellent! Now we'll use these to explore." If many errors: "Let's check labels together." "1/2 = ?" [Students: "0.5"] "1/4 = ?" [Students: "0.25"] "Please check yours and fix if needed." [Write on board in large letters:] **INVESTIGATION QUESTION:** **Ask:** How can we use these strips to PROVE that different fractions show the same amount? "That's what you'll explore in Part 2. Your strips are your TOOLS for discovery." **Quick check:** **Ask:** "Who can tell me: What question will you be answering?" [Call on student—ensure they understand the purpose] "Excellent! Let's investigate..." **Say:** "Keep your strips organized. You'll need them for Part 2."	- Look at teacher **Listen to teacher's framing:** Show strips when asked: • Hold up 1/2 strip • Hold up 1/4 strips **Pay attention to what comes next** **Read the Investigation Question on board:** "How can we use these strips to PROVE different fractions show the same amount?" **- Students read silently** - Some whisper: "Oh, so THAT'S what we're doing!" **Respond when teacher asks:** "Who can tell me: What question will you be answering?" **Volunteers raise hands and respond:** - Student 1: "We're going to prove that different fractions show the same amount" - Student 2: "We'll use the strips to prove they're equal" - Student 3: "We're answering how we can prove equivalence" **Class listens to peers' responses:** - Nod in agreement - Some clarify their own understanding - "Oh, I get it now!" **Understand the purpose:** - Strips are TOOLS for investigation - We're not just making strips but we're using them to PROVE something - The guiding question gives meaning to the task **Organize materials for Part 2:** - Keep strips organized on desk - Arrange neatly so they can be compared **Check labels one more time:** - "Are mine labeled correctly?"

	Before transition, quick prediction: **Ask whole class:** "Before we investigate, predict: How many 1/4 strips do you think will equal one 1/2 strip?" [Quick thumbs: 1, 2, 3, or 4?] **Say:** "Let's find out! Use your strips to PROVE your prediction."	- Self-correct if needed: "Oh, I wrote 0.4, should be 0.25" **Ready and motivated for Part 2:** - "Now we get to explore!" - "I want to see if different fractions really are equal" - "I'm going to use my strips to prove it!" - Curiosity activated about what they'll discover **Quick prediction (no writing):** - Most thumbs show "2" - Some show "4" (overgeneralizing) - Few show "1" (misconception) **Anticipation builds:** - "I think it's 2..." - "Wait, let me check with my strips!" - Ready to investigate with PURPOSE

Formative Checkpoint: Before moving to Part 2, check that:

Evidence of Readiness	**Evidence of Developing Understanding**	**Needs Additional Support**
• All groups have complete, labeled strips • Decimals are correctly labeled (especially 0.25 and 0.125) • Strips are cut accurately enough to align and compare • Students understand they will USE strips to discover equivalences	• Strips are complete but some labels incorrect • Cutting is rough but strips still usable	• Strips incomplete or severely mislabeled • Strips cut too inaccurately to compare

Scaffolding Strategies:
Students completing with ease

CHALLENGE 1: Prediction
Ask: "Before exploring, predict: Which three fractions will be equivalent? Write your predictions."
CHALLENGE 2: Pattern Finding
Ask: "Look at just the decimals: 0.5, 0.25, 0.125. What pattern do you notice? Can you predict what 1/16 would be?"
CHALLENGE 3: Create challenge question
Ask: "Create a question that would be HARD for another student. What equivalence would be tricky to prove?"
CHALLENGE 4: Explain Relationships
Ask: "Can you explain WHY 1/2 equals 0.5? What's the mathematical reason?"

If Students Struggle,

SCAFFOLD TYPE 1: Material Scaffolds
- Provide pre-cut strips to struggling groups, OR
- Have groups share sets (2 groups use one set of strips)

- Use teacher's demonstration set as reference

[RATIONALE: The mathematical exploration (Part 2) is more important than perfect strip creation. Don't let cutting difficulties prevent conceptual learning.]

SCAFFOLD TYPE 2: Instructions & Model
- Break down the task: "Let's just focus on creating the halves first. Once you have those, we'll move to quarters."
- Model think-aloud: "I'm cutting this strip. I'm making sure to stay on the line so it's accurate. Now I'm labeling it: This is 1/2, which equals 0.5."
- Provide visual reference: Keep a completed sample strip set visible for students to reference
- Provide sentence frames: "This strip shows ___ because ___"
- Check understanding: "Explain to me what 1/4 means in your own words."

SCAFFOLD TYPE 3: Organizational Scaffolds
Assign roles in groups: "One person cuts, & one person labels: but everyone discusses what goes on each strip."
- Provide step-by-step checklist:
☐ Cut all strips
☐ Label with fractions
☐ Label with decimals
☐ Organize on desk

SCAFFOLD TYPE 4 - Peer Support:
Strategic pairing: Pair struggling student with patient peer- Peer teaching: "Can you explain to your partner how you labeled your strips?"
Group share: Have successful group show their strategy to whole class

SCAFFOLD TYPE 5 - Emotional/Motivational Scaffolds
- Normalize struggle: "Creating these accurately is challenging! That's okay: take your time."
- Emphasize purpose: "These strips are going to help you discover something really cool about fractions and decimals. They're worth making carefully!"
- Celebrate progress: "I see you've finished the halves! That's great: now let's work on quarters."

SCAFFOLD TYPE 6 - Cognitive Support
- Simplify initially: "For now, only label the fractions. We'll add decimals in a few minutes."
- Connect to familiar: "Remember pizza slices? 1/2 means half the pizza. Same idea here."
- Check prerequisite understanding: "Before we label this 1/4, tell me: What does 1/4 mean?"

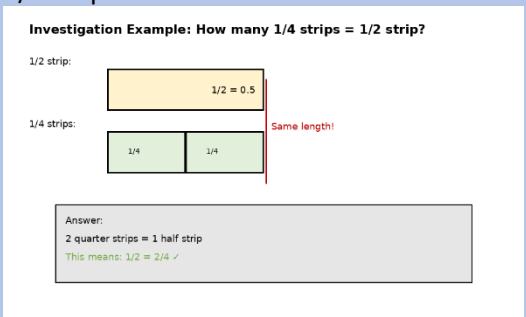
Teacher Note:
1. Is student struggling with MOTOR SKILLS (cutting)? → Provide pre-cut strips
2. Is student struggling with UNDERSTANDING (what to label)? → Use instructional scaffolds
3. Is student struggling with ORGANIZATION (overwhelmed)? → Break into smaller steps/Peer support
4. Is student struggling with MOTIVATION (doesn't see point)? → Emphasize purpose and connection

Time Management: This section can easily run over. Set a visible timer. If groups are taking too long, provide pre-cut strips or have them share sets.
Quality vs. Perfection: Strips don't need to be perfectly cut. They need to be accurate enough to compare. Don't let students spend too much time on aesthetics.

Support Struggling Groups: If a group is really struggling with cutting/labeling, provide them with a pre-made set so they can participate in Part 2 (the real learning). **Early Finishers:** Don't wait for everyone to finish perfectly. Once most groups are ready, move on. Early finishers can help others or preview Part 2.
Transition: "Great! Now you have the tools. Let's discover WHY different fractions can equal the same amount..."

1.4 Investigation Part 2: Exploring Equivalences (20-25 minutes)

Purpose: Students use physical fraction strips to discover equivalent fractions and connect them to decimals through hands-on exploration.
Activity: Students work through 8 guided questions, comparing strips to discover equivalences, matching fractions to decimals, and identifying patterns in fraction-decimal relationships.

