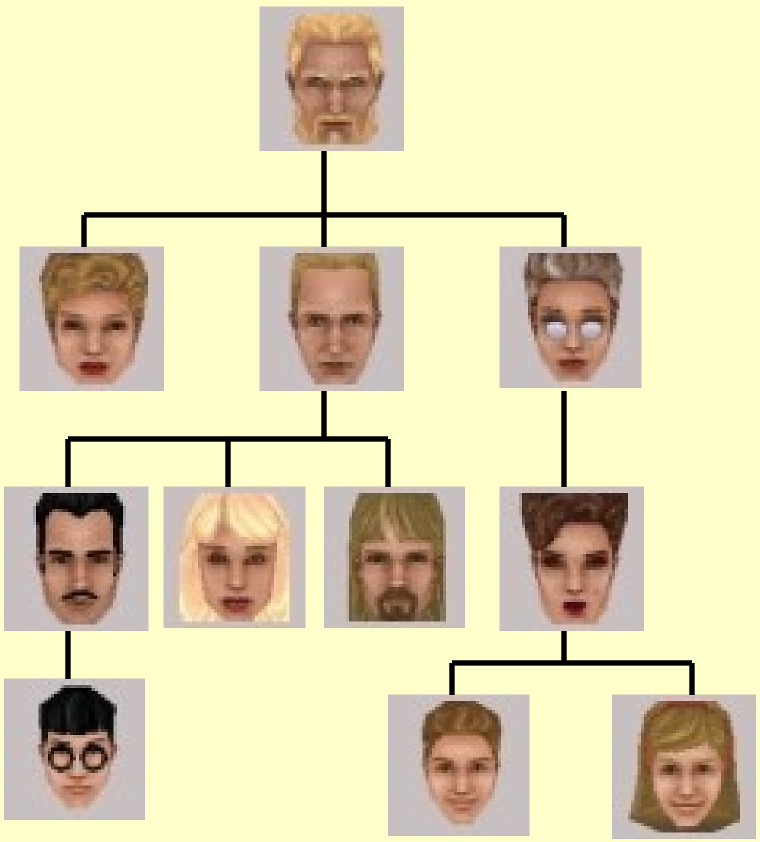
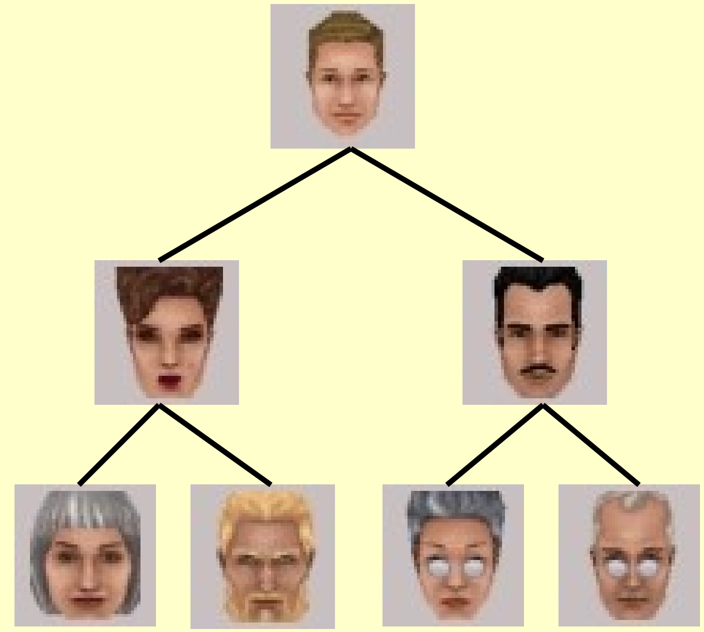
TREES

§1 Preliminaries

1. Terminology



Lineal Tree



Pedigree Tree
(binary tree)

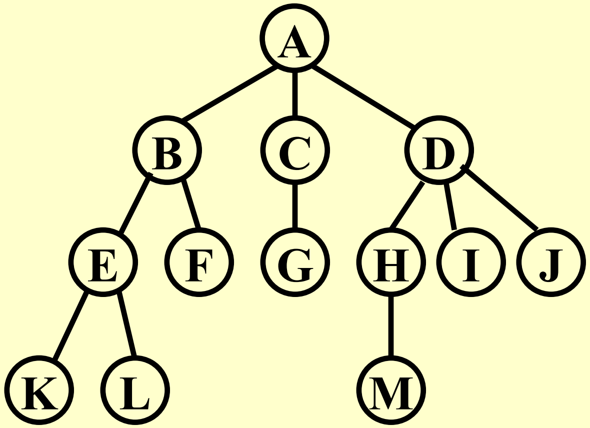
【 Definition 】 A **tree** is a collection of nodes. The collection can be empty; otherwise, a tree consists of

(1) a distinguished node r, called the **root**;

(2) and zero or more nonempty **(sub)trees** $T_1, \cdots, T_k$, each of whose roots are connected by a directed **edge** from r.

Note:

- Subtrees must not connect together. Therefore every node in the tree is the root of some subtree.
- There are $N - 1$ edges in a tree with N nodes.
- Normally the root is drawn at the top.



- **degree of a node ::=** number of subtrees of the node. For example, degree(A) = 3, degree(F) = 0.

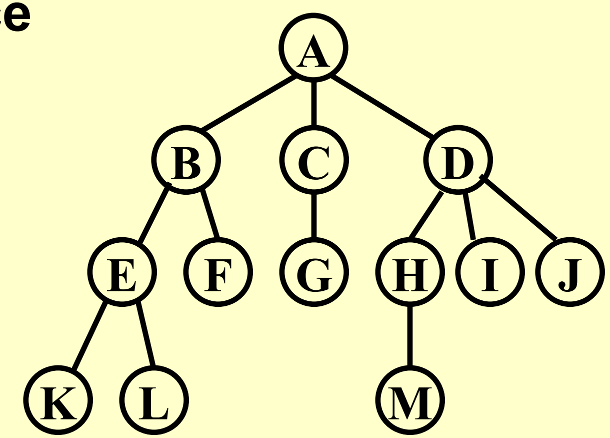
- **degree of a tree ::=** $\max_{node \in tree} \{ \text{degree}(node) \}$
  For example, degree of this tree = 3.

