# HASHING

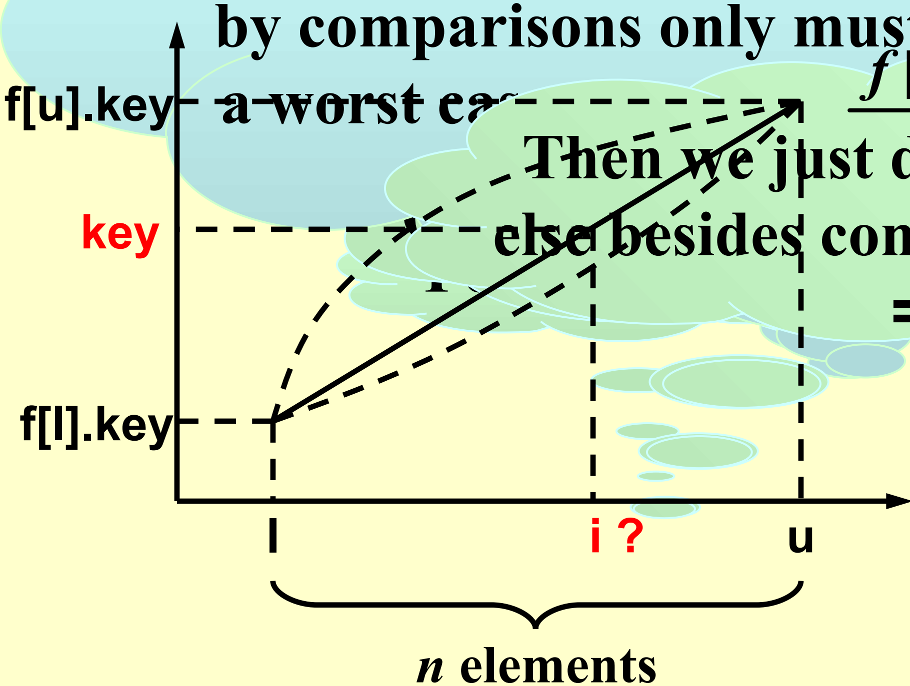
## Search by Formula

**Interpolation Search :**

Find key from a sorted list f [ l ].key, f [ l+1 ].key, ... , f [ u ].key.

There's a theorem saying that
Any algorithm that sorts
by comparisons only must have
a worst case ...

Then we just do something
else besides comparison.




$$\frac{f[u].key - f[l].key}{n} = \frac{key - f[l].key}{i - l}$$

$$\Rightarrow \quad i = l + \frac{(key - f[l].key)\cdot n}{f[u].key - f[l].key}$$

If ( f [ i ].key < key ) l = i ;

Else u = i ;

# §1 General Idea

**Symbol Table ( == Dictionary) ::= { < name, attribute > }**

**〖 Example 〗 In Oxford English dictionary**

name = since

attribute = a list of meanings
- M[0] = after a date, event, etc.
- M[1] = seeing that (expressing reason)
- ……

**〖 Example 〗 In a symbol table for a compiler**

name = identifier (e.g. int)

attribute = a list of lines that use the identifier, and some other fields

> **This is the worst disaster in California since I was elected.**
>
> *California Governor Pat Brown, discussing a local flood*

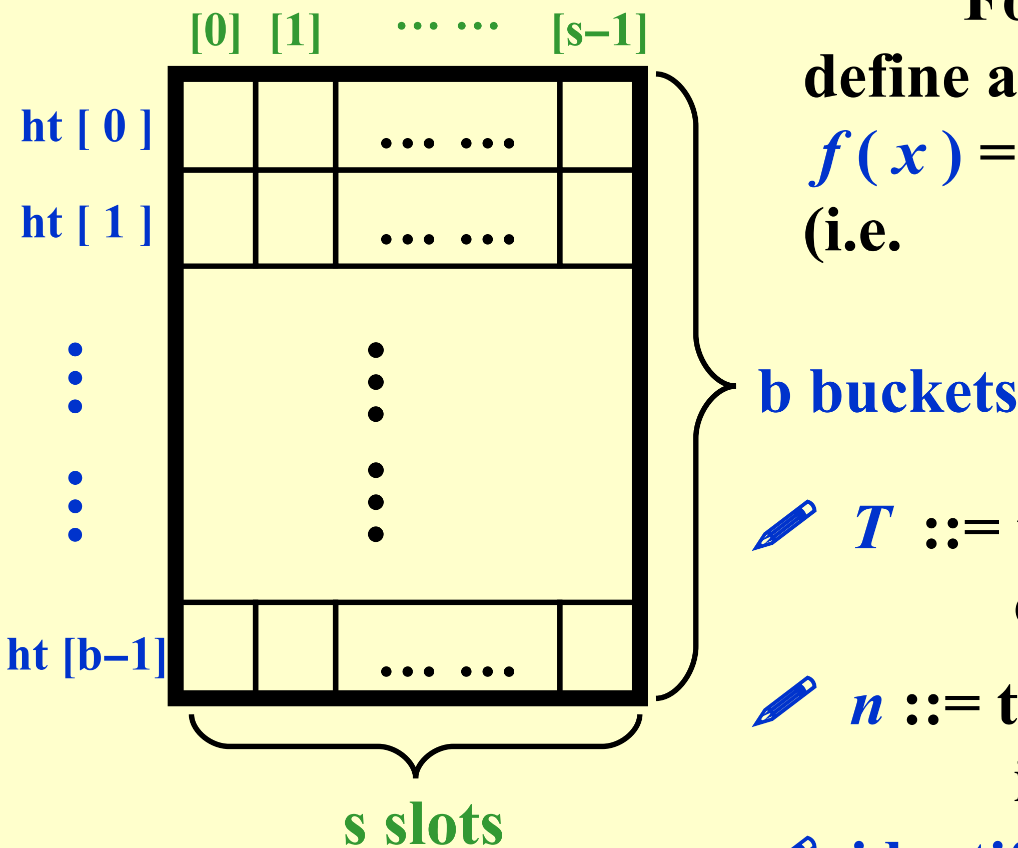
## ❖ Symbol Table ADT:

**Objects**: A set of name-attribute pairs, where the names are unique

**Operations**:

- ☞ SymTab Create(TableSize)
- ☞ Boolean IsIn(symtab, name)
- ☞ Attribute Find(symtab, name)
- ☞ SymTab Insert(symtab, name, attr)
- ☞ SymTab Delete(symtab, name)

## Hash Tables




For each identifier $x$, we define a **hash function** $f(x)$ = position of $x$ in ht[ ] (i.e. the index of the bucket that contains $x$ )

- $T$ ::= total number of distinct possible values for $x$
- $n$ ::= total number of identifiers in ht[ ]
- **identifier density** ::= $n$ / $T$
- **loading density** $\lambda$ ::= $n$ / ($s$ $b$)

- A **collision** occurs when we hash two nonidentical identifiers into the same bucket, i.e. $f(i_1) = f(i_2)$ when $i_1 \neq i_2$.
- An **overflow** occurs when we hash a new identifier into a full bucket.

Collision and overflow happen simultaneously if $s = 1$.

〖 Example 〗 Mapping $n = 10$ C library functions into a hash table ht[ ] with $b = 26$ buckets and $s = 2$.

Loading density $\lambda = 10 / 52 = 0.19$

To map the letters $a \sim z$ to $0 \sim 25$, we may define $f(x) = x[0] - \text{'}a\text{'}$

acos define float exp char

atan ceil floor clock ctime

| | Slot 0 | Slot 1 |
|---|---|---|
| 0 | acos | atan |
| 1 | | |
| 2 | char | ceil |
| 3 | define | |
| 4 | exp | |
| 5 | float | floor |
| 6 | | |
| …… | | |
| 25 | | |

Without overflow,

$$T_{search} = T_{insert} = T_{delete} = O(1)$$

## §2 Hash Function

**Properties of $f$:**

① $f(x)$ must be **easy** to compute and **minimizes** the number of **collisions**.

② $f(x)$ should be unbiased. That is, for any $x$ and any $i$, we have that Probability( $f(x) = i$ ) = $1 / b$. Such kind of a hash function is called a **uniform hash function**.

$f(x) = x \ \% \ TableSize$ ; /* if $x$ is an integer */

☹ What if *TableSize* = 10 and $x$'s are all end in zero?

☺ *TableSize* = **prime** number ---- good for random integer keys

$f(x) = (\Sigma\, x[i])$ % *TableSize* ; /* if $x$ is a string */

〖 Example 〗 *TableSize* = 10,007 and string length of $x \le 8$.

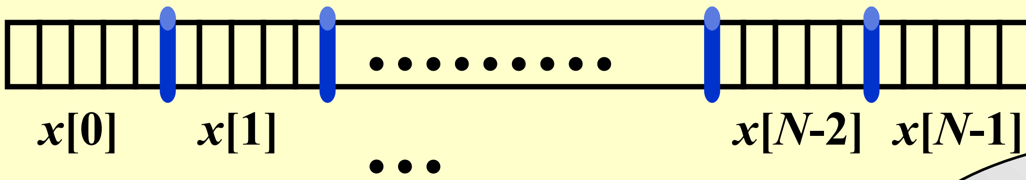
If $x[i] \in [0, 127]$, then $f(x) \in [0,$ 1016 $]$ ☹

$f(x) = (x[0] + x[1]*27 + x[2]*27^2)$ % *TableSize* ;

Total number of combinations = $26^3 = 17{,}576$

☹ Actual number of combinations < 3000

$$f(x) = (\Sigma\, x[N-i-1] * 32^i)\ \%\ TableSize\ ;$$




```
Index Hash3( const char *x, int TableSize )
{
            unsigned  int  HashVal = 0;
/* 1*/      while( *x != '\0' )
/* 2*/          HashVal = ( HashVal << 5 ) + *x++;
/* 3*/      return HashVal % TableSize;
}
```

Can you make it faster?