Time	Teacher Actions	Student Actions
0-3 min	**Say:** You have your strips which are your tools. You have questions on your worksheet. Take your time to think carefully and discover patterns. The goal is to understand WHY fractions are equivalent."Questions 3-7." **Set expectations for thoughtful work:** **Say:** "As you work: - Use your strips to TEST ideas by placing them side by side, compare them - THINK about what you notice and do not just rush through - DISCUSS with your partner and talk about what you're discovering - If you finish early, there are challenge questions to deepen your thinking" **Display time guideline** **Say:** "You have about 10 - 12 minutes for Questions 3-7. Work at a pace that lets you understand, not just finish." Point to Question 3: "Start here: How many 1/4 strips equal one 1/2 strip?" 	**Listen to teacher's message:** - Not a race - I can take time think - Goal is understanding, not just finishing - Use strips to discover and test ideas **Understand the approach:** - Work thoughtfully, not quickly - Discuss with partner - Think about WHY, not just WHAT **See time guideline:** - 17-20 minutes available - No pressure to rush - Can go deeper with challenge questions if finish early **Mental shift:** - "Okay, this is about discovering, not racing" - "I should think carefully" - "I can take my time" - “Ready to explore” - Get worksheet ready - Organize strips - Begin working on Question 3 **Understand task:** • Use physical strips • Place side by side • Count and compare • Write answer **Get organized:** • Strips spread out on desk

	Set expectation: "Work together as a group, but everyone write answers in their own notebook or worksheet." **Emphasize process over speed:** **Say:** "I care more about your THINKING than your speed. Show me you understand by explaining your discoveries. Start exploring!"	• Worksheet/notebook ready • Pencil ready Look at Question 3 on Activity Sheet: "How many 1/4 = 1/2?" **Begin working**
3-9 min	**Circulate among groups - Questions 3-5** Watch for key moments: **Question 3 (How many 1/4 = 1/2?):** Students placing two 1/4 strips under 1/2 "Oh! They match!" **Listen for and highlight productive math talk:** [Example dialogue you might hear (Q3 - How many 1/4 = 1/2?):] **If you hear surface-level talk, probe:** [Weak answer: "They're the same"] **Say:** "HOW do you know they're the same? Show me with your strips." [Weak answer: "Because the teacher said so"] **Say:** "But can YOU prove it? What does comparing the strips tell you?" **Question 4 (How many 1/8 = 1/2?):** Students counting four 1/8 strips "It takes 4 eighths!" **Question 5 (How many 1/8 = 1/4?):** Discovery: "Two eighths equal one quarter" **Ask probing questions:** • "Show me what you did" • "How do you know they're equal?" • "Can you explain why?"	**Work through Questions 3-5:** **Question 3:** • Pick up 1/2 strip • Pick up 1/4 strips • Place two 1/4s under 1/2 • Discovery: "They match!" • Write: 1/2 = 2/4 **Student A:** "I think it's two because half means two pieces" **Student B:** "Wait, let me check with the strips... [places strips side by side]" **Student A:** "Oh! They match! Two quarters equal one half!" **Student B:** "So 1/2 = 2/4. Should we write that?" - Quick answers without reasoning - One student doing all the work **Question 4:** • Get 1/2 strip • Line up 1/8 strips beneath • Count: "One, two, three, four" • Discovery: "Four eighths!" • Write: 1/2 = 4/8 **Question 5:** • Take 1/4 strip • Line up 1/8 strips • Count: "Two eighths" • Write: 1/4 = 2/8 • Notice pattern: "Half as many!" **Group discussion:**

	If struggling: "Show me your 1/2 strip. Now put your 1/4 strips underneath. How many fit?" **If rushing without using strips:** **Say:** "Wait - actually use your strips to check. Line them up." **Say:** "Yes! You can see they're equivalent!" **When you see students making discoveries, USE QUESTIONS to prompt articulation** **Ask:** students to TRANSLATE their discovery into words: - "What does this show you? Can you put into words what you're seeing?" - "What do you notice about the lengths?" - "Can you explain what this shows in a sentence?" - "How do you KNOW they're equivalent? What's your evidence?" - "Can you use the word 'because' to explain your thinking?"	• "Look! These match!" • "I found another way!" • Comparing answers • Helping each other **Student:** "Oh! When I put two 1/4 strips next to the 1/2 strip, they're exactly the same!" **Teacher asks:** "What does this show you?" **Student:** "It shows that 1/2 equals 2/4!" **Explaining evidence:** **Teacher asks:** "How do you KNOW they're equivalent?" **Student:** "Because they're the same length. Same length means same amount." **Connecting to decimals:** **Teacher asks:** "What about the decimals?" **Student:** "Well, 1/2 is 0.5, and 2/4 is also 0.5, so they're the same!" **Struggling to articulate:** **Student:** "Um... they match?" **Teacher asks:** "Yes! And what does matching tell us?" **Student:** "That... they're equal?"
	Teacher Note: Key Moments - The Discovery Moment (Q3-5) WHEN: Students place strips side by side WHAT KIDS MIGHT SAY: "They match!" (or) "They're the same length!" (or) "Oh! They're equal!" WHAT THIS SHOWS: Visual proof of equivalence is clicking and the students SEE that different fractions can show the same amount. WHAT TEACHER MIGHT SAY: "What does it mean when the strips match exactly?" "Can you put that discovery into a math sentence?" "So, you're telling me that 1/2 equals 2/4 because...?" WHY IT MATTERS: This is the conceptual breakthrough and physical evidence proves mathematical equivalence. **Highlight and celebrate!**	
	IMPORTANT NOTE (Teacher) During circulation, you're having BRIEF (30 seconds - 2 minutes) conversations with INDIVIDUALS or PAIRS. **This is NOT whole-class discussion.** **Your role here:** Coach/facilitator moving among working students, asking questions to push individual thinking deeper IN THE MOMENT. **What this looks like:** - Move from student to student or pair to pair	

	- Ask: "What are you thinking?" "How do you know?" "Why does that work?" - Brief interaction (30 sec - 2 min), then move on **- Mental note:** "I'll ask this student to share in Explain phase" **DO NOT:** - Stop the whole class for discussion (that's in Explain phase) - Have one pair explain to whole class (that's in Explain) - Organize themes on board (that's in Explain) **SAVE WHOLE-CLASS SHARING FOR EXPLAIN PHASE.**	
9-12 min	**Question 6: "Three ways to make 0.5"** This is KEY - encourages creativity **Listen for strong reasoning:** • "One 1/2 strip" • "Two 1/4 strips" • "Four 1/8 strips" **Advanced thinking:** • "One 1/4 and two 1/8s!" (combining!) **ADVANCED THINKING:** **What kids might say:** - "When you double the denominator, the pieces get half as big, so you need twice as many to make the same amount. That's why 1/2 equals 2/4 equals 4/8." - "I noticed a pattern: 0.5 is half of 1, and 0.25 is half of 0.5, so I predicted 0.125 would be half of 0.25, and it was!" - "The decimal 0.25 makes sense because 25 is one-fourth of 100, just like 1/4 means one out of four parts." What makes this STRONG: - Explains the mathematical WHY - Identifies and extends patterns - Connects multiple representations - Uses proportional reasoning If you hear this, celebrate: **Say:** "Wow! You found a way I didn't think of!" "That's sophisticated thinking! You explained WHY the pattern works, not just THAT it works. Can you share this with the class?" **If groups stuck:** "What does 0.5 equal as a fraction?" [1/2] "So how many ways can you show 1/2?" **DEVELOPING THINKING**	**Question 6: Three ways to make 0.5** **During exploration: Students reason at different levels** **STRONG REASONERS:** **Use evidence:** - "I know because when I line up my strips, they match exactly" **Make connections:** - "1/2 equals 0.5, and 2/4 also equals 0.5, so they're equivalent" **Identify patterns:** - "The decimals get cut in half each time" **Engage in discussion:** - Explain their reasoning when prompted- Reference their strips as evidence- Listen to peers and agree/disagree with reasons **DEVELOPING REASONERS:**

	What kids might say: - "They're the same because they match." - "I think it's 0.5 because that's what it says on the strip." - "The pattern is they're getting smaller." (observation, no explanation) **What makes this DEVELOPING:** - Correct observations but limited explanation - Relies on tool/authority rather than reasoning - Sees patterns but doesn't explain why **What teacher might say (push thinking further):** "You noticed they match—good! Now, WHY does matching lengths tell us they're equivalent?" "You're right it says 0.5. But can you explain WHY 1/2 equals 0.5?" **Teacher Note:**	**Make observations:** - "They match" - "They're the same" - "The numbers are getting smaller" **Respond to teacher probing:** -**Teacher:** "Why does matching matter? " **Student:** "Um... because... same length means same amount?" **Teacher:** "Yes! You're connecting length to amount. That's the key!" **Build understanding through discussion:** - Hear peers' explanations - **Strengthen their reasoning:** "Oh, so it's not just that they match but it's that matching shows they're equal!" **ALL STUDENTS participate when teacher facilitates discussion:** - Listen when teacher highlights peer thinking - Respond to "What do others think?" - Build on classmates' ideas - Practice articulating mathematical reasoning
	Teacher Note: **HOW TO ASK GOOD QUESTIONS TO REVEAL THINKING:** **Probing Questions:** - "How do you know that?" (ask for evidence) - "Why does that work?" (ask for reasoning) - "Can you explain your thinking?" (ask for articulation) - "What made you think of that?" (ask about process) **Deepening Questions:** - "Will that pattern always work? How do you know?" - "Can you connect this to what we learned earlier?" - "Is there another way to think about this?" **Facilitation Questions (to create discussion):** - "So-and-so said [Name]. What do others think about that?" - "Who has a different way of explaining this?" - "Does anyone want to add to what [name] said?" **FACILITATION MOVES: Creating Mathematical Discussion:** When you hear interesting thinking (strong OR developing):	

