AI6101
Introduction to AI and AI Ethics

Markov Decision Process

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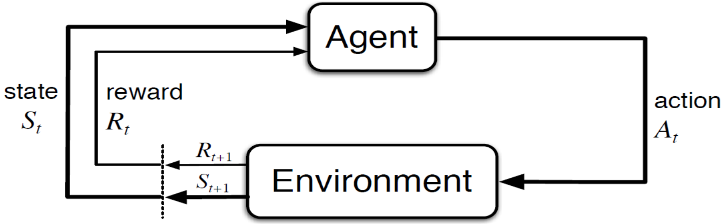
Lesson Outline

- Introduction
- Markov Decision Process
- Two methods for solving MDP
  - Value iteration
  - Policy iteration
- Temporal difference learning

Introduction

- We consider a framework for decision making under uncertainty
- Markov decision processes (MDPs) and their extensions provide an extremely general way to think about how we can act optimally under uncertainty
- For many medium-sized problems, we can use the techniques from this lecture to compute an optimal decision policy
- For large-scale problems, approximate techniques are often needed (more on these in later lectures), but the paradigm often forms the basis for these approximate methods

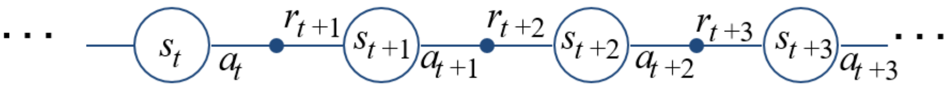
The Agent-Environment Interface



Agent and environment interact at discrete time steps: $t = 0,1,2,\ldots$

Agent:

1. observes state at step t: $s_t \in S$
2. Produces action at step t: $a_t \in A(s_t)$
3. Gets resulting reward: $r_{t+1} \in \Re$ and resulting next state: $s_{t+1} \in S$



Making Complex Decisions

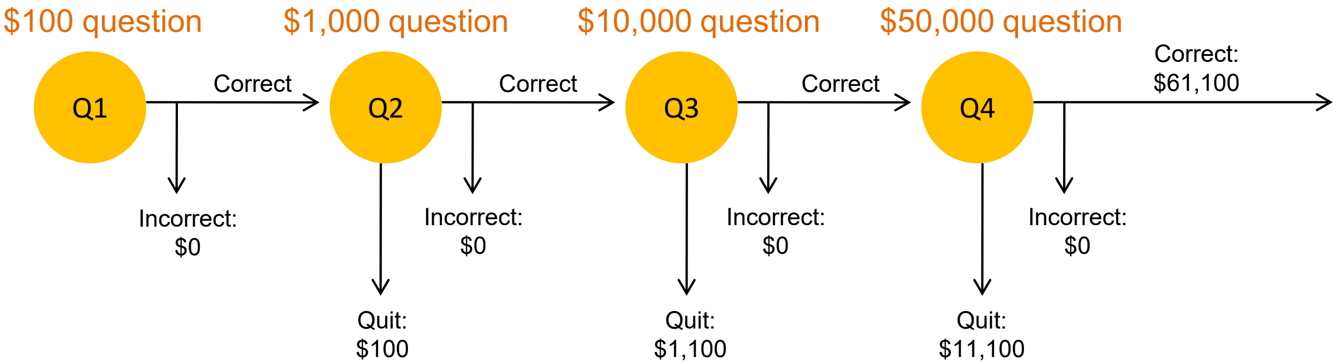
- Make a sequence of decisions
  - Agent's utility depends on a sequence of decisions
  - Sequential Decision Making
- Markov Property
  - Transition properties depend only on the current state, not on previous history (how that state was reached)
  - Markov Decision Processes