- **parent ::=** a node that has subtrees.

- **children ::=** the roots of the subtrees of a parent.

- **siblings ::=** children of the same parent.

- **leaf (terminal node) ::=** a node with degree 0 (no children).



- **path from n_1 to n_k ::=** a (unique) sequence of nodes $n_1, n_2, \ldots, n_k$ such that n_i is the parent of n_{i+1} for $1 \le i < k$.

- **length of path ::=** number of edges on the path.

- **depth of n_i ::=** length of the unique path from the root to n_i. Depth(root) = 0.

- **height of n_i ::=** length of the longest path from n_i to a leaf. Height(leaf) = 0, and height(D) = 2.

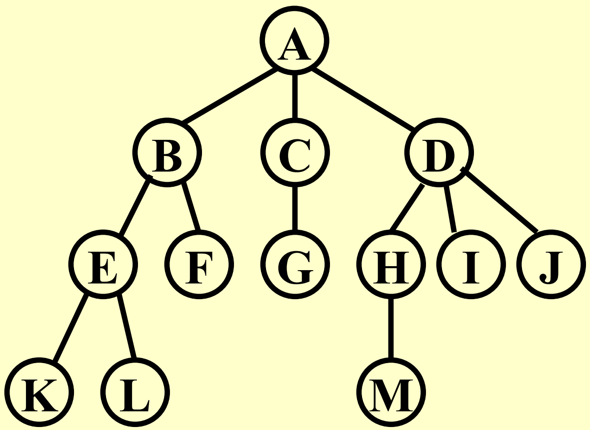
- **height (depth) of a tree ::=** height(root) = depth(deepest leaf).

- **ancestors of a node ::=** all the nodes along the path from the node up to the root.

- **descendants of a node ::=** all the nodes in its subtrees.

2. Implementation

❖ **List Representation**



(A)

(A (B, C, D))