<table>
<tr><td></td><td colspan="2">STEP 1 - Highlight it to the group: "Hold on, everyone. [Name] just said something interesting. [Name], can you repeat that for everyone?"
STEP 2 - Invite response: "What do others think about [Name]'s idea?"
"Does anyone want to build on what [Name] said?"
"Who agrees or disagrees? Why?"
STEP 3 - Encourage elaboration:"[Name], can you say more about that?"
"Can you show us with your strips what you mean?"
"Who can put [Name]'s idea into their own words?"</td></tr>
<tr><td></td><td colspan="2">Teacher Note: KEY MOMENT Creative Thinking (Q6)

WHEN: Students discover multiple ways to make 0.5
WHAT KIDS MIGHT SAY: "I found another way! 1/4 + 1/8 + 1/8 = 1/2!" (or) "You can do it with different combinations!" (or) "There's more than one answer!"
WHAT THIS SHOWS: Flexible thinking and students understand equivalence deeply enough to compose fractions in creative ways.
WHAT TEACHER MIGHT SAY: "Wow! You found a different combination.”” Can you show the class how 1/4 + 1/8 + 1/8 equals 1/2?" "Who else found a creative way to make 0.5?"
WHY IT MATTERS: This is advanced thinking! Highlight to class as example of mathematical creativity and deep understanding!! Celebrate!!</td></tr>
<tr><td>12-14 min</td><td>Question 7: "What fraction is 0.75?"

Connecting Fractions to Decimals [Deeper exploration begins (as promised in Introduction)]

What to listen for:
- "75 out of 100... so 75/100"
- "That's three quarters... 3/4!"
- Some may struggle to connect decimal to fraction

Probing questions to ask:
- "How did you figure out what fraction 0.75 represents?"
- "What does the 75 in 0.75 mean?"
- "Can you show me with your strips?"

Why this matters: This question shifts from discovering equivalence (Q3-6) to understanding the pattern in how fractions and decimals connect.

This is CHALLENGE - many need support

If struggling:
"Is 0.75 more or less than 0.5?"
[More]
"So you need more than half. Try three quarters..."</td><td>Question 7: What fraction is 0.75?

Thinking process (varies by student):

Strong thinkers: "0.75 = 75/100 = 3/4" (simplifies)
Developing thinkers: "0.75... maybe 3/4?" (uses strips to check)
Struggling thinkers: "I'm not sure..." (needs teacher support)

Discovery: Decimals represent fractions!

Challenge - need to think:
"0.75 is more than 0.5..."
"More than half..."

Try using strips:
• Start with 1/2 (0.5)
• Add another 1/4 (0.25)
• 0.5 + 0.25 = 0.75

Discovery:
• Count: Three 1/4 strips
• That's 3/4!
• Write: 0.75 = 3/4</td></tr>
</table>

	When they discover: Students place three 1/4 strips "That's 3/4 of the whole!" Extension for advanced: "Can you show 0.75 another way?" [Six 1/8 strips] "How did you know?" If stuck on decimal connection: "What's 0.25?" [1/4] "So 0.25 three times is...?" [0.75] "Which equals...?" [3/4] **Celebrate the "aha!" moments**	Some groups find: • Six 1/8 strips also = 0.75 • Write: 6/8 = 3/4 = 0.75 Feeling accomplished: • "We figured it out!" • "Now I understand!"
	Teacher Note: KEY MOMENT Challenge Success (Q7) WHEN: Students figure out 0.75 = ¾ WHAT KIDS MIGHT SAY: "I got it! 0.75!" (or) "I used my strips and patterns to figure it out!" (or) "This was hard but I did it!" WHAT THIS SHOWS: Students are applying strategies, using reasoning, and experiencing productive struggle leading to success WHAT TEACHER MIGHT SAY: "How did you figure that out? Walk me through your thinking." "What strategy did you use?" "You worked through a challenging problem and that's real mathematical work!" WHY IT MATTERS: Acknowledge the EFFORT and PROCESS, not just the answer. This builds **mathematical confidence and persistence.**	
14-25 min	**Questions 8-10: Pattern Discovery** **Question 8: Complete the table** Circulate - help with: • Finding equivalent fractions • Using strips to verify • Filling in decimals **Question 9: "What patterns do you notice?"** Listen for observations: • "Doubling makes equivalents" • "Same decimal = equivalent" • "Bigger denominator = smaller pieces" **Accept any mathematically accurate pattern** **Create productive cognitive conflict (select 1-2 advanced groups):** **[When group says "Doubling makes equivalents"]** **Probe:**	**Questions 8-10: Patterns** **Question 8: Fill in table** • Use strips to find equivalents • 1/4 = 2/8 • 3/4 = 6/8 • Fill in decimals using strips **Question 9: Notice patterns** Write observations: • "When you double both numbers, equivalent" • "Denominators of 2, 4, 8 are all related" • "Same decimal = same fraction" **Question 10: Predict 5/8** Think through it: • 4/8 = 0.5 (we know this) • 1/8 = 0.125 • So 5/8 = 0.5 + 0.125 = 0.625 Or: 5 ÷ 8 = 0.625

	Say: "If doubling works, does 1/4 = 2/10? Test it." [They discover it doesn't work] **Ask:** "So when DOES doubling create equivalents? What's the real rule?" **Question 10: "Predict 5/8 as decimal"** This is HARD - don't expect all to get it **Say:** "Question 10 is a challenge question. It's HARD—and that's the point! Hard problems help your brain grow. Here's how to approach it:" **Say:** "This question is tough. Some of you might figure it out, some might get part-way, and some might not get it at all. ALL of those outcomes are okay! What matters is that you TRY and you THINK." **Say:** "Here's what makes this hard: You can't just line up your strips to see the answer. You have to use PATTERNS and REASONING. That's higher-level thinking!" **Say:** "If you want to try it: - Look at the patterns you found in Questions 8-9 - Think about what you know about eighths - Use your 1/8 strip to help (how many eighths are in 5/8?) - Talk it through with your partner" **Say:** "If you get stuck: - That's NORMAL for challenge problems! - Write down what you tried and where you got stuck - We'll discuss strategies together in a few minutes" If they struggle: "What's 4/8?" [0.5] "What's 1/8?" [0.125] "So 5/8 = ?" [0.625] **Say:** "Remember: Struggling with a hard problem is how learning happens. If it feels difficult, your brain is working hard and that's good!" **Emphasize effort, not outcome:**	**Hear teacher's framing:** - This is HARD and that's okay! - Goal is to try and think, not necessarily to get it right - Struggle means learning **Different student responses:** **MOTIVATED STUDENTS:** - "Okay, a challenge! Let me try..." - "I'm going to use the patterns from the table" **- Think:** "If 1/8 is 0.125, and I have 5 of them..." **- Work through:** "0.125 + 0.125 + 0.125 + 0.125 + 0.125 = ?" - Some get it: "Oh! It's 0.625!" - Some get partially: "I think it has something to do with 0.125..." **UNCERTAIN STUDENTS:** - "This is hard... but I'll try" - **Talk to partner:** "What patterns did we find?" - **Attempt:** "Maybe I can add eighths?" - **Get stuck but document:** "I tried adding but I'm not sure if that's right" - **Think:** "I did my best and that's what matters" **STRUGGLING STUDENTS:** - "I don't know how to do this" - But feel okay because teacher said: "Struggle is normal" - **Try something:** "I'll at least look at my strips" - Or: "I'll write what I'm confused about" Don't feel defeated and feel encouraged to try ALL STUDENTS benefit from the framing: - Trying is valued over getting it right - Struggle is reframed as productive - Challenge is motivating, not discouraging

	"I'm going to be watching for: - Who takes a risk and tries, even if they're not sure - Who uses the patterns they discovered - Who talks through their thinking with a partner - NOT who gets the right answer fastest" **Note:** Questions 8-10 are extensions. Not all students will complete these. Focus on deep understanding of 3-7.	
	Teacher Note: KEY MOMENT Pattern Recognition (Q8-10) WHEN: Students articulate patterns in their own words WHAT KIDS MIGHT SAY: "Every time the denominator doubles, the decimal is half!" (or) "I see it going 0.5, 0.25, 0.125—each one is half!" (or) "The pattern is the pieces get smaller, so the decimal gets smaller too" WHAT THIS SHOWS: Mathematical thinking is developing and what students are seeing relationships, making generalizations, using mathematical language. WHAT TEACHER MIGHT SAY: "You articulated that pattern beautifully! Can you write it as a rule?" "That's sophisticated mathematical thinking!" "Does this pattern always work? How could we test it?" WHY IT MATTERS: Pattern recognition is core to mathematical reasoning. Hearing students articulate patterns shows they're **thinking like mathematicians**.	