Markov Decision Processes

- Components:
  - *Markov* **States** s, beginning with initial state s_0
  - **Actions** a
    - Each state s has actions $A(s)$ available from it
  - **Transition model** $P(s' \mid s, a)$
    - *assumption*: the probability of going to s' from s depends only on s and a and not on any other past actions or states
  - **Reward function** $R(s)$
- **Policy** $\pi(s)$: the action that an agent takes in any given state
  - The "solution" to an MDP

Game Show

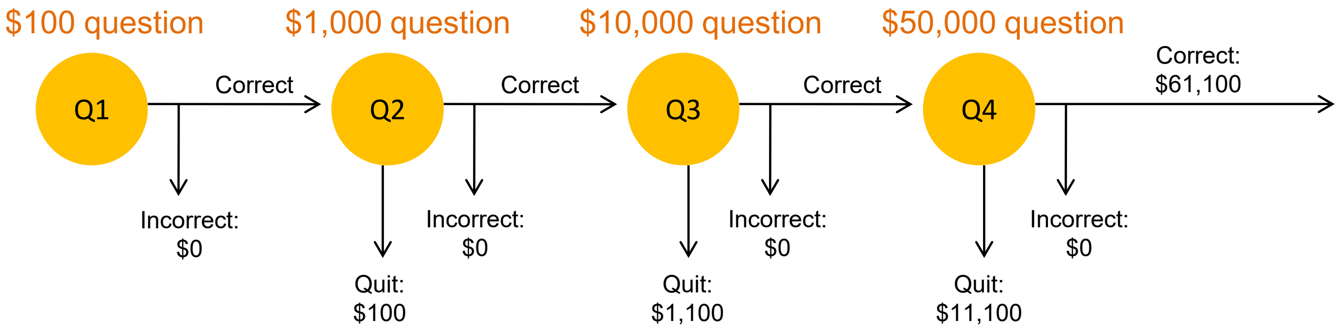
- A series of questions with increasing level of difficulty and increasing payoff
- Decision: at each step, take your earnings and quit, or go for the next question
  - If you answer wrong, you lose everything



Game Show

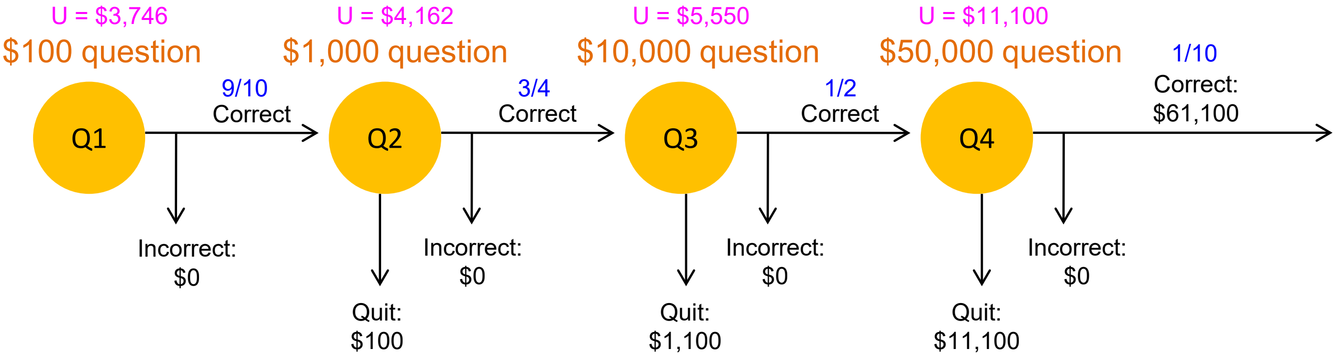
- Consider $50,000 question
  - Probability of guessing correctly: 1/10
  - Quit or go for the question?
- What is the expected payoff for continuing?

  0.1 * 61,100 + 0.9 * 0 = 6,110
- What is the optimal decision?