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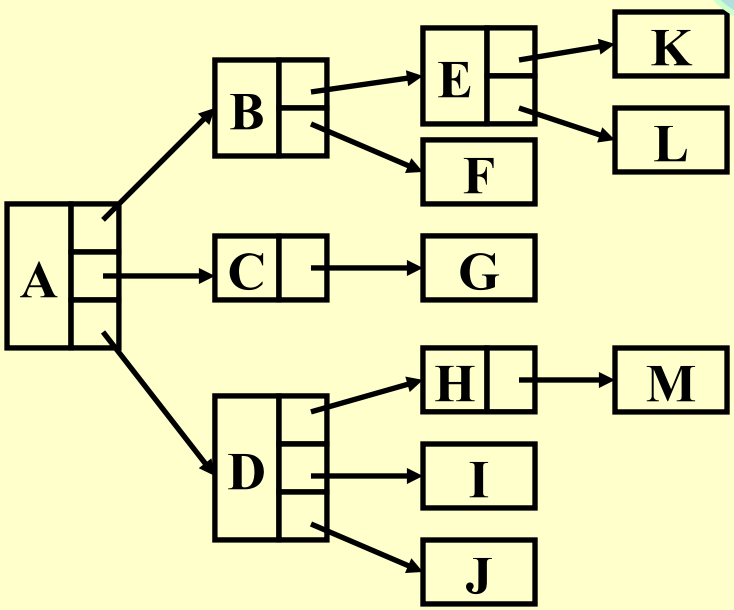
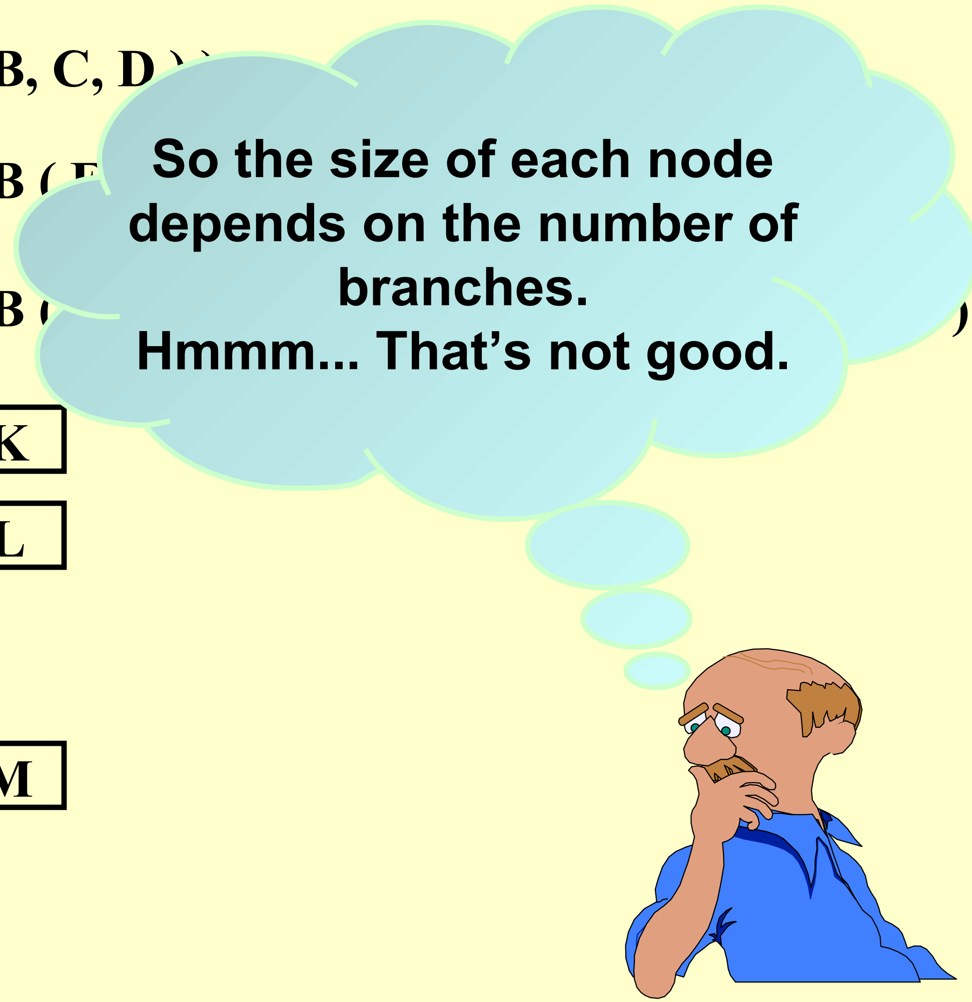
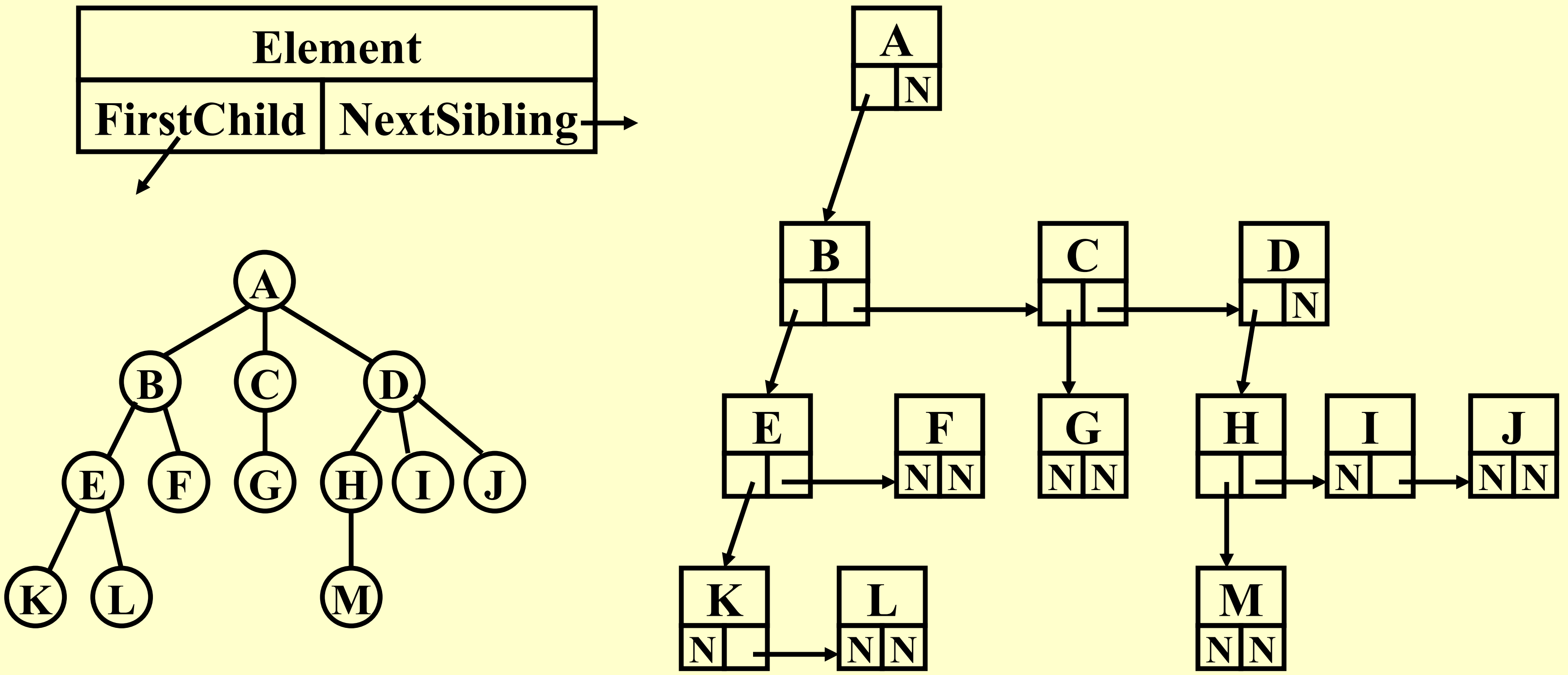