Formative Checkpoint: Look at student work on Questions 3-7. Students should be able to:

Evidence of Mastery:	**Evidence of Developing Understanding:**	**Needs Additional Support:**
• Use strips to physically show equivalences (1/2 = 2/4 = 4/8) • Explain WHY fractions are equivalent ("because they match in length" not just memorize rules) • Connect fractions to decimals (1/2 = 0.5, 3/4 = 0.75) • Show creativity in Question 6 (finding multiple ways) • Successfully solve Question 7 challenge (0.75 = 3/4)	• Use strips to physically show equivalences (1/2 = 2/4 = 4/8) • Explain WHY fractions are equivalent ("because they match in length" not just memorize rules) • Connect fractions to decimals (1/2 = 0.5, 3/4 = 0.75) • Show creativity in Question 6 (finding multiple ways) • Successfully solve Question 7 challenge (0.75 = 3/4)	• Cannot correctly use strips to show equivalences • Doesn't understand connection between physical matching and equality • Cannot connect fractions to decimals • Struggles with Questions 3-5 (basic equivalences)

Important Note: Questions 8-10 are extensions - not all students will complete these, and that's okay. Focus on deep understanding of Questions 3-7. The Challenge questions can be tried at home.

Common Issues & Scaffolding Solutions:

Students NOT USING STRIPS, just guessing:
Don't just say: "You need to use your strips. That's what they're for."
Instead, help them see WHY strips are essential:

Teacher approaches student who wrote "1/2 = 2/4" without using strips: "I see you wrote that 1/2 equals 2/4. How do you KNOW that's true?"
Student: "Um... I just think it is?"
Teacher: "Okay, so you have a theory. Now let's TEST your theory. Take out your 1/2 strip and your two 1/4 strips. Place them next to each other. What do you see?"
Student: [Places strips] "Oh! They match!"
Teacher: "So now do you KNOW or are you guessing?"
Student: "Now I know! The strips prove it!"
Teacher: "Exactly! The strips turn guesses into PROOF. That's why we use them. For every answer, I want you to show me the strip evidence."
Student discovers: Strips provide proof, not just pretty pictures. They're mathematical tools.

ONE STUDENT DOMINATING, others not engaged:

Don't just say: "Everyone needs to participate. Let others have a turn."
Instead, structure the work to require ALL members' thinking:
Teacher observes one student doing all the work while others watch:
Teacher: "Hold on. I'm going to give you a challenge. Each person in your group needs to explain ONE question to me using the strips.
[Student A's name], you explain Question 3.
[Student B's name], you'll explain Question 4.
[Student C's name], Question 5."

[Students look nervous: the quiet ones don't know the answers yet]

Teacher: "You have 3 minutes to help EACH OTHER. [Dominant student], your job is to make sure everyone can explain their question. Okay?."
This forces collaboration: dominant student must TEACH, not just do
After 3 minutes, return and ask each student to explain their question.

Teacher to whole group: "Why did I do this? What's the point of everyone explaining?"
Students discuss: "So everyone understands?" "So, one person doesn't do all the work?"
Teacher: "Yes! EVERYONE'S thinking matters. When you work in groups, your job is to make sure everyone understands, not just to finish fast."
Students discover: Collaboration means everyone understands, not just dividing tasks.

Group STUCK ON QUESTION 7 (0.75 as a fraction):

Don't just say: "Use your strips to figure out 0.75. Look at how many eighths equal 0.75."
Instead, guide them through reasoning step-by-step:

Teacher: "You're stuck on 0.75. Let's think about what you already know. What patterns did you find in Questions 8-9?"
Students: "Um... the decimals go 0.5, 0.25, 0.125..."
Teacher: "Good! Now, where does 0.75 fit in that pattern? Is it bigger or smaller than 0.5?"
Students: "Bigger!"
Teacher: "So if 0.5 is 1/2, what fractions are BIGGER than 1/2? Look at your strips—which ones are longer than the 1/2 strip?"
Students: [Look at strips] "Um... 3/4? And 5/8?"
Teacher: "Good thinking! So, 0.75 might be one of those. Now, how can you TEST which one? What if you tried to see how many eighths equal 0.75?"

Students: [Work it out] "6 eighths! And 6/8 simplifies to 3/4!"
Teacher: "You figured it out! Walk me through your reasoning and how did you know to try 3/4?"
Students discover: How to use what they know (patterns, comparisons) to figure out what they don't know. They built a solution path.

WORKING PACE:

Don't just say: "If you finish early, move on to the next question" OR "Wait for everyone."
Instead, differentiate with purpose:

FOR FAST FINISHERS:
Teacher: "You finished Questions 3-7? Excellent! Now here's your challenge: Can you create a HARDER version of Question 6? Instead of finding three ways to make 0.5, find four ways to make 0.75. Then test it and check if it actually works?"

FOR SLOWER WORKERS:
Say: "I see you're working carefully and that's good! Let me check: are you using your strips to test each answer?"
[If yes:] "Great! Keep going at your pace. Understanding is more important than speed."
[If no:] "Let me show you a strategy. For Question 3, instead of thinking in your head, physically place your 1/4 strips along the 1/2 strip. How many fit? Let the strips do the work for you."

FOR MIDDLE GROUP:
Say: "You're making good progress! When you get to Question 7, remember to use the patterns you found in Questions 8-9. They'll help you."

Students discover:
• Fast pace doesn't mean stop thinking but it means think DEEPER
• Slow pace is okay if you're being thorough
• Different students need different challenges

Transition: "Alright, explorers! Let's pause the hands-on work. You've made incredible discoveries today and I've seen you comparing strips, finding patterns, making predictions. Now it's time to step back and **make sense of everything** you learned."

Time	Teacher Action	Student Action
17 – 19 mins	**Transition instructions:** **Say:** - "Put your strips aside for now and keep them on your desk but not in your hands." - "Put down your pencils." - "Eyes up here, ears open. We're going to have a mathematical discussion." **Frame the purpose of Explain phase:**	**Students transition from hands-on to discussion mode:** - Put strips down (but keep them visible for reference) - Close or set aside worksheets - **Shift body language:** face teacher/board **Mental shift:** from "doing" to "thinking about what we did"

	Say: "For the past 20 minutes, you've been discovering. Now we're going to do something equally important: **make sense of those discoveries together**. I want to hear YOUR thinking, YOUR observations, YOUR big ideas."	- Ready to share verbally **Some students thinking:** - "What did I discover?" - "What patterns did I notice?" - "I'm ready to share my thinking"

2.6 Explain: Making sense of Fraction-Decimal Equivalence (20-25 minutes)

Purpose: Students verbally articulate their discoveries, collectively identify big ideas, formalize mathematical vocabulary, and reflect on conceptual change. This phase transforms individual exploration into shared understanding.
Activity: Students share discoveries in whole-class discussion, identify three key themes (equivalence, connection, patterns), reflect on "I used to think... Now I think," and co-construct mathematical understanding with teacher synthesis.