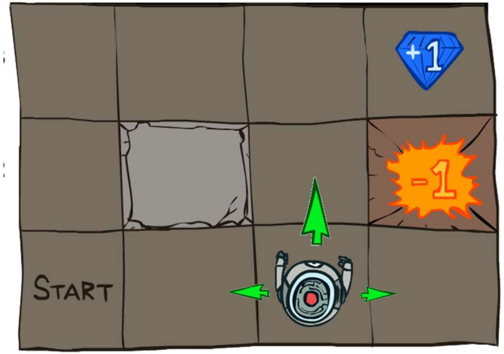


Game Show

- What should we do in Q3?
  - Payoff for quitting: $1,100
  - Payoff for continuing: 0.5 * $11,100 = $5,550
- What about Q2?
  - $100 for quitting vs. $4,162 for continuing
- What about Q1?

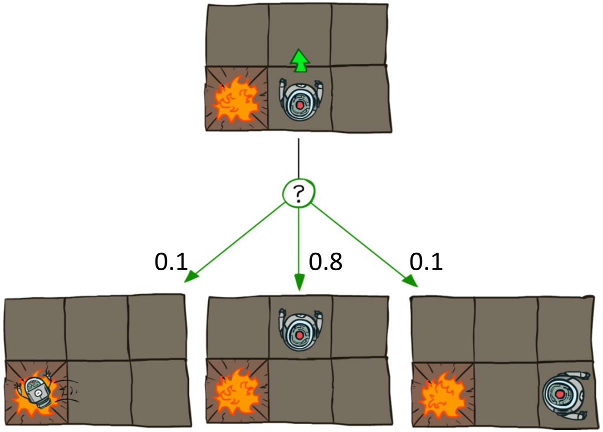


Grid World

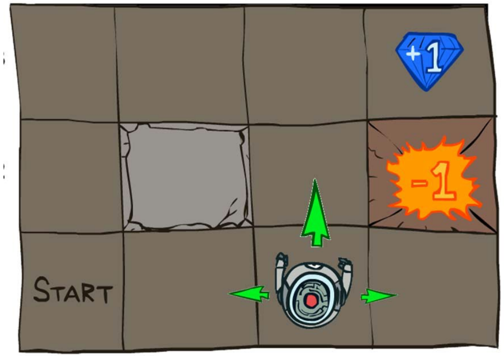


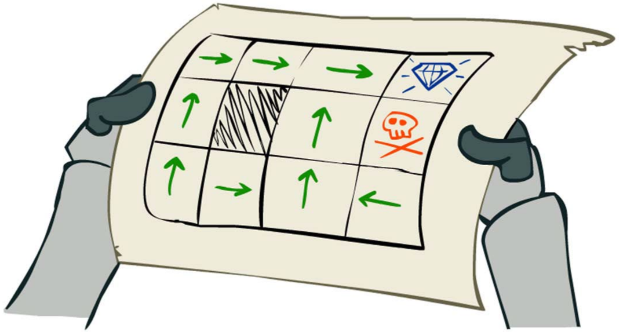
R(s) = -0.04 for every non-terminal state

Transition model:



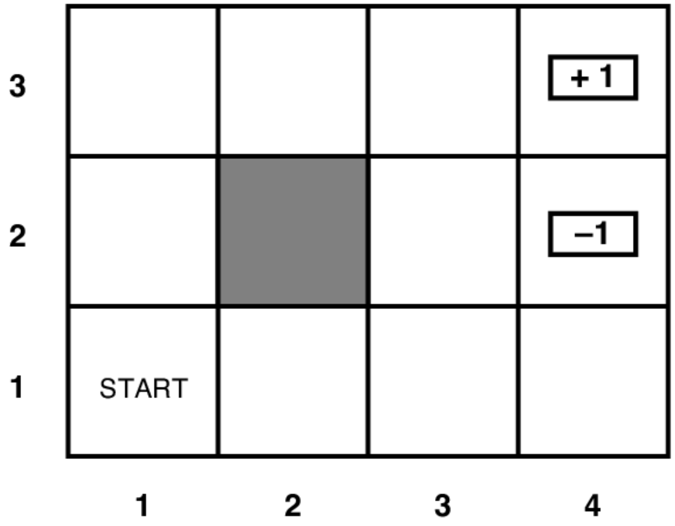
Goal: Policy



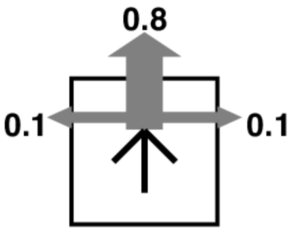


Source: P. Abbeel and D. Klein

Grid World

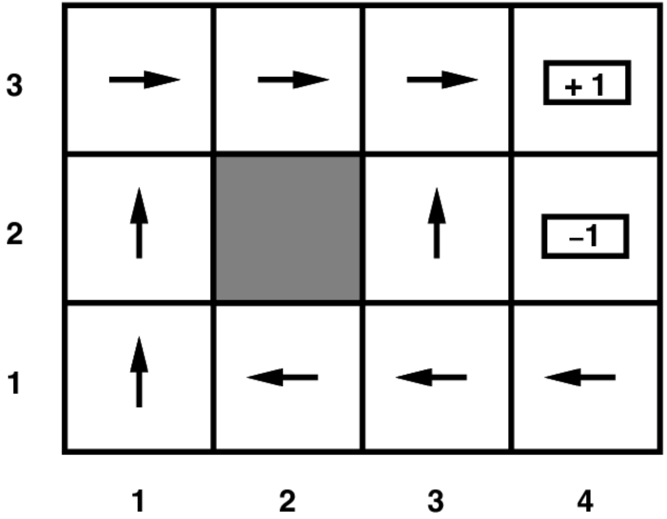


Transition model:



R(s) = −0.04 for every non-terminal state

Grid World



Optimal policy when R(s) = -0.04 for every non-terminal state

Solving MDPs

- MDP components:
  - **States** s
  - **Actions** a
  - **Transition model** $P(s' \mid s, a)$
  - **Reward function** $R(s)$
- The solution:
  - **Policy** $\pi(s)$ mapping from states to actions
  - How to find the optimal policy?

Maximising Expected Utility

- The optimal policy should maximise the *expected utility* over all possible state sequences produced by following that policy:

$$\sum_{\substack{\text{state sequences} \\ \text{starting from } s_0}} P(\text{sequence})U(\text{sequence})$$

- How to define the utility of a state sequence?
  - Sum of rewards of individual states
  - Problem: infinite state sequences
  - If finite, LP can be applied

Utilities of State Sequences

- Normally, we would define the utility of a state sequence as the sum of the rewards of the individual states
- **Problem:** infinite state sequences
- **Solution:** *discount* the individual state rewards by a factor γ between 0 and 1:

$$U([s_0, s_1, s_2, \ldots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \ldots$$

$$= \sum_{t=0}^{\infty} \gamma^t R(s_t) \leq \frac{R_{\max}}{1-\gamma} \qquad (0 < \gamma < 1)$$

  - Sooner rewards count more than later rewards
  - Makes sure the total utility stays bounded
  - Helps algorithms converge

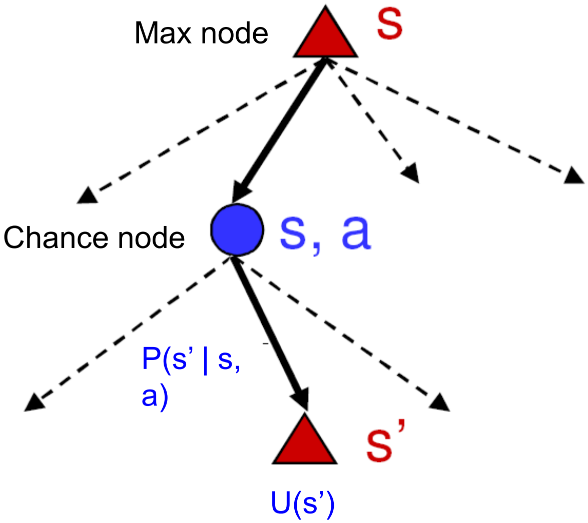
Utilities of States

- Expected utility obtained by policy π starting in state s:

$$U^{\pi}(s) = \sum_{\substack{\text{state sequences} \\ \text{starting from s}}} P(\text{sequence})U(\text{sequence})$$