Carefully select some characters from *x*.

☹ **If *x* is too long (e.g. street address), the early characters will be left-shifted out of place.**

# §3 Separate Chaining

## ---- keep a list of all keys that hash to the same value

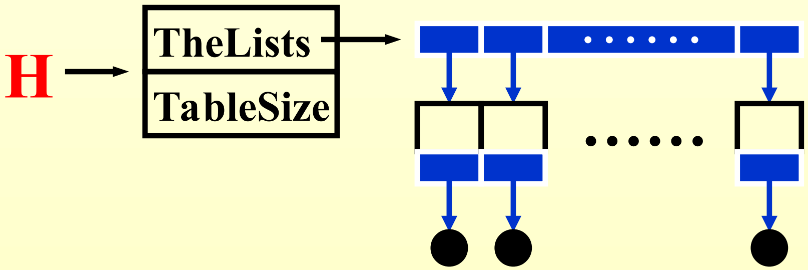
```
struct  ListNode;
typedef  struct  ListNode  *Position;
struct  HashTbl;
typedef  struct  HashTbl  *HashTable;

struct  ListNode {
        ElementType  Element;
        Position  Next;
};
typedef  Position  List;

/* List *TheList will be an array of lists, allocated later */
/* The lists use headers (for simplicity), */
/* though this wastes space */
struct  HashTbl {
        int  TableSize;
        List  *TheLists;
};
```

## ☞ Create an empty table

```
HashTable  InitializeTable( int TableSize )
{   HashTable  H;
    int  i;
    if ( TableSize < MinTableSize ) {
            Error( "Table size too small" );  return NULL;
    }
    H = malloc( sizeof( struct HashTbl ) );  /* Allocate table */
    if ( H == NULL )    FatalError( "Out of space!!!" );
    H->TableSize = NextPrime( TableSize );  /* Better be prime */
    H->TheLists = malloc( sizeof( List ) * H->TableSize );  /*Array of lists*/
    if ( H->TheLists == NULL )   FatalError( "Out of space!!!" );
    for( i = 0; i < H->TableSize; i++ ) {   /* Allocate list headers */
        H->TheLists[ i ] = malloc( sizeof( struct ListNode ) ); /* Slow! */
        if ( H->TheLists[ i ] == NULL )  FatalError( "Out of space!!!" );
        else    H->TheLists[ i ]->Next = NULL;
    }
    return  H;
}
```



## ☞ Find a key from a hash table

```
Position  Find ( ElementType Key, HashTable H )
{
    Position P;
    List L;

    L = H->TheLists[ Hash( Key, H->TableSize ) ];

    P = L->Next;
    while( P != NULL && P->Element != Key )  /* Probably need strcmp */
            P = P->Next;
    return P;
}
```

Your hash function

Identical to the code to perform a *Find* for general lists -- List ADT

# ☞ Insert a key into a hash table

```
void  Insert ( ElementType Key, HashTable H )
{
    Position   Pos, NewCell;
    List  L;
    Pos = Find( Key, H );
    if ( Pos == NULL ) {   /* Key is not found, then insert */
            NewCell = malloc( sizeof( struct ListNode ) );
            if ( NewCell == NULL )     FatalError( "Out of space!!!" );
            else {
                L = H->TheLists[ Hash( Key, H->TableSize ) ];
                NewCell->Next = L->Next;
                NewCell->Element = Key; /* Probably need strcpy! */
                L->Next = NewCell;
            }
    }
}
```

☹ Again?!

**Tip:** Make the TableSize about as large as the number of keys expected (i.e. to make the loading density factor $\lambda \approx 1$).

# §4 Open Addressing

---- find another empty cell to solve collision (avoiding pointers)

```
Algorithm: insert key into an array of hash table
{
   index = hash(key);
   initialize i = 0 ------ the counter of probing;
   while ( collision at index ) {
         index = ( hash(key) + f(i) ) % TableSize;
         if ( table is full )    break;
         else    i ++;
   }
   if ( table is full )
         ERROR ("No space left");
   else
         insert key at index;
}
```

Collision resolving function. $f(0) = 0$.

Tip: Generally $\lambda < 0.5$.

## 1. Linear Probing

$f(i) = i$; /* a linear function */

〖 E… 〗 M… C library functions into a … 6 buckets and $s$ = 1.

