
MODULE *hmac*

EXTENDS *Integers, Sequences, FiniteSets, TLC, Bitwise*

VARIABLES *message, key, sentHash, processedHash, equalityCheck, A, B, C, D, AA, BB, CC, DD, M, K*

BLOCK_SIZE $\triangleq 64$

RECURSIVE *shiftL*(-, -)
shiftL(*n*, *pos*) \triangleq
 IF *pos* = 0
 THEN *n*
 ELSE LET *double*(*z*) $\triangleq 2 * z$
 IN *shiftL*(*double*(*n*), *pos* - 1)

LeftRotate(*x*, *c*) \triangleq
 (*shiftL*(*x*, *c*) | *shiftR*(*x*, 32 - *c*)) % (2³²)

MD5(*m*) \triangleq
 LET
 ProcessChunk(*chunk*) \triangleq
 LET *P* $\triangleq [j \in 0 \dots 15 \mapsto \text{SubSeq}(m, (chunk - 1) * 512 + j * 32 + 1, (chunk - 1) * 512 + (j + 1) * 32)]$
 IN
 $\wedge AA' = A$
 $\wedge BB' = B$
 $\wedge CC' = C$
 $\wedge DD' = D$
 $\wedge \forall i \in 0 \dots 63 :$
 LET
 F \triangleq IF *i* $\in 0 \dots 15$ THEN (*B* & *C*) | ((*Not*(*B*)) & *D*)
 ELSE IF *i* $\in 16 \dots 31$ THEN (*D* & *B*) | ((*Not*(*D*)) & *C*)
 ELSE IF *i* $\in 32 \dots 47$ THEN ((*B* ^^ *C*) ^^ *D*)
 ELSE *C* ^^ (*B* | (*Not*(*D*)))
 g \triangleq IF *i* $\in 0 \dots 15$ THEN *i*
 ELSE IF *i* $\in 16 \dots 31$ THEN (5 * *i* + 1) % 16
 ELSE IF *i* $\in 32 \dots 47$ THEN (3 * *i* + 5) % 16
 ELSE (7 * *i*) % 16
 IN
 $\wedge F' = F + A + K[i] + P[g]$
 $\wedge A' = D$
 $\wedge D' = C$
 $\wedge C' = B$
 $\wedge B' = B + \text{LeftRotate}(F', S[i])$
 $\wedge A' = A + AA$
 $\wedge B' = B + BB$
 $\wedge C' = C + CC$
 $\wedge D' = D + DD$