Time	Teacher Actions	Student Actions
0-3 min	**Launch the discussion and solicit big ideas.** **Say:** "You've done incredible exploration working with your fraction strips. Now let's make sense of what you discovered. "Write on board (large, visible): **BIG IDEAS FROM TODAY** **Frame the discussion:** **Say:** "Think about everything you explored in the last 20 minutes. What did you DISCOVER? What did you LEARN? What surprised you?" **Give 30 seconds think time:** **Say:** "Take 30 seconds to think silently. What's ONE big idea you learned?" **[Wait - honor the think time]** **Invite sharing:** **Ask:** "Who wants to share a big idea you discovered?" **Listen and acknowledge (don't evaluate yet):**"Interesting! What else?"	**Transition to discussion mode** Sit up, look at teacher and board. Strips still on desk (for reference if needed) **Think individually (30 seconds):** • "What did I discover?" • "What was most important?" • "What surprised me?" **Different students thinking:** • "I learned that 1/2 equals 2/4" • "I found patterns in the decimals" • "I discovered fractions and decimals are connected" **Prepare to share:** Raise hands Ready to articulate their thinking
3-10 min	**Facilitate theme identification and organization** As students share discoveries, ORGANIZE them into themes on board.	**Share discoveries verbally** Multiple students volunteer and respond: **Student A:** "We found that 1/2 equals 2/4 equals 4/8"

	Listen for and categorize statements like: Teacher: [Write under THEME 1] "So you discovered different fractions can show the same amount. That's about equivalence! Teacher: [Add to THEME 1] "You're explaining HOW you proved equivalence through matching lengths. Teacher: [Write under THEME 2] "You discovered a connection between fractions and decimals! Teacher: [Write under THEME 3] "You noticed a pattern! Mathematical patterns help us predict. "**Board organization emerges:** **THEME 1: EQUIVALENCE** • Different fractions = same amount • 1/2 = 2/4 = 4/8 • Matching length proves equivalence **THEME 2: FRACTION-DECIMAL CONNECTION** • Same amount, different notation • 1/2 = 0.5 • 1/4 = 0.25 • 3/4 = 0.75 **THEME 3: PATTERNS** • Decimals halve: 0.5 → 0.25 → 0.125 • As denominators double, pieces get smaller • Patterns help us predict **BRIDGE TO REAL WORLD:** **Say:** "Let's think about WHY these matters outside math class. When does understanding fraction-decimal equivalence help you in real life?" **Call on 2-3 students:** [Student responses (authentic examples):] **Teacher affirms and extends:**	**Student B:** "When strips match, they show the same amount" **Student C:** "I discovered that 1/2 and 0.5 are the same thing" **Student D:** "There's a pattern and the decimals go 0.5, 0.25, 0.125" **Student E:** "We can prove fractions are equal by comparing strip lengths " **Listen to peers:** • Nod in agreement • **Build on ideas:** "Oh yeah, we found that too!" • **Make connections:** "That's like what I was going to say" - "When I shop, prices show ₹0.50 - that's the same as half a rupee" -"When I ask the shop keeper for 1/2 liter of oil he gives 500 ml packet" - "When I measure things, like 0.25 meters is the same as 1/4 meter"

	Say: "Exactly! Money, measurements - they ALL use this connection. That's mathematical power!" **Write on board:** 1/2 = 0.50 = half a rupee = 50 paise **Say:** "Four different ways to express the SAME quantity. You choose which makes sense for the situation." **Ask probing questions to deepen:** • "Can someone explain WHY matching lengths prove equivalence?" • "How exactly are fractions and decimals connected?" • "Why does that pattern work?" • "Who can add to what [Name] said?" **Facilitate peer-to-peer discussion:** "So-and-so said [X]. What do others think about that?" "Does everyone agree? Who has a different way to explain?" **As themes solidify:** "I'm hearing three BIG IDEAS emerging. Let's name them clearly."	**Respond to probing:** **Student:** "Because... if they're the same length, they show the same amount of the whole. " **Student:** "They're different ways to write the same number." **Watch themes emerge on board:** • "Oh, there are big categories!" • "Everything we discovered fits into these three ideas" • "I see how it all connects now" **Some students have 'aha' moments:** • "I didn't realize there were patterns!" • "Now I see why we did all that exploration" • "The strips helped me understand these ideas"
10 – 16 min	**Facilitate metacognitive reflection: "I used to think... Now I think..."** **Say:** "We've identified three big ideas. Now I want you to think about YOUR learning journey today." Write on board: **I used to think...Now I think...** **Explain the reflection:** **Say:** "Think about what you believed about fractions and decimals BEFORE today, maybe this morning or from previous classes. Has your thinking CHANGED because of what you explored today?" **Model it yourself first:**	**Individual reflection (1 minute)** **Think silently and deeply:** • "What did I believe before today?" • "What do I believe now?" • "How has my thinking changed?" **Different students' internal thoughts:** **Strong student:** • **Used to think:** "Fractions and decimals are separate" • **Now think:** "They're connected and represent same amounts in different notation" **Developing student:** • **Used to think:** "I didn't understand equivalence" • **Now think:** "When fractions have the same amount, they're equivalent"

	Say: "Let me share mine. I **used to think** fractions and decimals were two completely separate topics that we teach separately. **Now I think** they're deeply connected and they're two different notations for the same mathematical quantities. What we learned today is that understanding this connection helps students see numbers more flexibly." **Give 1 minute for individual thinking:** **Say:** "Take one minute. Think silently. What did you used to think? What do you think now?" **[Honor the silence: this is thinking time]** **Invite sharing:** **Say:** "Who would like to share their 'I used to think, now I think'?" **As students share, record key transformations:** **Teacher:** [write & say] "Decimals = another way to show fractions" **Teacher:** [write & say] "Different fractions can be equivalent" **Teacher:** [write & say] "Visual tools help prove equivalence" **Affirm transformations:** "These are powerful conceptual changes! Your thinking has grown today."	**Struggling student:** • **Used to think:** "Fractions are confusing" • **Now think:** "Strips help me see what fractions mean" **Verbal sharing:** **Students volunteer to share:** **Student 1:** "I used to think you couldn't compare fractions. Now I think strips make it easy." **Student 2:** "I used to think decimals were completely different from fractions. Now I think they're related." **Student 3:** "I used to think 1/2 and2/4 were different. Now I know they're the same amount." **Student 4:** "I used to be confused about equivalence. Now I understand it means same amount." **Listen to peers' transformations:** • See that others had similar misconceptions • **Feel validation:** "I'm not the only one who was confused!" • Celebrate learning growth together **Sense of accomplishment:** "My thinking really changed today!" "I learned something important"
16 – 23 min	**Synthesize and formalize understanding** **Say:** "Let me bring it all together. Here's what WE as a class discovered today." [Point to themes on board systematically] **Synthesize Theme 1:** **Say:** "First, we discovered EQUIVALENCE. Different fractions can represent the same amount. We proved this with our strips and when strips match in length, the fractions are equivalent. So, 1/2 = 2/4 = 4/8. Same amount, different names." **Formalize vocabulary:** **Say:** "The mathematical term for this is **equivalent fractions:** That is fractions	**Listen to synthesis** Watch teacher pulls themes together **See the big picture:** "Everything connects!" • "All our discoveries fit into these three ideas" • "This makes sense now" **Learn/reinforce vocabulary:** **Repeat:** "Equivalent fractions" **Understand:** Fractions that show same amount

	that represent the same quantity."[Write term on board, have students repeat] **Synthesize Theme 2:** **Say:** "Second, we discovered the FRACTION-DECIMAL CONNECTION. These aren't separate topics but they're two ways to write the same numbers. When we say 1/2 = 0.5, we're saying one-half and five-tenths are the same amount [write the quantities on the board]. The notation looks different, but the quantity is identical." **Synthesize Theme 3:** **Say:** "Third, we discovered PATTERNS. Mathematics is full of predictable relationships. You all noticed that as denominators double, the decimal values halve: 0.5 → 0.25 →0.125. These patterns aren't random but they're mathematical structure. Understanding patterns helps us predict and extend our knowledge." **Connect back to Quick Start:** **Say:** "Remember at the start of class when you predicted which chocolate bars were equal? Now you can PROVE it. That's what learning looks like: moving from guessing to knowing." **Preview next lesson:** **Say:** "Tomorrow we'll explore WHY these patterns work. We'll use place value to understand the relationship between fractions and decimals more deeply." **Affirm their work:** **Say:** "You did real mathematical work today by exploring, discovering, making sense. That's what mathematicians do. Be proud of your learning!"	**Make connections:** • Connect synthesis to own exploration • "Oh, that's what we found in Question 5!" • "The pattern we noticed was real!" **Connect to Quick Start:** • "Oh yeah, we were guessing then" • "Now we can prove it with strips" • **See learning journey:** guess → explore → know **Feel consolidation:** • Scattered discoveries now organized • "I understand what we learned today" • Feel prepared for next lesson **Sense of accomplishment:** • "We really learned something important" • "I did mathematical work today" • "I'm ready to use this knowledge" **Ready to transition to Close:** • Understanding is consolidated • Ready for exit ticket • Confident in learning