❖ FirstChild-NextSibling Representation

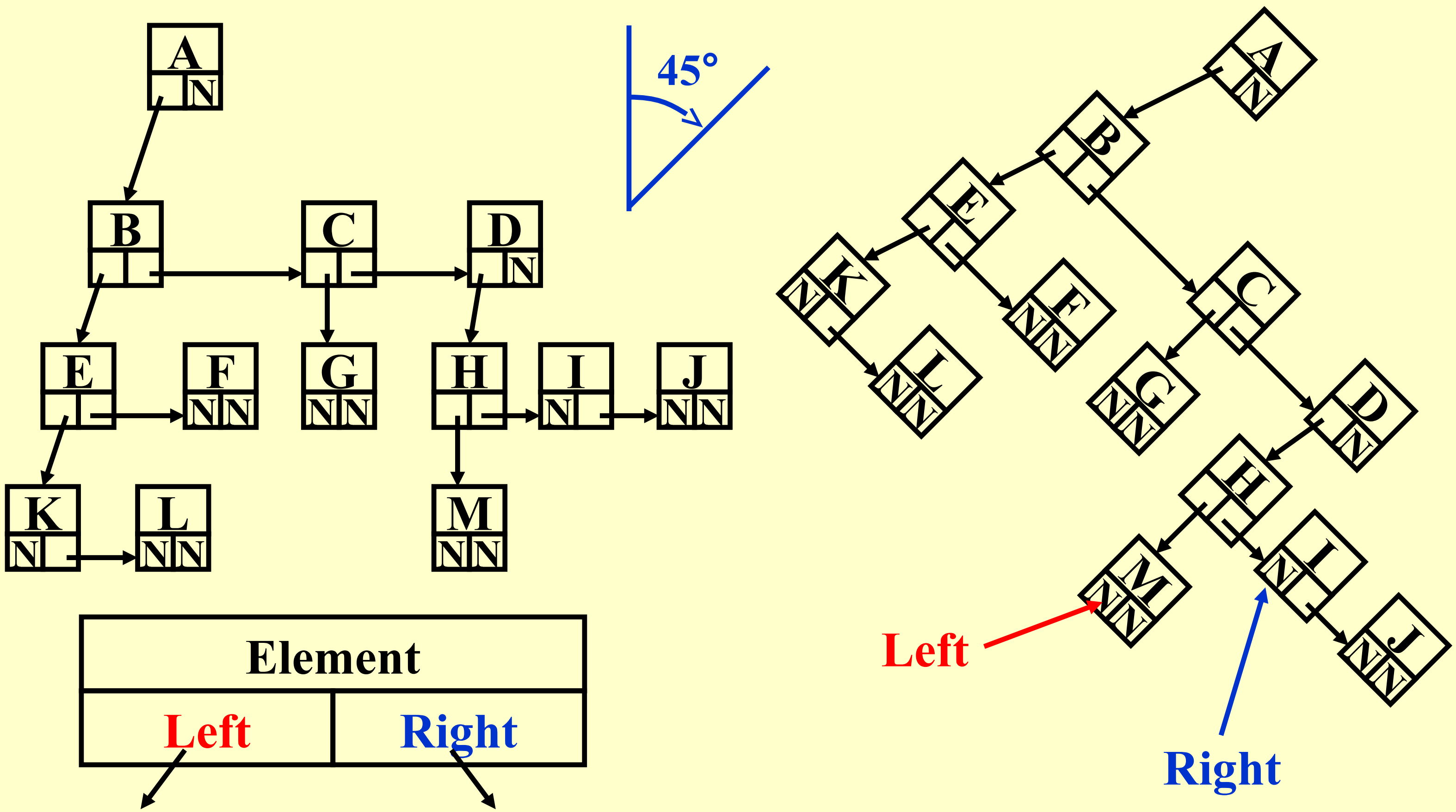


Note: The representation is not unique since the children in a tree can be of any order.

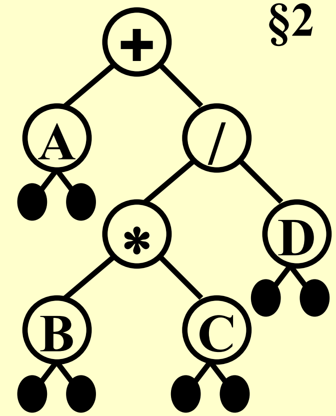
§2 Binary Trees

【 Definition 】 A binary tree is a tree in which no node can have more than two children.

Rotate the FirstChild-NextSibling tree clockwise by 45°.



❖ Expression Trees (syntax trees)

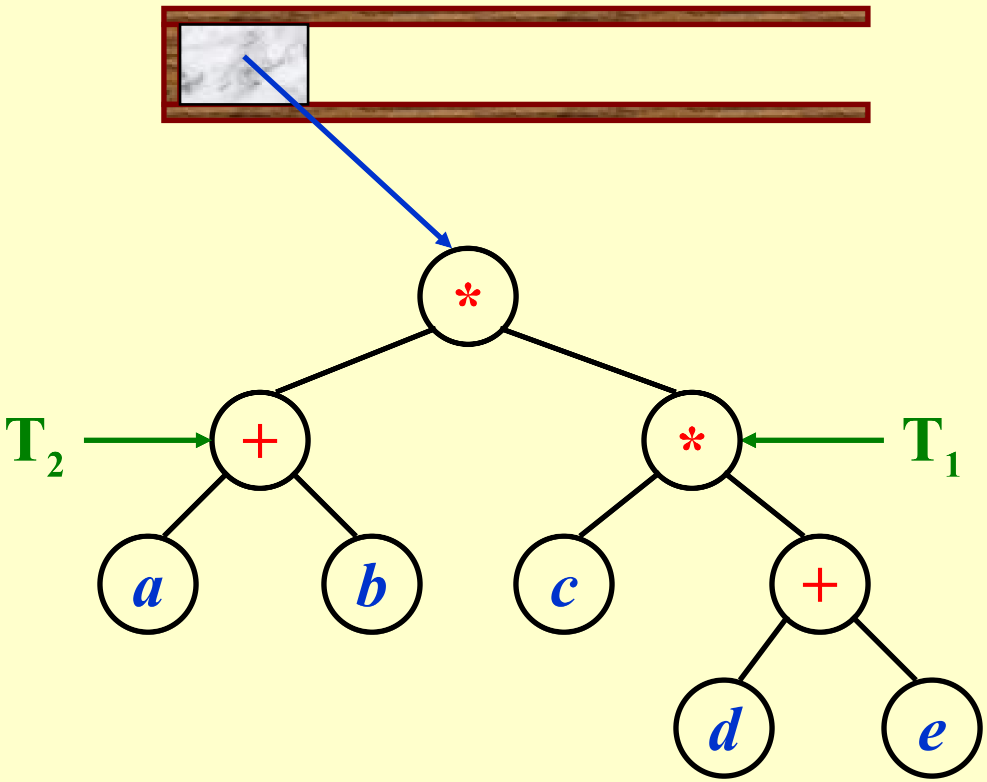


〖 Example 〗 Given an infix expression:

$$A + B * C / D$$

☞ Constructing an Expression Tree (from **postfix** expression)

