
MODULE *rsa*

EXTENDS *Integers, Sequences, FiniteSets*

VARIABLES *p, q, n, phi, e, d, m, c, message, ciphertext, plaintext*

Definicija prostih brojeva u ograničenom opsegu

$Prime \triangleq \{x \in 2 \dots 18 : \forall y \in 2 \dots (x-1) : x \% y \neq 0\}$

Definicija pomoćne funkcije za modularnu eksponencijaciju

$ModExpHelper(base, half_exp, mod, half_result) \triangleq$
 $(half_result * half_result) \% mod$

Definicija rekurzivne funkcije za modularnu eksponencijaciju

RECURSIVE $ModExp(-, -, -)$
 $ModExp(base, exp, mod) \triangleq$
 IF $exp = 0$ THEN 1
 ELSE
 IF $exp \% 2 = 0$ THEN
 $ModExpHelper(base, exp \div 2, mod, ModExp(base, exp \div 2, mod))$
 ELSE
 $(base * ModExp(base, exp - 1, mod)) \% mod$

Generisanje ključeva

$GenerateKeys \triangleq$
 $\wedge d' = \text{CHOOSE } x \in 1 \dots (phi - 1) : (e * x) \% phi = 1$
 $\wedge \text{UNCHANGED } \langle p, q, n, phi, e, m, c, plaintext, ciphertext, message \rangle$

Enkripcija

$Encrypt \triangleq$
 $\wedge c' = ModExp(m, e, n)$
 $\wedge \text{UNCHANGED } \langle p, q, n, phi, e, d, m, plaintext, ciphertext, message \rangle$

Dekripcija

$Decrypt \triangleq$
 $\wedge plaintext' = ModExp(c, d, n)$
 $\wedge \text{UNCHANGED } \langle p, q, n, phi, e, d, m, c, ciphertext, message \rangle$

Izlaz

$Output \triangleq$
 $\wedge ciphertext' = c$
 $\wedge message' = plaintext$
 $\wedge \text{UNCHANGED } \langle p, q, n, phi, e, d, m, c, plaintext \rangle$

Sledeće stanje sistema

$Next \triangleq$
 $\vee GenerateKeys$
 $\vee Encrypt$
 $\vee Decrypt$

∨ *Output*

Inicijalno stanje

$$Init \triangleq$$
$$\wedge p \in Prime$$
$$\wedge q \in Prime$$
$$\wedge p \neq q$$
$$\wedge n = p * q$$
$$\wedge phi = (p - 1) * (q - 1)$$
$$\wedge e \in 1 \dots (phi - 1)$$
$$\wedge \exists x \in 1 \dots (phi - 1) : (e * x) \% phi = 1$$
$$\wedge d = \text{CHOOSE } x \in 1 \dots (phi - 1) : (e * x) \% phi = 1$$
$$\wedge m \in 1 \dots (n - 1)$$
$$\wedge c = ModExp(m, e, n)$$
$$\wedge plaintext = ModExp(c, d, n)$$
$$\wedge ciphertext = c$$
$$\wedge \text{message} = \text{plaintext}$$

Specifikacija

$$Spec \triangleq$$
$$Init \wedge \Box[Next]_{\langle p, q, n, phi, e, d, m, c, plaintext, ciphertext, message \rangle}$$