FORMATIVE CHECKPOINT FOR EXPLAIN PHASE: How do you know students are making sense?		
Evidence of Strong Understanding	**Evidence of Developing Understanding**	**Needs Additional Support**
• Articulates discoveries clearly using mathematical language • Connects ideas across themes • Explains WHY, not just WHAT	• Shares observations but needs prompting for explanation	• Struggles to articulate discoveries • Relies on others' ideas without understanding

• Builds on peers' ideas • Shows metacognitive awareness in "I used to/now I think"	• Makes connections with teacher support • Can restate peers' ideas • Shows some awareness of learning change	• Cannot explain thinking • "I used to/now I think" is vague or missing
Teacher Note: For developing/struggling students during Explain: • Ask simpler, more concrete questions: "Show me with your strips what you discovered" • Partner with strong articulator: "Can you and [name] explain together?" • Revoice their partial ideas: "So you're saying that when strips match, they show...?" [student completes] • Validate attempts: "You're on the right track. Let's build on that."		
Transition: Say: "You've explored, you've discovered, and you've made sense of big mathematical ideas. Now I want to see if you can USE this understanding independently."		
Time	**Teacher Action**	**Student Action**
23 – 25 min	**Transition instructions:** **Say:** "Take out a clean piece of paper or turn to the exit ticket section in your notebook." "This is individual work and I want to see YOUR thinking." **Frame the exit ticket:** **Say:** "This isn't a test. It is a way for me to understand what you learned and what we might need to revisit tomorrow."	- Get paper/notebook ready - **Mental shift:** From discussion to individual work- Feel confident: "I can do this as we just talked about these ideas" **Ready to show learning** **Some students thinking:** - "I understand this and I can explain it" - "I'll use what we talked about in the Explain phase" - "Let me show what I learned"

2.7 Close (5 minutes)

Purpose: Individually assess student understanding of core concepts and provide teacher with formative data for next lesson planning.
Activity: Students independently complete exit ticket, engage in brief personal reflection, and teacher collects responses for conceptual understanding analysis.

Time	Teacher Actions	Student Actions
0-3 min	**Administer Exit Ticket** **Say:** "You've explored, discovered, and made sense of fraction-decimal equivalence today. Now I want to see YOUR individual understanding." Display exit ticket question on board (write large and clear): **EXIT TICKET** **Question:** Uma says that 6/8 and 3/4 are equivalent fractions. Arun disagrees and says they're different.	**Receive exit ticket task** **Read question on board carefully:** • Uma says 6/8= 3/4 • Arun says they're different • Who's correct?

	Who is correct? Use what you learned today to explain your answer. You can draw strips or use words. Show your thinking! **Set expectations:** **Say:** • "This is INDIVIDUAL work and no talking, no looking at partners" • "I want to see YOUR understanding" • "You have 3 minutes" • "Show me your thinking and explain WHY, not just what" **Clarify if needed:** If student asks "Can I draw strips?" → "Yes! Show me how you'd use strips to figure it out." If student asks "How long should my answer be?" → "Quality over length. I care that you EXPLAIN your reasoning." **Circulate silently:** • Don't answer math questions • Don't give hints • Observe: Who writes confidently? Who struggles? • Mental note: This is formative data **Time warnings:** • At 1.5 min: "You have 1.5 minutes left" • At 2.5 min: "30 seconds: finish your thought"	**Understand expectations:** • Work alone • Show thinking • Explain why **Write response individually (3minutes):** **Strong understanding- writes:** • "Uma is correct. 6/8 equals 3/4 because if you simplify 6/8 (divide both by 2), you get 3/4. • "I can prove it with strips. If I place six 1/8 strips next to three 1/4 strips, they match in length. Same length means same amount, so they're equivalent." • Shows conceptual understanding with explanation **Developing understanding- writes:** • "Uma is right 6/8 is the same as 3/4." • Correct answer but limited explanation • Draws strips correctly but doesn't explain what it proves **Needs support- writes:** • "Arun is correct because 6 and 8 are different from 3 and 4" • Incorrect answer, doesn't understand equivalence • "I don't know" or leave blank • Confuses fractions with whole numbers **All students:** • Work silently • Focus on explaining thinking • Some confident, some uncertain • Write for full 3 minutes
3-4 min	**Brief Individual Reflection** **Say:** "Pencils down. Don't talk yet. Just think." "Take one minute to think silently about today's lesson." **Think about:** • "What's one thing you understand now that you didn't understand this morning?"	**Individual quiet reflection** Stop writing Put pencil down Think silently **Internal thinking:** **Strong student:** • "I understand now that different fractions can show the same amount"

	• "How did the fraction strips help you learn?" • "What question do you still have?" **Honor the silence:** • Give full minute for thinking • Students don't share out (we did that in Explain phase) • This is personal, quiet reflection **Purpose:** Brief moment for students to consolidate learning individually. We already did collective metacognition in Explain and this is personal closure.	• "The strips made it so clear!" • "I'm curious about more complex fractions" **Developing student:** • "I get that 6/8 and 3/4 are equal" • "I still need to practice with other fractions" • "The strips helped me see it" **Struggling student:** • "I'm still confused about some parts" • "I understand strips match, but I'm not sure about decimals • "I need more practice" **Personal consolidation:** • Moment to process learning • No pressure to share • Individual sense-making
4-5 min	**Collect, Affirm, and Dismiss** **Say:** "Thank you for your thinking today. You did real mathematical work by exploring with your hands, thinking with your minds, and explaining with your words. That's what mathematicians do." **Collect exit tickets:** **Say:** "Please pass your exit tickets to the front." [Collect papers efficiently] Affirm the learning process (not outcomes): **Say:** "Today you:" • "Discovered equivalence through hands-on exploration" • "Made sense of fraction-decimal connections together" • "Reflected on how your thinking changed" "That's powerful learning. Be proud of your mathematical work today." **Brief preview:** **Say:** "Tomorrow we'll explore WHY these patterns work using place value." **Dismiss:** "You may pack up. Great work today!"	**Hear affirmation** **Feel validated:** • Not about being "right" • About thinking and learning • Process is valued **Pass exit tickets forward:** • Hand papers to front • Wait quietly **Listen to summary:** • Recall what we did today • Feel sense of accomplishment • "I really learned something" **Different student feelings:** **Confident:** • "I nailed that exit ticket!" • "I really get this now" • "I'm ready for tomorrow" **Developing:** • "I think I did okay" • "I understand some of it" • "I want to learn more" **Uncertain:** • "That was hard, but I tried" • "I hope I get more practice" • "I did my best" **Pack up and leave:** • Feeling affirmed • Sense of closure

	As students leave: • Smile, make eye contact **Affirm individuals:** **Say:** "Nice thinking today" • Keep exit tickets organized for sorting	• Ready for next lesson

Formative Checkpoint (From Exit Ticket): Exit Ticket Assessment Guide:

Teacher Action After Class:

STEP 1: Sort Exit Tickets by Conceptual Understanding (Not Just Correct/Incorrect). Analyze the DEPTH and TYPE of understanding shown.

Category 1: Strong understanding (Conceptual change has occurred)
What you see in exit ticket:
• Correct answer (Student 1 is correct)
• Clear explanation of WHY (uses equivalence concept)
• Evidence-based reasoning (strips, simplification, etc.)
• Uses mathematical vocabulary appropriately
Examples:
• "Uma is correct because 6/8 simplifies to 3/4. When I divide both numbers by 2, I get 3/4." • "I used strips. Six 1/8 pieces equal three 1/4 piece in length. Same length = same amount = equivalent fractions."