$$\begin{aligned}
& \text{digest} \triangleq \langle A, B, C, D \rangle \\
& \text{IN} \\
& \text{digest} \\
& \text{ExtendedKey} \triangleq \text{IF } \text{Len}(\text{key}) > \text{BLOCK_SIZE} \text{ THEN } \text{MD5}(\text{key}) \text{ ELSE } \text{Append}(\text{key}, \langle 0 \rangle^{(\text{BLOCK_SIZE} - \text{Len}(\text{key}))}) \\
& \text{ipad} \triangleq [i \in 1 \dots \text{Len}(\text{ExtendedKey}) \mapsto \text{ExtendedKey}[i] \oplus 54] \\
& \text{opad} \triangleq [i \in 1 \dots \text{Len}(\text{ExtendedKey}) \mapsto \text{ExtendedKey}[i] \oplus 92] \\
& \text{HashFunction}(m, k) \triangleq \\
& \quad \text{LET} \\
& \quad \quad \text{innerHash} \triangleq \text{MD5}(\text{ipad} \circ m) \\
& \quad \quad \text{resultHash} \triangleq \text{MD5}(\text{opad} \circ \text{innerHash}) \\
& \quad \text{IN} \\
& \quad \text{resultHash} \\
& \text{SendHash} \triangleq \\
& \quad \wedge \text{sentHash}' = \text{HashFunction}(\text{message}, \text{ExtendedKey}) \\
& \quad \wedge \text{UNCHANGED } \langle \text{message}, \text{key}, \text{processedHash}, \text{equalityCheck}, A, B, C, D, AA, BB, CC, DD, K, S, M \rangle \\
& \text{ProcessHash} \triangleq \\
& \quad \wedge \text{processedHash}' = \text{HashFunction}(\text{message}, \text{ExtendedKey}) \\
& \quad \wedge \text{UNCHANGED } \langle \text{message}, \text{key}, \text{sentHash}, \text{equalityCheck}, A, B, C, D, AA, BB, CC, DD, K, S, M \rangle \\
& \text{CompareHashes} \triangleq \\
& \quad \wedge \text{equalityCheck}' = (\text{sentHash} = \text{processedHash}) \\
& \quad \wedge \text{UNCHANGED } \langle \text{message}, \text{key}, \text{sentHash}, \text{processedHash}, A, B, C, D, AA, BB, CC, DD, K, S, M \rangle \\
& \text{Init} \triangleq \\
& \quad \wedge \text{message} = \langle 0, 6, 56, 78, 87, 12 \rangle \\
& \quad \wedge \text{key} = \langle 0, 6, 56, 78, 87, 12 \rangle \\
& \quad \wedge \text{sentHash} = \langle \rangle \\
& \quad \wedge \text{processedHash} = \langle \rangle \\
& \quad \wedge \text{equalityCheck} = \text{FALSE} \\
& \quad \wedge A = 13 \\
& \quad \wedge B = 17 \\
& \quad \wedge C = 19 \\
& \quad \wedge D = 23 \\
& \quad \wedge K = \langle \\
& \quad \quad 102, 205, 307, 410, \\
& \quad \quad 512, 615, 718, 820, \\
& \quad \quad 923, 1025, 1128, 1230, \\
& \quad \quad 1333, 1435, 1538, 1640, \\
& \quad \quad 1743, 1845, 1948, 2050, \\
& \quad \quad 2153, 2255, 2358, 2460, \\
& \quad \quad 2563, 2665, 2768, 2870, \\
& \quad \quad 2973, 3075, 3178, 3280, \\
& \quad \rangle
\end{aligned}$$

$$\begin{aligned}
& 3383, 3485, 3588, 3690, \\
& 3793, 3895, 3998, 4100, \\
& 4203, 4305, 4408, 4510, \\
& 4613, 4715, 4818, 4920, \\
& 5023, 5125, 5228, 5330, \\
& 5433, 5535, 5638, 5740, \\
& 5843, 5945, 6048, 6150, \\
& 6253, 6355, 6458, 6560 \\
& \rangle \\
& \wedge S = \langle \\
& \quad 7, 12, 17, 22, \\
& \quad 7, 12, 17, 22, \\
& \quad 7, 12, 17, 22, \\
& \quad 7, 12, 17, 22, \\
& \quad 5, 9, 14, 20, \\
& \quad 5, 9, 14, 20, \\
& \quad 5, 9, 14, 20, \\
& \quad 5, 9, 14, 20, \\
& \quad 5, 9, 14, 20, \\
& \quad 4, 11, 16, 23, \\
& \quad 4, 11, 16, 23, \\
& \quad 4, 11, 16, 23, \\
& \quad 4, 11, 16, 23, \\
& \quad 4, 11, 16, 23, \\
& \quad 6, 10, 15, 21, \\
& \quad 6, 10, 15, 21, \\
& \quad 6, 10, 15, 21, \\
& \quad 6, 10, 15, 21 \\
& \rangle \\
& \wedge AA = A \\
& \wedge BB = B \\
& \wedge CC = C \\
& \wedge DD = D \\
& \wedge M = \langle 0, 6, 56, 78, 87, 12 \rangle \\
& Next \triangleq \\
& \quad \vee SendHash \\
& \quad \vee ProcessHash \\
& \quad \vee CompareHashes \\
& Spec \triangleq \\
& \quad Init \wedge \Box [Next]_{\langle message, key, sentHash, processedHash, equalityCheck, A, B, C, D, AA, BB, CC, DD, M, K, S \rangle}
\end{aligned}$$