〖 Example 〗 $(a + b) * (c * (d + e)) = a\ b + c\ d\ e + * *$



☞ Tree Traversals —— visit each node exactly once

❖ Preorder Traversal

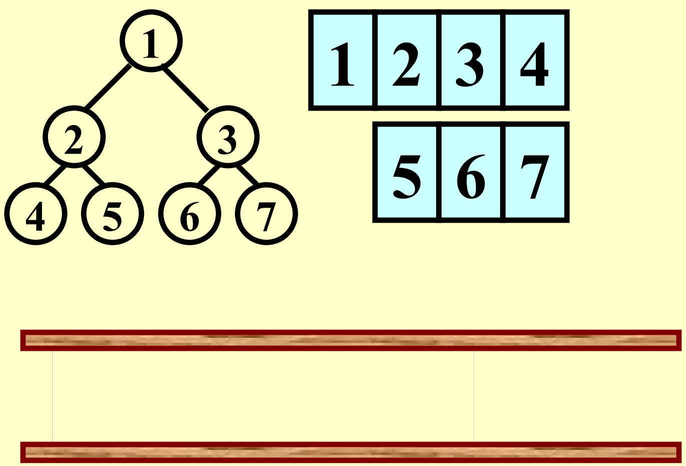
```
void  preorder ( tree_ptr  tree )
{  if  ( tree )   {
        visit ( tree );
        for (each child C of tree )
            preorder ( C );
    }
}
```

❖ Postorder Traversal

```
void  postorder ( tree_ptr  tree )
{  if  ( tree )   {
        for (each child C of tree )
            postorder ( C );
        visit ( tree );
    }
}
```

❖ Levelorder Traversal

```
void  levelorder ( tree_ptr  tree )
{   enqueue ( tree );
    while (queue is not empty) {
        visit ( T = dequeue ( ) );
        for (each child C of T )
            enqueue ( C );
    }
}
```



❖ Inorder Traversal

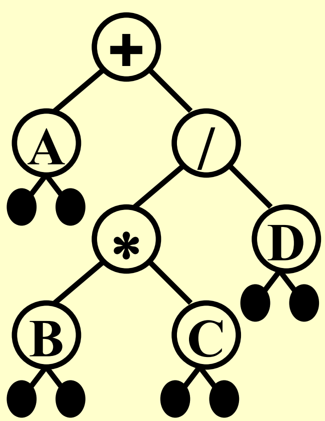
```
void  inorder ( tree_ptr  tree )
{  if  ( tree )   {
        inorder ( tree->Left );
        visit ( tree->Element );
        inorder ( tree->Right );
   }
}
```

Iterative Program

```
void  iter_inorder ( tree_ptr  tree )
{ Stack  S = CreateStack( MAX_SIZE );
  for ( ; ; )  {
    for ( ; tree; tree = tree->Left )
      Push ( tree, S ) ;
    tree = Top ( S );  Pop( S );
    if ( ! tree )  break;
    visit ( tree->Element );
    tree = tree->Right;   }
}
```

〖Example〗 Given an infix expression:

$$A + B * C / D$$

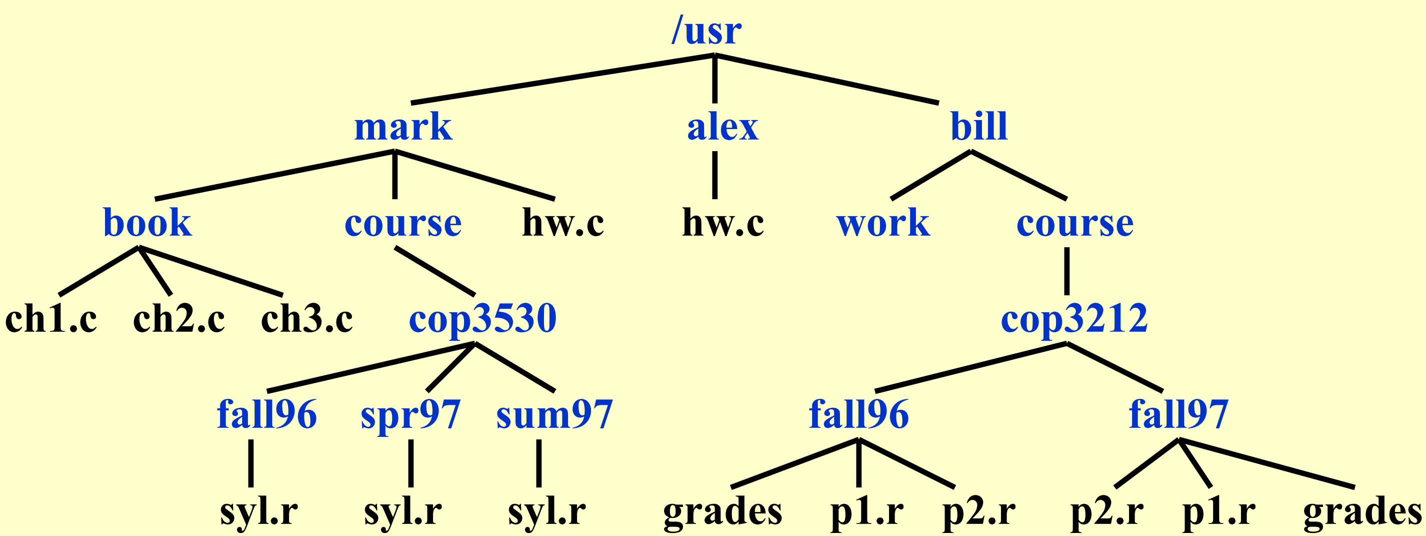


Then inorder traversal $\Rightarrow$ $A + B * C / D$

postorder traversal $\Rightarrow$ $A\ B\ C * D / +$

preorder traversal $\Rightarrow$ $+ A / * B\ C\ D$

〖 Example 〗 Directory listing in a hierarchical file system.