- The "true" utility of a state, denoted $U(s)$, is the expected sum of discounted rewards if the agent executes an *optimal* policy starting in state s
- Reminiscent of minimax values of states…

Finding the Utilities of States



- What is the expected utility of taking action $\boldsymbol{a}$ in state $\boldsymbol{s}$?

$$\sum_{s'} P(s' \mid s, a) U(s')$$

- How do we choose the optimal action?

$$\pi^*(s) = \arg\max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) U(s')$$

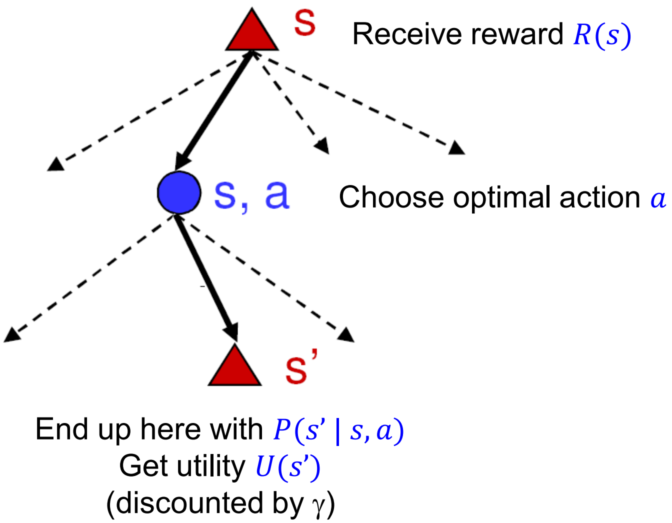
- What is the recursive expression for U(s) in terms of the utilities of its successor states?

$$U(s) = R(s) + \gamma \max_a \sum_{s'} P(s' \mid s, a) U(s')$$

The Bellman Equation

- Recursive relationship between the utilities of successive states:

$$U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U(s')$$



The Bellman Equation

- Recursive relationship between the utilities of successive states:

$$U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) U(s')$$

- For *N* states, we get *N* equations in *N* unknowns
  - Solving them solves the MDP
  - We could try to solve them through expectimax search, but that would run into trouble with infinite sequences
  - Instead, we solve them algebraically
  - Two methods: **value iteration** and **policy iteration**

Method 1: Value Iteration

- Start out with every $U(s) = 0$
- Iterate until convergence
  - During the *i*th iteration, update the utility of each state according to this rule:

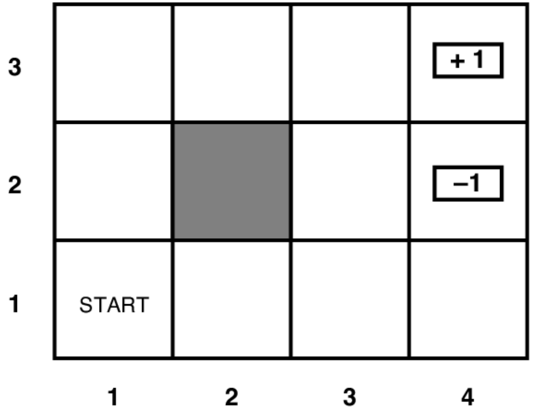
$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U_i(s')$$

- In the limit of infinitely many iterations, guaranteed to find the correct utility values
  - In practice, don’t need an infinite number of iterations…

Value Iteration

- What effect does the update have?