… small,

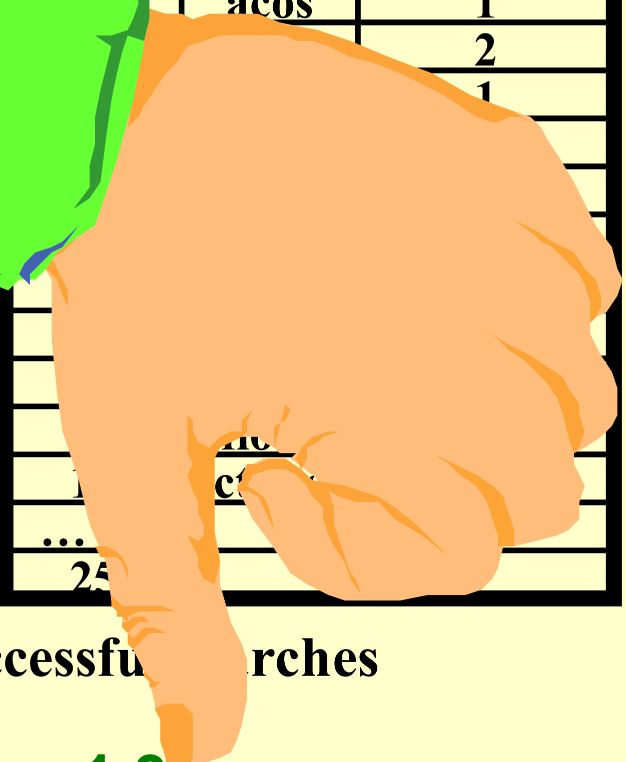
acos …

l co…

Cause *primary clustering*: any key that hashes into the cluster will add to the cluster after several attempts to resolve the collision.

Loa…

| …cket | $x$ | search time |
|---|---|---|
| | acos | 1 |
| | | 2 |
| | | 1 |
| … | | |
| 25… | | |



Average search time = 41 / 11 = 3.73

Analysis of the linear probing show that the expected number of probes

$$p = \begin{cases} \frac{1}{2}\left(1 + \frac{1}{(1-\lambda)^2}\right) & \text{for insertions and unsuccessfu… …rches} \\ \frac{1}{2}\left(1 + \frac{1}{1-\lambda}\right) & \text{for successful searches} \end{cases}$$

= 1.36

## 2. Quadratic Probing

$f(i) = i^2$; /* a quadratic function */

【Theorem】 If quadratic probing is used, and the table size is **prime**, then a new element can always be inserted if the table is **at least half empty**.

**Proof:** Just prove that the first $\lfloor TableSize/2 \rfloor$ alternative locations are all **distinct**. That is, for any $0 < i \neq j \leq \lfloor TableSize/2 \rfloor$, we have

$$( h(x) + i^2 ) \,\%\, TableSize \neq ( h(x) + j^2 ) \,\%\, TableSize$$

Suppose: $h(x) + i^2 = h(x) + j^2 \pmod{TableSize}$

then: $i^2 = j^2 \pmod{TableSize}$

$(i + j)(i - j) = 0 \pmod{TableSize}$

TableSize is prime $\Rightarrow$ either $(i + j)$ or $(i - j)$ is divisible by TableSize

**Contradiction !**

For any $x$, it has $\lceil TableSize/2 \rceil$ distinct locations into which it can go. If **at most** $\lfloor TableSize/2 \rfloor$ positions are taken, then an empty spot can always be found. ∎

**Note:** If the table size is a prime of the form $4k + 3$, then the quadratic probing $f(i) = \pm i^2$ can probe the entire table.

**Read Figures 7.15 - 7.16 for detailed representations and implementations of initialization.**

```
Position  Find ( ElementType Key, HashTable H )
{   Position  CurrentPos;
    int  CollisionNum;
    CollisionNum = 0;
    CurrentPos = Hash( Key, H->TableSize );
    while( H->TheCells[ CurrentPos ].Info != Empty &&
              H->TheCells[ CurrentPos ].Element != Key ) {
              CurrentPos += 2 * ++CollisionNum – 1;
              if ( CurrentPos >= H->TableSize )  CurrentPos - = H->TableSize;
    }
    return CurrentPos;
}
```

What is returned?

```
void  Insert ( ElementType Key, HashTable H )
{
   Position  Pos;
   Pos = Find( Key, H );
   if ( H->TheCells[ Pos ].Info != Legitimate ) { /* OK to insert here */
           H->TheCells[ Pos ].Info = Legitimate;
           H->TheCells[ Pos ].Element = Key; /* Probably need strcpy */
   }
}
```

**Question: How to delete a key?**

**Note:** ① Insertion will be seriously slowed down if there are too many **deletions intermixed with insertions**.

② Although primary clustering is solved, ***secondary clustering*** occurs – that is, keys that hash to the same position will probe the same alternative cells.