Unix directory

Listing format: files that are of depth d_i will have their names indented by d_i tabs.

```
/usr
    mark
        book
            Ch1.c
            Ch2.c
            Ch3.c
        course
            cop3530
                fall96
                    syl.r
                spr97
                    syl.r
                sum97
                    syl.r
        hw.c
    alex
        hw.c
    bill
        work
        course
            cop3212
                fall96
                    grades
                    p1.r
                    p2.r
                fall97
                    p2.r
                    p1.r
                    grades
```

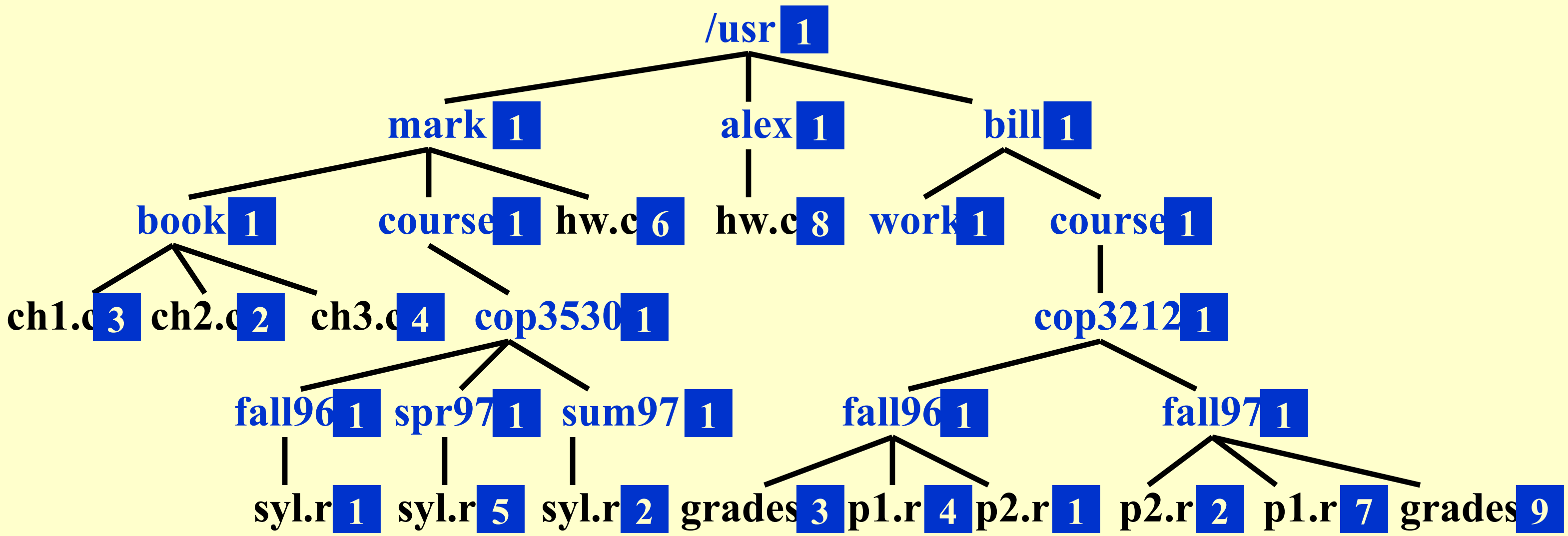
```
static void  ListDir ( DirOrFile D, int Depth )
{
   if  ( D is a legitimate entry )   {
      PrintName (D, Depth );
      if ( D is a directory )
         for (each child C of D )
            ListDir ( C, Depth + 1 );
   }
}
```

$$T(N) = O(N)$$

Note: Depth is an internal variable and must not be seen by the user of this routine. One solution is to define another interface function as the following:

```
void ListDirectory ( DirOrFile  D )
{          ListDir( D, 0 );              }
```

〖 Example 〗 Calculating the size of a directory.