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U_i(s')$$



Method 2: Policy Iteration

- Start with some initial policy π_0 and alternate between the following steps:
  - **Policy evaluation:** calculate $U^{\pi_i}(s)$ for every state s
  - **Policy improvement:** calculate a new policy π_{i+1} based on the updated utilities

$$\pi^{i+1}(s) = \arg\max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) U^{\pi_i}(s')$$

TD(Temporal difference) Prediction

Policy Evaluation (the prediction problem):

for a given policy *p*, compute the state-value function V^{π}

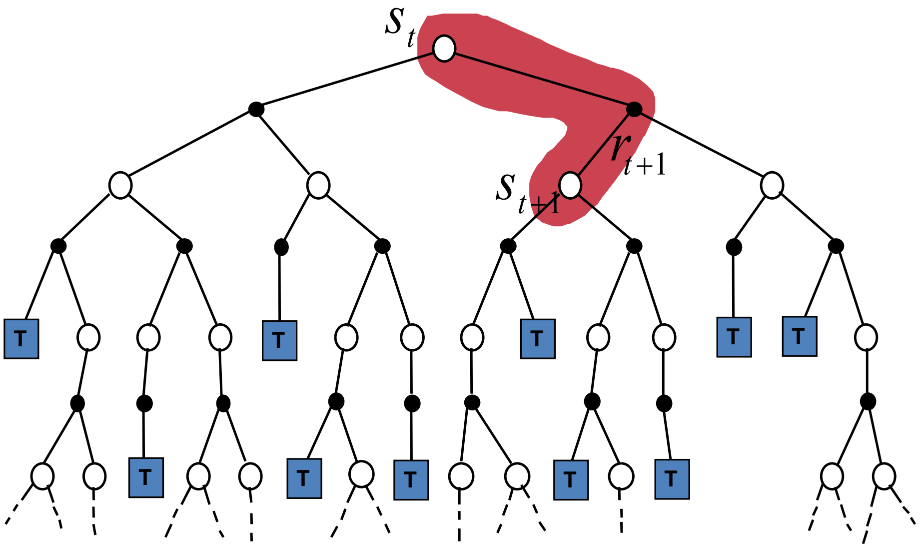
The simplest TD method, TD(0):

$$V(s_t) \leftarrow V(s_t) + \alpha \left[\underbrace{r_{t+1} + \gamma V(s_{t+1})}_{\text{target}} - V(s_t) \right]$$

target: an estimate of the return

Simplest TD Method

$$V(s_t) \leftarrow V(s_t) + \alpha\left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)\right]$$



Advantages of TD Learning

- TD methods do not require a model of the environment, only experience
- TD methods can be fully incremental
  - You can learn before knowing the final outcome
    - Less memory
    - Less peak computation
  - You can learn without the final outcome
    - From incomplete sequences

Further Reading [AAAI’18: http://www.ntu.edu.sg/home/boan/papers/AAAI18_Malmo.pdf]

We Won 2017 Microsoft Collaborative AI Challenge

- Collaborative AI
  - *How can AI agents learn to recognise someone’s intent (that is, what they are trying to achieve)?*
  - *How can AI agents learn what behaviours are helpful when working toward a common goal?*
  - *How can they coordinate or communicate with another agent to agree on a shared strategy for problem-solving?*

Further Reading [AAAI’18: http://www.ntu.edu.sg/home/boan/papers/AAAI18_Malmo.pdf]

- Microsoft Malmo Collaborative AI Challenge
  - *Collaborative mini-game, based on an extension “stag hunt”*
  - *Uncertainty of pig movement*
  - *Unknown type of the other agent*
  - *Detection noise (frequency 25%)*
- Our team HogRider won the challenge (out of more than 80 teams from 26 countries)
  - *learning + game theoretic reasoning + sequential decision making + optimisation*