MTH 212 - Real Analysis new lect 1c

Continuity of a function

Definition:

Let f be a function of a real variable x defined on the Euclidean space E. Then f is said to be continuous at the $x_0 \in D \subset E$ if given $\varepsilon > 0$ there is a number $\delta(\varepsilon) > 0$ such that

$$|f(x) - f(x_0)| < \varepsilon$$

whenever $|x - x_0| < \varepsilon$

In other words a function $f(x)$ is continuous if it has a limit L.

That is to say

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$|f(x) - L| < \varepsilon$

whenever L is a the limit of f(x)

This implies that

$|f(x) - L| < \varepsilon$

$\Rightarrow -\varepsilon < f(x) - L < \varepsilon$

$\Rightarrow L - \varepsilon < f(x) < L + \varepsilon$

$\Rightarrow f(x) \in \{L - \varepsilon, L + \varepsilon\}$