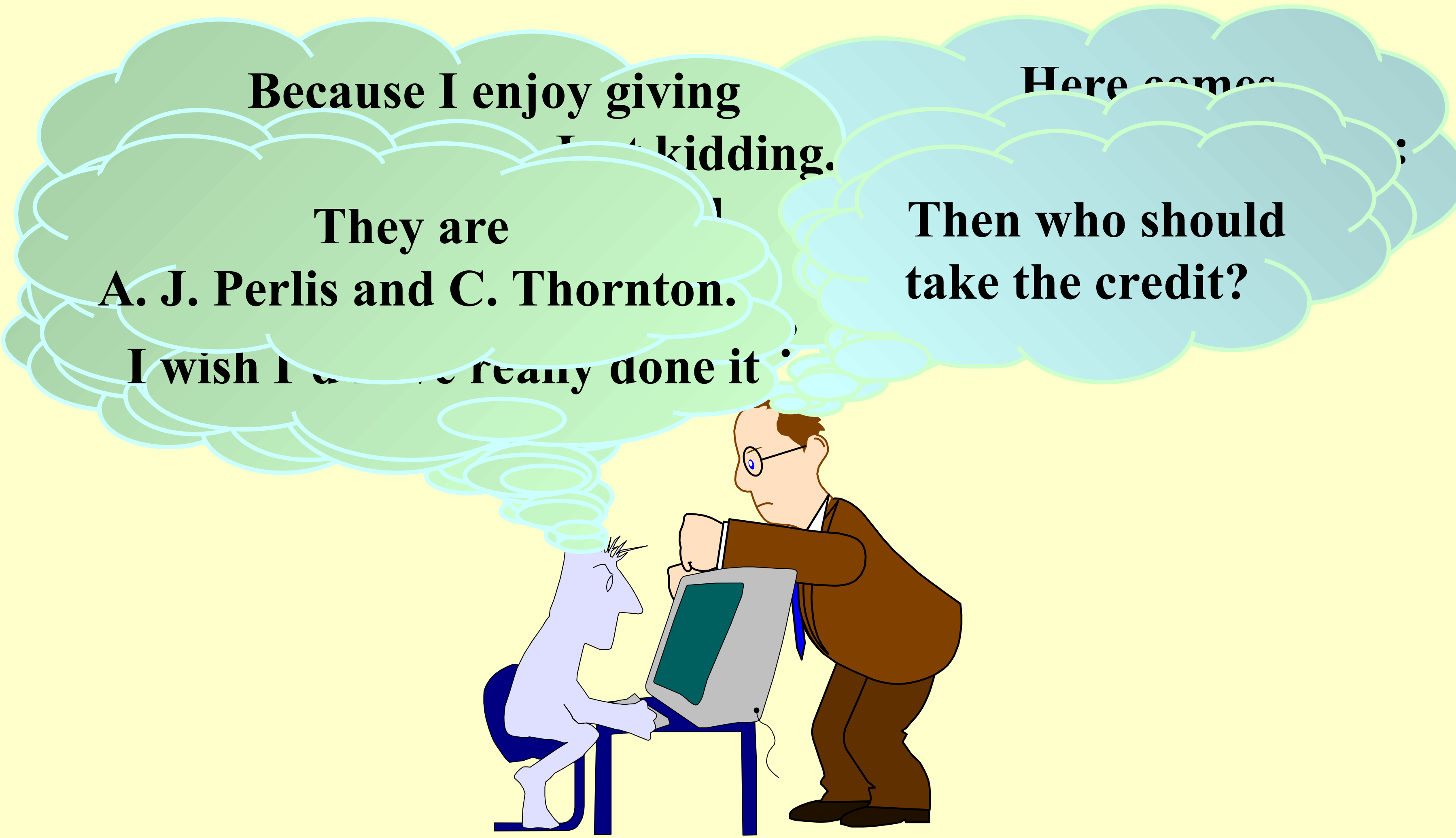


Unix directory with file sizes

```
static int  SizeDir ( DirOrFile D )
{
   int TotalSize;
   TotalSize = 0;
   if  ( D is a legitimate entry )   {
      TotalSize = FileSize( D );
      if ( D is a directory )
         for (each child C of D )
            TotalSize += SizeDir(C);
   } /* end if D is legal */
   return TotalSize;
}
```

$$T(N) = O(N)$$

❖ Threaded Binary Trees



Rule 1: If Tree->Left is null, replace it with a pointer to the inorder predecessor of Tree.

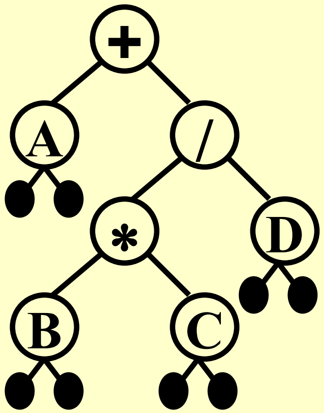
Rule 2: If Tree->Right is null, replace it with a pointer to the inorder successor of Tree.