What this means for next lesson:
✓ Ready to move forward to Lesson 2
✓ Can explore place value and decimal relationships
✓ May benefit from extension/challenge problems

Category 2: Developing understanding (Conceptual change in progress)
What you see in exit ticket:
• Correct answer BUT limited explanation
• Partial reasoning (mentions strips but doesn't explain what they prove)
• Correct visual representation without verbal explanation
• Beginning to use concepts but not fluently

Examples:
• "Uma is right. 6/8 equals 3/4." [No explanation why]
• "The strips match." [Doesn't explain what matching proves]
• Draws correct strip comparison but no written explanation
What this means for next lesson:
Can move to Lesson 2 BUT needs targeted support
• During Lesson 2, check in with these students specifically
• Ask probing questions: "How do you know? Why does that work?"
• Pair with strong explainers for peer learning
• May need small group re-teaching of equivalence concept

Category 3: Fragile understanding (Conceptual change not yet achieved - misconceptions persist)
What you see in exit ticket:

• Incorrect answer (says Arun is correct)
• Explanation reveals misconception about equivalence
• Treats fractions as separate whole numbers
• "I don't know" or blank response
• Random guessing with no reasoning

Examples:
•"Arun is right because 6 and 8 are bigger numbers than 3 and 4, so 6/8 is bigger."
•"They're different because the numbers are different."
• Leaves blank or writes "I'm confused"
What this means for next lesson:
NOT ready for Lesson 2 without intervention MUST provide additional support:
• Small group re-teaching before or during Lesson 2
• Use strips again to rebuild equivalence understanding
• One-on-one check-in to diagnose specific misconception
• Possibly need to slow down and may need two lessons on L1 content

STEP 2: Reflect on Learning Goals
[Return to the learning goals mentioned at the start of the lesson. Based on exit tickets AND observations throughout the lesson:]
Goal 1: Equivalence - Did students understand that different fractions can represent the same quantity?
Evidence to consider:
- Exit ticket: How many students explained equivalence correctly?
- Investigation Part 2: Did students discover that 1/2 = 2/4 = 4/8?
- Explain phase: Were the students able to articulate why matching strips prove equivalence?
Reflection questions:
- What percentage of students are in each category?
- What patterns do I see in misconceptions?
- Did my teaching effectively support conceptual change?
Goal 2: Fraction-decimal connection - Did students understand they're two notations for the same amount?
Evidence to consider:
- Exit ticket: Did any students reference decimals in their explanation?
- Investigation Part 2: Were students comfortable in labeling strips with both representation?
- Explain phase: Did students articulate the connection during theme identification?
Reflection questions:
- Is this connection solid, or still fragile?
- Do students see fractions and decimals as related, or separate?
- Do I need more emphasis on this tomorrow?

Goal 3: Visual proof - Did students use strips as mathematical evidence?
Evidence to consider:
- Exit ticket: Did students reference strips in their reasoning?
- Investigation Part 2: Did they use strips purposefully or randomly?
- Explain phase: Did students say things like "I know because the strips matched"?
Reflection questions:
- Do students see strips as PROOF tools or just manipulatives?
- Can they transfer this reasoning to other representations?

Goal 4: Patterns - Did students identify predictable relationships?

Evidence to consider:
- Investigation Part 2 Q (8-9): Did students complete the pattern table?
- Explain phase: Did anyone articulate the halving pattern?

Reflection questions:
- Was pattern recognition too rushed?
- Should I spend more time on this in Lesson 2?

STEP 3: Make instructional decisions for next lesson

Based on your sorting and reflection work on the decisions before moving to next lesson
IF: Majority (>60%) in Category 1 (Strong Understanding)
THEN: Move forward to Lesson 2 with confidence
ACTION: Plan small group pull-out for Categories 2 & 3 students
TIME: 5-10 min small group while others work independently

IF: Majority (>50%) in Category 2 (Developing Understanding)
THEN: Start Lesson 2 with brief review/reinforcement
ACTION: First 10 min of Lesson 2 (whole class) = "Let's review what we learned yesterday" Use different examples (Can be the exit ticket problem) to reinforce equivalence
TIME: Don't reteach whole lesson, just strategic reinforcement

IF: Significant group (>40%) in Category 3 (Fragile Understanding)
THEN: STOP. Don't move to Lesson 2 yet
ACTION: Next class period = re-teaching Lesson 1 content with different approach - Use different manipulatives (maybe 10x10 grids instead of strips) - More explicit direct instruction on equivalence - Slower pace, more practice
TIME: Full class period

STEP 4: Plan specific scaffolding & interventions

FOR CATEGORY 1 STUDENTS (Strong understanding):
- No intervention needed- Provide extension in Lesson 2: "Can you predict what 7/8 would be as a decimal?"- Use as peer tutors if appropriate

FOR CATEGORY 2 STUDENTS (Developing understanding):
- Check in during Lesson 2: "Can you explain why 2/10 equals 0.2?"- Provide sentence frames: "These fractions are equivalent because ____"-
Ask probing questions to push articulation- small group (5-10 min) if needed

FOR CATEGORY 3 STUDENTS (Fragile understanding):
- Small group re-teaching (required)
- Use concrete manipulatives again
- Diagnose specific misconception: Do they understand what fractions represent?
- May need one-on-one conversation- Consider: Do they have prerequisite gaps? (What IS a fraction?)

STEP 5: Document for continuity

Keep brief notes (Example): - "8 students need small group on equivalence concept"
- "Common misconception: treating fractions as separate whole numbers"
- "Most students solid on fraction-decimal matching"
- "Need more time on patterns in Lesson 2"
This documentation helps you (and any co-teacher) provide continuity across lessons.

LESSON ACTIVITY SHEET

Exploring Fraction-Decimal Equivalence

Name:	
Date:	
Group Members:	

Instructions: Today you will explore how fractions and decimals are equivalent using physical fraction strips. Work with your group, but everyone should write answers in their own notebook or on this sheet.

Part 1: Creating Your Fraction Strips (8-10 min)
What You Need:
• Fraction strip template
• Scissors
• Ruler

Step Number	Steps
1	Cut out all strips from your template: 1 whole strip 2 half strips (1/2) 4 quarter strips (1/4) 8 eighth strips (1/8)
2	Label each strip with fraction AND decimal: 1 whole = 1.0 1/2 = 0.5 1/4 = 0.25 1/8 = 0.125

IMPORTANT: Show your completed strips before continuing.

Part 2: Exploring Equivalences (17-20 min)
Important: Use your physical strips to answer these questions! Place them side-by-side and compare.

Question 3: How many 1/4 strips equal one 1/2 strip?
Answer: _____ quarter strips = _____ half strip
This means: 1/2 = _____/4

Question 4: How many 1/8 strips equal one 1/2 strip?
Answer: _____ eighth strips = _____ half strip
This means: 1/2 = _____/8

Question 5: How many 1/8 strips equal one 1/4 strip?
Answer: _____ eighth strips = _____ quarter strip
This means: 1/4 = _____/8

Question 6: Use your strips to show THREE different ways to make 0.5 (one half):

Way 1: _____

Way 2: _____

Way 3: _____

Question 7: What fraction is the same as 0.75? Use your strips to find out.

Answer: 0.75 = _____ (as a fraction)

Explain how your strips helped you:

_____

_____

Question 8: Complete this table using your fraction strips:

Fraction	Equivalent Fractions	Decimal
1/2	2/4, 4/8	0.5
1/4		0.25
3/4		
2/8		
6/8		

Question 9: What patterns do you notice in your table? Write at least TWO observations:

Pattern 1:

_____

_____

Pattern 2:

_____

_____

Question 10 (Hard): Based on your patterns, what would 5/8 be as a decimal?

My prediction: 5/8 = _____

How I figured this out:

_____

_____

CHALLENGE PROBLEMS: (If you finish early OR for homework 😊)

Problem 1: Pizza Party/Dosai Party 🍕
At a party, you ate 1/4 of a pizza and your friend ate 2/8 of the same pizza. Did you eat the same amount? Prove it using fractions AND decimals.

Answer:

Problem 2: Ribbon/fabric Cutting ✂️
You have a ribbon that is 1 meter long. You cut it into pieces that are each 1/8 meter. How many pieces do you get? What is 1/8 meter as a decimal?

Answer:

Problem 3: Pattern Discovery 🔍
We explored halves (1/2), quarters (1/4), and eighths (1/8). What comes next in this pattern? What would 1/16 be as a decimal? Explain your reasoning.

Answer:

Reflection:
What was the most interesting thing you discovered today?

_____

_____

✓ IMPORTANT INSTRUCTION: Keep your fraction strips safe! You'll need them for tomorrow's lesson.

APPENDICES (Teacher Guide)

APPENDIX A: ANSWER KEY

This appendix contains complete answers to all activities, sample student responses, common misconceptions, and assessment guidance.

Quick Start - Answers

Question 1: Rectangle A: 4/8 = 1/2 = 0.5
Rectangle B: 2/4 = 1/2 = 0.5
Rectangle C: 1/4 = 0.25

Question 2: Rectangles A and B show the same amount (both equal 1/2).