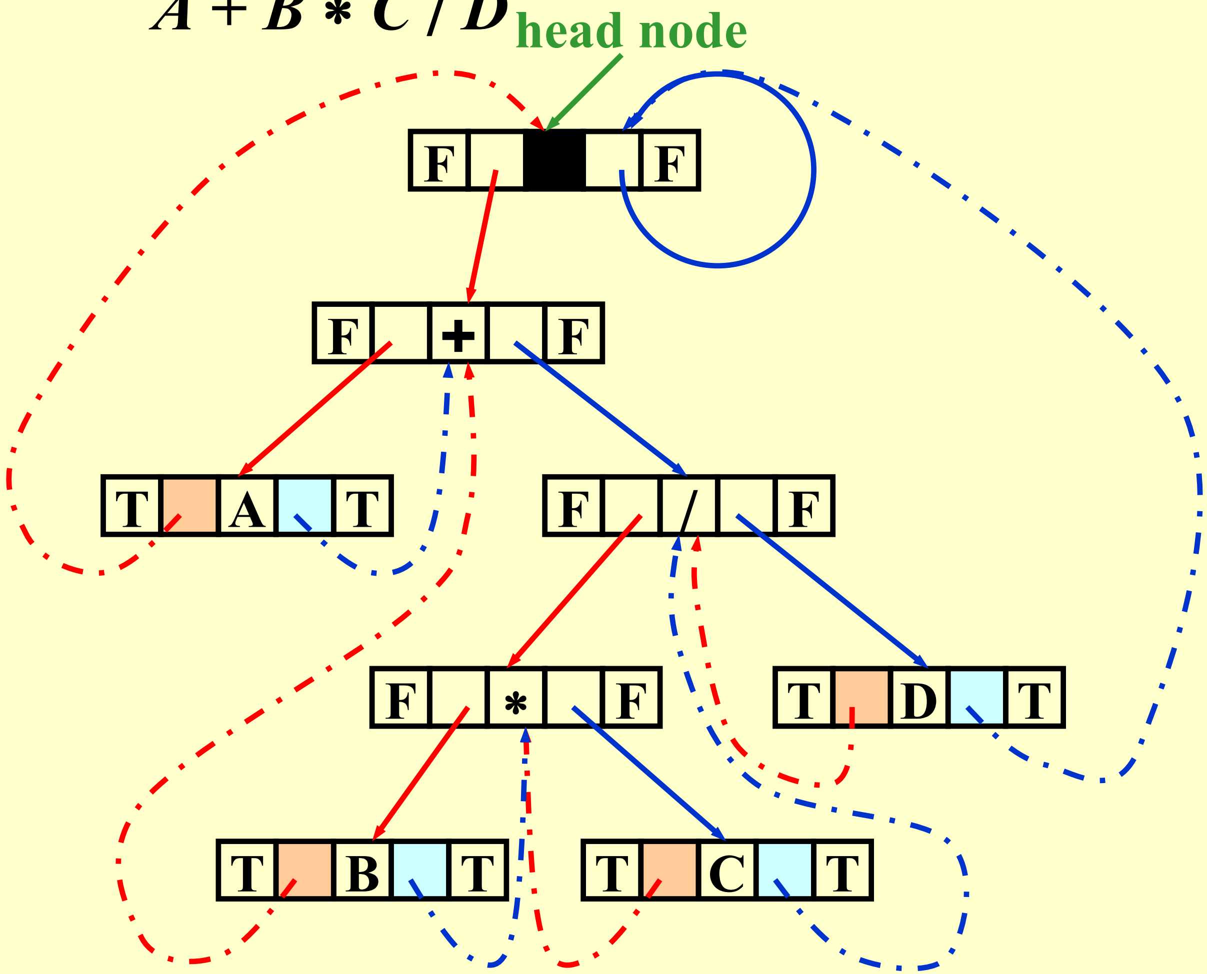
Rule 3: There must not be any loose threads. Therefore a threaded binary tree must have a head node of which the left child points to the first node.

```
typedef  struct  ThreadedTreeNode  *PtrTo  ThreadedNode;
typedef  struct  PtrToThreadedNode  ThreadedTree;
typedef  struct  ThreadedTreeNode {
     int                    LeftThread;   /* if it is TRUE, then Left */
     ThreadedTree           Left;      /* is a thread, not a child ptr.   */
     ElementType Element;
     int                    RightThread; /* if it is TRUE, then Right
   */
     ThreadedTree           Right;    /* is a thread, not a child ptr.   */
}
```

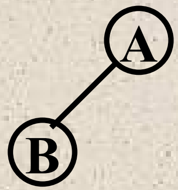
〖 Example 〗 Given the syntax tree of an expression (infix)

$A + B * C / D$

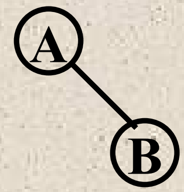




Note: In a tree, the order of children does not matter. But in a binary tree, left child and right child are different.

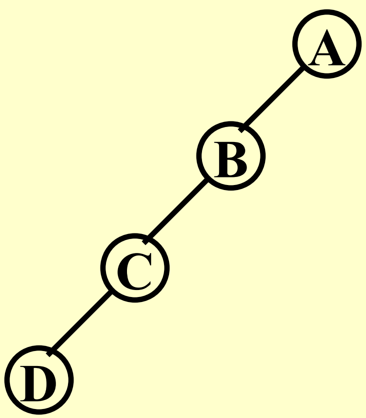


and

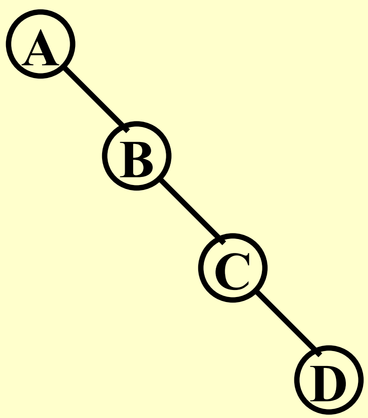


are two different binary trees.

Skewed Binary Trees

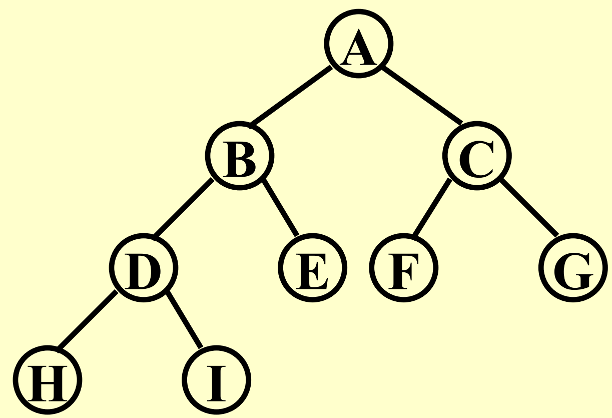


Skewed to the left



Skewed to the right

Complete Binary Tree



All the leaf nodes are on two adjacent levels

☞ Properties of Binary Trees

☑ The maximum number of nodes on level i is 2^{i-1}, i 1.
The maximum number of nodes in a binary tree of depth k is $2^k - 1$, k 1.

☑ For any nonempty binary tree, $n_0 = n_2 + 1$ where n_0 is the number of leaf nodes and n_2 the number of nodes of degree 2.

Proof: Let n_1 be the number of nodes of degree 1, and n the total number of nodes. Then

$$n = n_0 + n_1 + n_2 \quad ①$$

Let B be the number of branches. Then $n = B + 1$. ②

Since all branches come out of nodes of degree 1 or 2, we have $B = n_1 + 2\,n_2$. ③

$$\Rightarrow \qquad n_0 = n_2 + 1 \qquad \blacksquare$$

Bonus Problem 1

Path of Equal Weight

(2 points)

Due: Tuesday, January 7th, 2020 at 23:59pm

The problem can be found and submitted at
https://pintia.cn/