Investigation Part 2 - Answers

Question 3: 2 quarter strips = 1 half strip → 1/2 = 2/4
Question 4: 4 eighth strips = 1 half strip → 1/2 = 4/8
Question 5: 2 eighth strips = 1 quarter strip → 1/4 = 2/8
Question 6 - Three ways to make 0.5:
• One 1/2 strip
• Two 1/4 strips
• Four 1/8 strips
Advanced: One 1/4 + two 1/8 strips
Question 7: 0.75 = 3/4 = 6/8
Question 8: Pattern Discovery Table – Completed

Fraction	Equivalent Fractions	Decimal
1/2	2/4, 4/8	0.5
1/4	2/8	0.25
3/4	6/8	0.75
2/8	1/4	0.25
6/8	3/4	0.75

Question 9: Reflection (Descriptive)
Question 10: 5/8 = 0.625
Reasoning: 4/8 = 0.5, and 1/8 = 0.125, so 4/8 + 1/8 = 0.5 + 0.125 = 0.625

Challenge Problems – Answers

Problem 1 (Pizza):	**Problem 2 (Ribbon):**	**Problem 3 (Pattern):**
a) Yes b) 2/8 = 1/4 c) 0.25 = 0.25	a) 1/8 meter = 0.125 meters per piece b) 8 pieces c) 8 eighth-pieces = 1 meters	a) Each fraction/decimal is half the previous b) 1/16 = 0.0625 d) As denominator increases, decimal value decreases

Exit Ticket - Answer

1. Fraction: 6/8
2. Simplified: 6/8 = 3/4
3. Decimal: 0.75
4. Show with strips: Six 1/8 strips or three 1/4 strip

APPENDIX B: MATERIALS & PREPARATION

Materials List

Per Student	**Per Group** (3-4 students)	**Per Class**
Notebook, pencil, ruler	Fraction strip templates, scissors	Chart paper, markers, demo strips

Preparation Checklist

One Week Before:	**One Day Before:**	**Day Of (10 min before class):**
☐ Print fraction strip templates (1 per group) ☐ Print investigation worksheets (1 per student) ☐ Create teacher demonstration set	☐ Organize materials by group ☐ Prepare board/display ☐ Review lesson plan	☐ Set up demonstration area ☐ Arrange room for group work ☐ Display Quick Start questions ☐ Place materials accessible

Alternative materials (If resources limited)

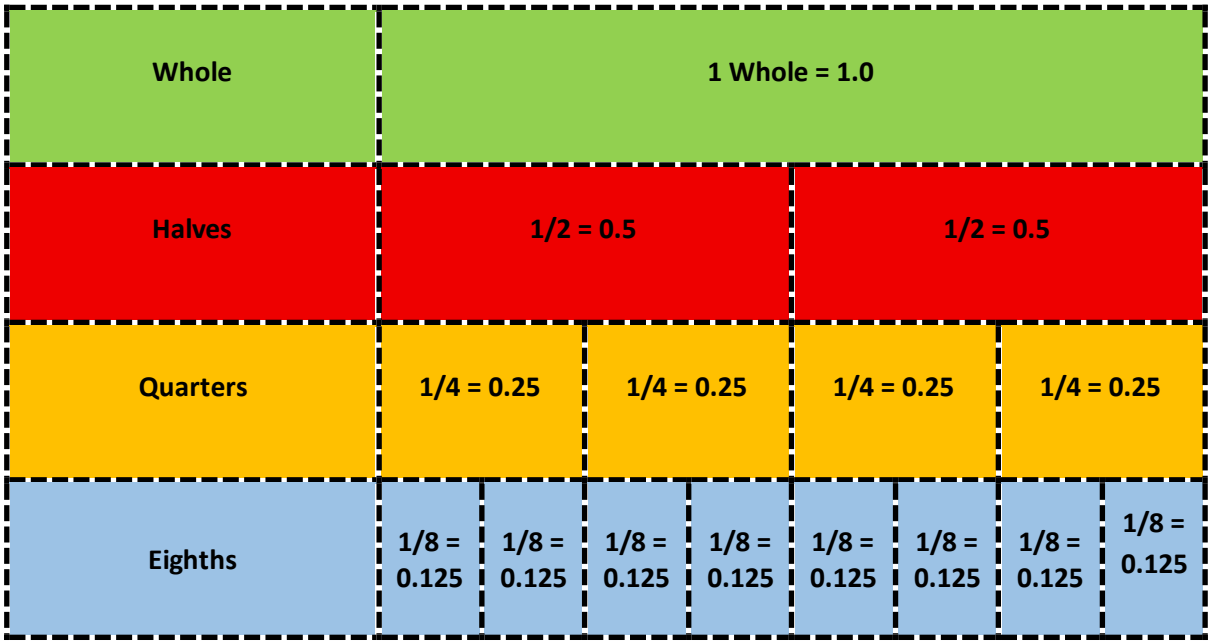
- **If NO printed templates:** Students draw and measure own strips using rulers (provide dimensions)
- **If NO scissors:** Pre-cut strips or students tear carefully

Material costs

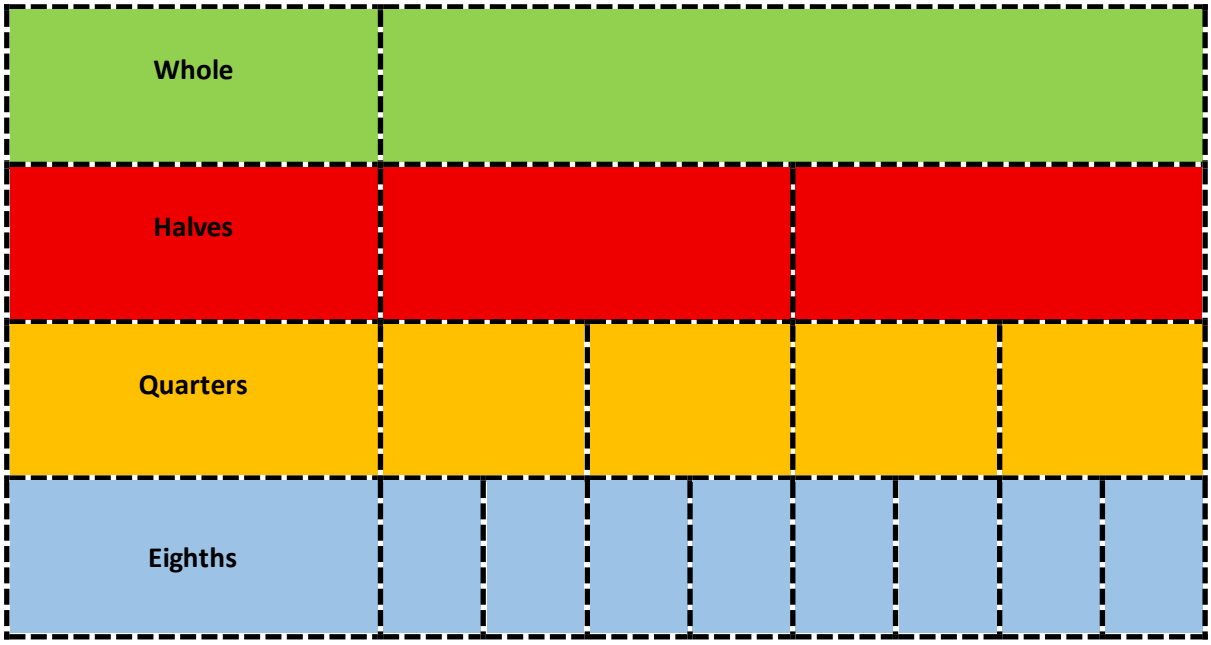
For class of 30 students (approximate):

- Printing (40 pages): ₹40-80
- Markers: ₹20
- Scissors (if buying): ₹100-200
- Total: ₹160-300 (one-time investment for multiple topics)

Teacher Demo: Fraction strip template



Student Handout: Fraction strip template



APPENDIX C: LANGUAGE SUPPORT

Bilingual Support (Tamil/English)

English	Tamil	Pronunciation
Fraction	பின்னம்	Pinnam
Decimal	தசம	Dasama
Equivalent	சமான	Samāṉa
Whole	முழு	Muḻu
Half	பாதி	Pāthi
Quarter	கால்	Kāl

APPENDIX D: PRINTABLE MATERIALS

Printable 1: Quick Start Handout	(Optional - can display on board instead) Shows three shaded rectangles with questions about fractions and equivalence.
Printable 2: Fraction Strip Template (Print on cardstock (thick gsm) if possible)	Template includes: (Teacher & Student templates) • 1 whole strip • 2 half strips (1/2 = 0.5) • 4 quarter strips (1/4 = 0.25) • 8 eighth strips (1/8 = 0.125) Instructions: Cut along dotted lines, ensure labels are clear.
Printable 3: Investigation Worksheet	Includes: Lesson 1 activity sheet, Challenge problems, Exit ticket